Flavour Physics (of quarks)

Part 4: Flavour Changing Neutral Currents

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Warwick Week Graduate Lectures

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Overview

Lecture 1: Flavour in the SM

- Flavour in the SM
- Quark Model History
- The CKM matrix

Lecture 2: Mixing and CP violation

- Neutral Meson Mixing (no CPV)
- B-meson production and experiments
- CP violation

Lecture 3: Measuring the CKM parameters

- Measuring CKM elements and phases
- Global CKM fits
- CPT and T-reversal
- Dipole moments

Lecture 4: Flavour Changing Neutral Currents (Today)

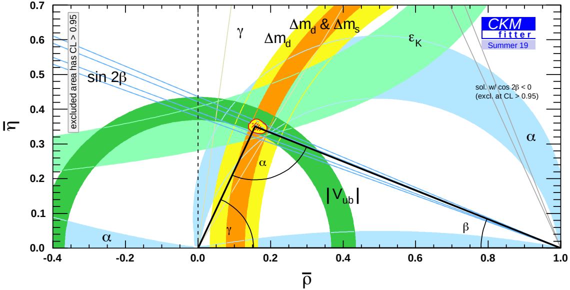
- Effective Theories
- New Physics in B mixing
- New Physics in rare $b \to s$ processes
- Lepton Flavour Violation

1. Recap

Recap

Last time we looked at

- Measurements of the CKM matrix elements
- Measurements of the CKM matrix phases
- Recall from Lecture 1 the lack of tree-level flavour-changing-neutral-currents (FCNCs) in the SM



2. Dipole Moments

Magentic dipole moments

- A "spinning" charge acts as a magnetic dipole with moment, μ, which gives an energy shift to an externally applied magnetic field

$$\Delta E = -\vec{\mu} \cdot \vec{B} \tag{1}$$

- The prediction of $g = 2$ (classically $g = 1$) was a big success of the Dirac equation
- In an external field A^{μ}

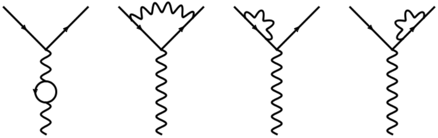
$$\left(\frac{1}{2m}(\vec{p} + e\vec{A}) + \frac{e}{2m}\vec{\sigma} \cdot \vec{B} - eA^{0} \right) \psi = E\psi \tag{2}$$

- The magnetic dipole moment μ is given by

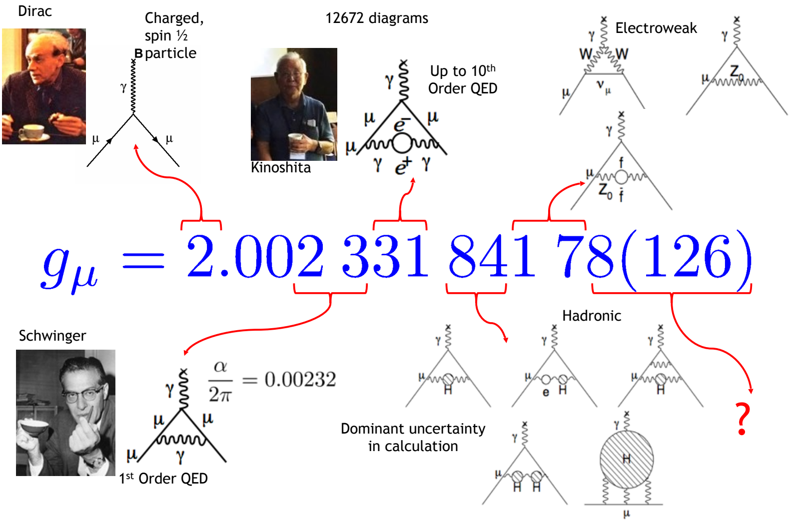
$$\vec{\mu} = -\frac{e}{2m}\vec{\sigma} = -g\frac{\mu_B}{\hbar}\vec{S} \tag{3}$$

- Receives corrections from higher order processes (*e.g.* at order α^2)

$$g = 2 + \frac{\alpha}{2\pi} + \mathcal{O}(\alpha^2)$$



Anomalous magnetic moment



Slide from Becky Chislett (via Tom Blake)

Anaomalous magnetic moments

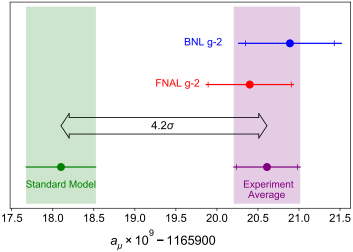
- $(g-2)_e$ is a powerful precision test of QED

$$(g-2)_e - (1159.652186 \pm 0.000004) \times 10^{-6}$$

- $(g-2)_\mu$ receives important Weak and QCD contributions. The latest experimental value from Brookhaven E821 and Fermilab $g-2$ experiments

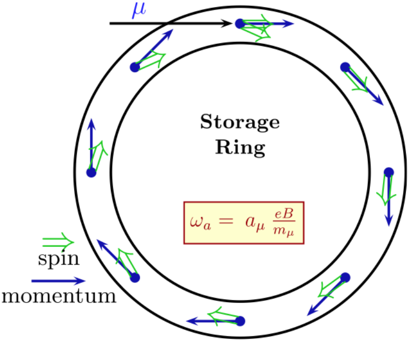
$$(g-2)_\mu = (116591810 \pm 43) \times 10^{-11} \text{ (Theory)}$$
$$(g-2)_\mu = (116592061 \pm 41) \times 10^{-11} \text{ (Experiment)}$$

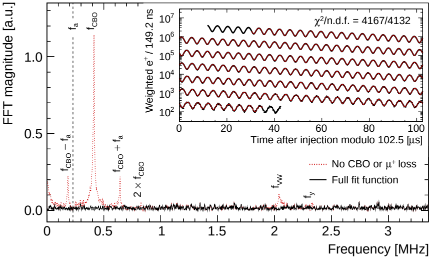
  from [arXiv:2104.03281] is 4.2σ from the SM expectation [arXiv:2006.04822]
- Is this a hint of a NP contribution to $(g-2)_\mu$ (review in [arXiv:0902.3360])?



The $g-2$ experiment

- Experiment at Fermilab aiming for $\sim 0.1 - 0.2$ppm precision
- The anomalous magnetic moment causes the spin to process at a different rate to the momentum vector
- Can use this procession to precisely measure $g-2$



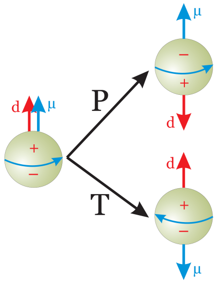


Electric dipole moments

- Classically, EDMs are a measure of the spatial separation of positive and negative charges in a particle
  - A finite EDM can only exist if the charge centres do not coincide
- EDMs can also be measured for fundamental particles (electron, muon, neutron *etc.*)
  - Can interpret this as a measure of the "sphericity" of the particle
- This is tested using the Zeeman effect
  - Look for a shift in energy levels under an external electrical field (analogous to the magenetic moment)

$$\Delta E = -\vec{d} \cdot \vec{E} \tag{4}$$

- A non zero EDM would violate T and P symmetries
  - Under T reversal, the MDM would change direction but the EDM would remain unchanged
  - Under P, the EDM would change direction but the MDM remains unchanged
- Violation of P and T implies CP violation



Electric dipole moments

- Electron EDM:
  - $d_e < 8.7 \times 10^{-29}$ [arXiv:1310.7534]
- Muon EDM:
  - $d_e < 1.9 \times 10^{-19}$ [arXiv:0811.1207]
- Neutron EDM:
  - $d_e < 3.0 \times 10^{-26}$ [arXiv:hep-ex/0602020]
- Probing incredibly small charge separation distances!

Strong CP problem

- The complicated nature of the QCD vacuum should give rise to a term in the Langrangian like

$$\mathcal{L}_\theta = \theta \frac{\alpha_s}{8\pi} F^{\mu\nu}_{\alpha} \tilde{F}_{\alpha,\mu\nu} \tag{5}$$

- This is both P and T-violating but C-conserving (hence CP-violating)
- This terms would also contribute to the neutron dipole moment, but experimentally we know this is very small

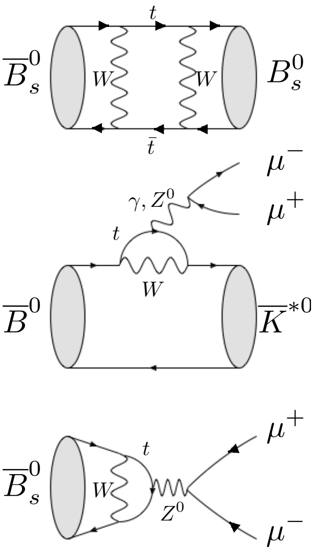
$$d_n \sim e \cdot \theta \cdot m_q / M_N^2 \Longrightarrow \theta \leq 10^{-9} \tag{6}$$

- This is incredibly small size of the θ parameter is (another) massive fine tuning problem (the so-called "strong CP problem")
- What mechanism forces θ to be so small?

Axion searches

- The Peccei-Quin solution to the strong CP problem is to introduce a U(1) symmetry that removes the strong CP problem by dynamically making θ small
- Spontaneous breaking of this symmetry is associated with a pseudo-Nambu-Goldstone boson (in analogy with the Higgs mechanism), the axion
- The axion can be a light particle that couples very weakly to known SM particles
- There are a large number of searches for axions produced in particle colliders (direct searches)
- Can also be detected by the presence of axions converting into photons in the presence of a strong magnetic field (*e.g.* the CAST experiment at CERN)

- FCNC processes can probe incredibly high mass scales (well beyond those directly accesible at the LHC)
  - If there are new particles at the TeV-scale, why don't they manifest themselves in FCNC processes (the so-called "flavour problem")
- There are two types of FCNC process:
  - $\Delta F = 2$: meson anti-meson mixing
  - $\Delta F = 1$: "rare decays" *e.g.* $B_s^0 \to \mu^+\mu^-$ or $B^0 \to K^{*0}\mu^+\mu^-$
- In the SM these processes are heavily suppressed
  - They are loop processes that are CKM suppressed and (depending on the process) can also be GIM suppressed and/or helicity suppressed



4. Effective Theories

Effective Theories

- In meson/baryon decays there is a clear separation of scales which we can "decouple"
  - b quark states have $m \sim 5$ GeV while particles in loops ($W^{\pm}$,t) have $m \sim 100$ GeV

$$m_w \gg m_b > \Lambda_{\text{QCD}} \tag{7}$$

- We want to study the physics of the mixing/decay at or below a scale, Λ_{NP}, in a theory which has contributions from particles at a scale below and above Λ_{NP}
- We can replace the full theory with an effective theory (which is renormalisable) valid at Λ

$$\mathcal{L}(\phi_L, \phi_H) \to \mathcal{L}(\phi_L) + \mathcal{L}_{\text{eff}} = \mathcal{L}(\phi_L) + \underbrace{\boxed{\sum_i \mathcal{C}_i \mathcal{O}_i(\phi_L)}}_{\text{operator product expansion}} \tag{8}$$

- In other words for interactions originating at a high scale (*i.e.* SM+NP) we get an effective matrix element

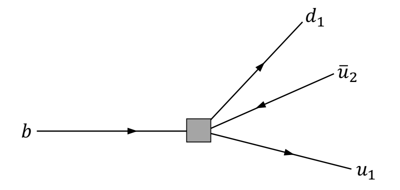
$$\langle f | H_{\text{eff}} | i \rangle = \sum_k \frac{1}{\Lambda^k} \sum_i \underbrace{\mathcal{C}_{k,i}}_{\substack{\text{short distance} \\ \text{contribution} \\ (\text{physics} > \Lambda)}} \underbrace{\langle f | \mathcal{O}_k | i \rangle |_{\Lambda}}_{\substack{\text{"Local operators"} \\ \text{long distance} \\ \text{contribution} \\ (\text{physics} < \Lambda)}} \tag{9}$$

- The so-called "Wilson coefficients" are independent of Λ

Effective Theories

Non-leptonic b decay

e.g. $b \to u\bar{c}s$



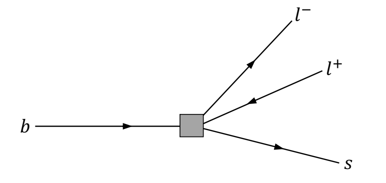
$$H_{\rm eff}(b \to u_1\bar{u}_2 d_1) = \frac{G_F}{\sqrt{2}} V_{u_1 b} V^*_{u_2 d_1} [\mathcal{C}_1 \mathcal{O}_1^{u_1\bar{u}_2 d_1} + \mathcal{C}_2 \mathcal{O}_2^{u_1\bar{u}_2 d_1}]$$

$$\mathcal{O}_1^{u_1\bar{u}_2 d_1} = (\bar{u}_1^\alpha \gamma_\mu (1-\gamma_5) b^\beta)(\bar{d}_1^\beta \gamma^\mu (1-\gamma_5) u_2^\alpha)$$

$$\mathcal{O}_2^{u_1\bar{u}_2 d_1} = (\bar{u}_1^\alpha \gamma_\mu (1-\gamma_5) b^\alpha)(\bar{d}_1^\beta \gamma^\mu (1-\gamma_5) u_2^\beta)$$

EW penguin

e.g. $b \to s\ell^+\ell^-$



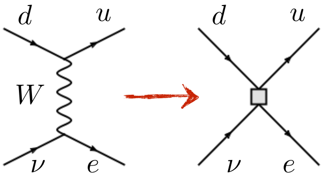
$$H_{\rm eff}(b \to s\ell^+\ell^-) = \frac{2G_F}{\sqrt{2}} V_{ts} V^*_{tb} \sum_{i=9,10} \mathcal{C}_i \mathcal{O}_i$$

$$\mathcal{O}_9 = (\bar{s}_L \gamma_\mu b_L)(\bar{\ell}\gamma^\mu \ell)$$

$$\mathcal{O}_{10} = (\bar{s}_L \gamma_\mu b_L)(\bar{\ell}\gamma^\mu \gamma_5 \ell)$$

$\mathcal{C} = \mathcal{C}_{\rm SM} + \mathcal{C}_{\rm NP}$ and is complex

- In the Fermi model of the weak interaction, the full electroweak Lagrangian (which was unknown at the time) is replaced by a low-energy theory (QED) plus a single operator with an effective coupling constant



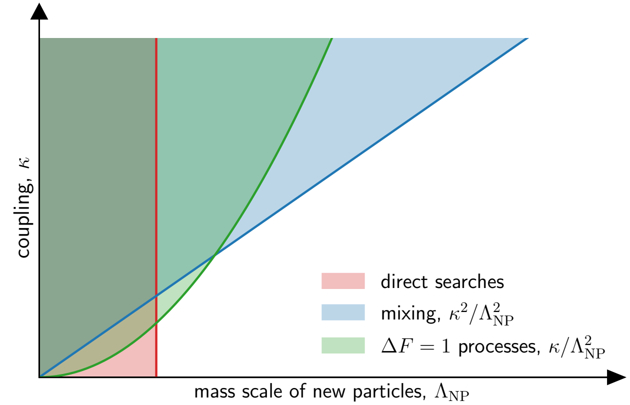
- At low energies the full theory can be replaced by a 4-fermion operator and a single coupling constant, G_F, as

$$\lim_{q^2 \to 0} \left(\frac{g^2}{m_W^2 - q^2} \right) = \frac{g^2}{m_W^2} \tag{10}$$

- The Lagrangian simplifies to

$$\mathcal{L}_{\rm EW} \to \mathcal{L}_{\rm QED} + \frac{G_F}{\sqrt{2}} (\overline{u}d)(e\overline{\nu}) \tag{11}$$

FCNC constraints

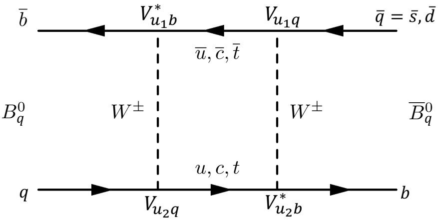


- In reality the direct searches do have *some* dependence on κ as you need a coupling to SM particles in order to produce the new particles in pp collisions

5. $\Delta F = 2$ processes (NP in B mixing)

GIM mechanism

- Take neutral B mixing diagram as an example



- Have an amplitude (summed over up-type quarks in the loop, u_1, u_2)

$$\mathcal{A}(B^0_q \to \overline{B}^0_q) = \sum_{u_1, u_2} (V^*_{u_1 b} V_{u_1 q})(V^*_{u_2 b} V_{u_2 q}) A_{u_1 u_2} \quad \text{where} \quad A_{u_1 u_2} \propto m_{u_1} m_{u_2} / m^2_W \tag{12}$$

- Inserting the known CKM constraint $V^*_{ub} V_{ud} + V^*_{cb} V_{cd} + V^*_{tb} V_{td} = 0$ gives

$$\mathcal{A}(B^0_q \to \overline{B}^0_q) = \sum_{u_1} (V^*_{u_1 b} V_{u_1 d} [V^*_{tb} V_{td} (A_{tu_1} - A_{uu_1}) + V^*_{cb} V_{cd} (A_{cu_1} - A_{uu_1})] \tag{13}$$

so for the B system the top totally dominates as $A_{tu_1} \gg A_{cu_1} \gg A_{uu_1}$

New physics in B mixing

- Introducing new physics at some higher scale, Λ_{NP}, with coupling, κ_{NP}

$$\mathcal{L}_{\mathrm{eff}} = \mathcal{L}_{\mathrm{SM}} + \sum_i \frac{\kappa_{\mathrm{NP}}^2}{\Lambda_{\mathrm{NP}}^{d-4}} \mathcal{O}_i^{(d)} \tag{14}$$

- With the SM contribution from the box diagram

$$\boxed{(V_{tb}^* V_{td})^2 \frac{g^4 m_t^2}{16\pi^2 m_W^4}}$$

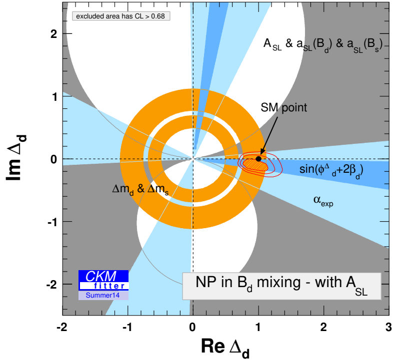
- and a NP contribution (at dimension 6)

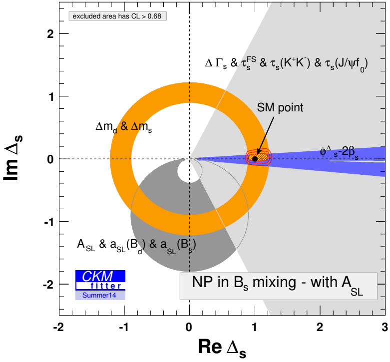
$$\boxed{\frac{\kappa_{\mathrm{NP}}^2}{\Lambda_{\mathrm{NP}}^2}}$$

- Quantify the NP contribution to B mixing with a multiplicative factor such that

$$M_{12} = M_{12,\text{SM}} \cdot \Delta_q \qquad (15)$$

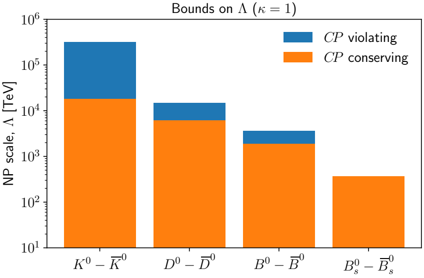
- Constraints provided by CKM fitter show that the result is consistent with the SM (*i.e.* $\mathcal{R}e(\Delta) = 1$ and $\mathcal{I}m(\Delta) = 0$)



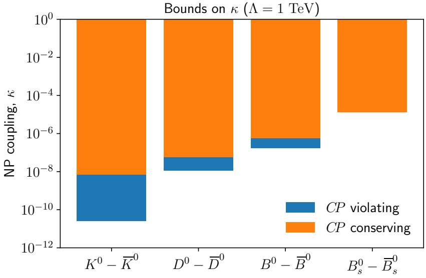


- So far everything shows consistency with the SM
- We can use this to set limits on the size of the NP scale (Λ) or coupling to SM (κ)

Plots produced using [arXiv:1002.0900]



Scale of NP if $\kappa = 1$



Size of κ if $\Lambda = 1$ TeV

Small couplings?

- New flavour violating sources (if there are any) must be highly tuned
  - Either come with a very small coupling constant
  - Or must have a very large mass
- For an $\mathcal{O}(1)$ effect:
  - generic tree-level

$$\kappa_{\rm NP} \sim 1 \qquad \longrightarrow \Lambda_{\rm NP} \gtrsim 10^4 \text{ TeV}$$

  - generic loop-order

$$\kappa_{\rm NP} \sim \frac{1}{(4\pi)^2} \qquad \longrightarrow \Lambda_{\rm NP} \gtrsim 10^3 \text{ TeV}$$

  - tree-level with "alignment"

$$\kappa_{\rm NP} \sim (y_t V_{ti}^* V_{tj})^2 \qquad \longrightarrow \Lambda_{\rm NP} \gtrsim 5 \text{ TeV}$$

  - loop-order with "alignment"

$$\kappa_{\rm NP} \sim \frac{(y_t V_{ti}^* V_{tj})^2}{(4\pi)^2} \qquad \longrightarrow \Lambda_{\rm NP} \gtrsim 0.5 \text{ TeV}$$

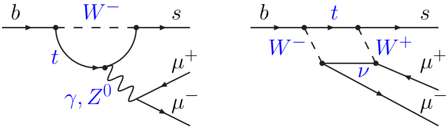
Minimal Flavour Violation

- One way of achieving small couplings is to build models that have a flavour structure which is "aligned" with the CKM matrix
  - Require that the Yukawa couplings are also the unique source of flavour breaking beyond the SM
- This is referred to as minimal flavour violation (MFV)
- The couplings to new particles are naturally supressed by the Hierarchy of CKM elements
- Clearly this massively degrades the sensitivty to finding it

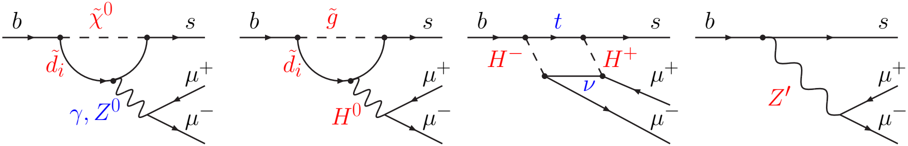
6. $\Delta F = 1$ processes (Rare B decays)

$\Delta F = 1$ FCNC decays

- FCNC transitions only occur at loop order (and beyond) in the SM
- The SM diagrams involve the charged current interaction ($W^{\pm}$)



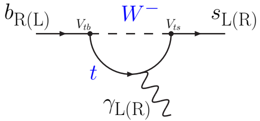
- New particles can also contribute (at either tree or loop level depending on the NP characteristics)



- The effect of the NP amplitudes can be to enhance (or suppress) decays, introduce new sources of CP violation or modify angular distributions of final-state particles (as their spin structure and coupling will be different to the SM)

Properties of $\Delta F = 1$ processes

- There are a large number of other observables that can be considered
- In the SM, photons from $b \to s\gamma$ decays are predominantly left-handed ($\mathcal{C}_7/\mathcal{C}_7' \sim m_b/m_s$) due to the charged current interaction



- The flavour structure of the SM implies that the rate of $b \to s$ processes is suppressed by $|V_{td}/V_{ts}|^2$ relative to $b \to s$ processes
- In the SM

$$\Gamma(B \to M\mu^+\mu^-) \approx \Gamma(B \to Me^+e^-)$$

  due to the universal couplings of the gauge bosons (except the Higgs) to the different lepton flavours (known as lepton universality). The only differences in the rate come down to phase-space considerations
- Direct lepton flavour violation is unobservable **in the SM** at any conceivable experiment due to the small size of the neutrino mass

The effective theory for rare $b \to s$ decays

- Can write an effective theory Hamiltonian as

$$H_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \frac{\alpha_e}{4\pi} \sum_i \mathcal{C}_i(\mu) \mathcal{O}_i(\mu) \tag{16}$$

Weak decay, $(1/m_W)^2$
CKM supprresion
Loop suppression, $(1/4\pi)^2$
Wilson coefficient (integrating out scales above μ)
Local operator with different Lorentz structure (vector, axial vector *etc.*)

- Then introduce new particles that give rise to corrections

$$\Delta H_{\text{eff}} = \frac{\kappa_{\text{NP}}}{\Lambda_{\text{NP}}^2} \mathcal{O}_{\text{NP}} \tag{17}$$

NP scale
local operator

- The constant κ can share some, all or none of the suppression of the SM process

Leptonic decay operators

- Have already seen some of the non-lepontic operators (and the $b \to s\ell^+\ell^-$ operators $\mathcal{O}_9$ and $\mathcal{O}_{10}$)

$$\mathcal{O}_7 = \frac{m_b}{e}\bar{s}\sigma^{\mu\nu}P_R b F_{\mu\nu}$$

EW penguin

$$\mathcal{O}_8 = g_s\frac{m_b}{e^2}\bar{s}\sigma^{\mu\nu}P_R T^\alpha b G^\alpha_{\mu\nu}$$

gluonic penguin

$$\mathcal{O}_9 = \bar{s}\gamma_\mu P_L b\bar{\ell}\gamma^\mu\ell$$

vector current

$$\mathcal{O}_{10} = \bar{s}\gamma_\mu P_L b\bar{\ell}\gamma^\mu\gamma_5\ell$$

axial-vector current

$$\mathcal{O}'_7 = \frac{m_b}{e}\bar{s}\sigma^{\mu\nu}P_L b F_{\mu\nu}$$

$$\mathcal{O}'_8 = g_s\frac{m_b}{e^2}\bar{s}\sigma^{\mu\nu}P_L T^\alpha b G^\alpha_{\mu\nu}$$

$$\mathcal{O}'_9 = \bar{s}\gamma_\mu P_R b\bar{\ell}\gamma^\mu\ell$$

$$\mathcal{O}'_{10} = \bar{s}\gamma_\mu P_R b\bar{\ell}\gamma^\mu\gamma_5\ell$$

right handed currents
(suppressed in the SM)

NP operators

- Scalar and pseudo-scalar operators (*e.g.* from Higgs penguins)

$$\mathcal{O}_S = \overline{s}P_R b\bar{\ell}\ell, \qquad \mathcal{O}'_S = \overline{s}P_L b\bar{\ell}\ell$$
$$\mathcal{O}_P = \overline{s}P_R b\bar{\ell}\gamma_5\ell, \qquad \mathcal{O}'_P = \overline{s}P_L b\bar{\ell}\gamma_5\ell$$

- Tensor operators

$$\mathcal{O}_T = \overline{s}\sigma_{\mu\nu} b\bar{\ell}\sigma^{\mu\nu}\ell, \qquad \mathcal{O}'_T 5 = \overline{s}\sigma_{\mu\nu} b\bar{\ell}\sigma^{\mu\nu}\ell$$

- All of these are vanishingly small in the SM
- **In principle one could also introduce LFV versions of every operator**

Generic $\Delta F = 1$ process

- In the effective theory we then have

$$\mathcal{A}(B \to f) = V_{tb}^* V_{tq} \sum_i \mathcal{C}_i(M_W) U(\mu, m_W) \underbrace{\langle f|\mathcal{O}_i(\mu)|B\rangle}_{\text{had. mat. elem,}} \tag{18}$$

- For **inclusive processes** the sum over exclusive states is related to the quark level decays

$$\mathcal{B}(B \to X_s\gamma) = \mathcal{B}(b \to s\gamma) + \mathcal{O}(\Lambda_{\rm QCD}^2/m_B^2) \tag{19}$$

- For **exclusive processes** we need to compute form-factors / decay constants
- In leptonic decays the matrix element can be factorised into a leptonic current and a B meson decay consant, f_{B_q}

$$\langle \ell^+\ell^-|j_\ell j_q|B_q\rangle = \langle \ell^+\ell^-|j_\ell|0\rangle\langle 0|j_q|B_q\rangle \approx \boxed{\langle \ell^+\ell^-|j_\ell|0\rangle \cdot f_{B_q}} \tag{20}$$

- In semi-leptonic decays the matrix element can be factorised into a leptonic current times a form-factor

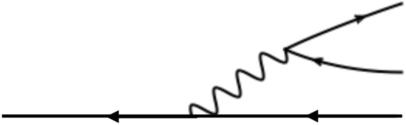
$$\langle \ell^+\ell^- M|j_\ell j_q|B_q\rangle = \langle \ell^+\ell^-|j_\ell|0\rangle\langle M|j_q|B_q\rangle \approx \boxed{\langle \ell^+\ell^-|j_\ell|0\rangle \cdot F(q^2) + \mathcal{O}(^{\Lambda_{\rm QCD}}/_{m_B})} \tag{21}$$

although, due to hadronic contributions, this factorisation is not exact

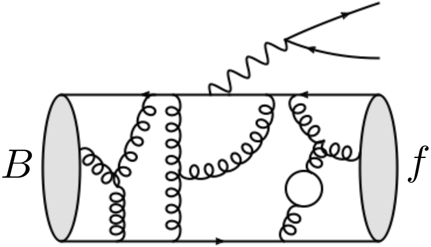
Form-factors

- Alas, we never have free quarks so we need to compute hadronic matrix elements (form-factors and decay constants) which relate us back to a real life mesonic or baryonic decay system
- This is the non-perturbative regime of QCD *i.e.* very difficult (and very nasty) to estimate
- Fortunately there have been considerable recent developments (last 10-20 years) which do provide us the tools to make some calculations in different kinematic regimes

What we can do



Real life

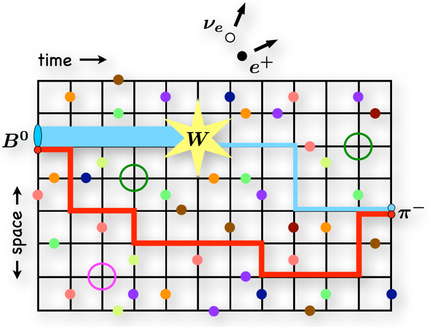


Theoretical QCD calculation tools

- Lattice QCD
  - Non-perturbative approach to QCD using a discretised system of points in space and time
  - As the lattice becomes infinitely large and the spacing infinitely small the continuum of QCD is reached
- Light-Cone-Sum-Rules (LCSR)
  - Exploit parton-hadron duality to compute form-factors and decay constants
- Operator product expansions (OPE)
  - Match physics at relevant scales
- Heavy quark expansion
  - Exploit the heaviness of the b quark, $m_b \gg \Lambda_{\text{QCD}}$
- QCD factorisation
  - Light quark has large energy in the meson decay frame
  - *e.g.* in $B \to \pi$ decays, quarks in the π have high energy in the B rest frame
- Soft Collinear Effective Theory
  - Model the system as highly energetic quarks interacting with soft collinear gluons
- Chrial perturbation theory

Lattice QCD

- Theory developments have been rapid over the past decade
- Takes huge scale collaborations (for theorists anyway)
- Lattice QCD is a numerical approach to non-perturbative calculations
  - Recall the QCD Lagrangian has massless gluons and nearly massless quarks
  - There is a strong coupling $\implies$ non-perturbative
- Perform the Feynman path integral in Euclidean space on the "lattice" (space-time grid) using Markov Chain MC (MCMC)
  - Correlation lengths $\rightarrow$ masses
  - Amplitudes $\rightarrow$ matrix elements



7. FCNC Experimental Results

FCNC Experimental Results

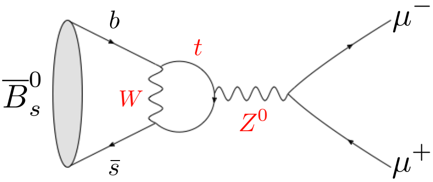
- Will mainly focus on recent measurements of B decay processes, predominantly involving $b \to s$ transitions
- These are some of the less well tested (only recently had sufficient samples of B decays for many of these measurements
- FCNC decays of charm and strange can also be studied however the GIM mechanism is much more effective (*i.e.* there is a larger natural cancellation) for them
  - For the charm mesons the masses and mass differences are small (*i.e.* $m_c - m_s$)
  - For strange the top contribution is considerably suppressed relative to the B decays because $V_{ts} \ll V_{tb}$
- These are some of the arguments that make B physics so compelling (at least to some)

- $B^0_s \to \mu^+\mu^-$ is the golden channel for study of FCNC decays
- It is highly suppressed in the SM
  1. Loop suppressed
  2. CKM suppressed
  3. Helicity suppressed (pseudo-scalar B to two spin-$\frac{1}{2}$ muons)

SM process

with neutral current (axial-vector)

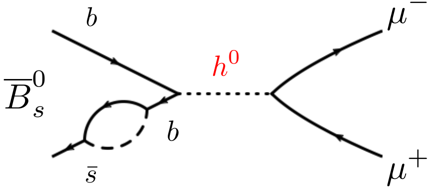
There is also a contribution from $W^{\pm}$ box diagrams



Possible NP process

with scalar operators

No helicity suppression *e.g.* SUSY at high $\tan(\beta)$



- Nice and clean because only one operator contributes in the SM

$$\mathcal{O}_{10} = (\bar{s}\gamma_\mu b)(\bar{\mu}\gamma^\mu\gamma_5\mu) \tag{22}$$

- The branching fraction in the SM is

Decay constant $\langle 0|\bar{s}\gamma^\mu\gamma_5 b|B\rangle = i f_B p^\mu$

$$\mathcal{B}(B_s^0 \to \mu^+\mu^-) = \boxed{|V_{tb}^* V_{ts}|^2} \frac{G_F^2 \alpha_e^2}{16\pi^3 \Gamma_H} M_B M_\mu^2 \boxed{f_B^2} \sqrt{1 - \frac{4M_\mu^2}{M_B^2}} |\mathcal{C}_{10}(m_b)|^2 \boxed{\left(\frac{M_\mu^2}{M_B^2}\right)}$$

CKM factors

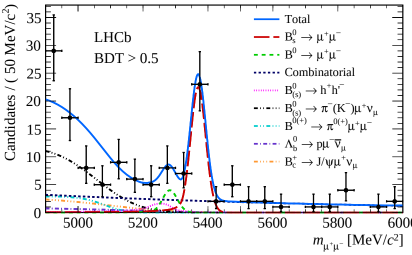
helicity suppression

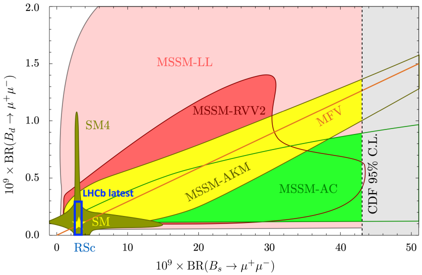
- Beyond the SM

$$\frac{\mathcal{B}(B_s^0 \to \mu^+\mu^-)_{\rm NP}}{\mathcal{B}(B_s^0 \to \mu^+\mu^-)_{\rm SM}} = \frac{1}{|\mathcal{C}_{\rm SM}|^2} \left[\left(1 - 4\frac{m_\mu^2}{m_B^2}\right) \left| \frac{m_B}{2m_\mu}(\mathcal{C}_S - \mathcal{C}_S') \right|^2 + \left| \frac{m_B}{2m_\mu}(\mathcal{C}_P - \mathcal{C}_P') + (\mathcal{C}_{10} - \mathcal{C}_{10}') \right|^2 \right]$$

$B_s^0 \to \mu^+\mu^-$ experimental results

- Observation is the end of a long road of searches
- $B_s^0 \to \mu^+\mu^-$ ($B^0 \to \mu^+\mu^-$) now observed at $> 7\sigma$ ($\sim 3\sigma$). Both are consistent with the SM predictions
- No sign of NP here (unfortunately) but this does set some very strong constraints on many models





Photon polarisation

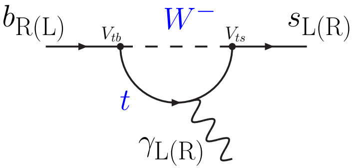
- In radiative B decays allows both

$$b_L \to s_R \gamma_R \tag{23}$$

$$b_R \to s_L \gamma_L \tag{24}$$

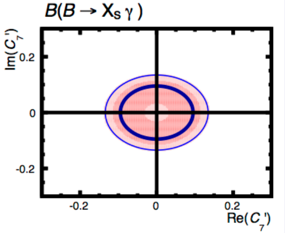
- However the charged current interaction **only** couples to left-handed quarks
- Need a helicity flip (boost into suitable frame) to either the b or s quark
- The right-handed contribution is therefore suppressed by

$$\frac{\mathcal{A}(b_L \to s_R \gamma_R)}{\mathcal{A}(b_R \to s_L \gamma_L)} \sim \frac{m_s}{m_b}$$

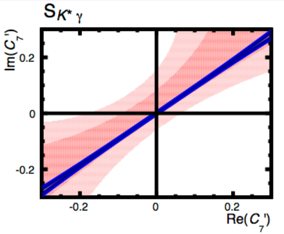


- Constraints on right-handed currents in $b \to s\gamma$ decays
- Results are consistent with the LH polarisation expected in the SM

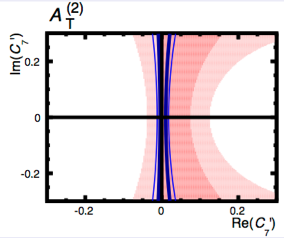
Inclusive branching fraction



Time-dependent CP violation in $B \to [K_S^0 \pi^0]\gamma$



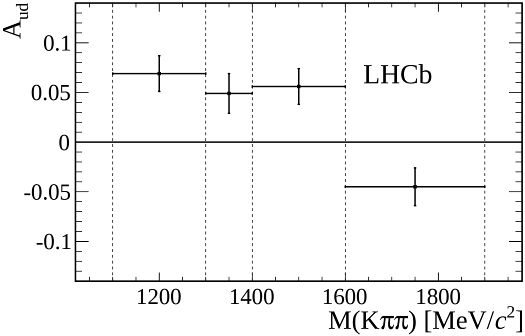
Angular distribution of $B \to K^* e^+ e^-$



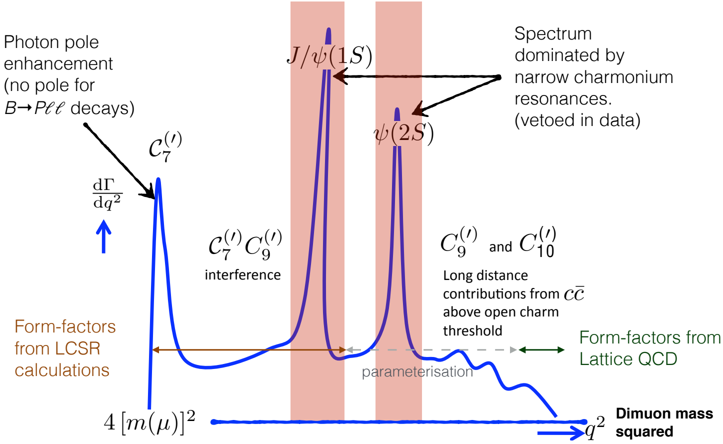
Is the photon polarised?

- Yes, in $B^+ \to K^+\pi^-\pi^+\gamma$ decays the photon has a preferred direction with respect to the $K^+\pi^-\pi^+$ decay plane
- This can only happen if the photon is polarised

[arXiv:1402.6852]



- A very important class of decays for FCNC limits are $b \to s\ell^+\ell^-$ transitions
- Understanding distributions with respect to the invariant mass of the di-lepton spectrum (q^2) is vital

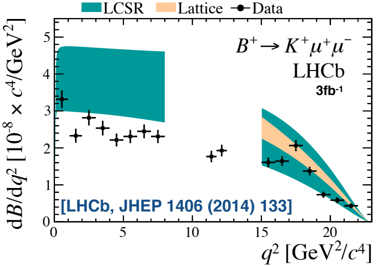


slide from Tom Blake

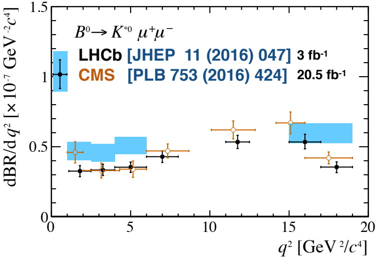
Branching fractions in $b \to s\mu^+\mu^-$

- The LHCb (and CMS) Run 1 datasets already have precise measurements of differential branching fractions with at least comparable precision to the SM theory expectations

[arXiv:1403.8044]

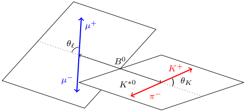


[arXiv:1606.04731], [arXiv:1507.08126]

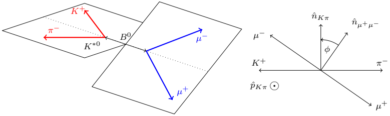


- SM predictions have large theory uncertainties from the hadronic form-factors (of which there are 3 for $B^\pm \to K^\pm$ and 7 for $B \to K^*$)
- Details of theory predictions in [arXiv:1111.2558], [arXiv:1306.0434] and [arXiv:1411.3161]

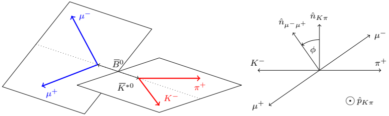
- We have a four-body final state (as $K^{*0} \to K^+\pi^-$)
  - The angular distribution provides many observables that are sensitive to new physics
  - The branching fraction might not be affected (or affected at a very small level) however angular distributions can be affected by different spin structure of NP particles
  - For example, at low q^2, the angle between the two decays planes, ϕ, is sensitive to the photon polarisation
- The four-body system is described by three decay angles (defined in the helicity basis) and the dimuon invariant mass squared, q^2
  - ϕ angle between the two decay planes in the B rest-frame
  - θ_ℓ, θ_K angle between the B momentum in the B frame and the $K\pi$ or $\ell^+\ell^-$ momentum in their decay frame



(a) θ_K and θ_ℓ definitions for the B^0 decay



(b) ϕ definition for the B^0 decay



(c) ϕ definition for the $\overline{B}^0$ decay

The $B^0 \to K^{*0}\mu^+\mu^-$ angular distribution

- A rather complex angular distribution with many observables (which depend on form-factors for the $B \to K^*$ transition plus the Wilson coefficients
- The CP-averaged angular decay rate (where $\Omega = (\theta_K, \theta_\ell, \phi)$) is

$$\frac{1}{d(\Gamma+\bar{\Gamma})/dq^2} \frac{d^3(\Gamma+\bar{\Gamma})}{d\vec{\Omega}}\bigg|_P = \frac{9}{32\pi}\Big[\tfrac{3}{4}(1-\boxed{F_L})\sin^2\theta_K + \boxed{F_L}\cos^2\theta_K$$
$$+\tfrac{1}{4}(1-\boxed{F_L})\sin^2\theta_K\cos 2\theta_\ell$$
$$-\boxed{F_L}\cos^2\theta_K cos2\theta_\ell + S_3\sin^2\theta_K\sin^2\theta_\ell\cos 2\theta$$
$$+S_4\sin 2\theta_K\sin 2\theta_\ell\cos\phi + \boxed{S_5}\sin 2\theta_K\sin\theta_\ell\cos\phi$$
$$+\tfrac{4}{3}\boxed{A_{FB}}\sin^2\theta_K\cos\theta_\ell + S_7\sin 2\theta_K\sin\theta_\ell\sin\phi$$
$$+S_8\sin 2\theta_K\sin 2\theta_\ell\sin\phi + S_9\sin^2\theta_K\sin^2\theta_\ell\sin 2\phi\Big]$$

$\boxed{F_L}$	fractional longitudinal polarisation of the K^{*0}
$\boxed{A_{FB}}$	forward-backward asymmetry of the dilepton system
$\boxed{S_5}$	particularly sensitive to $\mathcal{C}_9$

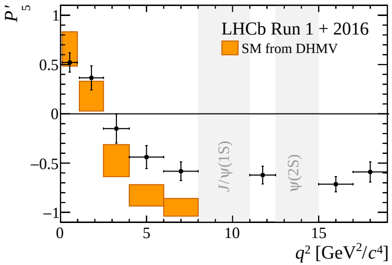
Form-factor “free” observables

- Several experiments have produced such an angular analysis (LHCb is the most sensitive)
- In QCD factorisation / SCET there are only two form factors
  - One is associated with A_0 and the other with $A_{\parallel}$ and $A_{\perp}$
- Can then construct ratios of observables which are independent of the form-factors (at least to leading order) *e.g.*

$$P_5' = S_5/\sqrt{F_L(1-F_L)}$$

[arXiv:2003.04831]

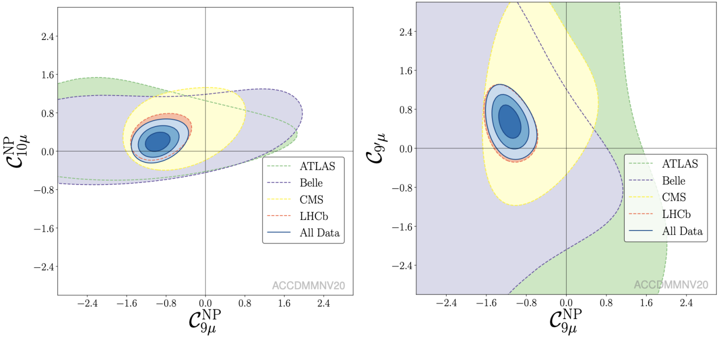
- Historically there has been quite a bit of tension between predictions and measurement of P_5'. In the latest LHCb measurement ([arXiv:]) this specific tension is a bit reduced but there remains an overall considerable tension with the SM (arising from discrepancies in P_5' and A_{FB} and F_L)



Global fits

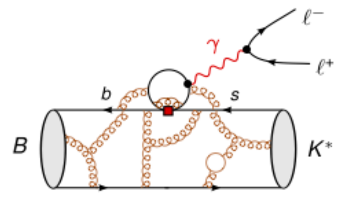
- These measurements then lead to some very nice interpretations in terms of the Wilson coefficients with global fits to $b \to s$ data
- Note a general pattern of consistency between experiments/measurements **and** data seems to favour a modified vector coupling ($C_9^{NP} \neq 0$) at $\sim 4 - 5\sigma$ (if you entirely trust the theory assumptions

[arXiv:1903.09578] (updated 2020)



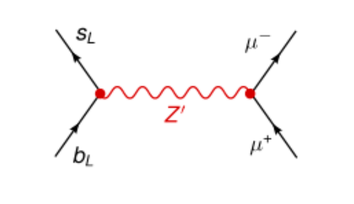
Interpretation of global fits

Pessimist's view point



- A vector-like contribution could point to a problem with our understanding of QCD
- *e.g.* are we correctly estimating the contribution from charm loops that produce dimuon pairs via a virtual photon?

Optimist's view point



- Vector-like contribution could come from a new tree-level contribution (*e.g.* Z' with $m \sim O(1)$ TeV)
- A Z' should also give effects elsewhere (*e.g.* particularly in mixing, which it doesn't) so a challenge for model builders who need to suppress this

Which one are you?

Further work is needed from both experiment and theory to establish what is going on here

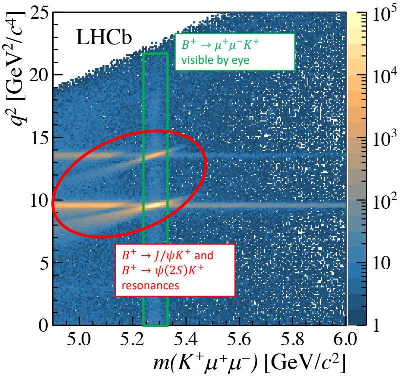
- In the SM ratios like

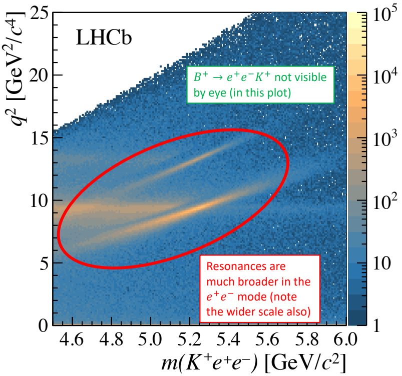
$$R_K = \frac{\int d\Gamma(B^+ \to K^+\mu^+\mu^-)/dq^2 \cdot dq^2}{\int d\Gamma(B^+ \to K^+e^+e^-)/dq^2 \cdot dq^2} \tag{25}$$

  should only differ from unity by phase space
- The dominant SM processes couple equally to the different lepton flavour (apart from the Higgs)
- Incredibly theoretically clean since hadronic uncertainties cancel in the ratio (they have the same hadronic matrix element). The only consideration is from small electroweak corrections as q^2 approaches 0
- Experimentally these are much more challenging, primarily due to differences in muon/electron reconstruction
  - In particular Bremsstrahlung radiation from the electrons
  - LHCb does not have a high resolution ECAL
  - Electron efficiency is much poorer than muon efficiency at LHCb (trigger and reconstruction)

$B^+ \to K^+\ell^+\ell^-$ candidates

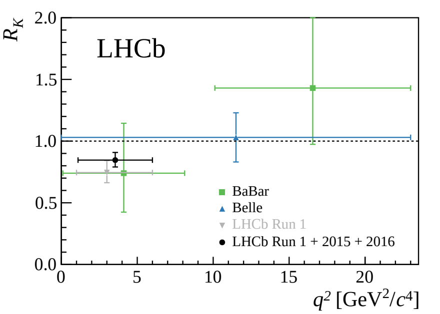
- Have to correct electrons for energy loss due to Bremmstrahlung (look for ECAL clusters (*i.e.* photons) associated with the electron track
- This is successful to some extent but even after Bremmstrahlung recovery there are significant differences in mass resolution between the dielectron and dimuon final states





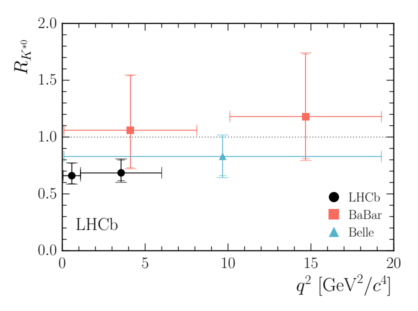
R_K from $B^\pm \to K^\pm \ell^+ \ell^-$

[arXiv:1903.09252]



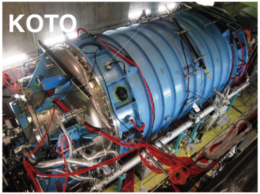
$R_{K^{*0}}$ from $B^0 \to K^{*0} \ell^+ \ell^-$

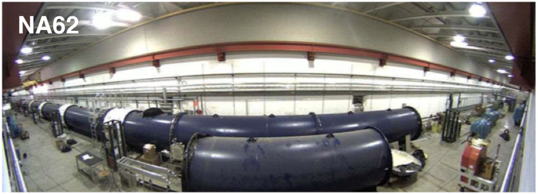
[arXiv:1705.05802]



Rare kaon decays

- Two new rare kaon decay experiments
  - KOTO at J-PARC, searching for $K^0_{\rm L} \to \pi^0 \nu\overline{\nu}$
  - NA62 at CERN, searching for $K^+ \to \pi^+ \nu\overline{\nu}$
- The advantage (theoretically) of final states with neutrinos is that there is no contribution from quark loops involving light quarks (which can annihilate to produce charged leptons *e.g.* charm loops)
- The challenge experimentally is these are incredibly rare (and contain just one charged track in the final state)





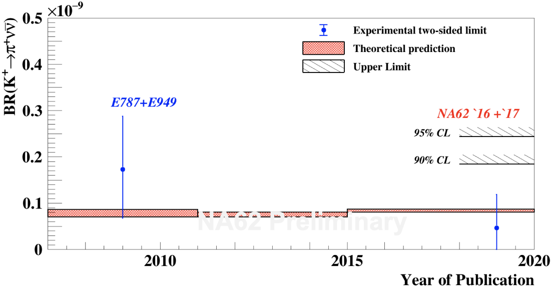
NA62

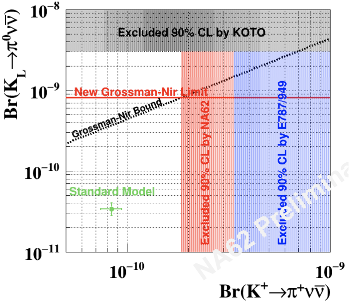
- Aim to collect a dataset of $\sim 100\ K^+ \to \pi^+ \nu \bar{\nu}$ decays
- Currently have ~ 3 events in analysed data (2016+2017) giving

$$\mathcal{B}(K^+ \to \pi^+ \nu \bar{\nu}) = (4.7^{+7.2}_{-4.7}) \times 10^{-11}$$

i.e. consistent with zero

- Also search for lepton number violating $K^\pm \to \pi^\mp \ell^\pm \ell^\pm$ decays



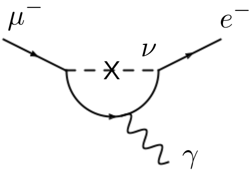


8. Lepton Flavour Violation

- Essentially forbidden in the SM by the smallness of the neutrino mass

$$\mathcal{B}(\mu \to e\gamma) \propto \frac{m_\nu^4}{m_W^4} \sim 10^{-54} \tag{26}$$

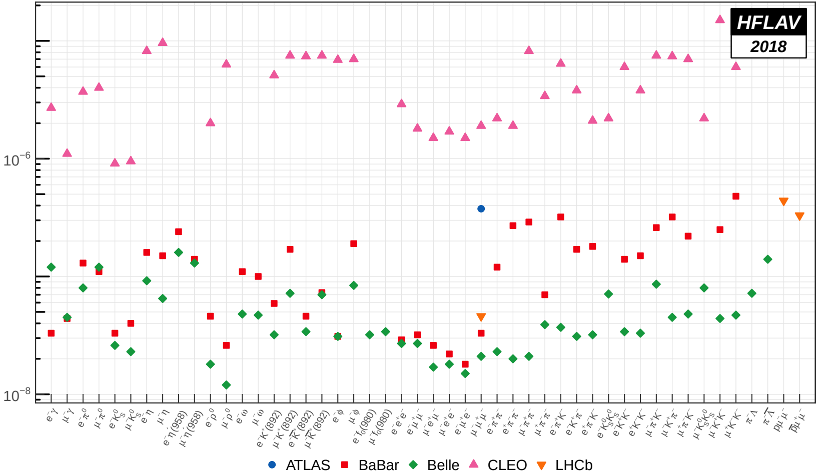
  - Very powerful null test of the SM
- **Any** visible signal is a clear sign of New Physics



- Different signatures include
  1. $\mu \to e\gamma$ at rest (MEG at PSI, Mu2E at PSI)
  2. $\mu \to 3e$ (Mu3e at PSI)
  3. μ conversion in field of Au nucleus (SINDRUM II as PSI)

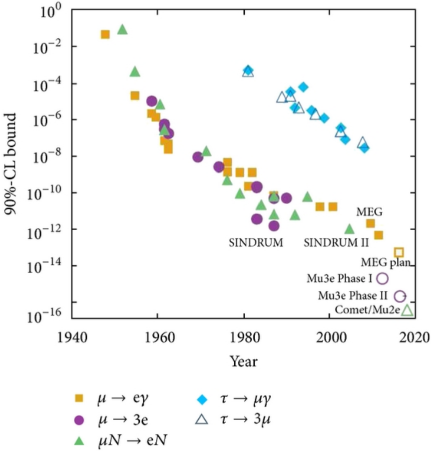
- A large number of experimental signatures
- Global summary (averages) provided by HFLAV

90% CL upper limits on τ LFV decays



Charged LFV future

- Data taking has begun at MEG-II (aiming for $O(10^{-14})$)
- New $\mu \to 3e$ experiment (Mu3e) at PSI
- Two new conversion experiments (Mu2e) at PSI and (COMET) at J-PARC
- Expect improvements for LFV τ decays from Belle 2



9. Recap

New Physics?

- We have seen in these lectures the incredible success of the CKM matrix as a predictive tool for properties of flavour decays
- Our various measurements which constrain the CKM picture are all consistent with the SM predictions
- However, there are some very tantalising hints that could suggest New Physics
  - Tension in V_{ub} (and to a lesser extent V_{cb})
  - Enhancement / tension in $B \to D^{(*)}\tau\nu_\tau$
  - Anomalies in $B \to K^{(*)}\ell^+\ell^-$ decays
  - Muon g-2

  } all at $\gtrsim 3\sigma$
- These should all be resolved in the next 5-10 years
- **It's an exciting time to be a flavour physicist!**

Recap

In this lecture we have covered

- Effective theories
- Flavour Changing Neutral Current processes
- Experimental constraints on new particles in $\Delta F = 1$ and $\Delta F = 2$ FCNCs
- Minimal Flavour Violation
- Lepton Flavour Violation
- Future Flavour Violation Experiments

End of Lecture 4

GAME OVER

Thanks for playing (listening)!