Flavour Physics (of quarks)

Part 3: Measuring the CKM parameters

Matthew Kenzie

Warwick Week Graduate Lectures

July 2020

Overview

Lecture 1: Flavour in the SM

- Flavour in the SM
- Quark Model History
- The CKM matrix

Lecture 2: Mixing and CP violation

- Neutral Meson Mixing (no CPV)
- B-meson production and experiments
- CP violation

Lecture 3: Measuring the CKM parameters (Today)

- Measuring CKM elements and phases
- Global CKM fits
- CPT and T-reversal
- Dipole moments

Lecture 4: Flavour Changing Neutral Currents

- Effective Theories
- New Physics in B mixing
- New Physics in rare $b \to s$ processes
- Lepton Flavour Violation

1. Recap

Recap

Homework from last time

Why is it that down type neutral mesons contain the anti-quark species but up type contain the quark?

For example:

- $B^0 = (\bar{b}d)$, $B_s^0 = (\bar{b}, s)$, $K^0 = (\bar{s}d)$
- $\overline{B}^0 = (b\bar{d})$, $\overline{B}_s^0 = (b, \bar{s})$, $\overline{K}^0 = (s\bar{d})$

But:

- $D^0 = (c\bar{u})$
- $\overline{D}^0 = (\bar{c}u)$

- Hypercharge:

$$Y = B + S + C + B' + T' \tag{1}$$

- Electric charge:

$$Q = I_3 + Y/2 \tag{2}$$

Recap

- Last time we discussed neutral meson mixing and all three types of CPV
- Saw the "master" equations for neutral meson decays which are characterised by

$$\lambda_f = \frac{q}{p}\frac{\bar{A}_f}{A_f}$$

  - CPV in decay (the only type possible for a charged initial state) $[|\bar{A}_{\bar{f}}/A_f| \neq 0]$
  - CPV in mixing $[|q/p| \neq 1]$
  - CPV in the interference between mixing and decay $[|\arg(\lambda_f) \neq 0]$
- We got two important expressions which we will see again today
  1. The direct (time-integrated) CP asymmetry arising when we have two amplitudes with different strong (δ) and weak (ϕ) phases and magnitude ratio (r):

$$\mathcal{A}_{CP} = \frac{2r\sin(\delta)\sin(\phi)}{1 + r^2 + 2r\cos(\delta)\cos(\phi)} \tag{3}$$

  2. The general time-dependent CP asymmetry for a neutral meson to a CP-eigenstate

$$\mathcal{A}_{CP}(t) = \frac{C_f\cos(\Delta m t) - S_f\sin(\Delta m t)}{\cosh(\frac{1}{2}\Delta\Gamma t) + D_f\sinh(\frac{1}{2}\Delta\Gamma t)} \tag{4}$$

where $$C_f = \frac{1-|\lambda_f|^2}{1+|\lambda_f|^2}, \quad D_f = \frac{2\mathcal{R}e(\lambda_f)}{1+|\lambda_f|^2}, \quad S_f = \frac{2\mathcal{I}m(\lambda_f)}{1+|\lambda_f|^2} \tag{5}$$

- Recall the CKM matrix which governs quark weak transitions

CKM exhibits a clear hierarchy

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \sim \begin{pmatrix} 1 & 0.2 & 0.004 \\ 0.2 & 1 & 0.04 \\ 0.008 & 0.04 & 1 \end{pmatrix}$$

experimentally
determined values

Commonly represented in the Wolfenstein parametrisation

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

4 $\mathcal{O}(1)$ real parameters (A, λ, ρ, η)

Recap

- Wolfenstein parameterisation ensures that

$$\bar{\rho} + i\bar{\eta} = -(V_{ud}V_{ub}^*)/(V_{cd}V_{cb}^*) \tag{6}$$

  is phase convention independent and CKM matrix written in $(A, \lambda, \bar{\rho}, \bar{\eta})$ is unitary to all orders in λ

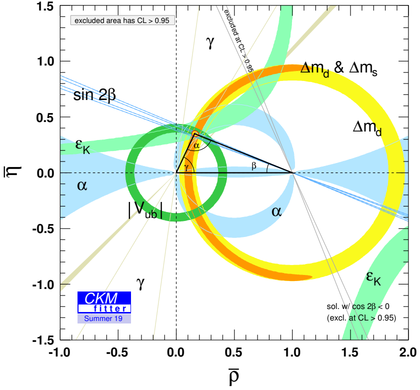
$$\bar{\rho} = \rho(1 - \lambda^2/2 + \dots) \quad \text{and} \quad \bar{\eta} = \eta(1 - \lambda^2/2 + \dots) \tag{7}$$

- The amount of CP violation in the SM is equivalent to asking how big is η relative to ρ.
- There are many experimental observables (9 element magnitudes and 4 phases) we can measure to over-constrain the CKM picture.

- Unitarity gives 6 constraints for off-diagonals represented as triangles in $(\bar{\rho}, \bar{\eta})$ space

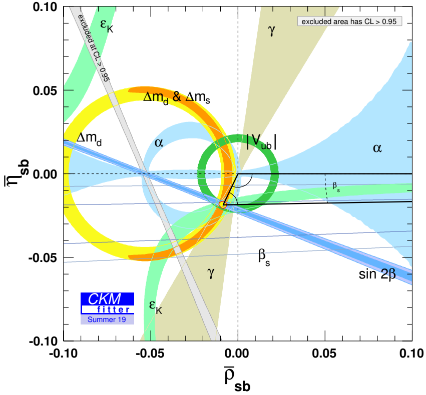
The (B^0) Unitarity Triangle

$$\bar{\rho}_{(db)} + i\bar{\eta}_{(db)} = -(V_{ud}V_{ub}^*)/(V_{cd}V_{cb}^*)$$



The B_s^0 Unitarity Triangle

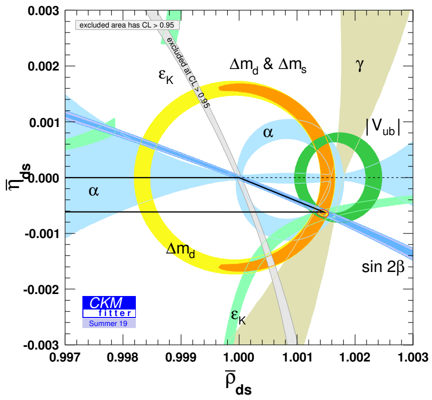
$$\bar{\rho}_{sb} + i\bar{\eta}_{sb} = -(V_{us}V_{ub}^*)/(V_{cs}V_{cb}^*)$$



- Unitarity gives 6 constraints for off-diagonals represented as triangles in $(\bar{\rho},\bar{\eta})$ space

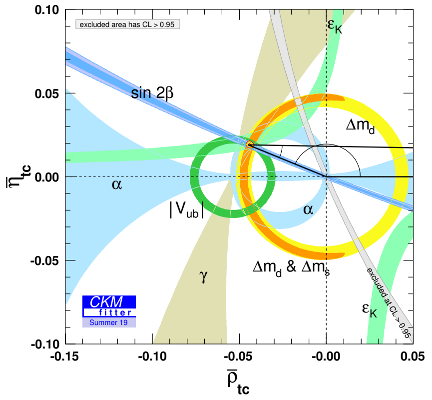
The K Unitarity Triangle

$$\bar{\rho}_{(ds)} + i\bar{\eta}_{(ds)} = -(V_{ud}V_{us}^*)/(V_{cd}V_{cs}^*)$$



The tc Unitarity Triangle

$$\bar{\rho}_{tc} + i\bar{\eta}_{tc} = -(V_{td}V_{cd}^*)/(V_{ts}V_{cs}^*)$$

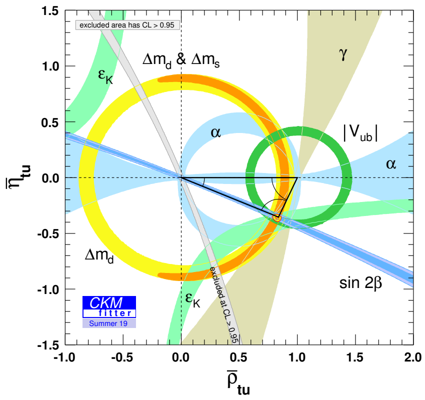


CKM Unitarity Triangles

- Unitarity gives 6 constraints for off-diagonals represented as triangles in $(\bar{\rho}, \bar{\eta})$ space

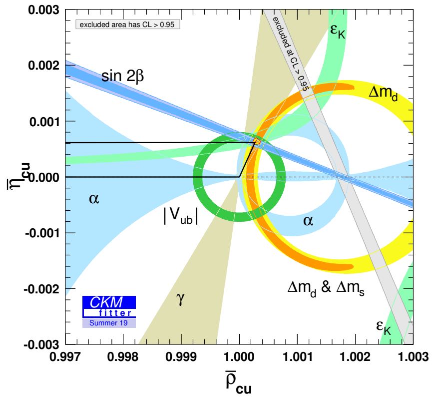
The tu Unitarity Triangle

$$\bar{\rho}_{(tu)} + i\bar{\eta}_{(tu)} = -(V_{td}V^*_{ud})/(V_{ts}V^*_{us})$$



The D Unitarity Triangle

$$\bar{\rho}_{cu} + i\bar{\eta}_{cu} = -(V_{cd}V^*_{ud})/(V_{cs}V^*_{us})$$

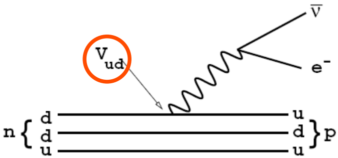


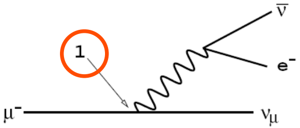
2. Measuring CKM matrix element magnitudes

Measuring V_{ud}

- Compare rates of neutron, n^0, and muon, μ^-, decays
- The ratio is proportional to $|V_{ud}|^2$
- $|V_{ud}| = 0.947417 \pm 0.00021$
- $|V_{ud}| \approx 1$

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$





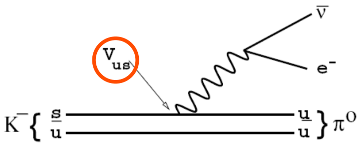
$$\frac{d\Gamma(n \to pe^-\overline{\nu}_e)}{dx_p} = \frac{G_F^2 m_n^2}{192\pi^2} |V_{ud}|^2 f(q^2)^2 \left(x_p^2 - 4\frac{m_p^2}{m_n^2} \right)^{3/2}, \quad \text{where} \quad x_p = \frac{2E_p}{m_n}$$

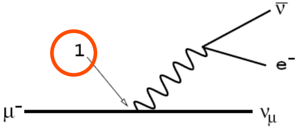
Measuring CKM matrix elements

Measuring V_{us}

- Compare rates of kaon, K^-, and muon, μ^-, decays
- The ratio is proportional to $|V_{us}|^2$
- $|V_{us}| = 0.2248 \pm 0.0006$
- $|V_{us}| \approx \sin(\theta_C) \approx \lambda$

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$



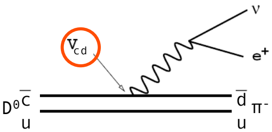


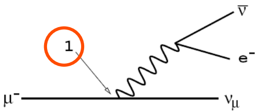
$$\frac{d\Gamma(\overline{K}^0 \to \pi^+ e^- \overline{\nu}_e)}{dx_\pi} = \frac{G_F^2 m_K^2}{192\pi^2} |V_{us}|^2 f(q^2)^2 \left(x_\pi^2 - 4\frac{m_\pi^2}{m_K^2} \right)^{3/2}, \quad \text{where} \quad x_\pi = \frac{2E_\pi}{m_K}$$

Measuring V_{cd} and V_{cs}

- Early measurements used neutrino DIS
- Now use semi-leptonic charm decays, $D^0 \to \pi^- \ell^+ \nu_\ell$ (V_{cd}) and $D^0 \to K^- \ell^+ \nu_\ell$ (V_{cs})
- $|V_{cd}| = 0.220 \pm 0.005$
- $|V_{cs}| = 0.995 \pm 0.016$
- $|V_{cd}| \approx \sin(\theta_C) \approx \lambda$
- $|V_{cs}| \approx 1$

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

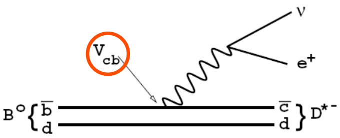


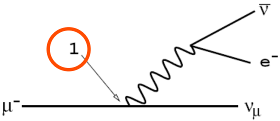


Measuring V_{cb}

- Compare rates of $B^0 \to D^{*-}\ell^+\nu_\ell$ and muon decays
- Ratio is proportional to $|V_{cb}|^2$
- $|V_{cb}| = 0.0405 \pm 0.0013$
- $|V_{cd}| \approx \sin^2(\theta_C) \approx \lambda^2$

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$



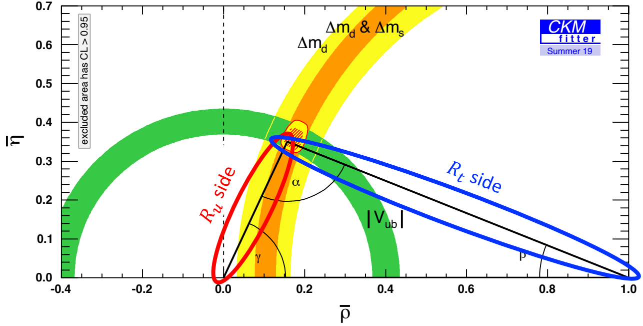


$$\frac{d\Gamma(b \to u_\alpha \ell^- \overline{\nu}_\ell)}{dx} = \frac{G_F^2 m_b^5}{192\pi^2}|V_{\alpha b}|^2 \left(2x^2\left(\frac{1-x-\xi}{1-x}\right)^2\left(3-2x+\xi+\frac{2\xi}{1-x}\right)\right)$$

$$\text{where} \quad \alpha = u, c, \quad \xi = \frac{m_\alpha^2}{m_b^2}, \quad x = \frac{2E_l}{m_b}$$

Measuring CKM matrix elements

- The sides of *the* (B^0) unitarity triangle are constrained by
  - The ratio V_{ub}/V_{cb} for the left side (known sometimes as R_u)
  - The ratio $\Delta m_d/\Delta m_s$ for the right side (known sometimes as R_t)
- Sometimes called "UT constraints from CP-conserving quantities



Measurements of V_{ub}

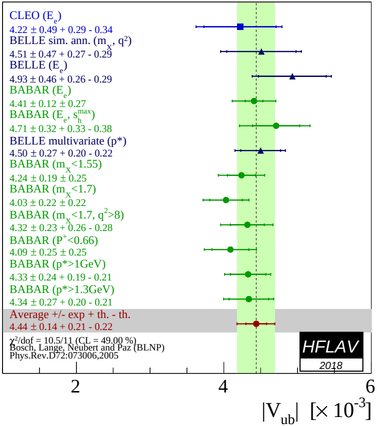
- There are three ways to determine V_{ub}
  1. "Inclusive" decays of $b \to u\ell^- \overline{\nu}_\ell$
     - Of course there are no bare quarks so we are really looking at a sum of exclusive decays of the form $B_{(s)}^{0(-)} \to \pi^{0(-)}\ell^-\overline{\nu}_\ell X$
  2. "Exclusive" decays *e.g.* $\overline{B}^0 \to \pi^+\ell^-\overline{\nu}_\ell$
  3. Leptonic "annhilation" decays *e.g.* $B^+ \to \ell^+\nu_\ell$
- These each come with various requirements on theory (form factors) and the results have historically been rather inconsistent
  - This is typical in flavour physics
  - Is the discrepancy a theory issue, an experimental issue or New Physics (or some combination)?

$$\begin{pmatrix} V_{ud} & V_{us} & \color{red}{V_{ub}} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

- Measure the sum of all processes containing $b \to u\ell^-\overline{\nu}_\ell$
  - Just think about what this means and how hard this is to achieve

[arXiv:1909.12524]

- Experimentally this is incredibly challenging due to backgrounds from the dominant $b \to c$ semileptonic decays
- These backgrounds are reduced by either
  - Cutting on the mass of the X_u system or
  - Cutting on the lepton energy (use the end-point to reject X_c)
- Essential to have a hermetic detector (need to resolve the neutral) so can only be done at Belle and BaBar



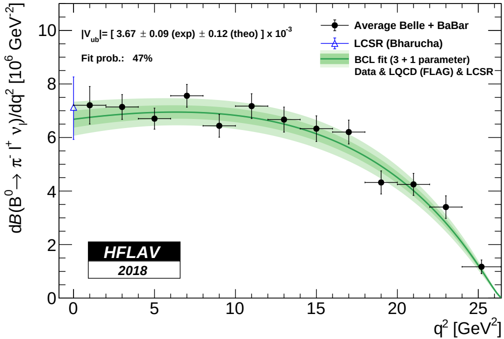
- It is the mass or end-point cuts which then introduce large theory uncertainties
  - Need to estimate how much of the X_u phase space is being removed by these cuts

- Determined by fitting the decay rate seen by BaBar and Belle in *e.g.* $B^0 \to \pi^- \ell^+ \nu_\ell$

$$\frac{d\Gamma}{dq^2} = |V_{ub}|^2 \frac{G_F^2}{192\pi^3 m_B^3} \lambda(m_B, m_\pi, q^2)^{3/2} |f_+(q^2)|^2$$

- Much more straightfoward experimentally but more challenging for the theory
  - Have a dependence on form-factors, $f_+(q^2)$, for the $B \to \pi$ transition
  - Use Lattice QCD calculations

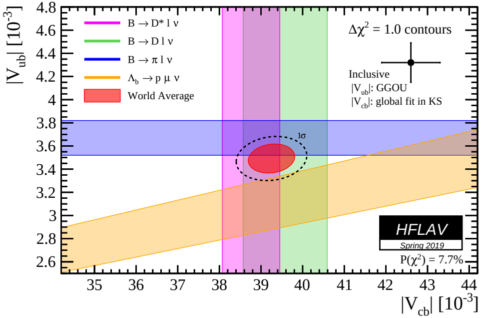
[arXiv:1909.12524]



- LHCb has also pioneered an approach with the $\Lambda_b^0 \to p\mu^-\overline{\nu}_\mu$ decay
- Take the ratio with $\Lambda_b^0 \to \Lambda_c^+\mu^-\overline{\nu}_\mu$ to get $|V_{ub}|/|V_{cb}|$
- Requires the form factor ratio, R_{FF}, from the Lattice

$$\frac{\mathcal{B}(\Lambda_b^0 \to p\mu^-\overline{\nu}_\mu)}{\mathcal{B}(\Lambda_b^0 \to \Lambda_c^+\mu^-\overline{\nu}_\mu)} = \frac{|V_{ub}|^2}{|V_{cb}|^2} R_{FF}$$

- The global average exhibits a considerable tension between inclusive and exclusive

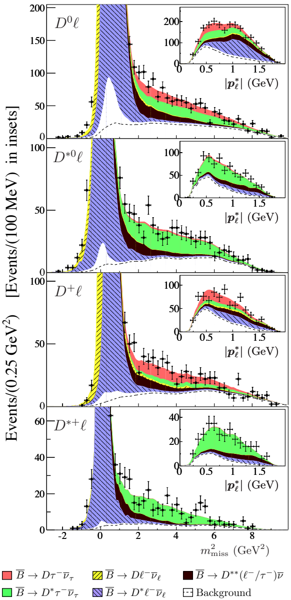


[arXiv:1909.12524]

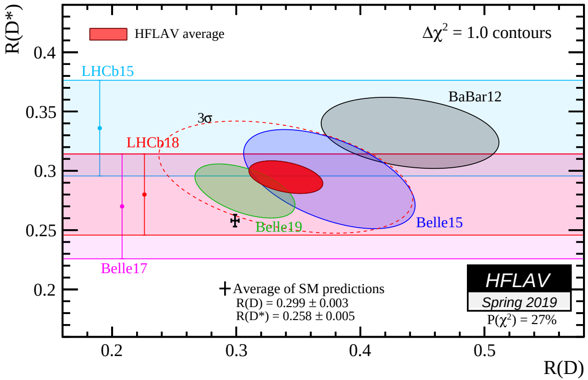
- Another interesting tension has been found between experiment and theory in $B \to D^{(*)}\tau\nu_\tau$ decays

$$\mathcal{R}_{D^{(*)}} = \frac{\Gamma(\overline{B} \to D^{(*)}\tau^-\overline{\nu}_\tau)}{\Gamma(\overline{B} \to D^{(*)}\ell^-\overline{\nu}_\ell)} \tag{8}$$

- Very difficult experimentally due to the presence of neutrinos / missing energy in the final state
- Also complicated by "feed-down" from D^* mode into D mode



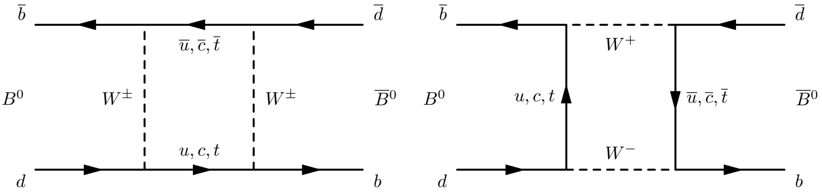
- Combining measurements from the B-factories and LHCb
- Find a tension with the SM predictions although this has somewhat decreased with recent updates from LHCb and Belle
- SM predictions require form-factor calculations - [arXiv:1606.08030], [arXiv:1703.05330], [arXiv:1707.09509], [arXiv:1707.09977]



- There is no top decay but can obtain indirect measurements from the loops which appear in B^0 and B_s^0 mixing

- $|V_{ts}| = 0.0082 \pm 0.0006$
- $|V_{td}| = 0.0400 \pm 0.0027$

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$



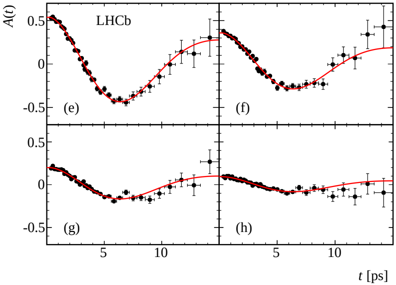
- Ratio of frequencies for B^0 and B_s^0:

$$\frac{\Delta m_s}{\Delta m_d} = \frac{m_{B_s^0}}{m_{B^0}} \frac{f_{B_s^0}^2}{f_{B^0}^2} \frac{B_{B_s^0}^2}{B_{B^0}} \frac{|V_{ts}|^2}{|V_{td}|^2} = \frac{m_{B_s^0}}{m_{B^0}} \xi^2 \frac{|V_{ts}|^2}{|V_{td}|^2} \tag{9}$$

- B^0 and B_s^0 oscillation frequencies (which we use to get constraints on V_{td} and V_{ts}) measured at LEP, Tevatron, B-factories and LHCb
- Most precise measurements now come from LHCb

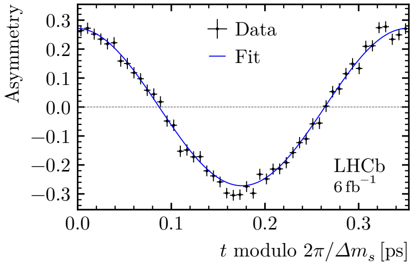
Δm_d from $B^0 \to D^{(*)-}\mu^+\nu_\mu X$

[arXiv:1604.03475]



Δm_s from $B_s^0 \to D_s^-\pi^+$

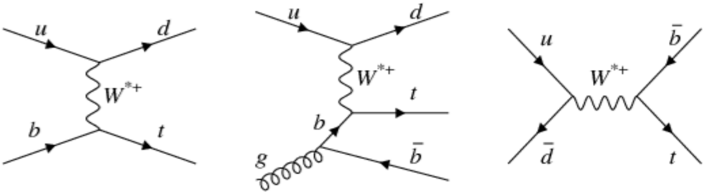
[arXiv:2104.04421]



Measuring V_{tb}

- Use single top production at the Tevatron
- Ratio is proportional to $|V_{tb}|^2$
- $|V_{tb}| = 1.009 \pm 0.0031$

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$



$$R = \frac{\mathcal{B}(t \to Wb)}{BR(t \to Wq)} = \frac{|V_{tb}|^2}{\sum_q |V_{tq}|^2}$$

Measuring CKM matrix elements

- These measurements have all been for the **magnitudes** of the CKM elements
  - Developed over a long period of time using several experiments

$$\begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}| & |V_{ts}| & |V_{tb}| \end{pmatrix} = \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}$$

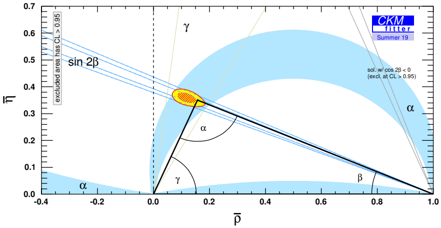
$$\lambda \approx \sin(\theta_C) = \sin(\theta_{12}) \approx 0.22$$

- These give no information on the phase(s)
  - Let's now consider measurements of this imaginary part
  - To find the imaginary part we need CPV

3. Measuring CKM matrix angles

Measuring CKM matrix phases

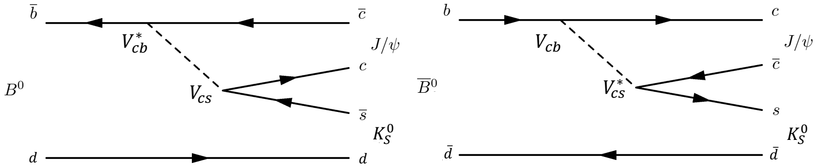
Amplitude	Rel. magnitude	phase
$b \to c$	Dominant	0
$b \to u$	Supressed	γ
$t \to d$	Time-dependent	2β
$t \to s$	Time-dependent	$-2\beta_s$



- γ in interference between $b \to u$ and $b \to c$ transitions
- β in interference between B^0 mixing and decay
- $\beta_s \approx \phi_s$ in interference between B_s^0 mixing and decay
- α arises in the interference between different $b \to u$ transitions

$$V_{CKM} = \begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}|e^{-i\gamma} \\ -|V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}|e^{-i\beta} & -|V_{ts}|e^{-i\beta_s} & |V_{tb}| \end{pmatrix} + \mathcal{O}(\lambda^5)$$

- Arises in the interference between $B^0 \to f_{CP}$ and $B^0 \to \overline{B}^0 \to f_{CP}$
- The golden mode is $B^0 \to J/\psi K^0_{\rm S}$ because the master equations (see Lecture 2) simplify considerably
  1. For a B^0 we have no (or at least negligible) CPV in mixing
  $$\left|\frac{q}{p}\right| \approx 1$$
  2. For the $J/\psi K^0_{\rm S}$ we have a CP-even final state so $f = \bar{f}$ therefore
  $$\lambda_f \equiv \frac{q}{p}\frac{\bar{A}_f}{A_f} = \frac{q}{p}\frac{\bar{A}_{\bar{f}}}{A_{\bar{f}}} \equiv \lambda_{\bar{f}}$$
  3. The B^0 and $\overline{B}^0$ amplitudes to f are (almost) identical (can you think what makes them unequal?)
  $$|A_f| = |\bar{A}_f|$$



- Recall from the master equations (Lecture 2) that

$$C_f = \frac{1-|\lambda_f|^2}{1+|\lambda_f|^2}, \quad D_f = \frac{2\mathcal{R}e(\lambda_f)}{1+|\lambda_f|^2}, \quad S_f = \frac{2\mathcal{I}m(\lambda_f)}{1+|\lambda_f|^2}$$

- Giving a time-dependent asymmetry of

$$\mathcal{A}_{CP}(t) = \frac{\Gamma_{X^0\to f}(t) - \Gamma_{\overline{X}^0\to f}(t)}{\Gamma_{X^0\to f}(t) + \Gamma_{\overline{X}^0\to f}(t)} = \boxed{\frac{C_f\cos(\Delta m t) - S_f \sin(\Delta m t)}{\cosh(\frac{1}{2}\Delta\Gamma t) + D_f \sinh(\frac{1}{2}\Delta\Gamma t)}} \tag{10}$$

- In the case of $B^0 \to J/\psi K^0_{\rm S}$ this hugely simplifies as $|\lambda_f| = 1$ and $\Delta\Gamma = 0$ so that

$$\boxed{\mathcal{A}_{CP}(t) = -\mathcal{I}m(\lambda_f)\sin(\Delta m t)} \tag{11}$$

CKM angle β

- Looking into more detail at what λ_f is in the case of $B^0 \to J/\psi K^0_S$

$$\lambda_{J/\psi K^0_S} = \left(\frac{q}{p}\right)_{B^0} \frac{\bar{A}_{J/\psi K^0_S}}{A_{J/\psi K^0_S}} = \left(\frac{q}{p}\right)_{B^0} \frac{\bar{A}_{J/\psi K^0}}{A_{J/\psi K^0}} \left(\frac{p}{q}\right)_{K^0} \tag{12}$$

$$= -\underbrace{\left(\frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*}\right)}_{B^0 \text{ mixing}} \underbrace{\left(\frac{V_{cb} V_{cs}^*}{V_{cb}^* V_{cs}}\right)}_{B^0 \to J/\psi K^0} \underbrace{\left(\frac{V_{cs} V_{cd}^*}{V_{cs}^* V_{cd}}\right)}_{K^0 \text{ mixing}} \tag{13}$$

$$= -e^{-2i\beta} \tag{14}$$

it's a useful exercise to show this using the equations from Lecture 2

- So that the time-dependent asymmetry is

$$\boxed{\mathcal{A}_{CP}(t) = \pm\sin(2\beta)\sin(\Delta m t)} \tag{15}$$

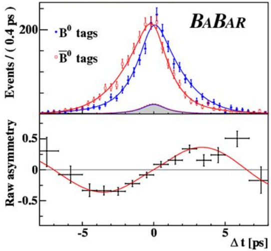
the $\pm$ is for CP-even (*e.g.* $J/\psi K^0_L$) or CP-odd (*e.g.* $J/\psi K^0_S$) final states

- A theoretically and experimentally clean signature
- Also has a relatively large branching fraction, $O(10^{-4})$

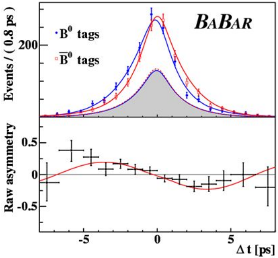
CKM angle β

[arXiv:0902.1708]

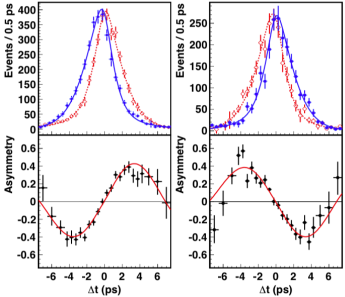
$(c\bar{c})\ K_S$ (CP odd) modes



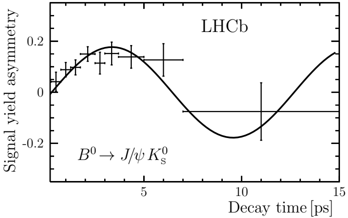
J/ψ K_L (CP even) mode

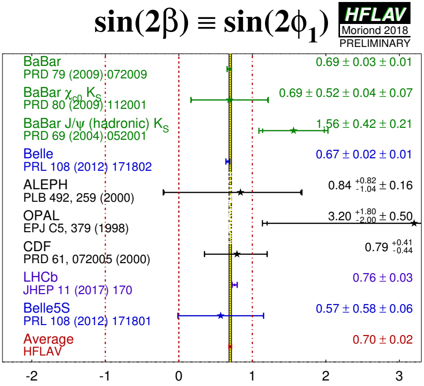


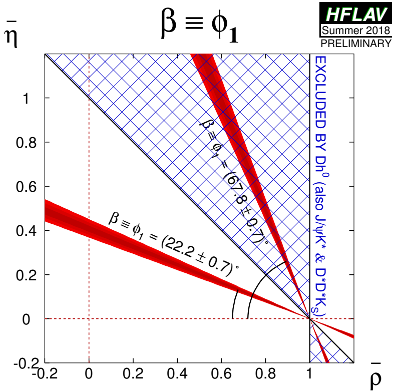
[arXiv:1201.4643]



[arXiv:1709.03944]







$$\sin(2\beta) = 0.699 \pm 0.017$$

$$\beta = (22.2 \pm 0.7)^{\circ}$$

CKM angle β_s

- The B_s^0 analogue of β (recall the squeezed B_s^0 unitarity triangle)
- Use $B_s^0 \to J/\psi\phi$ which is a spectator quark $d \leftrightarrow s$ switch for $B^0 \to J/\psi K_S^0$
  - There are four main differences:

	$B^0 \to J/\psi K_S^0$	$B_s^0 \to J/\psi\phi$
1. CKM element	V_{td}	V_{ts}
2. $\Delta\Gamma$	~ 0	~ 0.1
3. Final state (spin)	$K^0 : s = 0$	$\phi : s = 1$
4. Final state (K)	K^0 mixing	-

- Recall from the master equations the time-dependent CP asymmetry

$$\mathcal{A}_{CP}(t) = \frac{\Gamma_{X^0\to f}(t) - \Gamma_{\overline{X}^0\to f}(t)}{\Gamma_{X^0\to f}(t) + \Gamma_{\overline{X}^0\to f}(t)} = \frac{2C_f\cos(\Delta m t) - 2S_f\sin(\Delta m t)}{2\cosh(\frac{1}{2}\Delta\Gamma t) + 2\sinh(\frac{1}{2}\Delta\Gamma t)} \quad (16)$$

- We still have one dominant amplitude so $A_f \approx A_{\bar{f}} \implies |\lambda_f| \approx 1 \implies C_f \approx 0$ so

$$\mathcal{A}_{CP}(t) = \boxed{\frac{-\mathcal{I}m(\lambda_{J/\psi\phi})\sin(\Delta m t)}{\cosh(\frac{1}{2}\Delta\Gamma t) + \mathcal{R}e(\lambda_{J/\psi\phi})\sinh(\frac{1}{2}\Delta\Gamma t)}} \quad (17)$$

CKM angle β_s

- Looking into more detail at what λ_f is in the case of $B_s^0 \to J/\psi\phi$

$$\lambda_{J/\psi\phi} = \left(\frac{q}{p}\right)_{B_s^0} \left(\eta_{J/\psi\phi} \frac{\bar{A}_{J/\psi\phi}}{A_{J/\psi\phi}}\right) \tag{18}$$

$$= (-1)^l \left(\frac{V_{tb}^* V_{ts}}{V_{tb} V_{ts}^*}\right) \left(\frac{V_{cb} V_{cs}^*}{V_{cb}^* V_{cs}}\right) \tag{19}$$

$$= (-1)^l e^{-2i\beta_s} \tag{20}$$

η represents the CP-eigenvalue

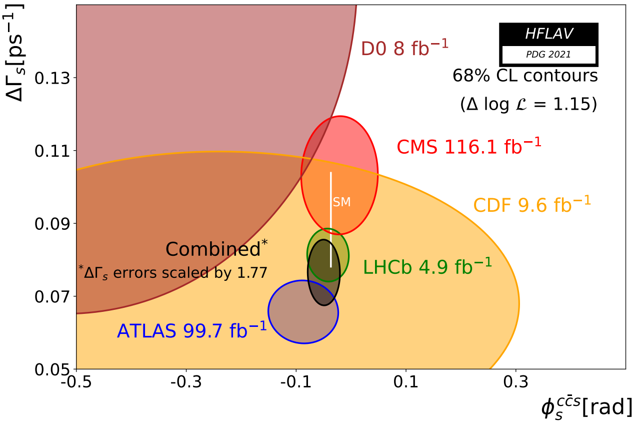
- Because we have two vectors in the final state there are three amplitudes to consider (as opposed to the one amplitude for $B^0 \to J/\psi K_S^0$)

$A_{||} \quad (\uparrow\uparrow) \quad l = 2$ | $A_{\perp} \quad (\uparrow\rightarrow) \quad l = 1$ | $A_0 \quad (\uparrow\downarrow) \quad l = 0$

- Thus the time-dependent asymmetry becomes

$$\mathcal{A}_{CP}(t) = \boxed{\frac{-\eta \sin(2\beta_s) \sin(\Delta m t)}{\cosh(\frac{1}{2}\Delta\Gamma t) + \eta \cos(2\beta_s) \sinh(\frac{1}{2}\Delta\Gamma t)}} \tag{21}$$

at least it does for each polarisation amplitude independently (the interference between the amplitudes is slightly more complicated)



$$\phi_s^{c\bar{c}s} = -0.050 \pm 0.019$$

- But watch out for polluting penguin contributions!

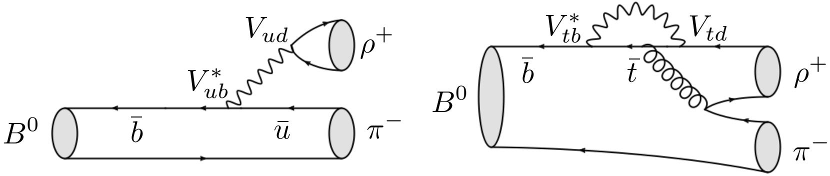
$$\phi_s = -2\beta_s + \delta\phi^{\mathrm{SM}} + \phi^{\mathrm{NP}} \qquad (22)$$

- Following a similar logic to that of $B^0 \to J/\psi K^0_S$ for β one finds that α arises in the time-dependent asymmetry for modes containing a $b \to u\bar{u}d$ transition
  - For example $B^0 \to \pi^+\pi^-$ or $B^0 \to \rho^+\rho^-$
- Recalling the master equations with $\Delta\Gamma = 0$
- Nominally we should have $C_f = 0$ and $S_f = \sin(2\alpha)$ to give

$$\mathcal{A}_{CP}(t) = \pm\sin(2\alpha)\sin(\Delta m t) \tag{23}$$

exactly equivalent to the extraction of β

- However, in this case there is a **non-negligible contribution from penguin decays** of $b \to d\bar{u}u$
  - Similar in magnitude to the $b \to u\bar{u}q$ transition but has a different weak phase
  - Therefore $C \neq 0$ and $S \neq \pm\sin(2\alpha)$
  - How do we deal with the penguin contamination?

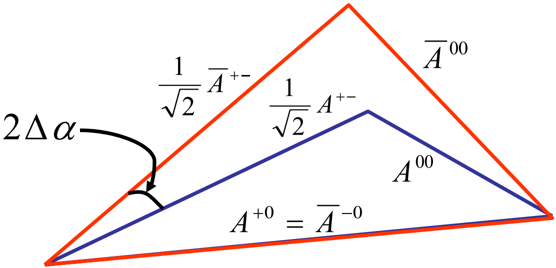


- The contributions from the penguin amplitudes can be accounted for using an "isopsin analysis"
  - Relate the amplitudes for isospin partners

$$A^{+-} \text{ for } B^0 \to \pi^+\pi^-, \quad A^{+0} \text{ for } B^+ \to \pi^+\pi^0, \quad A^{00} \text{ for } B^0 \to \pi^0\pi^0, \tag{24}$$

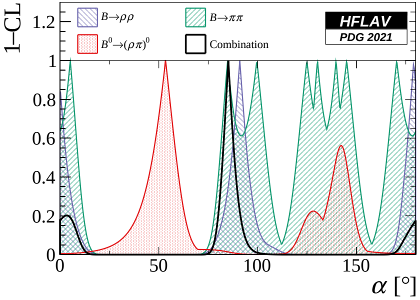
  - There is no penguin contribution to A^{+0} and $\bar{A}^{-0}$ because $\pi^\pm\pi^0$ is a pure isospin-2 state and the QCD-penguin ($\Delta I = 1/2$) only contributes to the isospin-0 final states
  - Obtain isospin triangle relations

$$A^{+0} = \frac{1}{\sqrt{2}}A^{+-} + A^{00}, \text{ and } \bar{A}^{-0} = \frac{1}{\sqrt{2}}\bar{A}^{+-} + \bar{A}^{00} \tag{25}$$



Add in the related $B \to \rho\rho$ modes

- These are vectors (not scalars like the πs) so do not have a fixed CP-eigenvalue
- However it is found that these decays are almost entirely longitudinally polarised (so approximately CP-even)
- Much easier to reconstruct, have a much higher branching fraction and have much smaller penguin contributions (triangles are flatted) so have better sensitivity and reduced ambiguities



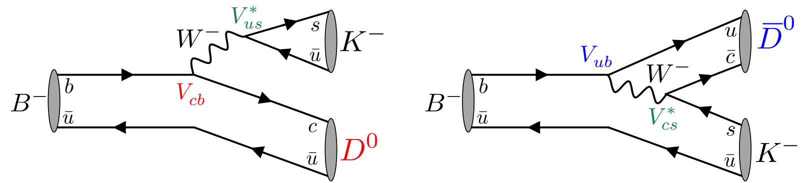
Add the $B \to \rho\pi$ system

- Get a pentagonal (rather than triangular) isospin relation
- The relative amplitudes of $\rho^+\pi^-$, $\rho^-\pi^+$ and $\rho^0\pi^0$ can all be determined from Dalitz analysis of $B^0 \to \pi^+\pi^-\pi^0$

CKM angle γ

- γ is the phase between $V_{ub}^{*}V_{ud}$ and $V_{cb}^{*}V_{cd}$
  - Require interference between $b \to cW$ and $b \to uW$ to access it
  - No dependence on CKM elements involving the top
  - **Can be measured using tree level B decays**
- The "textbook" case is $B^{\pm} \to \overset{(-)}{D}{}^{0} K^{\pm}$:
  - Transitions themselves have different final states (D^0 and $\bar{D}^0$)
  - Interference occurs when D^0 and $\bar{D}^0$ decay to the same final state f

Reconstruct the $D^0/\bar{D}^0$ in a final state accessible to both to acheive interference

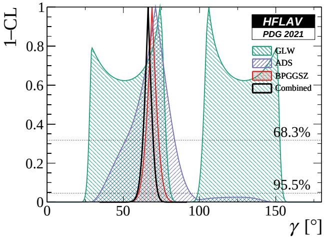


- **The crucial feature of these (and similar) decays is that the D^0 can be reconstructed in several different final states** [all have same weak phase γ]

Categorise decays sensitive to γ depending on the $\overset{(-)}{D}{}^0 \to f$ final state

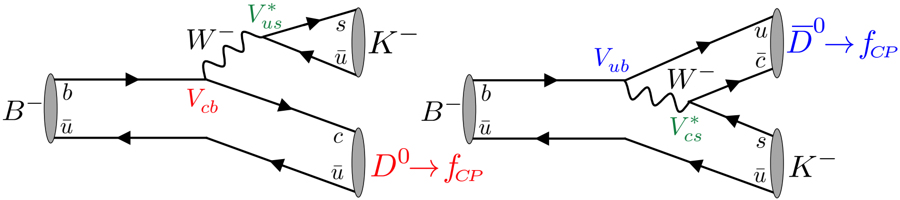
Optimal sensitivity is only acheived when combining them all together

- GLW
  - CP eigenstates e.g. $D \to KK$, $D \to \pi\pi$
  - [Phys. Lett. B253 (1991) 483]
  - [Phys. Lett. B265 (1991) 172]
- ADS
  - CF or DCS decays e.g. $D \to K\pi$
  - [Phys. Rev. D63 (2001) 036005]
  - [Phys. Rev. Lett. 78 (1997) 3257]
- BPGGSZ
  - 3-body final states e.g. $D \to K^0_S\pi\pi$
  - [Phys. Rev. D68 (2003) 054018]
- TD (Time-dependent)
  - Interference between mixing and decay e.g. $B^0_s \to D^-_s K^+$ [phase is $(\gamma - 2\beta_s)$]
  - Penguin free measurement of ϕ_s?
- Dalitz
  - Look at 3-body B decays with D^0 or $\overline{D}^0$ in the final state, e.g. $B^0 \to \overline{D}^0 K^+\pi^-$
  - [Phys. Rev. D79 (2009) 051301]



- Use the $B^{\pm} \to \overset{(-)}{D}{}^{0} K^{\pm}$ case as an example:
  - **Consider only D decays to CP eigenstates, f_{CP}**
  - Favoured: $b \to c$ with strong phase δ_F and weak phase ϕ_F
  - Supressed: $b \to u$ with strong phase δ_S and weak phase ϕ_S

Favoured: Supressed:



Subsequent amplitude to final state f_{CP} is:

$$B^- : A_f = |F| e^{i(\delta_F - \phi_F)} + |S| e^{i(\delta_S - \phi_S)} \quad (26)$$

$$B^+ : \bar{A}_f = |F| e^{i(\delta_F + \phi_F)} + |S| e^{i(\delta_S + \phi_S)} \quad (27)$$

because strong phases (δ) don't change sign under CP while weak phases (ϕ) do

γ with CP eigenstates (GLW)

- Can define the sum and difference of rates with B^+ and B^-

Rate difference and sum

$$|\bar{A}_{\bar{f}}|^2 - |A_f|^2 = 2|F||S|\sin(\delta_F - \delta_S)\sin(\phi_F - \phi_S) \quad (28)$$

$$|\bar{A}_{\bar{f}}|^2 + |A_f|^2 = |F|^2 + |S|^2 + 2|F||S|\cos(\delta_F - \delta_S)\cos(\phi_F - \phi_S) \quad (29)$$

- Choose $r_B = \frac{|S|}{|F|}$ (so that $r < 1$) and use strong phase difference $\delta_B = \delta_F - \delta_S$
- γ is the weak phase difference $\phi_F - \phi_S$
- Subsequently have two **experimental observables** which are

GLW CP asymmetry

$$\mathcal{A}_{CP} = \frac{\pm 2 r_B \sin(\delta_B)\sin(\gamma)}{1 + r_B^2 \pm 2 r_B \cos(\delta_B)\cos(\gamma)}$$

GLW total rate

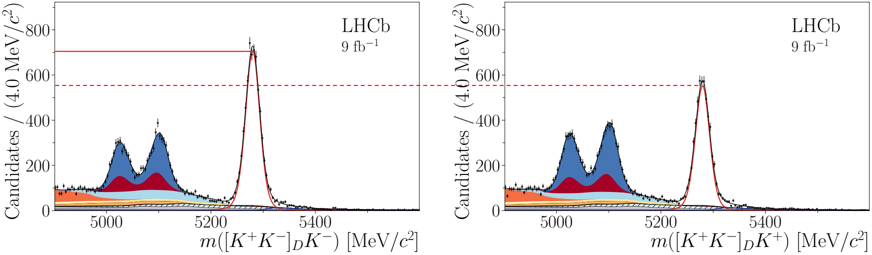
$$\mathcal{R}_{CP} = 1 + r_B^2 \pm 2 r_B \cos(\delta_B)\cos(\gamma)$$

- The $+(-)$ sign corresponds to CP-even (-odd) final states
- Note that r_B and δ_B (ratio and strong phase difference of favoured and supressed modes) are different for each B decay
- **The value of γ is shared by all such decays**

GLW observables

$$A_{CP} = \frac{\pm 2 r_B (2F^+ + 1) \sin(\delta_B) \sin(\gamma)}{1 + r_B^2 \pm 2 r_B (2F^+ + 1) \cos(\delta_B) \cos(\gamma)} \quad (28)$$

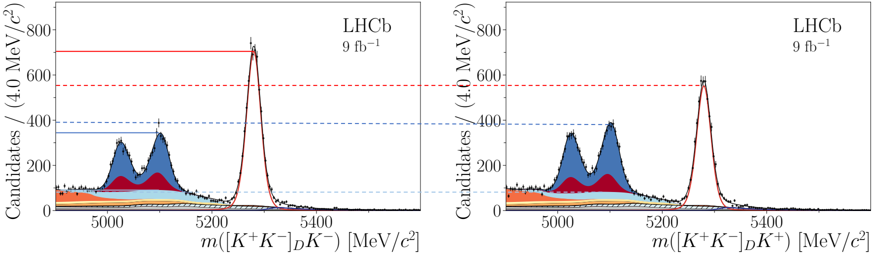
$$R_{CP} = 1 + r_B^2 \pm 2 r_B (2F^+ + 1) \cos(\delta_B) \cos(\gamma) \quad (29)$$



GLW observables

$$A_{CP} = \frac{\pm 2 r_B (2F^+ + 1) \sin(\delta_B) \sin(\gamma)}{1 + r_B^2 \pm 2 r_B (2F^+ + 1) \cos(\delta_B) \cos(\gamma)} \quad (28)$$

$$R_{CP} = 1 + r_B^2 \pm 2 r_B (2F^+ + 1) \cos(\delta_B) \cos(\gamma) \quad (29)$$

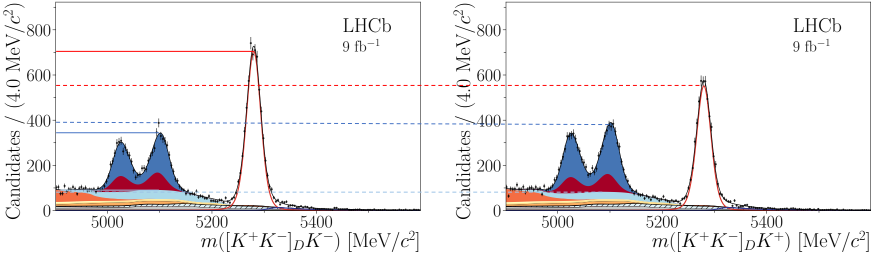


- **LHCb has recently extracted GLW observables from partially reconstructed $B^- \to D^{*0} K^-$ in the same fit** - [arXiv:2012.09903]

GLW observables

$$A_{CP} = \frac{\pm 2 r_B (2F^+ + 1) \sin(\delta_B) \sin(\gamma)}{1 + r_B^2 \pm 2 r_B (2F^+ + 1) \cos(\delta_B) \cos(\gamma)} \quad (28)$$

$$R_{CP} = 1 + r_B^2 \pm 2 r_B (2F^+ + 1) \cos(\delta_B) \cos(\gamma) \quad (29)$$

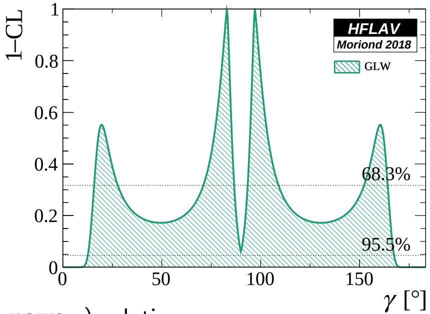


- **LHCb has recently extracted GLW observables from partially reconstructed $B^- \to D^{*0} K^-$ in the same fit** - [arXiv:2012.09903]
- Can extend to quasi-CP-eigenstates ($D^0 \to K K \pi^0$) if fraction of CP content, F^+, is known

GLW observables

$$A_{CP} = \frac{\pm 2 r_B (2F^+ + 1) \sin(\delta_B) \sin(\gamma)}{1 + r_B^2 \pm 2 r_B (2F^+ + 1) \cos(\delta_B) \cos(\gamma)} \quad (28)$$

$$R_{CP} = 1 + r_B^2 \pm 2 r_B (2F^+ + 1) \cos(\delta_B) \cos(\gamma) \quad (29)$$



- Multiple (**but very narrow**) solutions
- Require knowledge of F^+ from charm friends

γ with CF and DCS decays (ADS)

- A 2-body D decay to final state f accesible to both D^0 and $\overline{D}^0$ can be
  - Cabibbo-favoured (CF) - $D^0 \to \pi^- K^+$
  - Doubly-Cabibbo-supressed (DCS) - $\overline{D}^0 \to \pi^- K^+$
- Introduces 2 new hadronic parameters:
  - r_D - ratio of magnitudes for D^0 and $\overline{D}^0$ decay to f
  - δ_D - relative phase for D^0 and $\overline{D}^0$ decay to f
- Gives a modified asymmetry and rate defintion

ADS asymmetry

$$\mathcal{A}_{ADS} = \frac{2 r_D r_B \sin(\delta_B + \delta_D) \sin(\gamma)}{r_D^2 + r_B^2 + 2 r_B r_D \cos(\delta_B + \delta_D) \cos(\gamma)}$$

ADS ratio

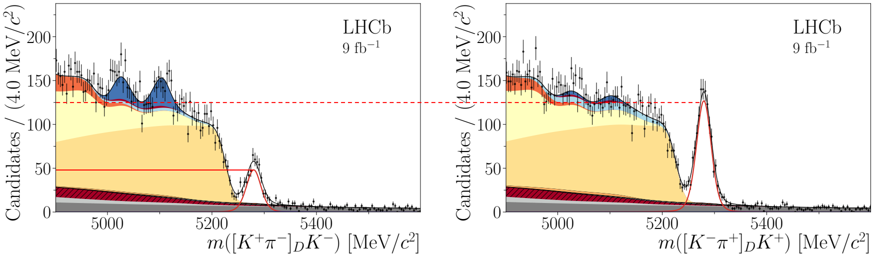
$$\mathcal{R}_{ADS} = \frac{|\bar{A}_{\bar{f}}|^2 + |A_f|^2}{|\bar{A}_f|^2 + |A_{\bar{f}}|^2} = r_B^2 + r_D^2 + 2 r_B r_D \cos(\delta_B + \delta_D) \cos(\gamma)$$

- Hadronic parameters r_D and δ_D can be de independently determined (using CLEO data and HFAG averages)

ADS observables

$$A_{ADS} = \frac{2r_B r_D \sin(\delta_B + \delta_D)\sin(\gamma)}{r_B^2 + r_D^2 + 2r_B r_D \cos(\delta_B + \delta_D)\cos(\gamma)} \quad (28)$$

$$R_{ADS} = r_B^2 + r_D^2 + 2r_B r_D \cos(\delta_B + \delta_D)\cos(\gamma) \quad (29)$$

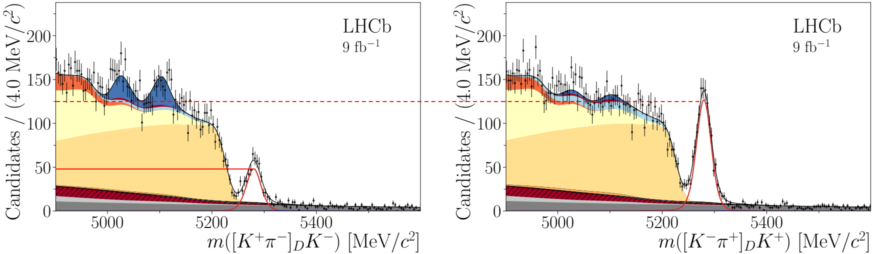


- **Much harder to extract partially reconstructed observables because of $B_s^0 \to D^{(*)0} K^+ \pi^-$ backgrounds.**

ADS observables

$$A_{ADS} = \frac{2r_B r_D \sin(\delta_B + \delta_D)\sin(\gamma)}{r_B^2 + r_D^2 + 2r_B r_D \cos(\delta_B + \delta_D)\cos(\gamma)} \quad (28)$$

$$R_{ADS} = r_B^2 + r_D^2 + 2r_B r_D \cos(\delta_B + \delta_D)\cos(\gamma) \quad (29)$$

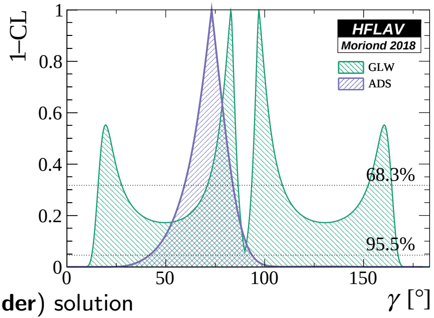


- **Much harder to extract partially reconstructed observables because of $B_s^0 \to D^{(*)0} K^+ \pi^-$ backgrounds.**
- Can extend to multibody-DCS-decays ($D^0 \to K\pi\pi^0$) if dilution from interference, κ_D, is known

ADS observables

$$A_{ADS} = \frac{2r_B r_D \sin(\delta_B + \delta_D)\sin(\gamma)}{r_B^2 + r_D^2 + 2r_B r_D \cos(\delta_B + \delta_D)\cos(\gamma)} \quad (28)$$

$$R_{ADS} = r_B^2 + r_D^2 + 2r_B r_D \cos(\delta_B + \delta_D)\cos(\gamma) \quad (29)$$



- A single (**yet broader**) solution
- Require knowledge of r_D, δ_D, κ_D from charm friends

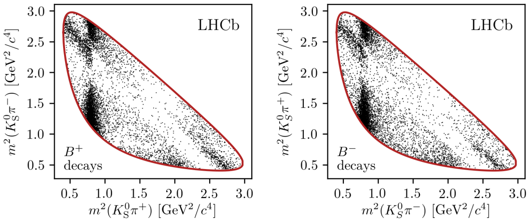
γ with 3-body self-conjugate states (BPGGSZ)

- Now get additional sensitivity over the 3-body phase space
- Idea is to perform a GLW/ADS type analysis across the D decay phase space
- For example $D^0 \to K^0_S \pi^+ \pi^-$ has contributions from
  - Singly-Cabibbo-suppressed decay $D^0 \to K^0_S \rho^0$
  - Doubly-Cabibbo-suppressed decay $D^0 \to K^{*+} \pi^-$
  - Interference between them enhances sensitivity and resolves ambiguities in γ

BPGGSZ observables (partial rate as function of Dalitz position)

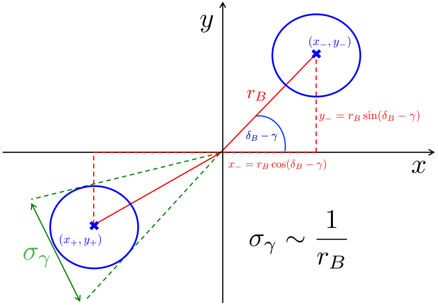
$$d\Gamma_{B^\pm}(\mathbf{x}) = A^2_{(\pm,\mp)} + r_B^2 A^2_{(\mp,\pm)} + 2A_{(\pm,\mp)}A_{(\mp,\pm)} \left[r_B \cos(\delta_B \pm \gamma)\cos(\delta_{D(\pm,\mp)}) + r_B \sin(\delta_B \pm \gamma)\sin(\delta_{D(\pm,\mp)})\right] \quad (30)$$

[arXiv:2010.08483]



BPGGSZ observables (partial rate as function of Dalitz position)

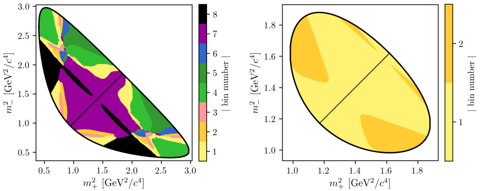
$$d\Gamma_{B^\pm}(\mathbf{x}) = A^2_{(\pm,\mp)} + r_B^2 A^2_{(\mp,\pm)} + 2A_{(\pm,\mp)}A_{(\mp,\pm)} \Big[\underbrace{r_B\cos(\delta_B \pm \gamma)}_{x_\pm}\underbrace{\cos(\delta_{D(\pm,\mp)})}_{c_i} + \underbrace{r_B\sin(\delta_B \pm \gamma)}_{y_\pm}\underbrace{\sin(\delta_{D(\pm,\mp)})}_{s_i}\Big] \quad (31)$$



- $x_\pm + iy_\pm = r_B e^{i(\delta_B \pm \gamma)}$
- **Uncertainty on γ is inversely proportional to central value of hadronic unknown!!**
- Fluctuation in nuisance parameter = fluctuation in error on parameter of interest!

[arXiv:2010.08483]

- Consider both $D \to K_S^0\pi\pi$ and $D \to K_S^0 KK$ decays
- Divide up the Dalitz space into $2N$ symmetric bins chosen to optimise sensitivity to γ



Decay amplitude is a superposition of supressed and favoured contributions

$$A_B(m_-^2, m_+^2) \propto A_D(m_-^2, m_+^2) + r_B e^{i(\delta_B - \gamma)} A_{\overline{D}}(m_-^2, m_+^2)$$

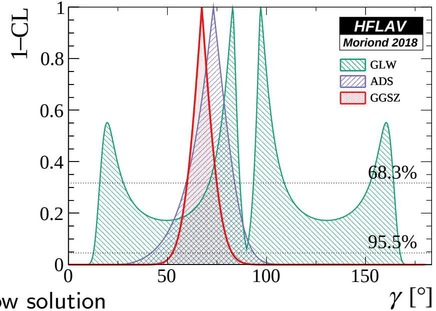
Expected number of B^+ (B^-) events in bin i

$$N_{\pm i}^{+} = h_{B+}\left[F_{\mp i} + (x_+^2 + y_+^2)F_{\pm i} + 2\sqrt{F_i F_{-i}}(x_+ c_{\pm i} - y_+ s_{\pm i})\right]$$
$$N_{\pm i}^{-} = h_{B-}\left[F_{\pm i} + (x_-^2 + y_-^2)F_{\mp i} + 2\sqrt{F_i F_{-i}}(x_- c_{\pm i} - y_- s_{\pm i})\right]$$

- $N_{\pm i}^{\pm}$ - events in each bin
- $F_{\pm i}$ - from $B \to D^{*\pm}\mu^{\mp}\nu_\mu X$
- c_i, s_i - from CLEO-c (QC $D^0\overline{D}^0$) measurements
- $h_{B\pm}$ - overall normalisation

Expected number of B^+ (B^-) events in bin i

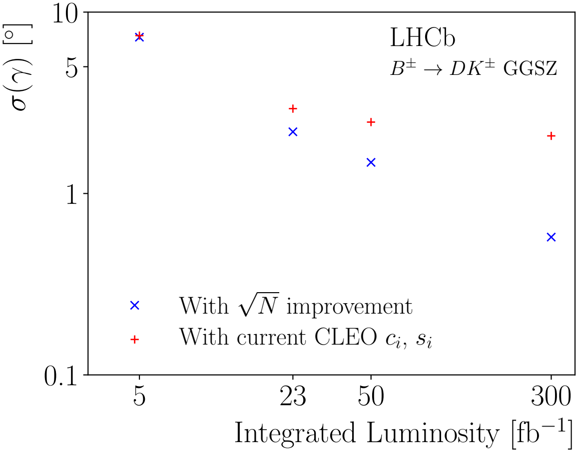
$$N^+_{\pm i} = h_{B^+}\left[F_{\mp i} + (x_+^2 + y_+^2)F_{\pm i} + 2\sqrt{F_i F_{-i}}(x_+ c_{\pm i} - y_+ s_{\pm i})\right]$$
$$N^-_{\pm i} = h_{B^-}\left[F_{\pm i} + (x_-^2 + y_-^2)F_{\mp i} + 2\sqrt{F_i F_{-i}}(x_- c_{\pm i} - y_- s_{\pm i})\right]$$



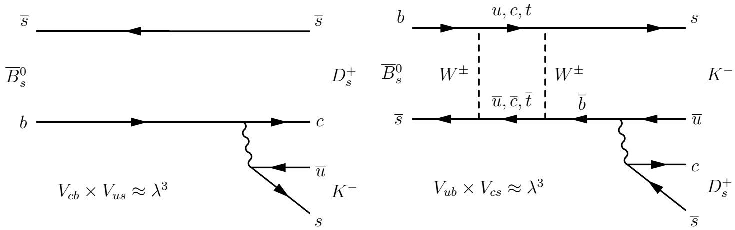
- A single and narrow solution
- Require knowledge of $c_{\pm i}$ and $s_{\pm i}$ from charm friends

A comment on BPGGSZ systematics

- Sensitivty to γ starts to degrade due to dependence on input from charm sector
- Measurements from BES-III (Beijing) will be vital to achieve ultimate precision on γ



- B_s^0 and $\overline{B}_s^0$ can both decay to same final state $D_s^{\mp} K^{\pm}$ (one via $b \to cW$, the other via $b \to uW$)
- Intereference acheived by neutral B_s^0 mixing (requires knowledge of $-2\beta_s \equiv \phi_s$)
  - Weak phase difference is $(\gamma - 2\beta_s)$



- Requires tagging the initial B_s^0 flavour
- Requires a time-dependent analysis to observe the meson oscillations
- Fit the decay-time-dependent decay rates
- Also requires knowledge of Γ_s, $\Delta\Gamma_s$, Δm_s

The time-dependent method with $B_s^0 \to D_s^{\mp} K^{\pm}$

- Recall the master equations (and equivalents for CP conjugate final state $\bar{f}$)

Time-dependent decay rate for initial B_s^0 or $\overline{B}_s^0$ at $t = 0$

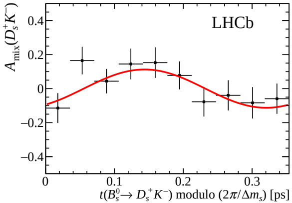
$$\frac{\mathrm{d}\Gamma_{B_s^0 \to f}(t)}{\mathrm{d}t} \propto e^{-\Gamma_s t} \Big[\cosh\left(\frac{\Delta\Gamma_s t}{2}\right) + D_f \sinh\left(\frac{\Delta\Gamma_s t}{2}\right) + C_f \cos\left(\Delta m_s t\right) - S_f \sin\left(\Delta m_s t\right) \Big]$$

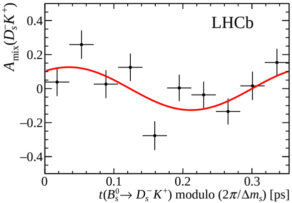
$$\frac{\mathrm{d}\Gamma_{\overline{B}_s^0 \to f}(t)}{\mathrm{d}t} \propto e^{-\Gamma_s t} \Big[\cosh\left(\frac{\Delta\Gamma_s t}{2}\right) + D_f \sinh\left(\frac{\Delta\Gamma_s t}{2}\right) - C_f \cos\left(\Delta m_s t\right) + S_f \sin\left(\Delta m_s t\right) \Big]$$

Time-dependent rate asymmetry

$$\mathcal{A}_{CP}(t) = \frac{\Gamma_{\overline{B}_s^0 \to f}(t) - \Gamma_{B_s^0 \to f}(t)}{\Gamma_{\overline{B}_s^0 \to f}(t) + \Gamma_{B_s^0 \to f}(t)} = \frac{S_f \sin(\Delta m_s t) - C_f \cos(\Delta m_s t)}{\cosh(\frac{\Delta\Gamma_s t}{2}) + D_f \sinh(\frac{\Delta\Gamma_s t}{2})}$$

- Fit for decay-time-dependent asymmetry



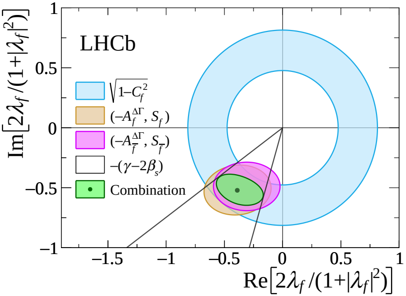


Variable definitions

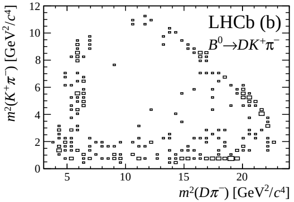
$$C_f = -C_{\bar{f}} = \frac{1 - r_B^2}{1 + r_B^2}$$

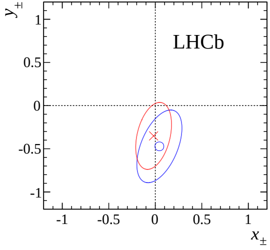
$$D_{f(\bar{f})} = \frac{-2r_B \cos(\gamma - 2\beta_s \mp \delta_B)}{1 + r_B^2}$$

$$S_{f(\bar{f})} = \frac{\pm 2r_B \sin(\gamma - 2\beta_s \mp \delta_B)}{1 + r_B^2}$$



- Study Dalitz structure of 3-body B **decays** with $B^0 \to DK^+\pi^-$
  - In principle has excellent sensitivity to γ
  - "GW method"? (Gershon-Williams - [arXiv:0909.1495])
- Get multiple interfering resonances which increase sensitivity to γ
  - $D^*{}_0(2400)^-$, $D^*{}_2(2460)^-$, $K^*(892)^0$, $K^*(1410)^0$, $K^*{}_2(1430)^0$
- Fit B decay Dalitz Plot for cartesian parameters (similar to BPGGSZ except for the B not the D)
  - $D \to K^+K^-$, $D \to \pi^+\pi^-$ - GLW-Dalitz (done by LHCb - [arXiv:1602.03455])
  - $D \to K^\pm\pi^\mp$ - ADS-Dalitz (problematic backgrounds from $B^0_s \to DK^\pm\pi^\mp$)
  - $D \to K^0_S\pi^+\pi^-$ - BPGGSZ-Dalitz (double Dalitz!)





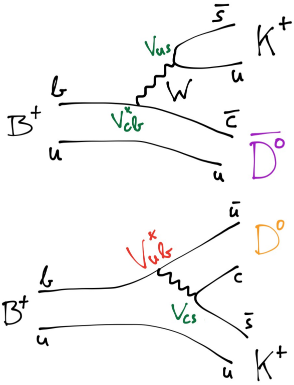
Different *B* decays

$B^\pm \to DK^\pm$ $r_B^{DK},\ \delta_B^{DK}$

$B^\pm \to D^*K^\pm$ $r_B^{D^*K},\ \delta_B^{D^*K}$

$B^\pm \to DK^{*\pm}$ $r_B^{DK^*},\ \delta_B^{DK^*}$

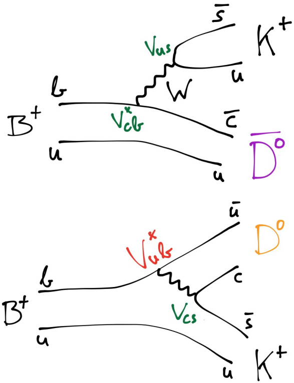
$B^0 \to DK^{*0}$ $r_B^{DK^{*0}},\ \delta_B^{DK^{*0}}$



Building up sensitivity

Different *B* decays →

$B^{\pm} \to DK^{\pm}$	$B^{\pm} \to D^{*}K^{\pm}$	$B^{\pm} \to DK^{*\pm}$	$B^{0} \to DK^{*0}$
$r_B^{DK},\ \delta_B^{DK}$	$r_B^{D^*K},\ \delta_B^{D^*K}$	$r_B^{DK^*},\ \delta_B^{DK^*}$	$r_B^{DK^{*0}},\ \delta_B^{DK^{*0}}$



$D \to hh$	
$D \to hh\pi^0$	F^+
$D \to hhhh$	F^+
$D \to hh'$	$r_D,\ \delta_D$
$D \to hh'\pi^0$	$r_D,\ \delta_D,\ \kappa_D$
$D \to hh'hh$	$r_D,\ \delta_D,\ \kappa_D$
$D \to K_S hh$	$c_i,\ s_i$

Different *D* decays ↓

MANY NUISANCE PARAMETERS

			Highest Statistics	Poorer sensitivity			High potential (Dalitz structure of B)		Low stats (multibody B)
Method		**B Decay** / **D Decay**	$B^- \to D^0 K^-$	$B^- \to D^0 K^{*-}$ $[K^{*-} \to K^0_S \pi^-]$	$B^- \to D^{*0} K^-$ $[D^{*0} \to D^0 \pi^0],[D^{*0} \to D^0 \gamma]$		$B^0 \to D^0 K^+ \pi^-$		$B^- \to D^0 K^- \pi^+ \pi^-$
					part-rec	full-rec	K^{*0} res	Dalitz	
GLW	(+)	$D^0 \to K^+ K^-$	$5\,\text{fb}^{-1}$	$5\,\text{fb}^{-1}$	$5\,\text{fb}^{-1}$	•	$3\,\text{fb}^{-1}$(•)	$3\,\text{fb}^{-1}$	$3\,\text{fb}^{-1}$
		$D^0 \to \pi^+ \pi^-$	$5\,\text{fb}^{-1}$	$5\,\text{fb}^{-1}$	$5\,\text{fb}^{-1}$	•	$3\,\text{fb}^{-1}$(•)	$3\,\text{fb}^{-1}$	$3\,\text{fb}^{-1}$
		$D^0 \to K^+ K^- \pi^0$	$3\,\text{fb}^{-1}$(•)	-	-	-	-	-	-
		$D^0 \to \pi^+ \pi^- \pi^0$	$3\,\text{fb}^{-1}$	-	-	-	-	-	-
		$D^0 \to K^+ K^- \pi^+ \pi^-$	•	-	-	-	-	-	-
		$D^0 \to \pi^+ \pi^- \pi^+ \pi^-$	$3\,\text{fb}^{-1}$(•)	$5\,\text{fb}^{-1}$	•	•	•	-	-
	(−)	$D^0 \to K^0_S \pi^0$	•	-	-	-	-	-	-
ADS		$D^0 \to K^+ \pi^-$	$3\,\text{fb}^{-1}$(•)	$5\,\text{fb}^{-1}$	•	•	$3\,\text{fb}^{-1}$(•)	•	$3\,\text{fb}^{-1}$
		$D^0 \to K^+ \pi^- \pi^0$	$3\,\text{fb}^{-1}$	-	-	-	-	-	-
		$D^0 \to K^+ \pi^- \pi^+ \pi^-$	$3\,\text{fb}^{-1}$(•)	$5\,\text{fb}^{-1}$	•	•	•	-	-
GGSZ		$D^0 \to K^0_S \pi^+ \pi^-$	$5\,\text{fb}^{-1}$	•	-	•	$3\,\text{fb}^{-1}$(•)	•	•
		$D^0 \to K^0_S K^+ K^-$	$5\,\text{fb}^{-1}$	•	-	•	$3\,\text{fb}^{-1}$(•)	•	•
		$D^0 \to K^0_S \pi^+ \pi^- \pi^0$	•	-	-	-	-	-	-
		$D^0 \to K^0_S K^+ K^- \pi^0$	•	-	-	-	-	-	-

KEY: •: (update) in progress

•: requires input from Charm sector (r_D,δ_D, κ_D)

NOTE: TD result with $B^0_s \to D^-_s K^+$ $3\,\text{fb}^{-1}$(•)

TD result with $B^0 \to D^- \pi^+$ $3\,\text{fb}^{-1}$

GLS result from $B^- \to D^0 K^-$ with $D^0 \to K^0_S K^\pm \pi^\mp$ $3\,\text{fb}^{-1}$(•)

Working on $B^- \to D^0 K^{*-}$ with $K^{*-} \to K^- \pi^0$ •

LHCb Input Status

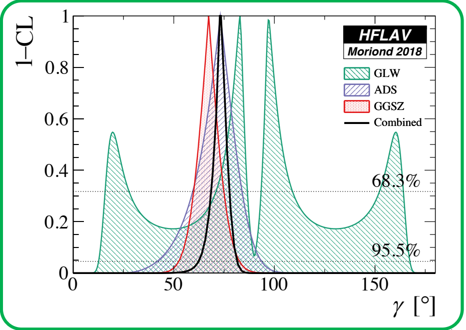
		Highest Statistics	Poorer sensitivity		High potential (Dalitz structure of B)	Low stats (multibody B)
Method	**B Decay** / **D Decay**	$B^- \to D^0K^-$	$B^- \to D^0K^{*-}$ $[K^{*-} \to K^0_S\pi^-]$	$B^- \to D^{*0}K^-$ $[D^{*0} \to D^0\pi^0],[D^{*0} \to D^0\gamma]$	$B^0 \to D^0K^+\pi^-$	$B^- \to D^0K^-\pi^+\pi^-$
GLW (+)	$D^0 \to K^+K^-$	$5\,\text{fb}^{-1}$				
GLW (+)	$D^0 \to \pi^+\pi^-$	$5\,\text{fb}^{-1}$				
GLW (+)	$D^0 \to K^+K^-\pi^0$	$3\,\text{fb}^{-1}$(•)				
GLW (+)	$D^0 \to \pi^+\pi^-\pi^0$	$3\,\text{fb}^{-1}$	-	-		
GLW (+)	$D^0 \to K^+K^-\pi^+\pi^-$	•	-	-		
GLW (+)	$D^0 \to \pi^+\pi^-\pi^+\pi^-$	$3\,\text{fb}^{-1}$(•)	$5\,\text{fb}^{-1}$	•		
GLW (−)	$D^0 \to K^0_S\pi^0$	•	-	-		
ADS	$D^0 \to K^+\pi^-$	$3\,\text{fb}^{-1}$(•)	$5\,\text{fb}^{-1}$	•		
ADS	$D^0 \to K^+\pi^-\pi^0$		-	-		
ADS	$D^0 \to K^+\pi^-\pi^+\pi^-$	[illegible] (•)				
GGSZ	$D^0 \to K^0_S\pi^+\pi^-$	$5\,\text{fb}^{-1}$				
GGSZ	$D^0 \to K^0_SK^+K^-$	$5\,\text{fb}^{-1}$				
GGSZ	$D^0 \to K^0_S\pi^+\pi^-\pi^0$	•				
GGSZ	$D^0 \to K^0_SK^+K^-\pi^0$	•	-	-		

GLW

These guys drive the sensitivity

GGSZ

These guys resolve the ambiguities



KEY: •: (update) in progress

•: requires input from Charm sector (r_D,δ_D, κ_D)

NOTE: TD result with $B^0_s \to D^-_s K^+$ $3\,\text{fb}^{-1}$(•)

TD result with $B^0 \to D^-\pi^+$ $3\,\text{fb}^{-1}$

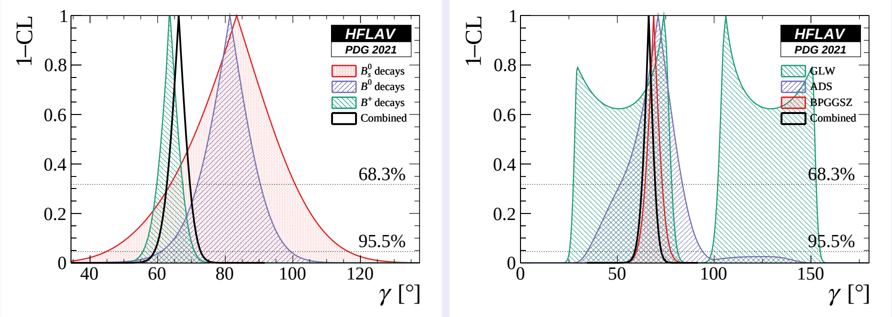
GLS result from $B^- \to D^0K^-$ with $D^0 \to K^0_SK^\pm\pi^\mp$ $3\,\text{fb}^{-1}$(•)

Working on $B^- \to D^0K^{*-}$ with $K^{*-} \to K^-\pi^0$ •

Combined constraints on γ

World Average (HFLAV) - [PDG 2021]

$$\gamma = (66.2^{+3.4}_{-3.6})^\circ$$



Indirect constraints are: $\gamma = (65.3^{+1.0}_{-2.5})^\circ$

4. CKM constraints from kaon decays

- CPV first observed in 2π decays of $K^0_{\rm L}$ mesons
  - Is this just mixing induced or is it direct CPV also (*i.e.* CPV in decay)?
  - For the CPV in kaon mixing we introduce the complex parameter ϵ such that

$$|K^0_{\rm S}\rangle = \frac{1}{\sqrt{1+|\epsilon|^2}}\left(|K_1\rangle + \epsilon|K_2\rangle\right) \quad \text{and} \quad |K^0_{\rm L}\rangle = \frac{1}{\sqrt{1+|\epsilon|^2}}\left(|K_2\rangle + \epsilon|K_1\rangle\right) \tag{32}$$

- If CPV is only mixing induced then we expect $K^0_{\rm L} : K^0_{\rm S}$ amplitude ratios to be equivalent for neutral and charged final states (*i.e.* $\eta_{00} = \eta_{+-}$) where

$$\eta_{00} = \frac{\mathcal{A}(K^0_{\rm L} \to \pi^0\pi^0)}{\mathcal{A}(K^0_{\rm S} \to \pi^0\pi^0)}, \quad \eta_{+-} = \frac{\mathcal{A}(K^0_{\rm L} \to \pi^+\pi^-)}{\mathcal{A}(K^0_{\rm S} \to \pi^+\pi^-)}. \tag{33}$$

- But we also see evidence for CPV in kaon decay (via semileptonic decays)

$$\delta \equiv \mathcal{A}_{CP}(K^0_{\rm L} \to \ell^+\nu_\ell\pi^-) \tag{34}$$

- Can then summarise CPV in the kaon system using two parameters, (ϵ, ϵ') where

$$\eta_{00} = \epsilon - 2\epsilon' \tag{35}$$

$$\eta_{11} = \epsilon + \epsilon' \tag{36}$$

$$\delta = \frac{2\mathcal{R}e(\epsilon)}{1+|\epsilon|^2} \tag{37}$$

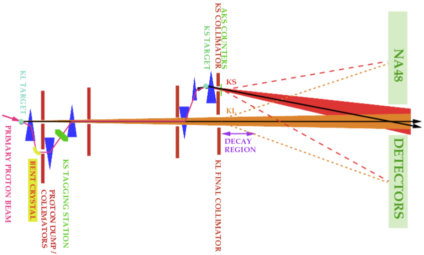
NA48 experiment

- Established that $\mathcal{R}e(\epsilon'/\epsilon) \neq 0$ by NA48 at CERN and KTEV in Japan
- NA48 is a fixed target experiment in CERN's North Area
- Measure the double ratio of $\pi^0\pi^0$ and $\pi^+\pi^-$ decays from $K^0_{\rm L}$ and $K^0_{\rm S}$

$$R = \frac{|\eta_{00}|^2}{|\eta_{+-}|^2} \approx 1 - 6\mathcal{R}e\left(\frac{\epsilon'}{\epsilon}\right) \tag{38}$$
$$= (13.7 \pm 2.5 \pm 1.8) \times 10^{-4}$$

- Now replaced by NA62 an even more sensitive kaon physics experiment looking for very rare kaon decays

Simultaneous beams of $K_{\rm S}$ and $K_{\rm L}$ from separate targets



5. Status of CKM matrix global fits

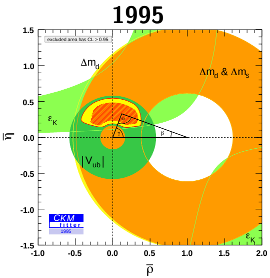
Putting all the constraints together

- All of these separate measurements can be put together to over-constrain the CKM picture
- This is incredibly powerful because we can attack the (ρ, η) vertex of the unitarity triangle in several ways

World Averages are performed by several groups

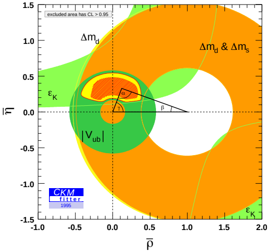
- CKMfitter (frequentist)
  - http://ckmfitter.in2p3.fr/
- UTFit (Bayesian)
  - http://www.utfit.org/UTfit/
- Heavy Flavour Averaging Group (HFLAV)
  - https://hflav.web.cern.ch/
- Particle Data Group (PDG)
  - http://pdg.lbl.gov/

The CKM fit

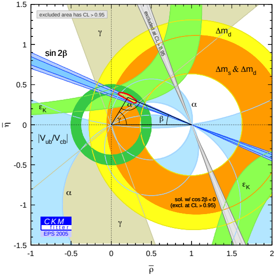


- Before the B-factories and LHC the CKM picture was not even established

1995

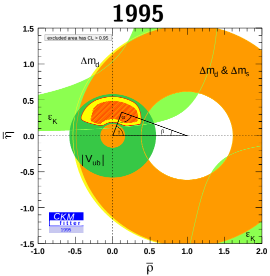


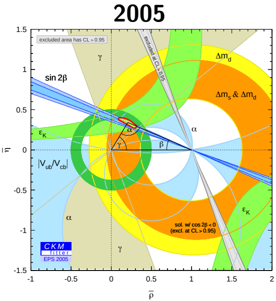
2005

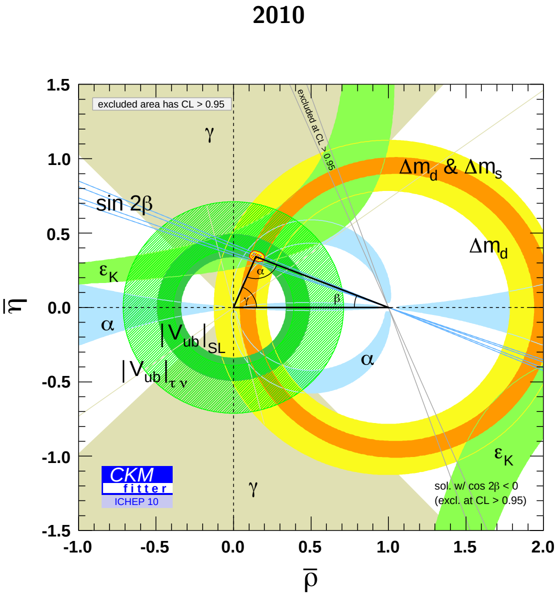


- With data from the Tevatron and B-factories the CKM picture is verified
- When adding the LHC it now becomes a suite of precision physics measurements

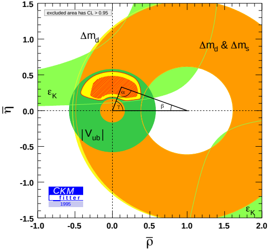
The CKM fit


1995
excluded area has CL > 0.95
Δm_d
Δm_d & Δm_s
ε_K
|V_ub|
α
β
γ
CKM fitter 1995
ρ̄
η̄

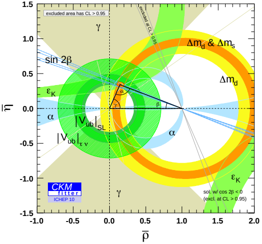

2005
excluded area has CL > 0.95
excluded at CL > 0.95
γ
sin 2β
Δm_d
Δm_s & Δm_d
ε_K
α
β
|V_ub/V_cb|
sol. w/ cos 2β < 0
(excl. at CL > 0.95)
CKM fitter EPS 2005
ρ̄
η̄


2010
excluded area has CL > 0.95
excluded at CL > 0.95
γ
sin 2β
Δm_d & Δm_s
Δm_d
ε_K
α
β
|V_ub|_SL
|V_ub|_τν
sol. w/ cos 2β < 0
(excl. at CL > 0.95)
CKM fitter ICHEP 10
ρ̄
η̄

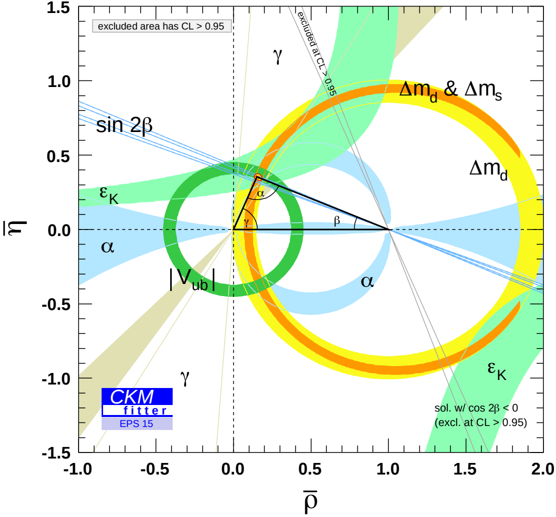
1995



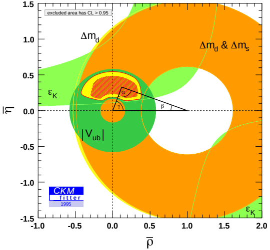
2010



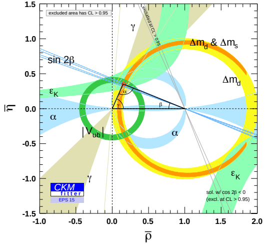
2015



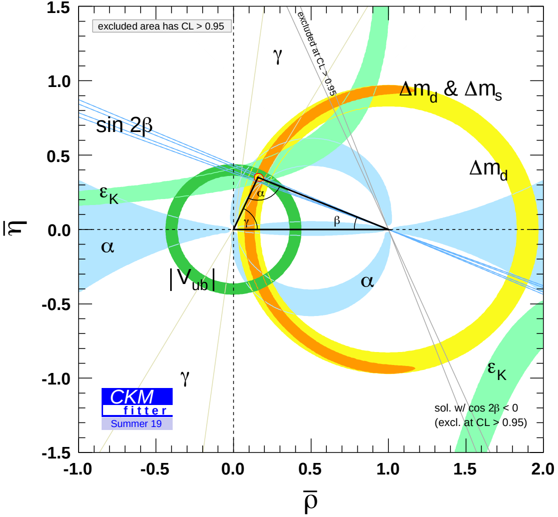
1995



2015

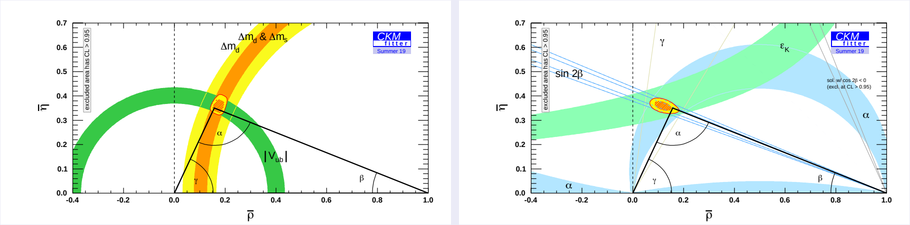


2019

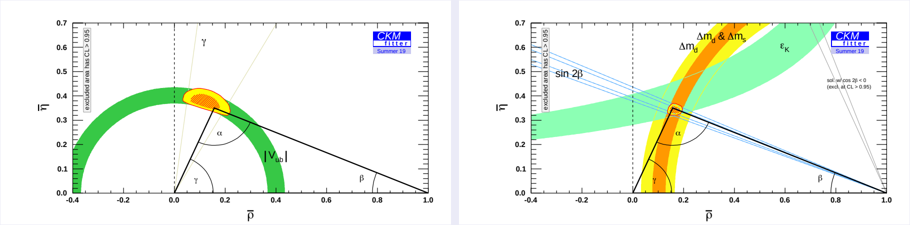


The CKM fit

Comparison between CP-conserving (lengths of sides) and CP-violating (angles)



Comparison between tree-level (γ, V_{ub}) and loop-level ($\alpha, \beta, \Delta m, \epsilon$)



6. CPT and T-reversal

CPT theorem

- It is not possible to write a quantum field theory that is Lorentz invariant, with a Hermitian Hamiltonian $H = H^\dagger$, that violates the product of CPT
  - *i.e.* one in which measurements are not invariant under position translations and Lorentz boosts of the system
- There are several important consquences that CPT invariance implies
  1. Mass and lifetime of particles and antiparticles are identical
  2. Quantum numbers of antiparticles are opposite those of particles
  3. Integer spin particles obey Bose-Einstein statistics and half-integer spin particle obey Fermi-Dirac statistics
- Time reversal symmetry translates $t \to -t$
  - Obviously we can't test this experimentally (cannot run an experiment backwards in time)
  - However if CP is violated and the product CPT is conserved then T must also be violated

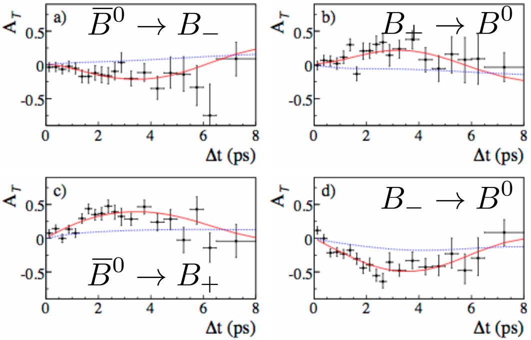
T violation in the B system

- This can actually be tested in the B system
- A generalisation of the $\sin(2\beta)$ analysis
- Identify the flavour of the B by tagging the other B in the event and in addition separate the events by CP-odd ($J/\psi K^0_\mathrm{S}$) and CP-even ($J/\psi K^0_\mathrm{L}$) final states
- A T reversal violation would appear as a difference in the rates between

$$\overline{B}^0(t_1) \to B_-(t_2) \quad \text{and} \quad B_-(t_1) \to \overline{B}^0(t_2)$$

- T violation has been observed by BaBar ([arXiv:1207.5832])

$$\Delta S_T^+ = -1.37 \pm 0.15$$
$$\Delta S_T^- = 1.17 \pm 0.21$$



7. Dipole Moments

Magentic dipole moments

- A "spinning" charge acts as a magnetic dipole with moment, μ, which gives an energy shift to an externally applied magnetic field

$$\Delta E = -\vec{\mu} \cdot \vec{B} \tag{39}$$

- The prediction of $g = 2$ (classically $g = 1$) was a big success of the Dirac equation
- In an external field A^μ

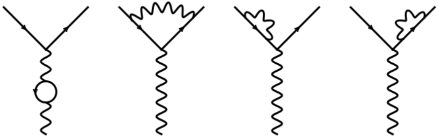
$$\left(\frac{1}{2m}(\vec{p} + e\vec{A}) + \frac{e}{2m}\vec{\sigma} \cdot \vec{B} - eA^0 \right) \psi = E\psi \tag{40}$$

- The magnetic dipole moment μ is given by

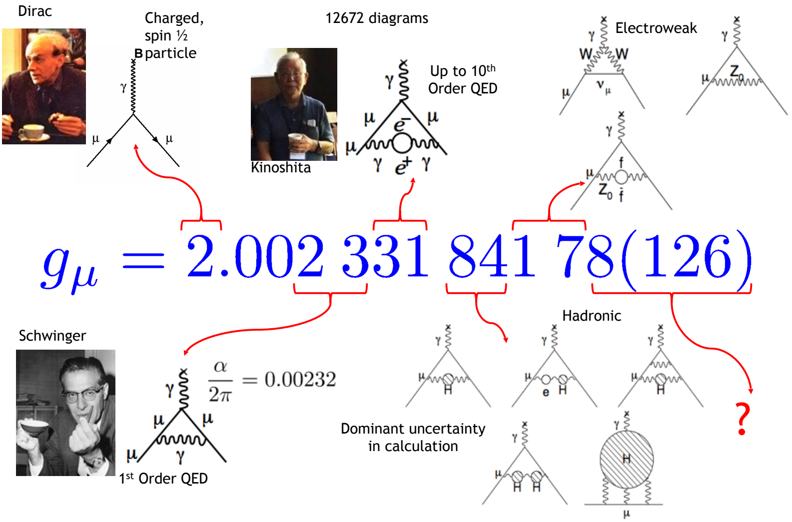
$$\vec{\mu} = -\frac{e}{2m}\vec{\sigma} = -g\frac{\mu_B}{\hbar}\vec{S} \tag{41}$$

- Receives corrections from higher order processes (*e.g.* at order α^2)

$$g = 2 + \frac{\alpha}{2\pi} + \mathcal{O}(\alpha^2)$$



Anomalous magnetic moment



Slide from Becky Chislett (via Tom Blake)

Anaomalous magnetic moments

- $(g-2)_e$ is a powerful precision test of QED

$$(g-2)_e - (1159.652186 \pm 0.000004) \times 10^{-6}$$

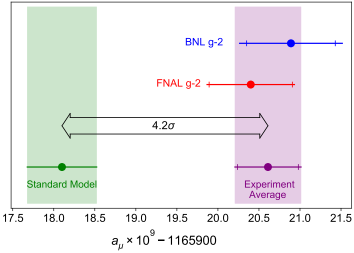
- $(g-2)_\mu$ receives important Weak and QCD contributions. The latest experimental value from Brookhaven E821 and Fermilab $g-2$ experiments

$$(g-2)_\mu = (116591810 \pm 43) \times 10^{-11} \text{ (Theory)}$$

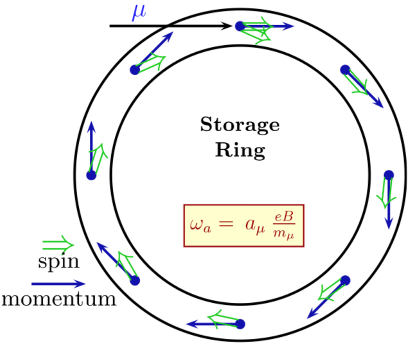
$$(g-2)_\mu = (116592061 \pm 41) \times 10^{-11} \text{ (Experiment)}$$

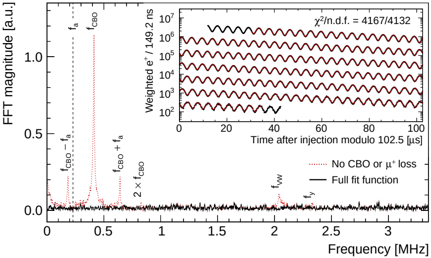
from [arXiv:2104.03281] is 4.2σ from the SM expectation [arXiv:2006.04822]

- Is this a hint of a NP contribution to $(g-2)_\mu$ (review in [arXiv:0902.3360])?



- Experiment at Fermilab aiming for $\sim 0.1 - 0.2$ppm precision
- The anomalous magnetic moment causes the spin to process at a different rate to the momentum vector
- Can use this procession to precisely measure $g-2$

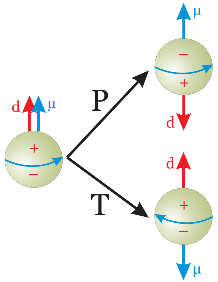




- Classically, EDMs are a measure of the spatial separation of positive and negative charges in a particle
  - A finite EDM can only exist if the charge centres do not coincide
- EDMs can also be measured for fundamental particles (electron, muon, neutron *etc.*)
  - Can interpret this as a measure of the "sphericity" of the particle
- This is tested using the Zeeman effect
  - Look for a shift in energy levels under an external electrical field (analogous to the magenetic moment)

$$\Delta E = -\vec{d} \cdot \vec{E} \quad (42)$$

- A non zero EDM would violate T and P symmetries
  - Under T reversal, the MDM would change direction but the EDM would remain unchanged
  - Under P, the EDM would change direction but the MDM remains unchanged
- Violation of P and T implies CP violation



Electric dipole moments

- Electron EDM:
  - $d_e < 8.7 \times 10^{-29}$ [arXiv:1310.7534]
- Muon EDM:
  - $d_e < 1.9 \times 10^{-19}$ [arXiv:0811.1207]
- Neutron EDM:
  - $d_e < 3.0 \times 10^{-26}$ [arXiv:hep-ex/0602020]
- Probing incredibly small charge separation distances!

Strong CP problem

- The complicated nature of the QCD vacuum should give rise to a term in the Langrangian like

$$\mathcal{L}_\theta = \theta \frac{\alpha_s}{8\pi} F_\alpha^{\mu\nu} \tilde{F}_{\alpha,\mu\nu} \tag{43}$$

- This is both P and T-violating but C-conserving (hence CP-violating)
- This terms would also contribute to the neutron dipole moment, but experimentally we know this is very small

$$d_n \sim e \cdot \theta \cdot m_q / M_N^2 \implies \theta \leq 10^{-9} \tag{44}$$

- This is incredibly small size of the θ parameter is (another) massive fine tuning problem (the so-called "strong CP problem")
- What mechanism forces θ to be so small?

Axion searches

- The Peccei-Quin solution to the strong CP problem is to introduce a U(1) symmetry that removes the strong CP problem by dynamically making θ small
- Spontaneous breaking of this symmetry is associated with a pseudo-Nambu-Goldstone boson (in analogy with the Higgs mechanism), the axion
- The axion can be a light particle that couples very weakly to known SM particles
- There are a large number of searches for axions produced in particle colliders (direct searches)
- Can also be detected by the presence of axions converting into photons in the presence of a strong magnetic field (*e.g.* the CAST experiment at CERN)

8. Recap

Recap

In this lecture we have covered

- Recap of the CKM matrix and unitarity triangles
- Measurements of the CKM matrix element magnitudes
  - In particular the sides of the unitarity triangle
  - The tension between inclusive and exclusive measurements of V_{ub}
- Measurements of the CKM matrix angles
  - The angles α, β, γ and ϕ_s
- CP violation in the kaon system
- Global constraints on the CKM matrix and unitarity triangle(s)
- T violation and CPT
- Electric and magentic dipole moments

End of Lecture 3