

Flavour Physics (of quarks)

Part 2: Mixing and CP violation

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Warwick Week Graduate Lectures

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Lecture 1: Flavour in the SM

- ▶ Flavour in the SM
- ▶ Quark Model History
- ▶ The CKM matrix

Lecture 2: Mixing and CP violation (Today)

- ▶ Neutral Meson Mixing (no CPV)
- ▶ B -meson production and experiments
- ▶ CP violation

Lecture 3: Measuring the CKM parameters

- ▶ Measuring CKM elements and phases
- ▶ Global CKM fits
- ▶ CPT and T -reversal
- ▶ Dipole moments

Lecture 4: Flavour Changing Neutral Currents

- ▶ Effective Theories
- ▶ New Physics in B mixing
- ▶ New Physics in rare $b \rightarrow s$ processes
- ▶ Lepton Flavour Violation

1. Recap

Recap

- ▶ Last time we introduced the role of flavour in the SM
- ▶ We saw how measurements of meson decays led to the predictions and subsequent discoveries of strange, charm, beauty and top decays
- ▶ We saw how various meson and baryon states are built out of the constituent quarks
- ▶ We introduced the CKM matrix (much more on that in the next two lectures)

Homework from last time

1. Can you explain the 2:1 ratio:

$$\sigma(p + p \rightarrow d + \pi^+) : \sigma(p + n \rightarrow d + \pi^0) = 2 : 1?$$

2. What do the spin-1 and spin-3/2 multiplets look like?

Homework for next time

Why is it that down type neutral mesons contain the anti-quark species but up type contain the quark?

For example:

▶ $B^0 = (\bar{b}d)$, $B_s^0 = (\bar{b}, s)$, $K^0 = (\bar{s}d)$

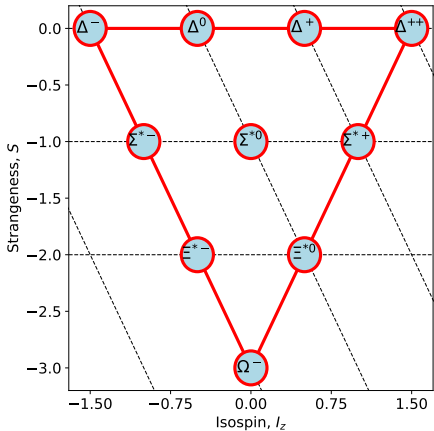
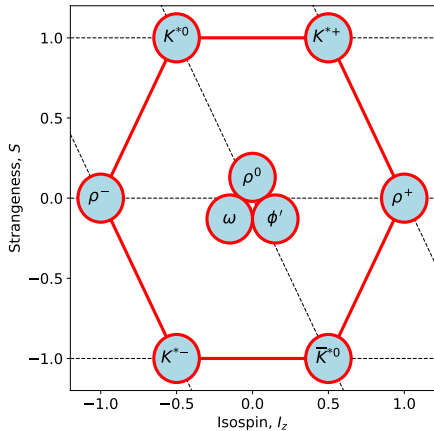
▶ $\bar{B}^0 = (b\bar{d})$, $\bar{B}_s^0 = (b, \bar{s})$, $\bar{K}^0 = (s\bar{d})$

But:

▶ $D^0 = (c\bar{u})$

▶ $\bar{D}^0 = (\bar{c}u)$

Higher resonance multiplets



Recap

- ▶ Recall the CKM matrix which governs quark weak transitions

CKM exhibits a clear hierarchy

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \sim \begin{pmatrix} 1 & 0.2 & 0.004 \\ 0.2 & 1 & 0.04 \\ 0.008 & 0.04 & 1 \end{pmatrix}$$

experimentally
determined values

Commonly represented in the Wolfenstein parametrisation

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1 - \lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \lambda^2/2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

4 $\mathcal{O}(1)$ real parameters (A, λ, ρ, η)

- ▶ Wolfenstein parameterisation ensures that

$$\bar{\rho} + i\bar{\eta} = -(V_{ud}V_{ub}^*)/(V_{cd}V_{cb}^*) \quad (1)$$

is phase convention independent and CKM matrix written in $(A, \lambda, \bar{\rho}, \bar{\eta})$ is unitary to all orders in λ

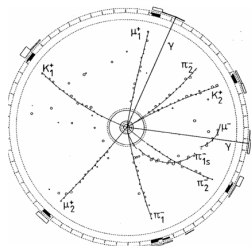
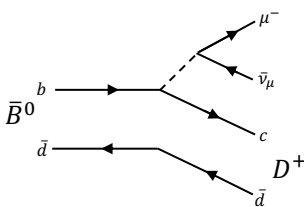
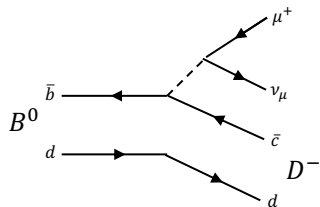
$$\bar{\rho} = \rho(1 - \lambda^2/2 + \dots) \quad \text{and} \quad \bar{\eta} = \eta(1 - \lambda^2/2 + \dots) \quad (2)$$

- ▶ The amount of CP violation in the SM is equivalent to asking
→ how big is η relative to ρ .

2. Neutral Meson Mixing (no CPV)

Neutral Meson Mixing

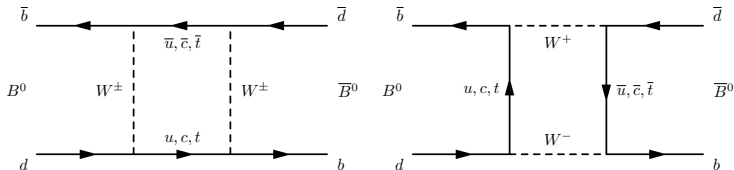
- ▶ In 1987 the ARGUS experiment observed coherently produced $B^0 - \bar{B}^0$ pairs and observed them decaying to **same sign leptons**
- ▶ How is this possible?
 - ▶ Semileptonic decays “tag” the flavour of the initial state



- ▶ The only explanation is that $B^0 - \bar{B}^0$ can oscillate
- ▶ Rate of mixing is large \rightarrow top quark must be heavy

Neutral Meson Mixing

- ▶ In the SM occurs via box diagrams involving a charged current (W^\pm) interaction
- ▶ Weak eigenstates are not the same as the physical mass eigenstates
 - ▶ The particle and antiparticle flavour states (via CPT theorem) have equal and opposite charge, identical mass and identical lifetimes
 - ▶ But the mixed states (*i.e.* the physical B_L^0 and B_H^0) can have $\Delta m, \Delta\Gamma \neq 0$



- ▶ In the SM we have four possible neutral meson states
 - ▶ K^0, D^0, B^0, B_s^0 (mixing has been observed in all four)
 - ▶ Although they all have rather different properties (as we will see in a second)

- ▶ A single particle system evolves according to the time-dependent Schrödinger equation

$$i\frac{\partial}{\partial t}|X(t)\rangle = \mathcal{H}|X(t)\rangle = \left(M - i\frac{\Gamma}{2}\right)|X(t)\rangle \quad (3)$$

- ▶ For neutral mesons, mixing leads to a coupled system

$$i\frac{\partial}{\partial t} \begin{pmatrix} |B^0\rangle \\ |\bar{B}^0\rangle \end{pmatrix} = \mathcal{H} \begin{pmatrix} |B^0\rangle \\ |\bar{B}^0\rangle \end{pmatrix} = \left(\mathbf{M} - i\frac{\mathbf{\Gamma}}{2}\right) \begin{pmatrix} |B^0\rangle \\ |\bar{B}^0\rangle \end{pmatrix} \quad (4)$$

$$= \begin{pmatrix} M_{11} - i\Gamma_{11}/2 & M_{12} - i\Gamma_{12}/2 \\ M_{12}^* - i\Gamma_{12}^*/2 & M_{22} - i\Gamma_{22}/2 \end{pmatrix} \begin{pmatrix} |B^0\rangle \\ |\bar{B}^0\rangle \end{pmatrix} \quad (5)$$

where

$$M_{12} = \frac{1}{2M}\mathcal{A}(B^0 \rightarrow \bar{B}^0) = \langle \bar{B}^0 | \mathcal{H}(\Delta B = 2) | B^0 \rangle \quad (6)$$

Coupled meson system

- ▶ To start with we will neglect CP -violation in mixing (approximately the case for all four neutral meson species)
- ▶ Neglecting CP -violation, the physical states are an equal mixture of the flavour states

$$|B_L^0\rangle = \frac{|B^0\rangle + |\bar{B}^0\rangle}{2}, \quad |B_H^0\rangle = \frac{|B^0\rangle - |\bar{B}^0\rangle}{2}$$

with mass and width differences

$$\Delta\Gamma = \Gamma_H - \Gamma_L = 2|\Gamma_{12}|, \quad \Delta M = M_H - M_L = 2|M_{12}|$$

so that the physical system evolves as

$$i\frac{\partial}{\partial t} \begin{pmatrix} |B_L^0\rangle \\ |B_H^0\rangle \end{pmatrix} = \mathcal{H} \begin{pmatrix} |B_L^0\rangle \\ |B_H^0\rangle \end{pmatrix} = \left(\mathbf{M} - i\frac{\mathbf{\Gamma}}{2} \right) \begin{pmatrix} |B_L^0\rangle \\ |B_H^0\rangle \end{pmatrix} \quad (7)$$

$$= \begin{pmatrix} M_L - i\Gamma_L/2 & 0 \\ 0 & M_H - i\Gamma_H/2 \end{pmatrix} \begin{pmatrix} |B_L^0\rangle \\ |B_H^0\rangle \end{pmatrix} \quad (8)$$

- ▶ Solving the Schrödinger equation gives the time evolution of a pure state $|B^0\rangle$ or $|\bar{B}^0\rangle$ at time $t = 0$

$$\begin{aligned}|B^0(t)\rangle &= g_+(t)|B^0\rangle + \frac{q}{p}g_-(t)|\bar{B}^0\rangle \\ |\bar{B}^0(t)\rangle &= g_+(t)|\bar{B}^0\rangle + \frac{p}{q}g_-(t)|B^0\rangle\end{aligned}\quad (9)$$

where

$$\begin{aligned}g_+(t) &= e^{-iMt}e^{-\Gamma t/2} \left[\cosh\left(\frac{\Delta\Gamma t}{4}\right) \cos\left(\frac{\Delta mt}{2}\right) - i \sinh\left(\frac{\Delta\Gamma t}{4}\right) \sin\left(\frac{\Delta mt}{2}\right) \right] \\ g_-(t) &= e^{-iMt}e^{-\Gamma t/2} \left[-\sinh\left(\frac{\Delta\Gamma t}{4}\right) \cos\left(\frac{\Delta mt}{2}\right) + i \cosh\left(\frac{\Delta\Gamma t}{4}\right) \sin\left(\frac{\Delta mt}{2}\right) \right]\end{aligned}\quad (10)$$

and $M = (M_L + M_H)/2$ and $\Gamma = (\Gamma_L + \Gamma_H)/2$

- ▶ No CP -violation in mixing means that $|p/q| = 1$ (and thus we have equal admixtures)

- ▶ Using Eq. (10) flavour remains unchanged (+) or will oscillate (−) with probability

$$|g_{\pm}(t)|^2 = \frac{e^{-\Gamma t}}{2} \left[\cosh\left(\frac{\Delta\Gamma t}{2}\right) \pm \cos(\Delta m t) \right] \quad (11)$$

- ▶ With no CP violation in the mixing, the time-integrated mixing probability is

$$\frac{\int |g_{-}(t)|^2 dt}{\int |g_{-}(t)|^2 dt + \int |g_{+}(t)|^2 dt} = \frac{x^2 + y^2}{2(x^2 + 1)} \quad (12)$$

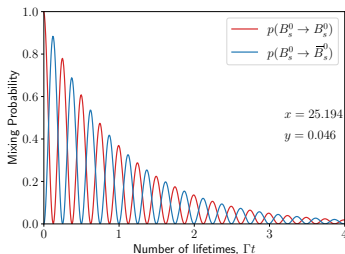
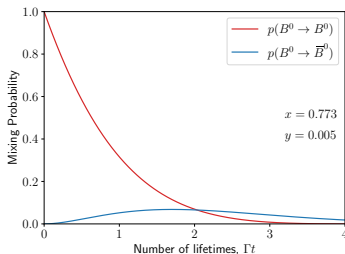
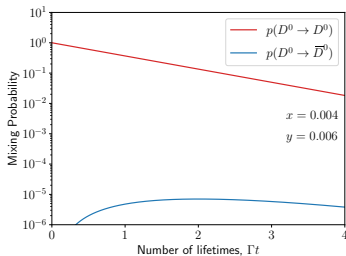
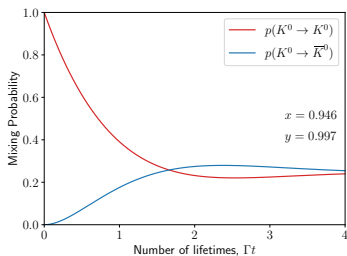
where

$$x = \frac{\Delta m}{\Gamma} \quad \text{and} \quad y = \frac{\Delta\Gamma}{2\Gamma} \quad (13)$$

- ▶ The four different neutral meson species which mix have very different values of (x, y) and therefore very different looking time evolution properties

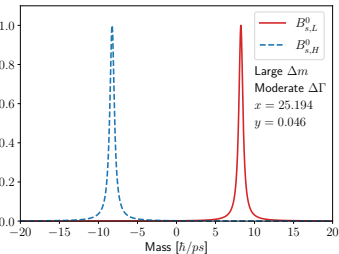
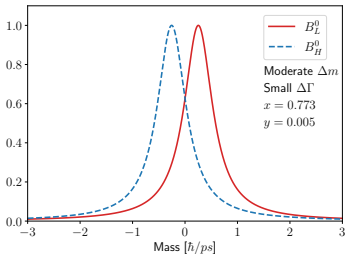
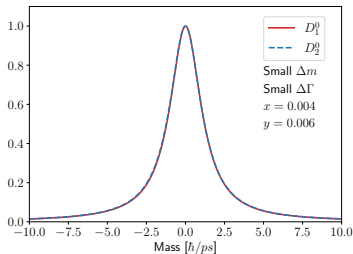
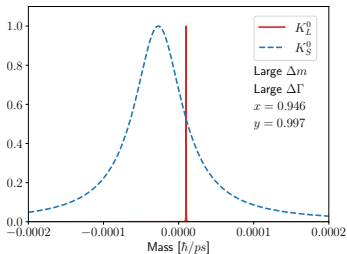
Neutral Meson Mixing

$$|g_{\pm}(t)|^2 = \frac{e^{-\Gamma t}}{2} \left[\cosh\left(\frac{\Delta\Gamma t}{2}\right) \pm \cos(\Delta m t) \right] \quad (14)$$



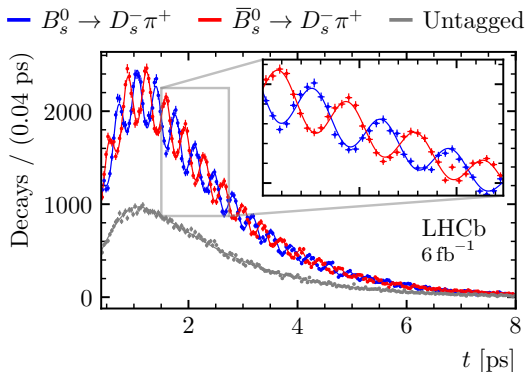
Neutral Meson Mixing

► Mass and width differences of the neutral meson mixing systems



Neutral Meson Mixing

- ▶ Very nice demonstration of the B_s^0 oscillation from the LHCb experiment
- ▶ Seen in $B_s^0 \rightarrow D_s^- \pi^+$ decays
- ▶ Tag the flavour of the initial state at **production** and compare to the flavour at decay (the $D_s^- \pi^+$ final state tags the decaying flavour)
- ▶ Why is this so different from the plot on the previous slide (damped oscillation and turn on at low values)?

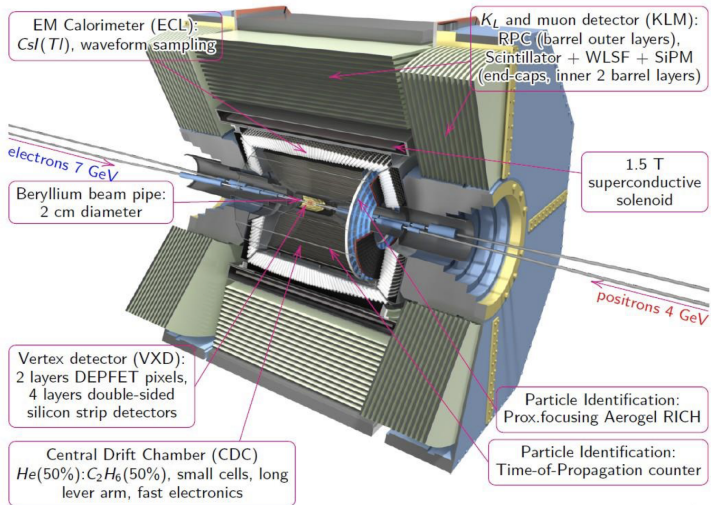


3. B -meson production and experiments

B -factories at the $\Upsilon(4S)$

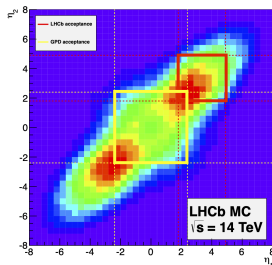
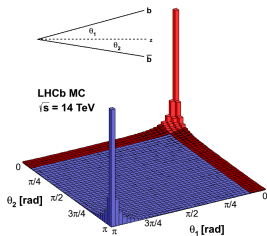
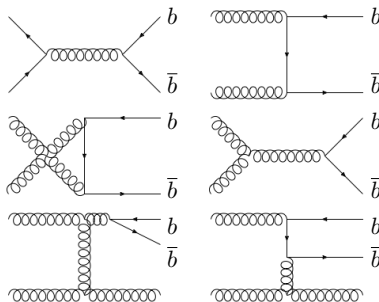
- ▶ Asymmetric e^+e^- colliders
- ▶ Produce excited $\Upsilon(4S)$ resonance (10.58 GeV) which decays strongly and produces a coherent pair of $B^0\bar{B}^0$ (50%) or B^+B^- pair (50%) moving in the lab frame
 - ▶ **BaBar** produced $\sim 500\text{M } B\bar{B}$ pairs in $\sim 530\text{fb}^{-1}$ of data from 9 GeV and 3.1 GeV beams at SLAC
 - ▶ **Belle** produced $\sim 770\text{M } B\bar{B}$ pairs in $\sim 710\text{fb}^{-1}$ of data from 8 GeV and 3.5 GeV beams at KEK
 - ▶ **Belle-II** expected to produce up to $\sim 50\text{B } B\bar{B}$ pairs in $\sim 50\text{ab}^{-1}$ of data
- ▶ Very clean environments but notice that the B_s^0 is not in range of the $\Upsilon(4S)$ resonance. This requires specific running at the $\Upsilon(5S)$.
 - ▶ In comparison to LHCb, $B\bar{B}$ pairs are not produced at high boost which makes resolution of B_s^0 oscillations impossible at B -factories
- ▶ Because B mesons are produced in pairs from a known resonance you get very high flavour tagging power and very good resolution for missing energy (*i.e.* final state neutrals)
- ▶ For Belle-II to achieve desired luminosity requires incredible squeezing of the beam (target is $8 \times 10^{35}\text{cm}^{-2}\text{s}^{-1}$ which is $40 \times$ Belle)

Belle-II Experiment

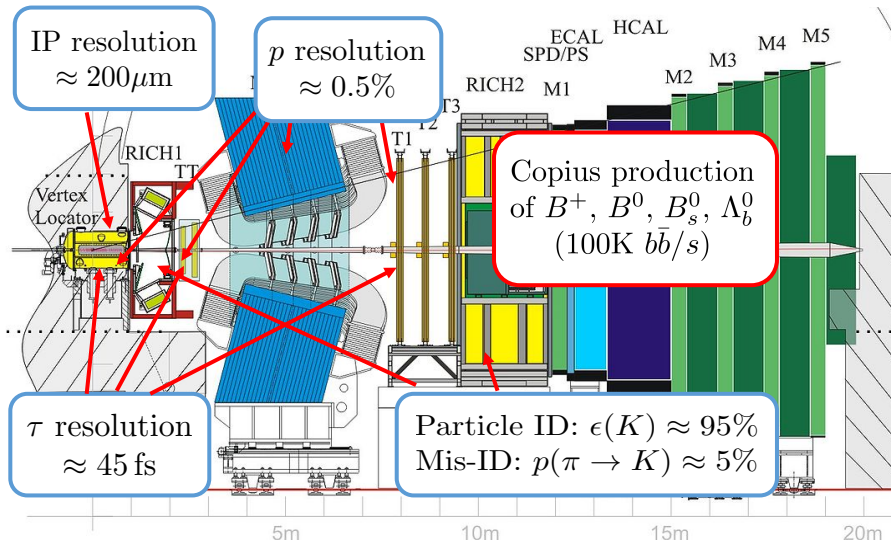


B-production at the LHC

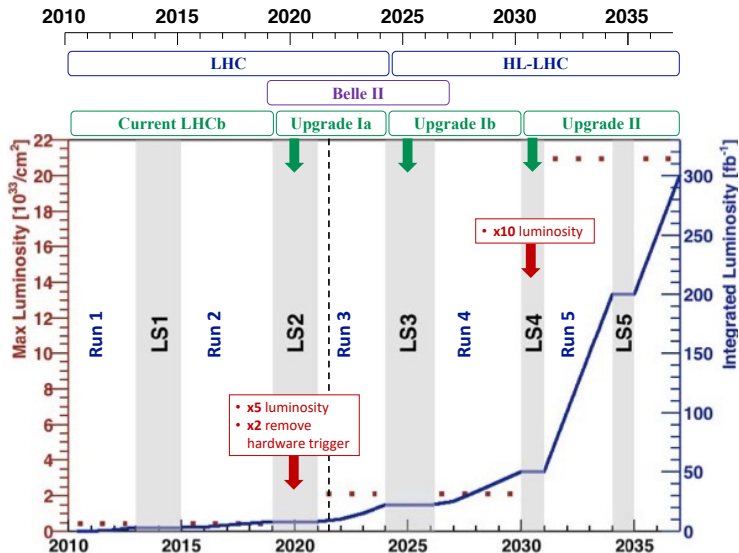
- ▶ The LHC is predominantly a gluon collider
- ▶ b -quarks are produced in pairs and predominantly in the forward region with a very large boost
 - ▶ Hence the very forward geometry of LHCb
- ▶ The very large boost and very high quality vertexing makes decay time measurements much easier
 - ▶ Can resolve the very rapid B_s^0 oscillations



The LHCb detector

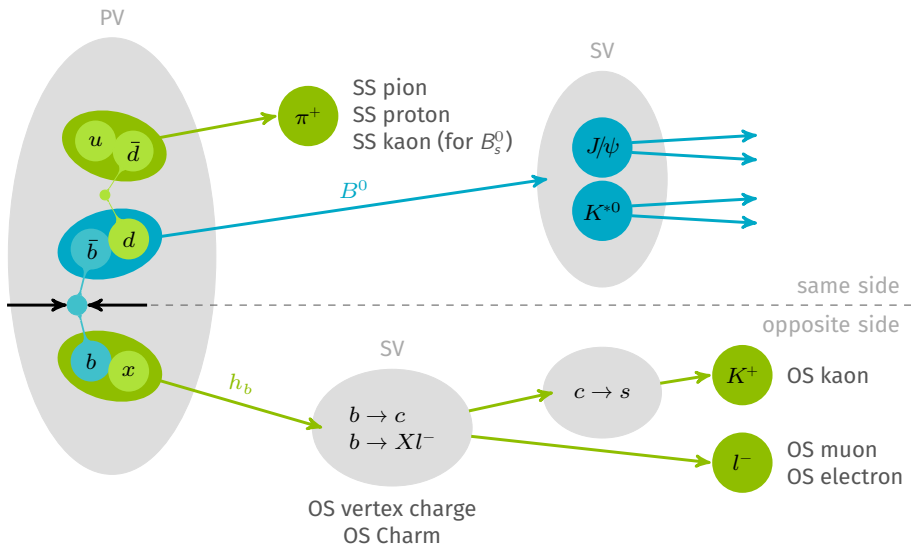


The LHCb upgrades



► COVID has pushed back future schedule by (at least) one year

Flavour Tagging at the LHC



Dalitz plot formalism

- ▶ For a nice overview of this, take a look at Sec. 2 of [\[arXiv:1711.09854\]](#)
- ▶ Provides a nice method and visualisation of 3-body decays, e.g. $B \rightarrow XYZ$
- ▶ The n -body decay rate is

$$d\Gamma = \frac{(2\pi)^4}{2M} |\mathcal{M}|^2 d\phi(p_1, p_2, \dots, p_n) \quad (15)$$

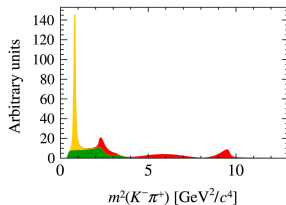
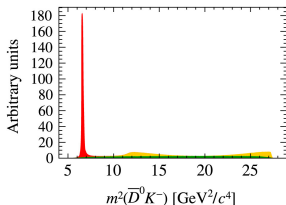
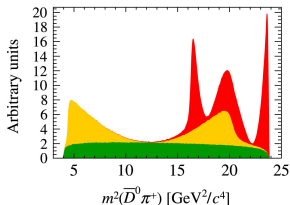
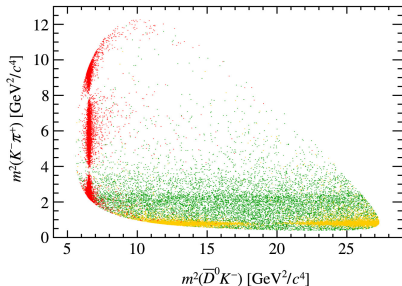
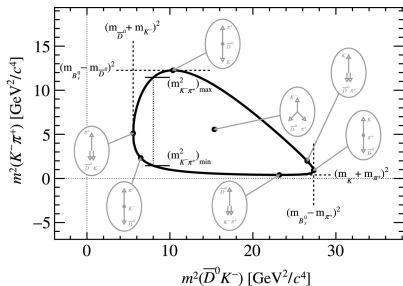
- ▶ So for a 3-body decay

$$d\Gamma = \frac{1}{(2\pi)^3} \frac{1}{32M^3} |\overline{\mathcal{M}}|^2 dm_{12}^2 dm_{23}^2 \quad (16)$$

- ▶ Note how 3-body phase-space is flat in the Dalitz plot
- ▶ Resonances appear as bands in the Dalitz plot where **The number of “lobes” in the Dalitz plot is related to the particle spin**
 - ▶ **Spin-0 “scalar” contributions** have 1 lobe
 - ▶ **Spin-1 “vector” contributions** have 2 lobes
 - ▶ **Spin-2 “tensor” contributions** have 3 lobes

Dalitz plot formalism

- ▶ Example shown for a $B^0 \rightarrow \bar{D}^0 K^- \pi^+$ decay



4. CP violation

Measuring CP violation

1. Need at least two interfering amplitudes
 2. Need two phase differences between them
 - ▶ One CP conserving (“strong”) phase difference (δ)
 - ▶ One CP violating (“weak”) phase difference (ϕ)
- ▶ If there is only a single path to a final state, f , then we cannot get direct CP violation
- ▶ If there is only one path we can write the amplitudes for decay as

$$\mathcal{A}(B \rightarrow f) = A_1 e^{i(\delta_1 + \phi_1)}$$

$$\mathcal{A}(\bar{B} \rightarrow \bar{f}) = A_1 e^{i(\delta_1 - \phi_1)}$$

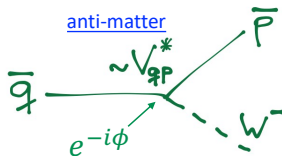
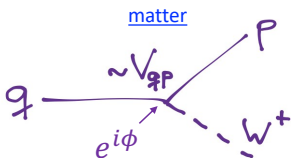
- ▶ Which gives an asymmetry of

$$\mathcal{A}_{CP} = \frac{|\mathcal{A}(\bar{B} \rightarrow \bar{f})|^2 - |\mathcal{A}(B \rightarrow f)|^2}{|\mathcal{A}(\bar{B} \rightarrow \bar{f})|^2 + |\mathcal{A}(B \rightarrow f)|^2} = 0 \quad (17)$$

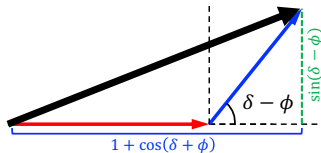
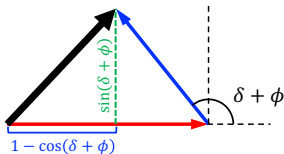
- ▶ In order to observe CP -violation we need a second amplitude.
- ▶ This is often realised by having interfering tree and penguin amplitudes

Measuring CP -violation

- ▶ We measure **quark couplings** which have a **complex phase**
- ▶ This is only visible when there are two amplitudes



- ▶ Below we represent two amplitudes (**red** and **blue**) with the same magnitude = 1
 - ▶ The strong phase difference is, $\delta = \pi/2$
 - ▶ The weak phase difference is, $\phi = \pi/4$



$$\Gamma(B \rightarrow f) = |A_1 + A_2 e^{i(\delta + \phi)}|^2$$

$$\Gamma(\bar{B} \rightarrow \bar{f}) = |A_1 + A_2 e^{i(\delta - \phi)}|^2$$

Measuring (direct) CP -violation

- ▶ Introducing the second amplitude we now have

$$\mathcal{A}(B \rightarrow f) = A_1 e^{i(\delta_1 + \phi_1)} + A_2 e^{i(\delta_2 + \phi_2)} \quad (18)$$

$$\mathcal{A}(\bar{B} \rightarrow \bar{f}) = A_1 e^{i(\delta_1 - \phi_1)} + A_2 e^{i(\delta_2 - \phi_2)} \quad (19)$$

- ▶ Which gives an asymmetry of

$$\mathcal{A}_{CP} = \frac{|\mathcal{A}(\bar{B} \rightarrow \bar{f})|^2 - |\mathcal{A}(B \rightarrow f)|^2}{|\mathcal{A}(\bar{B} \rightarrow \bar{f})|^2 + |\mathcal{A}(B \rightarrow f)|^2} \quad (20)$$

$$= \frac{4A_1 A_2 \sin(\delta_1 - \delta_2) \sin(\phi_1 - \phi_2)}{2A_1^2 + 2A_2^2 + 4A_1 A_2 \cos(\delta_1 - \delta_2) \cos(\phi_1 - \phi_2)} \quad (21)$$

$$= \frac{2r \sin(\delta) \sin(\phi)}{1 + r^2 + 2r \cos(\delta) \cos(\phi)} \quad (22)$$

where $r = A_1/A_2$, $\delta = \delta_1 - \delta_2$ and $\phi = \phi_1 - \phi_2$

- ▶ This is only non-zero if the amplitudes have **different** weak **and** strong phases
- ▶ This is CP -violation in decay (often called “direct” CP violation).
 - ▶ This is the only possible route of CP violation for a charged initial state
 - ▶ We will see now that for a neutral initial state there are other ways of realising CP violation

Classification of CP violation

- ▶ First let's consider a generalised form of a neutral meson, X^0 , decaying to a final state, f
- ▶ There are four possible amplitudes to consider

$$\begin{aligned} A_f &= \langle f|X^0\rangle & \bar{A}_f &= \langle f|\bar{X}^0\rangle \\ A_{\bar{f}} &= \langle \bar{f}|X^0\rangle & \bar{A}_{\bar{f}} &= \langle \bar{f}|\bar{X}^0\rangle \end{aligned}$$

- ▶ Define a complex parameter, λ_f (**not** the Wolfenstein parameter, λ)

$$\lambda_f = \frac{q}{p} \frac{\bar{A}_f}{A_f}, \quad \bar{\lambda}_f = \frac{1}{\lambda_f}, \quad \lambda_{\bar{f}} = \frac{q}{p} \frac{\bar{A}_{\bar{f}}}{A_{\bar{f}}}, \quad \bar{\lambda}_{\bar{f}} = \frac{1}{\lambda_{\bar{f}}}$$

Classification of CP violation

Can realise CP violation in three ways:

1. CP violation in decay

- ▶ For a charged initial state this is only the type possible

$$\Gamma(X^0 \rightarrow f) \neq \Gamma(\bar{X}^0 \rightarrow \bar{f}) \implies \left| \frac{\bar{A}_f}{A_f} \right| \neq 1 \quad (23)$$

2. CP violation in mixing

$$\Gamma(X^0 \rightarrow \bar{X}^0) \neq \Gamma(\bar{X}^0 \rightarrow X^0) \implies \left| \frac{p}{q} \right| \neq 1 \quad (24)$$

3. CP violation in the interference between mixing and decay

$$\Gamma(X^0 \rightarrow f) \neq \Gamma(X^0 \rightarrow \bar{X}^0 \rightarrow f) \implies \arg(\lambda_f) = \arg\left(\frac{q}{p} \frac{\bar{A}_f}{A_f}\right) \neq 0 \quad (25)$$

- ▶ We just saw an example of CP violation in decay
- ▶ Let's extend our formalism of neutral mixing, Eqs. (9–13), to include CP violation

Neutral meson mixing with CP violation

- ▶ Allowing for CP violation, $M_{12} \neq M_{12}^*$ and $\Gamma_{12} \neq \Gamma_{12}^*$
- ▶ The physical states can now be unequal mixtures of the weak states

$$\begin{aligned} |B_L^0\rangle &= p|B^0\rangle + q|\bar{B}^0\rangle \\ |B_H^0\rangle &= p|B^0\rangle - q|\bar{B}^0\rangle \end{aligned} \quad (26)$$

where

$$|p|^2 + |q|^2 = 1$$

- ▶ The states now have mass and width differences

$$|\Delta\Gamma| \approx 2|\Gamma_{12}| \cos(\phi), \quad |\Delta M| \approx 2|M_{12}|, \quad \phi = \arg(-M_{12}/\Gamma_{12}) \quad (27)$$

- ▶ We'll see some examples of this later
- ▶ Now to equip ourselves with the formalism for a generalised meson decay

Generalised Meson Decay Formalism

The probability that state X^0 at time t decays to f at time t

- ▶ contains terms for *CPV* in **decay**, **mixing** and **the interference between the two**

$$\Gamma_{X^0 \rightarrow f}(t) = |A_f|^2 \left(|g_+(t)|^2 + |\lambda_f|^2 |g_-(t)|^2 + 2\mathcal{R}e[\lambda_f g_+^*(t) g_-(t)] \right) \quad (28)$$

$$\Gamma_{X^0 \rightarrow \bar{f}}(t) = |\bar{A}_{\bar{f}}|^2 \left| \frac{q}{p} \right|^2 \left(|g_-(t)|^2 + |\lambda_{\bar{f}}|^2 |g_+(t)|^2 + 2\mathcal{R}e[\lambda_{\bar{f}} g_+(t) g_-^*(t)] \right) \quad (29)$$

$$\Gamma_{\bar{X}^0 \rightarrow f}(t) = |A_f|^2 \left| \frac{p}{q} \right|^2 \left(|g_-(t)|^2 + |\lambda_f|^2 |g_+(t)|^2 + 2\mathcal{R}e[\lambda_f g_+(t) g_-^*(t)] \right) \quad (30)$$

$$\Gamma_{\bar{X}^0 \rightarrow \bar{f}}(t) = |\bar{A}_{\bar{f}}|^2 \left(|g_+(t)|^2 + |\lambda_{\bar{f}}|^2 |g_-(t)|^2 + 2\mathcal{R}e[\lambda_{\bar{f}} g_+^*(t) g_-(t)] \right) \quad (31)$$

where the **mixing probabilities** are as before

$$|g_{\pm}(t)|^2 = \frac{e^{-\Gamma t}}{2} \left[\cosh\left(\frac{\Delta\Gamma t}{2}\right) \pm \cos(\Delta m t) \right] \quad (32)$$

$$g_+^* g_-^{(*)} = \frac{e^{-\Gamma t}}{2} \left[\sinh\left(\frac{\Delta\Gamma t}{2}\right) \pm i \sin(\Delta m t) \right] \quad (33)$$

Generalised Meson Decay Formalism

- ▶ From the above we get the “master equations” for neutral meson decay

$$\Gamma_{X^0 \rightarrow f}(t) = |A_f|^2 (1 + |\lambda_f|^2) \frac{e^{-\Gamma t}}{2} \left[\cosh\left(\frac{1}{2}\Delta\Gamma t\right) + C_f \cos(\Delta m t) + D_f \sinh\left(\frac{1}{2}\Delta\Gamma t\right) - S_f \sin(\Delta m t) \right] \quad (34)$$

$$\Gamma_{\bar{X}^0 \rightarrow f}(t) = |A_f|^2 \left| \frac{p}{q} \right|^2 (1 + |\lambda_f|^2) \frac{e^{-\Gamma t}}{2} \left[\cosh\left(\frac{1}{2}\Delta\Gamma t\right) - C_f \cos(\Delta m t) + D_f \sinh\left(\frac{1}{2}\Delta\Gamma t\right) + S_f \sin(\Delta m t) \right] \quad (35)$$

where

$$C_f = \frac{1 - |\lambda_f|^2}{1 + |\lambda_f|^2}, \quad D_f = \frac{2\mathcal{R}e(\lambda_f)}{1 + |\lambda_f|^2}, \quad S_f = \frac{2\mathcal{I}m(\lambda_f)}{1 + |\lambda_f|^2} \quad (36)$$

- ▶ and equivalents for the CP conjugate final state \bar{f}
- ▶ **The time-dependent CP asymmetry is** (for non- CP -eigenstates there are two)

$$\mathcal{A}_{CP}(t) = \frac{\Gamma_{X^0 \rightarrow f}(t) - \Gamma_{\bar{X}^0 \rightarrow f}(t)}{\Gamma_{X^0 \rightarrow f}(t) + \Gamma_{\bar{X}^0 \rightarrow f}(t)} = \frac{2C_f \cos(\Delta m t) - 2S_f \sin(\Delta m t)}{2 \cosh\left(\frac{1}{2}\Delta\Gamma t\right) + 2D_f \sinh\left(\frac{1}{2}\Delta\Gamma t\right)} \quad (37)$$

Specific Meson Formalism

- ▶ In the B^0 system $\Delta\Gamma \sim 0$

$$\Gamma_{X^0 \rightarrow f}(t) = |A_f|^2 (1 + |\lambda_f|^2) \frac{e^{-i\Gamma t}}{2} \left[\begin{array}{l} + C_f \cos(\Delta mt) \\ - S_f \sin(\Delta mt) \end{array} \right] \quad (38)$$

$$\Gamma_{\bar{X}^0 \rightarrow f}(t) = |A_f|^2 \left| \frac{p}{q} \right| (1 + |\lambda_f|^2) \frac{e^{-i\Gamma t}}{2} \left[\begin{array}{l} - C_f \cos(\Delta mt) \\ + S_f \sin(\Delta mt) \end{array} \right] \quad (39)$$

- ▶ The time-dependent CP asymmetry is

$$\mathcal{A}_{CP}(t) = \frac{\Gamma_{X^0 \rightarrow f}(t) - \Gamma_{\bar{X}^0 \rightarrow f}(t)}{\Gamma_{X^0 \rightarrow f}(t) + \Gamma_{\bar{X}^0 \rightarrow f}(t)} = C_f \cos(\Delta mt) - S_f \sin(\Delta mt) \quad (40)$$

Specific Meson Formalism

- ▶ In the D^0 system Δm and $\Delta\Gamma$ are both small

$$\Gamma_{X^0 \rightarrow f}(t) = |A_f|^2 (1 + |\lambda_f|^2) \frac{e^{-i\Gamma t}}{2} \left[\begin{array}{cc} 1 & + C_f \\ + D_f \frac{1}{2} \Delta\Gamma t & - S_f \Delta m t \end{array} \right] \quad (41)$$

$$\Gamma_{\bar{X}^0 \rightarrow f}(t) = |A_f|^2 \left| \frac{p}{q} \right| (1 + |\lambda_f|^2) \frac{e^{-i\Gamma t}}{2} \left[\begin{array}{cc} 1 & - C_f \\ + D_f \frac{1}{2} \Delta\Gamma t & + S_f \Delta m t \end{array} \right] \quad (42)$$

- ▶ The time-dependent CP asymmetry is

$$\mathcal{A}_{CP}(t) = \frac{\Gamma_{X^0 \rightarrow f}(t) - \Gamma_{\bar{X}^0 \rightarrow f}(t)}{\Gamma_{X^0 \rightarrow f}(t) + \Gamma_{\bar{X}^0 \rightarrow f}(t)} = \boxed{\frac{C_f - S_f \Delta m t}{1 + \frac{1}{2} D_f \Delta\Gamma t}} \quad (43)$$

Specific Meson Decay Formalism

- ▶ With no tagging of flavour we see no asymmetry (just get the sum)

$$\Gamma_{X^0 \rightarrow f}(t) = |A_f|^2 (1 + |\lambda_f|^2) \frac{e^{-i\Gamma t}}{2} \left[\cosh\left(\frac{1}{2}\Delta\Gamma t\right) + D_f \sinh\left(\frac{1}{2}\Delta\Gamma t\right) \right] \quad (44)$$

$$\Gamma_{\bar{X}^0 \rightarrow f}(t) = |A_f|^2 \left| \frac{p}{q} \right| (1 + |\lambda_f|^2) \frac{e^{-i\Gamma t}}{2} \left[\cosh\left(\frac{1}{2}\Delta\Gamma t\right) + D_f \sinh\left(\frac{1}{2}\Delta\Gamma t\right) \right] \quad (45)$$

- ▶ The time-dependent CP asymmetry is

$$\mathcal{A}_{CP}(t) = \frac{\Gamma_{X^0 \rightarrow f}(t) - \Gamma_{\bar{X}^0 \rightarrow f}(t)}{\Gamma_{X^0 \rightarrow f}(t) + \Gamma_{\bar{X}^0 \rightarrow f}(t)} = \boxed{0} \quad (46)$$

	K^0	K^+	Λ^0	D^0	D^+	D_s^+	Λ_c^+	B^0	B^+	B_s^0	Λ_b^0
CP violation in mixing	✓✓	-	-	✗	-	-	-	✗	-	✗	-
CP violation in interference	✓	-	-	✗	-	-	-	✓✓	-	✓✓	-
CP violation in decay	✓	✗	✗	✓✓	✗	✗	✗	✓✓	✓✓	✓	✓

KEY:

- ✓✓ Strong evidence ($> 5\sigma$)
- ✓ Some evidence ($> 3\sigma$)
- ✗ Not seen
- Not possible

5. Recap

In this lecture we have covered

- ▶ Neutral Meson Mixing (without CPV)
 - ▶ Time evolution of coupled systems
 - ▶ Differences in mixing parameters between neutral meson states
- ▶ B -meson production and experiments / techniques
 - ▶ B -factories and Belle 2
 - ▶ LHCb
 - ▶ Flavour Tagging
 - ▶ Dalitz analysis
- ▶ CP violation
 - ▶ CP violation types
 - ▶ The “master” equations for generalised meson decays

End of Lecture 2