Flavour Physics (of quarks)

Part 1: Flavour in the SM

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Warwick Week Graduate Lectures

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The mandatory early joke slide

Fig. 1. My teaching responsibilities for this term.

Overview

Lecture 1: Flavour in the SM (Today)

- Flavour in the SM
- Quark Model History
- The CKM matrix

Lecture 2: Mixing and CP violation

- Neutral Meson Mixing (no CPV)
- B-meson production and experiments
- CP violation

Lecture 3: Measuring the CKM parameters

- Measuring CKM elements and phases
- Global CKM fits
- CPT and T-reversal
- Dipole moments

Lecture 4: Flavour Changing Neutral Currents

- Effective Theories
- New Physics in B mixing
- New Physics in rare $b \to s$ processes
- Lepton Flavour Violation

Reading Material

- I have provided a short document containing an overview of the course and a reading list which can be found on the indico event page https://indico.cern.ch/event/1037445/
- I've also put a copy of it on my warwick page (along with these slides) https://warwick.ac.uk/fac/sci/physics/staff/academic/kenzie
- Most of the material for these slides comes from one of the sources on that reading list

Many thanks to **Tom Blake**, Tim Gershon, Niels Tuning, Mitesh Patel, Monika Blanke and Gino Isidori for inspiration, ideas and outright plagiarism

Quiz link for Lecture 1. Click here.

What is flavour physics?

WIKIPEDIA
The Free Encyclopedia

Flavour (particle physics)

From Wikipedia, the free encyclopedia
(Redirected from Flavour physics)

In particle physics, **flavour** or **flavor** refers to the *species* of an elementary particle. The Standard Model counts six flavours of quarks and six flavours of leptons. They are conventionally parameterized with *flavour quantum numbers* that are assigned to all subatomic particles. They also can be described by some of family symmetries proposed for the quark-lepton generations.

"The term flavor was first used in particle physics in the context of the quark model of hadrons. It was coined in 1971 by Murray Gell-Mann and his student at the time, Harald Fritzsch, at a Baskin-Robbins ice-cream store in Pasadena. Just as ice cream has both color and flavor so do quarks."

RMP 81 (2009) 1887

Flavour in particle physics

Flavour quantum numbers

- Isospin: **I** or I_3
- Charm: C
- Strangeness: S
- Topness: T
- Bottomness: B'

Related quantum numbers

- Baryon number: B
- Lepton number: L
- Weak isospin: **T** or T_3
- Electric charge: Q
- X-charge: X

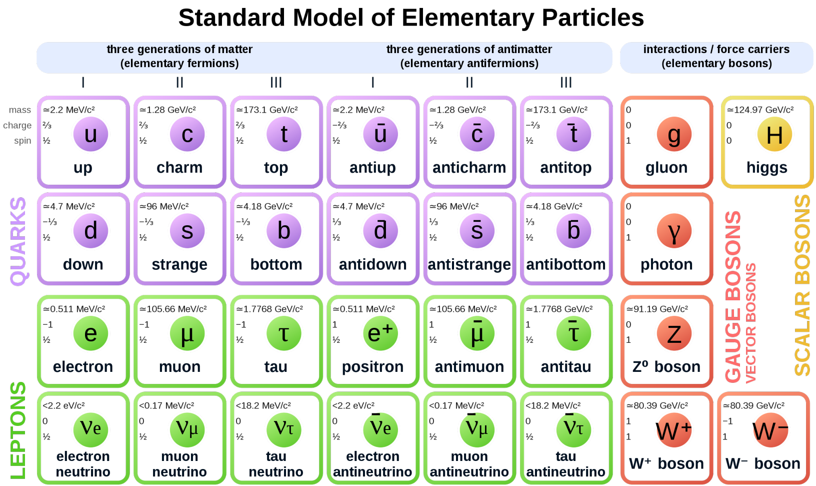
Combinations

- Hypercharge: Y
  - $Y = (B + S + C + B' + T)$
  - $Y = 2\,(Q - I_3)$
- Weak hypercharge: Y_W
  - $Y_W = 2\,(Q - T_3)$
  - $X + 2Y_W = 5\,(B - L)$

Flavour mixing

- CKM matrix
- PMNS matrix
- Flavour complementarity

V • T • E


Standard Model of Elementary Particles
three generations of matter (elementary fermions)
three generations of antimatter (elementary antifermions)
interactions / force carriers (elementary bosons)
I II III I II III
mass
charge
spin
≃2.2 MeV/c² ⅔ ½ u up
≃1.28 GeV/c² ⅔ ½ c charm
≃173.1 GeV/c² ⅔ ½ t top
≃2.2 MeV/c² −⅔ ½ ū antiup
≃1.28 GeV/c² −⅔ ½ c̄ anticharm
≃173.1 GeV/c² −⅔ ½ t̄ antitop
0 0 1 g gluon
≃124.97 GeV/c² 0 0 H higgs
≃4.7 MeV/c² −⅓ ½ d down
≃96 MeV/c² −⅓ ½ s strange
≃4.18 GeV/c² −⅓ ½ b bottom
≃4.7 MeV/c² ⅓ ½ d̄ antidown
≃96 MeV/c² ⅓ ½ s̄ antistrange
≃4.18 GeV/c² ⅓ ½ b̄ antibottom
0 0 1 γ photon
≃0.511 MeV/c² −1 ½ e electron
≃105.66 MeV/c² −1 ½ μ muon
≃1.7768 GeV/c² −1 ½ τ tau
≃0.511 MeV/c² 1 ½ e⁺ positron
≃105.66 MeV/c² 1 ½ μ̄ antimuon
≃1.7768 GeV/c² 1 ½ τ̄ antitau
≃91.19 GeV/c² 0 1 Z Z⁰ boson
<2.2 eV/c² 0 ½ νe electron neutrino
<0.17 MeV/c² 0 ½ νμ muon neutrino
<18.2 MeV/c² 0 ½ ντ tau neutrino
<2.2 eV/c² 0 ½ ν̄e electron antineutrino
<0.17 MeV/c² 0 ½ ν̄μ muon antineutrino
<18.2 MeV/c² 0 ½ ν̄τ tau antineutrino
≃80.39 GeV/c² 1 1 W⁺ W⁺ boson
≃80.39 GeV/c² −1 1 W⁻ W⁻ boson
QUARKS
LEPTONS
GAUGE BOSONS
VECTOR BOSONS
SCALAR BOSONS

Parameters of the Standard Model

- 3 gauge couplings
- 2 Higgs parameters

Flavour Parameters

- 6 quark masses
- 3 quark mixing angles + 1 phase [CKM matrix]
- 3 (+3) lepton masses
- (3 lepton mixing angles + 1 phase) [PMNS matrix]

() = with Dirac neutrino masses

These lectures cover the flavour physics of quarks and I will not discuss neutrinos (much)

- See Steve Boyd's lectures for more

Aspects of flavour physics

- Families / generations
  - 3 pairs of quarks
  - 3 pairs of leptons
  - Why? Do we know this for sure?
- Clear (and not so clear) hierarchies
  - $m(t) > m(c) > m(u)$
  - $m(b) > m(s) > m(d)$
  - $m(\tau) > m(\mu) > m(e)$
  - $m(\nu_\tau) > m(\nu_\mu) > m(\nu_e)$?
- Mixing and couplings
  - Hierarchy in (quark/lepton) mixings?
  - Universality
  - (no) flavour changing neutral current (FCNC)
- Symmetry (violation)
  - P / C / CP/ T violation
  - Baryon asymmetry of the universe
  - Lepton flavour violation / universality?
- Unification

What's with neutrinos?

- Parity violation / chirality
  - Neutrinos are only left-handed
  - Anti-neutrinos are only right-handed
- **BUT NOT** massless!
  - What happened to right-handed neutrinos?
- New Physics?
- Probe of Grand Unification?

See Steve Boyd's lectures for more.

2. Flavour in the SM

Flavour in the SM

A brief theoretical interlude which we will flesh out with some history afterwards

- Particle physics can be described to excellent precision by a relatively straightforward and very beautiful theory (we all know and love the SM):

$$\mathcal{L}_{\mathrm{SM}} = \mathcal{L}_{\mathrm{Gauge}}(A_a, \psi_i) + \mathcal{L}_{\mathrm{Higgs}}(\phi, A_a, \psi_i) \quad (1)$$

- It contains:
  - Gauge terms that deal with the free fields and their interactions via the strong and electroweak interactions
  - Higgs terms that give rise to the masses of the SM fermions and weak bosons

- The Gauge part of the Lagrangian is well verified

$$\mathcal{L}_{\text{Gauge}} = \sum_j i\bar{\psi}_j \not{D} \psi_j - \sum_a \frac{1}{4g_a^2} F^a_{\mu\nu} F^{\mu\nu,a} \tag{2}$$

- Parity is violated by electroweak interactions
- Fields are arranged as left-handed doublets and right-handed singlets

$$\psi = \boxed{Q_L, u_R, d_R, c_R, s_R, t_R, b_R} \text{ quarks} \tag{3}$$

$$\boxed{L_L, e_R, \mu_R, \tau_R} \text{ leptons} \tag{4}$$

with

$$Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}, \begin{pmatrix} c_L \\ s_L \end{pmatrix}, \begin{pmatrix} t_L \\ b_L \end{pmatrix} \text{ and } L_L = \begin{pmatrix} e_L \\ \nu_{eL} \end{pmatrix}, \begin{pmatrix} \mu_L \\ \nu_{\mu L} \end{pmatrix}, \begin{pmatrix} \tau_L \\ \nu_{\tau L} \end{pmatrix} \tag{5}$$

- The Lagrangian is invariant under a specific set of symmetry groups: $\mathrm{SU}(3)_c \times \mathrm{SU}(2)_\mathrm{L} \times \mathrm{U}(1)_\mathrm{Y}$

Quark Gauge Couplings

- Without the Higgs we have flavour universal gauge couplings equal for all three generations (huge degeneracy)

$$\mathcal{L}_{\text{quarks}} = \sum_{j}^{3} \underbrace{i\bar{Q}_j \not{D}_Q Q_j}_{\text{left-handed doublets}} + \underbrace{i\bar{U}_j \not{D}_U U_j + i\bar{D}_j \not{D}_D D_j}_{\text{right-handed singlets}} \quad (6)$$

leptons have been omitted for simplicity

- with the covariant derivatives

$$\begin{aligned} D_{Q,\mu} &= \partial_\mu + ig_s \lambda_\alpha G^\alpha_\mu + ig\sigma_i W^i_\mu + iY_Q g' B_\mu \\ D_{U,\mu} &= \partial_\mu + ig_s \lambda_\alpha G^\alpha_\mu + iY_U g' B_\mu \\ D_{D,\mu} &= \partial_\mu + ig_s \lambda_\alpha G^\alpha_\mu + iY_D g' B_\mu \end{aligned}$$

and $Y_Q = 1/6$, $Y_U = 2/3$, $Y_D = -1/3$

Yukawa couplings

- In order to realise fermion masses we introduce "Yukawa couplings"
- This is rather ad-hoc. It is necessary to understand the data but is not stable with respect to quantum corrections (the Hierarchy problem).
- By doing this we introduce flavour non-universality via the Yukawa couplings between the Higgs and the quarks

$$\mathcal{L}_{\text{Yukawa}} = \sum_{i,j}^{3} (-\bar{Q}^i_L Y^{ij}_U \tilde{H} u^j_R - \bar{Q}^i_L Y^{ij}_D H d^j_R + h.c.) \quad (7)$$

leptons have been omitted for simplicity

- Replace H by its vacuum expectation value, $\langle H \rangle = (0, \nu)^T$, and we obtain the quark mass terms

$$\sum_{i,j}^{3} (-\bar{u}^i_L m^{ij}_U u^j_R - \bar{d}^i_L m^{ij}_D d^j_R) \quad (8)$$

with the quark mass matrices given by $m_A = \nu Y_A$ with $A = (U, D, L)$

Diagonalising the mass matrices

- Quark mass matrices, m_U, m_D, m_L, are 3×3 complex matrices in "flavour space" with *a priori arbitary values*.
  - We can diagonalise them via a field redefintion

$$u_L = \hat{U}_L u_L^m, \;\; u_R = \hat{U}_R u_R^m, \;\; d_L = \hat{D}_L d_L^m, \;\; d_R = \hat{D}_R d_R^m \tag{9}$$

  - such that in the mass eigenstate basis the matrices are diagonal

$$m_U^{\text{diag}} = \hat{U}_L^\dagger m_U \hat{U}_R, \quad m_D^{\text{diag}} = \hat{D}_L^\dagger m_D \hat{D}_R \tag{10}$$

- The right-handed SU(2) singlet is invariant but recall the left-handed SU(2) doublet gives rise to terms like

$$\frac{g}{\sqrt{2}} \bar{u}_L^i \gamma_\mu W^\mu d_L^i \tag{11}$$

- In the mass basis this then becomes

$$\frac{g}{\sqrt{2}} \bar{u}_L^i \underbrace{\hat{U}_L^{\dagger ij} \hat{D}_L^{jk}}_{\hat{V}_{\text{CKM}}} \gamma_\mu W^\mu d_L^k \tag{12}$$

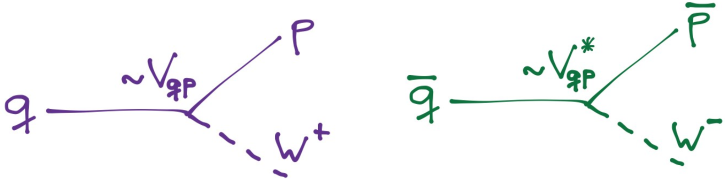
This combination, $\hat{V}_{\text{CKM}} = \hat{U}_L^{\dagger ij} \hat{D}_L^{jk}$, **is the** physical CKM matrix and generates flavour violating charged current interactions. It is complex and unitary, $VV^\dagger = \mathbb{1}$

- CKM matrix transforms the mass eigenstate basis to the flavour eigenstate basis
  - and brings with it a rich variety of observable phenomena

mass eigenstates $\neq$ weak eigenstates

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix} \quad (13)$$

- The up-type quark to down-type quark transition probability proportional to the squared magnitude of the CKM matrix elements, $|V_{ij}|^2$



$$\frac{g}{\sqrt{2}} \bar{u}_{Li} V_{ij} \gamma_\mu W^{\mu+} d_{Lj}$$

Lepton and baryon number conservation

- The gauge part of the SM Lagrangian is invariant under U(3) symmetries of the left-handed doublets and right-handed singlets if the fermions are massless

$$\mathcal{L}_{\text{Gauge}} = \sum_j i\bar{\psi}_j \not{D} \psi_j - \sum_a \frac{1}{4g_a^2} F^a_{\mu\nu} F^{\mu\nu,a}$$

- These U(3) symmetries are broken by the Yukawa terms. The only remaining symmetries correspond to lepton number and baryon number conservation
- These are "accidental" symmetries, coming from the particle content, rather than being explicitly imposed

We will return to the CKM matrix and CKM metrology later!

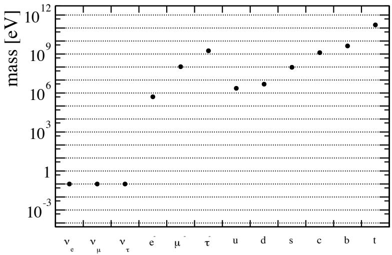
Why is flavour important?

- Most of the free parameters in the SM are related to the flavour sector
- The flavour sector provides the only source of CP-violation in the SM
- Flavour changing neutral current processes can probe mass scales well beyond those directly accessible at the LHC
  - If there are new particles at the TeV-scale, why don't they manifest themselves in FCNC processes (called the flavour problem)?

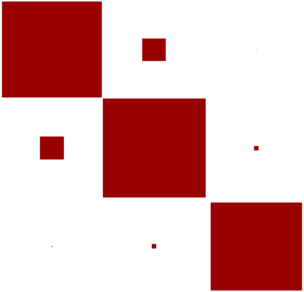
Puzzles in flavour

- Why are there so many parameters and why do they have the values they do?
- Why do we have a flavour structure with 3 generations
  - As we will see shortly, we know that we need ≥ 3 generations to get CP-violation. Are there more generations to discover? If not why exactly 3?
- Why do the quarks have a flavour structure that exhibits both smallness **and** hierarchy?
  - Why is the neutrino sector so different (neither small nor hierarchical)?

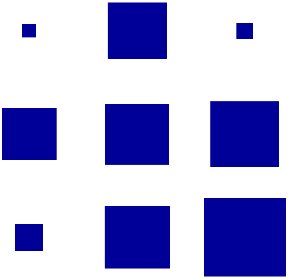
- Large hierarchy in scale between the masses of the fermions
- Equivalent to having a large hierarchy in the Yukawa couplings
- Why / how is this hierarchy so large and why is $y_t \sim 1$?



CKM matrix for the quark sector



PMNS matrix for the neutrino sector



3. Quark Model History

Isospin

- What's the difference between a proton (p) and a neutron (n^0)?
  - They have similar masses
  - They have a similar strong coupling
  - Just have a different charge
- In 1932 Heisenberg proposed that (p, n^0) are members of an isospin doublet
  - Can be treated as the same particle with different isospin projections

$$p : (I, I_z) = (1/2, +1/2), \quad n : (I, I_z) = (1/2, -1/2)$$

  - The pions can be arranged as an isospin triplet

$$\pi^+ : (I, I_z) = (1, +1), \quad \pi^0 : (I, I_z) = (1, 0), \quad \pi^- : (I, I_z) = (1, -1)$$

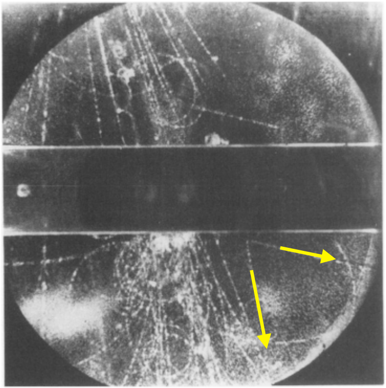
  - Isospin is conserved in strong interactions
  - Isospin is violated in weak interactions
- We now know this is not the correct model (it's not an exact symmetry) but it's still a very useful concept
  - It works because $m_u \sim m_d < \Lambda_{\mathrm{QCD}}$ and can be used to predict interaction rates:

$$\sigma(p + p \to d + \pi^+) : \sigma(p + n \to d + \pi^0) = 2 : 1$$

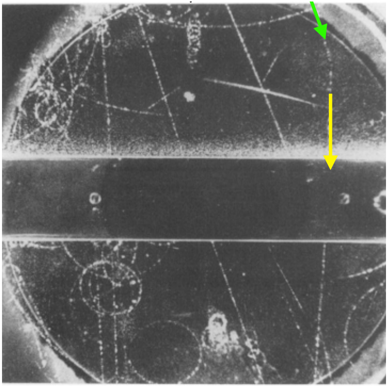
can you explain this 2:1 ratio?

- In 1947 Rochester and Butler observed two new particles with mass ~ 500 MeV and long lifetimes
  - Neutral particle (no track) $\rightarrow$ two charged pions
  - Charged particle (track) $\rightarrow$ charged pion + something
  - Long lifetimes, $O(10^{-10}s)$, so dubbed "strange"

$K^0_S \rightarrow \pi^+\pi^-$



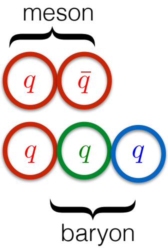
$K^+ \rightarrow \mu^+\nu_\mu$

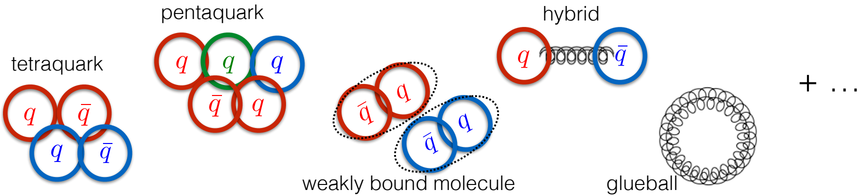


The Quark Model

- Many new particles (a "zoo") discovered in the 60s
- Gell-Mann, Nishijima and Ne'eman introduced the quark "model" (u, d, s) which could elegantly categorise them (the "eight-fold way" - flavour SU(3) symmetry)
- Gell-Mann and Pais
  - Strangeness conserved in strong interactions (production)
  - Strangeness violated in weak interactions (decay)

- Can only make colour neutral objects
  - Quark anti-quark mesons ($q\bar{q}$) or three quark baryons (qqq). Nearly all known states fall into one of these two categories
  - Can also build colour neutral states containing more quarks (e.g. 4 or 5 quark states). Only quite recently confirmed (and still not entirely understood).





The Quark Model

Volume 8, number 3 PHYSICS LETTERS 1 February

A SCHEMATIC MODEL OF BARYONS AND MESONS *

M. GELL-MANN
California Institute of Technology, Pasadena, California

Received 4 January 1964

If we assume that the strong interactions of baryons and mesons are correctly described in terms of the broken "eightfold way" [1-3], we are tempted to look for some fundamental explanation of the situation. A highly promised approach is the purely dynamical "bootstrap" model for all the strongly interacting particles within which one may try to derive isotopic spin and strangeness conservation and broken eightfold symmetry from self-consistency alone [4]. Of course, with only strong interactions, the orientation of the asymmetry in the unitary space cannot be specified; one hopes that in some way the selection of specific components of the F-spin by electromagnetism and the weak interactions determines the choice of isotopic spin and hypercharge directions.

Even if we consider the scattering amplitudes of strongly interacting particles on the mass shell only and treat the matrix elements of the weak, electromagnetic, and gravitational interactions by means ber $n_t - n_{\bar{t}}$ would be zero for all known baryons and mesons. The most interesting example of such a model is one in which the triplet has spin $\frac{1}{2}$ and $z = -1$, so that the four particles d^-, s^-, u^0 and b^0 exhibit a parallel with the leptons.

A simpler and more elegant scheme can be constructed if we allow non-integral values for the charges. We can dispense entirely with the basic baryon b if we assign to the triplet t the following properties: spin $\frac{1}{2}$, $z = -\frac{1}{3}$, and baryon number $\frac{1}{3}$. We then refer to the members $u^{\frac{2}{3}}$, $d^{-\frac{1}{3}}$, and $s^{-\frac{1}{3}}$ of the triplet as "quarks" [6] q and the members of the anti-triplet as anti-quarks $\bar{q}$. Baryons can now be constructed from quarks by using the combinations (qqq), $(qqqq\bar{q})$, etc., while mesons are made out of $(q\bar{q})$, $(qq\bar{q}\bar{q})$, etc. It is assuming that the lowes baryon configuration (qqq) gives just the representations **1**, **8**, and **10** that have been observed, while the lowest meson configuration $(q\bar{q})$ similarly give just **1** and **8**.

SU(2) flavour mixing

- Four possible combinations from two quarks (u and d)

$$u\overline{u}, d\overline{d}, u\overline{d}, \overline{u}d$$

- Under SU(2) symmetry the π^0 and η states are members of an isospin triplet and singlet respectively

$$\pi^0 = \frac{1}{\sqrt{2}}(u\overline{u} - d\overline{d}), \quad \eta = \frac{1}{\sqrt{2}}(u\overline{u} + d\overline{d})$$

SU(3) flavour mixing

- Introducing the strange quark (under SU(3) symmetry) we now have an octuplet and a singlet

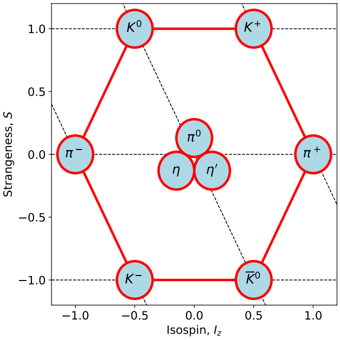
$$\pi^0 = \frac{1}{\sqrt{2}}(u\overline{u} - d\overline{d}), \quad \eta_1 = \frac{1}{\sqrt{3}}(u\overline{u} + d\overline{d} + s\overline{s}), \quad \eta_8 = \frac{1}{\sqrt{6}}(u\overline{u} + d\overline{d} - 2s\overline{s})$$

- The physical states involve a further mixing

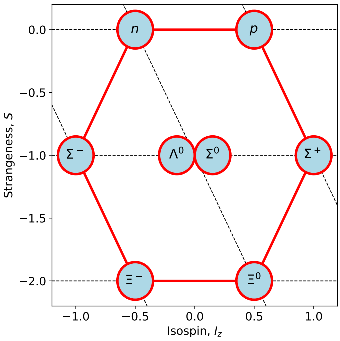
$$\eta = \eta_1 \cos\theta + \eta_8 \sin\theta, \quad \eta' = -\eta_1 \sin\theta + \eta_8 \cos\theta$$

- Can elegantly categorise states by isospin (up/downess) and strangeness
- Also get the excited states which can be categorised in the same way

Spin-0 Mesons



Spin-1/2 Baryons



Homework

- What is the quark content of these states?
- Do you know the spin-1 (spin-3/2) states?

- Compare rates of:

  $s \to u$: $K^+ \to \mu^+ \nu_\mu$ $(\Lambda^0 \to p\pi^-,\ \Sigma^+ \to n e^+ \nu_e)$

  $d \to u$: $\pi^+ \to \mu^+ \nu_\mu$ $(n \to p e^+ \nu_e)$

  - Apparent that $s \to u$ transitions are suppressed by a factor ~ 20
- Cabibbo (1963) suggested that "down-type" is some ad-mixture of d and s
  - The first suggestion of quark mixing
  - Physical state is an admixture of flavour states

$$\begin{pmatrix} u \\ d' \end{pmatrix} = \begin{pmatrix} u \\ d\cos(\theta_C) + s\sin(\theta_C) \end{pmatrix} \tag{14}$$

- The mixing angle is determined experimentally to be $\sin(\theta_C) = 0.22$.

- Cabibbo's solution opened up a new experimental problem
  - $K^+ \to \mu^+ \nu_\mu$ had been seen but not $K^0_{\rm L} \to \mu^+ \mu^-$
    - $\mathcal{B}(K^0_{\rm L} \to \mu^+\mu^-) \approx 7 \times 10^{-9}$
    - $\mathcal{B}(K^0_{\rm L} \to e^+e^-) \approx 1 \times 10^{-11}$
  - $K^+ \to \pi^0 \mu^+ \nu_\mu$ had been seen but not $K^0_{\rm L} \to \pi^0 \mu^+ \mu^-$
    - $\mathcal{B}(K^0_{\rm L} \to \pi^0\mu^+\mu^-) \approx 1 \times 10^{-10}$
- If the doublet of the weak interaction is the one Cabibbo suggested, Eq. (14), then one can have neutral currents

$$J^0_\mu = \bar{d}'\gamma_\mu(1-\gamma_5)d' \tag{15}$$

which introduces tree level FCNCs (which we don't see)

- Glashow, Iliopoulos and Maiani (1970) provided a solution by adding a second doublet

$$\begin{pmatrix} c \\ s' \end{pmatrix} = \begin{pmatrix} c \\ -d\sin(\theta_C) + s\cos(\theta_C) \end{pmatrix} \tag{16}$$

  - This exactly cancels the term above, Eq. (15)
  - Thus FCNC contributions are suppressed via loops

GIM suppression

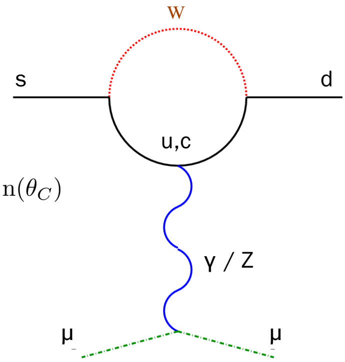
- Consider the $s \to d$ transition required for $K^0_{\rm L} \to \mu^+\mu^-$
- Given that $m_u, m_c \ll m_W$

$$\begin{aligned}\mathcal{A} &\approx V_{us}V_{ud}^* + V_{cs}V_{cd}^* \\ &= \sin(\theta_C)\cos(\theta_C) - \cos(\theta_C)\sin(\theta_C) \\ &= 0\end{aligned}$$

- Indeed 2×2 unitarity implies that

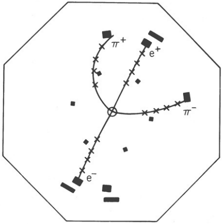
$$V_{us}V_{ud}^* + V_{cs}V_{cd}^* = 0$$

- **Predicts the existence of the charm quark:**
  - Kaon mixing
  - Low branching fractions for FCNC decays



Observation of the J/ψ

- Experimental evidence for the charm quark came in 1974.
- Discovery of charmonium (J) at Brookhaven in $p\mathrm{Be} \to \mathrm{e}^+\mathrm{e}^-\mathrm{X}$.
- Discovery of charmonium (ψ) at SLAC in $e^+e^- \to (\mathrm{hadrons}), \mathrm{e}^+\mathrm{e}^-, \mu^+\mu^-$



EW LETTERS 2 DECEMBER 197

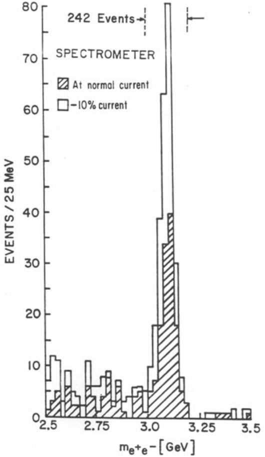
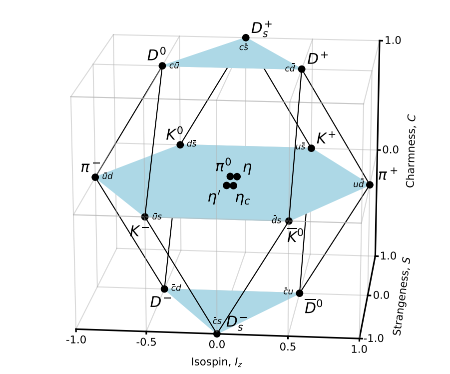


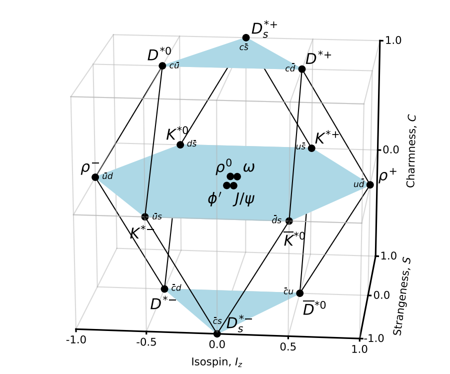
FIG. 2. Mass spectrum showing the existence of J. sults from two spectrometer settings are plotted wing that the peak is independent of spectrometer rents. The run at reduced current was taken two nths later than the normal run.

Mesons

Spin-0

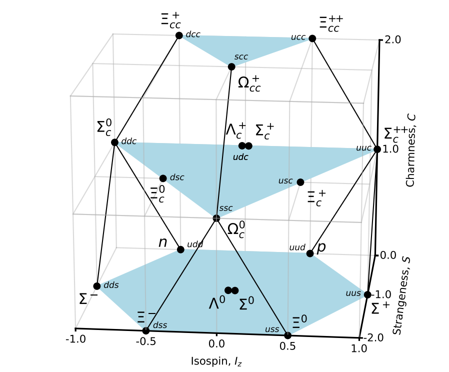


Spin-1

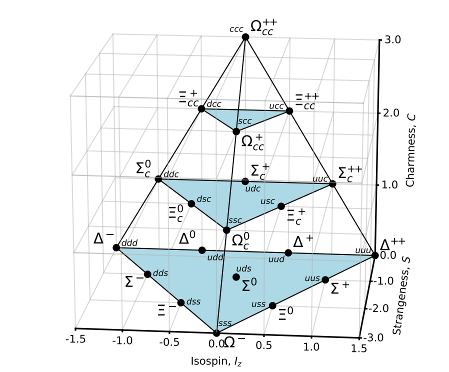


Baryons

Spin-1/2



Spin-3/2

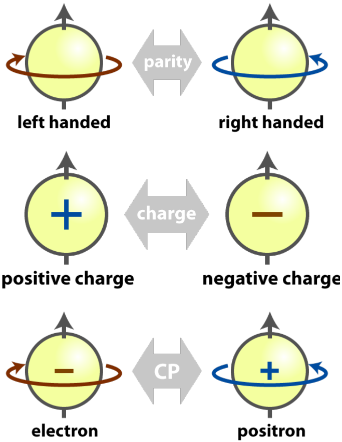


Parity violation

- Two decays were found for charged strange mesons
  - $\theta \to \pi^+\pi^0$
  - $\tau \to \pi^+\pi^-\pi^+$
- The $\theta - \tau$ puzzle
  - Masses and lifetimes of θ and τ are the same
  - But 2π and 3π final states have the opposite parity
- The resolution is that θ and τ are the same particle, K^+, and parity is violated in the decay

C and P

- Prior to 1956 it was thought that the laws of physics were invariant under parity, P, (*i.e.* a mirrored reflection)
  - Shown to be violated in β decays of Co-60 by C. S. Wu (following an idea by T. D. Lee and C. N. Yang)
- Now known that parity, P, is maximally violated in weak decays
  - There are no right-handed neutrinos
- Charge, C, is also maximally violated in weak decays
  - There is no left-handed anti-neutrino
- The product CP is conserved (Landau 1957) and distinguishes absolutely between matter and antimatter
- The product CPT is conserved in any Lorentz invariant gauge field theory



Neutral Kaon Mixing

- Ignoring CP-violation, in the neutral kaon system the two physical (mass/lifetime) states are admixtures of the strangeness (flavour) states

$$|K_1\rangle = \frac{|K^0\rangle - |\overline{K}^0\rangle}{\sqrt{2}} \quad \text{and} \quad |\mathrm{K}_2\rangle = \frac{|\mathrm{K}^0\rangle + |\overline{\mathrm{K}}^0\rangle}{\sqrt{2}} \tag{17}$$

under parity, P, and charge conjugation, C, the flavour states transform as

$$\mathcal{P}|K^0\rangle = -|K^0\rangle, \quad \mathcal{C}|K^0\rangle = |\overline{K}^0\rangle \quad \text{and} \quad \mathcal{CP}|K^0\rangle = -|\overline{K}^0\rangle. \tag{18}$$

- For the physical states

$$\mathcal{P}|K_{1,2}\rangle = -|K_{1,2}\rangle, \quad \mathcal{C}|K_{1,2}\rangle = \mp|K_{1,2}\rangle \quad \text{and} \quad \mathcal{CP}|K_{1,2}\rangle = \pm|K_{1,2}\rangle. \tag{19}$$

i.e. they are eigenstates of P, C and CP as well.

- What does this tell us about their decays?
  - $\pi^+\pi^-$ has $P = +1$, $C = +1$, $CP = +1$ - shorter lived $K_1 = K^0_\mathrm{S}$
  - $\pi^+\pi^-\pi^0$ has $P = -1$, $C = +1$, $CP = -1$ - longer lived $K_2 = K^0_\mathrm{L}$
- If CP is preserved K^0_L decay to two pions should be forbidden

CP-violation

- In 1964 Christensen, Cronin, Fitch and Turlay observed 2π decays of the K_2 (K^0_{L}) meson

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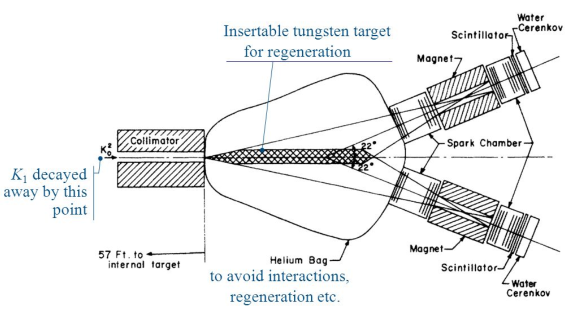
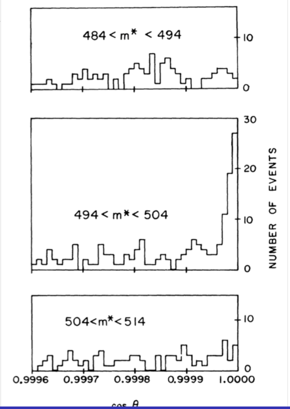
EVIDENCE FOR THE 2π DECAY OF THE $K_2{}^0$ MESON*†

J. H. Christenson, J. W. Cronin,‡ V. L. Fitch,‡ and R. Turlay§
Princeton University, Princeton, New Jersey

(Received 10 July 1964)





CP violation can be explained by the CKM mechanism

CKM mechanism

- In 1973 Kobayashi and Maskawa introduce the CKM mechanism to explain CP-violation
- As we will see this requires a third generation of quark and so they predict the existence of b and t quarks

CP Violation in the Renormalizable Theory of Weak Interaction

Makoto Kobayashi, Toshihide Maskawa (Kyoto U.)

Feb 1973 - 6 pages

Prog.Theor.Phys. 49 (1973) 652-657

Also in *Lichtenberg, D. B. (Ed.), Rosen, S. P. (Ed.): Developments In The Quark Theory Of Hadrons, Vol. 1*, 218-223.

DOI: 10.1143/PTP.49.652

KUNS-242

Abstract (Oxford Journals)

In a framework of the renormalizable theory of weak interaction, problems of CP-violation are studied. It is concluded that no realistic models of CP-violation exist in the quartet scheme without introducing any other new fields. Some possible models of CP-violation are also discussed.

4. The CKM Matrix

Parameters of the CKM matrix

- 3×3 complex matrix
  - 18 parameters
- Unitary
  - 9 parameters (3 mixing angles, 6 complex phases)
- Quark fields absorb 5 of these (unobservable) phases
- Left with:
  - 3 mixing angles $(\theta_{12}, \theta_{23}, \theta 13)$
  - one complex phase (δ) which gives rise to CP-violation in the SM

The CKM Matrix

$$V_{\mathrm{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

- A highly predictive theory

- Absorbing quark phases can be done because under a quark phase transformation

$$u_L^i \to e^{i\phi_u^i} u_L^i, \qquad d_L^i \to e^{i\phi_d^i} d_L^i \tag{20}$$

and a simultaneous rephasing of the CKM matrix ($V_{jk} \to e^{i(\phi_j - \phi_k)} V_{jk}$)

$$V_{\rm CKM} \to \begin{pmatrix} e^{i\phi_u} & & \\ & e^{i\phi_c} & \\ & & e^{i\phi_t} \end{pmatrix} \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} e^{i\phi_d} & & \\ & e^{i\phi_s} & \\ & & e^{i\phi_b} \end{pmatrix} \tag{21}$$

the charged current $J^\mu = \bar{u}_{Li} V_{ij} \gamma^\mu d_{Lj}$ is left invariant

- So all additional quark phases are rephased to be relative to just one

Degrees of freedom in an N generation CKM matrix

Number of generations	**2**	**3**	N
Number of real parameters	4	9	N^2
Number of imaginary parameters	4	9	N^2
Number of constraints ($VV^\dagger = \mathbb{1}$)	-4	-9	$-N^2$
Number of relative quark phases	-3	-5	$-(2N-1)$
Total degrees of freedom	1	4	$(N-1)^2$
Number of Euler angles	1	3	$N(N-1)/2$
Number of *CP* phases	0	1	$(N-1)(N-2)/2$

CKM parameterisations

- The standard form is to express the CKM matrix in terms of three rotation matrices and one CP-violating phase (δ)

$$V_{\mathrm{CKM}} = \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}}_{\text{2nd and 3rd gen. mixing}} \underbrace{\begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{+i\delta} & 0 & c_{13} \end{pmatrix}}_{\text{1st and 3rd gen. mixing + CPV phase}} \underbrace{\begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\text{1st and 2nd gen. mixing}} \quad (22)$$

$$= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{-i\delta} & -c_{13}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \quad (23)$$

where

$$c_{ij} = \cos(\theta_{ij}) \quad \text{and} \quad s_{ij} = \sin(\theta_{ij})$$

CKM parameterisations

- Emprically $s_{12} \sim 0.2$, $s_{23} \sim 0.04$, $s_{13} \sim 0.004$
- CKM matrix exhibits a very clear hierarchy
- The so-called Wolfenstein parameterisation exploits this
- Expand in powers of $\lambda = \sin(\theta_{12})$
- Use four real parameters which are all $\sim O(1)$, (A, λ, ρ, η)

The CKM Wolfenstein parameterisation

$$V_{\text{CKM}} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4) \quad (24)$$

- The CKM matrix is almost diagonal
  - Provides strong constraints on NP models in the flavour sector
- Have seen already that quark masses also exhibit a clear hierarchy
- The flavour hierarchy problem
  - Where does this structure come from?

CKM Unitarity Constraints

- The unitary nature of the CKM matrix provides several constraints, $VV^\dagger = \mathbb{1}$
- The ones for off-diagonal elements consist of three complex numbers summing to 0
  - Hence why these are often represented as triangles in the real / imaginary plane (see next slide)

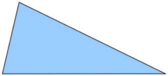
Constraints along diagonal	Constraints off-diagonal
$\|V_{ud}\|^2 + \|V_{us}\|^2 + \|V_{ub}\|^2 = 1$	$V_{ud}V_{us}^* + V_{cd}V_{cs}^* + V_{td}V_{ts}^* = 0$
$\|V_{cd}\|^2 + \|V_{cs}\|^2 + \|V_{cb}\|^2 = 1$	$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$
$\|V_{td}\|^2 + \|V_{ts}\|^2 + \|V_{tb}\|^2 = 1$	$V_{us}V_{ub}^* + V_{cs}V_{cb}^* + V_{ts}V_{tb}^* = 0$
$\|V_{ud}\|^2 + \|V_{cd}\|^2 + \|V_{td}\|^2 = 1$	$V_{ud}V_{cd}^* + V_{us}V_{cs}^* + V_{ub}V_{cb}^* = 0$
$\|V_{us}\|^2 + \|V_{cs}\|^2 + \|V_{ts}\|^2 = 1$	$V_{ud}V_{td}^* + V_{us}V_{ts}^* + V_{ub}V_{tb}^* = 0$
$\|V_{ub}\|^2 + \|V_{cb}\|^2 + \|V_{tb}\|^2 = 1$	$V_{cd}V_{td}^* + V_{cs}V_{ts}^* + V_{cb}V_{tb}^* = 0$

CKM Unitarity Triangles and the Jarlskog Invariant

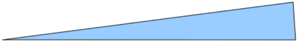
- The off-diagonal constraints can be represented as triangles in the complex plane

$$V_{ud}V_{us}^* + V_{cd}V_{cs}^* + V_{td}V_{ts}^* = 0$$
$$\lambda \quad + \quad \lambda \quad + \quad \lambda^5$$

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$$
$$\lambda^3 \quad + \quad \lambda^3 \quad + \quad \lambda^3$$



$$V_{us}V_{ub}^* + V_{cs}V_{cb}^* + V_{ts}V_{tb}^* = 0$$
$$\lambda^4 \quad + \quad \lambda^2 \quad + \quad \lambda^2$$



- All the triangles have the equivalent area (known as the Jarlskog invariant), $J/2$
- J is a phase convention independent measure of CP-violation in the quark sector

$$|J| = \mathcal{I}m(V_{ij}V_{kl}V_{kj}^*V_{il}^*) \quad \text{for } i \neq k \text{ and } j \neq k \tag{25}$$

- In the standard notation

$$J = c_{12}c_{13}^2c_{23}s_{12}s_{23}s_{13}\sin(\delta) \tag{26}$$

- The small size of the Euler angles means J (and CP-violation) is small in the SM

The matter-antimatter asymmetry

- From CMB measurements by WMAP and Plank

$$\frac{n_B - n_{\bar{B}}}{n_\gamma} \approx 6 \times 10^{-10} \tag{27}$$

- In the early hot universe we expect annihilation (upon expansion and cooling) to give

$$n_B \approx n_{\bar{B}} \approx n_\gamma \tag{28}$$

- The matter-antimatter imbalance is certainly small but far too large to be explained by electroweak baryogenesis.
- But CP-violation in the quark sector is too small because of the size of the mixing angles and the large hierarchy of quark masses.

Sakharov (1967) conditions:

- required for a matter dominated universe from a symmetric initial state

1. Baryon number violation
2. C and CP violation
3. Interactions out of thermal equilibrium

- If we start with equal amounts of matter (M) and antimatter ($\bar{M}$)
- And assume there are only two possible decay modes:
  - $M \to A$ (baryon number N_A) with probability p
  - $M \to B$ (baryon number N_B) with probability $(1-p)$
  - $\bar{M} \to \bar{A}$ (baryon number $-N_A$) with probability $\bar{p}$
  - $\bar{M} \to \bar{B}$ (baryon number $-N_B$) with probability $1-\bar{p}$
- Generated baryon asymmetry:

$$\Delta N_{\text{tot}} = N_A p + N_B(1-p) - N_A\bar{p} - N_B(1-\bar{p}) \quad (29)$$

$$= (p-\bar{p})(N_A - N_B) \quad (30)$$

- To have $\Delta N_{\text{tot}} \neq 0$ requires **both** $p \neq \bar{p}$ **and** $N_A \neq N_B$
- *i.e.* need baryon number violation **and** CP violation
- Even then, the system needs to be out of thermal equilibrium otherwise

$$\Gamma(A \to B + C) = \Gamma(B + C \to A) \quad (31)$$

and the asymmetry is destroyed as soon as it's created

Jarlskog Invariant and BAU

- We can estimate the size of the BAU from CP-violation in the quark sector using the Jarlskog invariant

$$\frac{n_B - n_{\bar{B}}}{n_\gamma} \approx \frac{n_B}{n_\gamma} \sim \frac{J \times P_u \times P_d}{M^{12}} \tag{32}$$

where

$$J = \cos(\theta_{12})\cos(\theta_{23})\cos^2(\theta_{13})\sin(\theta_{12})\sin(\theta_{23})\sin(\theta_{13})\sin(\delta) \tag{33}$$

$$P_u = (m_t^2 - m_c^2)(m_c^2 - m_u^2)(m_t^2 - m_u^2) \tag{34}$$

$$P_d = (m_b^2 - m_s^2)(m_s^2 - m_d^2)(m_b^2 - m_d^2) \tag{35}$$

$$M = \text{mass scale} \tag{36}$$

- Take the mass scale as the electroweak scale - $O(100\,\text{GeV})$
- Generates an asymmetry of $O(10^{-17}) \ll$ than the cosmological observation of $O(10^{-10})$

Thus CP-violation in the quark sector cannot explain the observed matter-antimatter asymmetry of the universe

Where is the rest of the CP-violation?

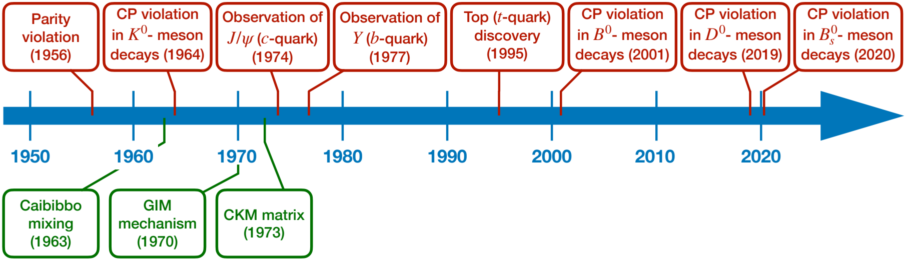
- SM insufficient to describe the BAU
- A large asymmetry requires
  - New sources of CP violation
  - At higher energy scales
- Where might this be?
  - Quark sectors
    - Discrepancies with CKM predictions
  - Lepton sector
    - CP violation in the neutrino sector
  - New Physics
    - New forces, extra dimensions, lepto-quarks, Z', $W'^{\pm}$
    - Many flavour observables are sensitive to generic additions to the SM

Prospects for Flavour Physics

- Historically provide evidence before the energy frontier
  - GIM mechanism before discovery of charm
  - CKM mechanism before discovery of bottom and top
  - Neutral currents before the Z
  - Electroweak precision before the Higgs
- Very sensitive to loop processes
  - Massive virtual particles
  - SM contributions heavily suppressed (or not allowed)
  - Flavour changing neutral currents
  - Penguin decays (CPV from interference between tree and loop)
  - Lepton flavour universality

5. Recap

Recap



Parity violation (1956)
CP violation in K^0- meson decays (1964)
Observation of J/ψ (c-quark) (1974)
Observation of Υ (b-quark) (1977)
Top (t-quark) discovery (1995)
CP violation in B^0- meson decays (2001)
CP violation in D^0- meson decays (2019)
CP violation in B_s^0- meson decays (2020)
1950
1960
1970
1980
1990
2000
2010
2020
Caibibbo mixing (1963)
GIM mechanism (1970)
CKM matrix (1973)

Recap

In this lecture we have covered

- What is (and what is not) flavour physics
- Flavour in the SM
- The Quark Model in the SM
  - Isospin
  - Strangeness
  - Cabibbo Mixing
  - The GIM mechanism
  - P and CP violation
- The CKM matrix
  - CKM parameterisations and hierarchy
  - Unitarity triangles
  - The Jarlskog invariant and the Matter-antimatter asymmetry

End of Lecture 1