Useful things to know about accelerators – part I

C. T. Rogers
ISIS
Rutherford Appleton Laboratory

Science and Technology Facilities Council

Accelerators - A Window on Nature

- Particle accelerators provide the source for most high energy physics experiments
  - Provide high luminosity, high energy beams for colliders
  - Provide high brightness beams for secondary particle production
  - Also key technology for life sciences, engineering, chemistry
- How do they work?
  - How can we get to high energy?
  - How can we keep the beam in the accelerator?
  - How can we get to high luminosity?
- What are the main HEP facilities in the world today?
- What might HEP facilities look like in the future?

Accelerator Components

- Most accelerators share similar components
- Main components of an accelerator
  - Bending – dipoles
  - Focussing – quadrupoles
  - Acceleration - RF cavities
- Also
  - Vacuum
  - Diagnostics
  - Targets for secondary particle production
- First Lecture: Derive basic theory of accelerator physics
- Second Lecture: Discuss accelerator equipment and techniques

Lorentz force law

- Fundamental equation for particles moving through fields

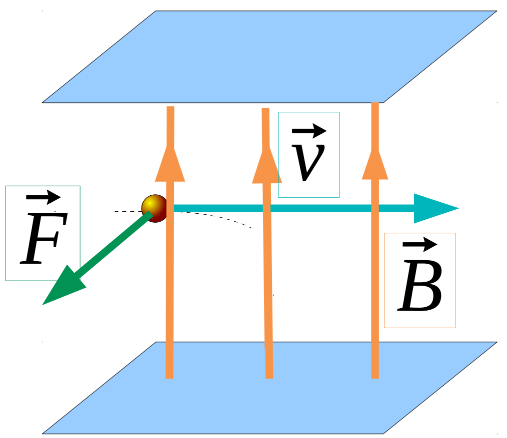
Force Charge Velocity Magnetic Field Electric Field

$$\vec{F} = q\,\vec{v} \times \vec{B} + q\,\vec{E} \qquad \text{(eq. 1)}$$

- Magnetic force is perpendicular to velocity
  - Magnetic field conserves energy
- Electric force is weaker by factor velocity
  - Magnets are better for bending and focussing

Magnetic Rigidity and Bending

- Simplest magnet - “dipole”
  - Uniform magnetic field perpendicular to beam direction



Lorentz force (eq. 1) + centripetal motion:

$$qvB=\frac{pv}{\rho}$$

Radius

Rearranging:

$$\underbrace{B\rho}_{}=\frac{p}{q}$$

Magnetic Rigidity

  - Constant force → constant curvature → circular motion
  - Magnetic rigidity parameterises momentum
  - Charge-to-mass ratio important when accelerating multiple particle species

Worked example – LHC

- If we wanted to accelerate, say, 7 TeV particles, what bending radius is required?
- Maximum dipole field around 8.3 T

$$B\rho = \frac{p}{q}$$

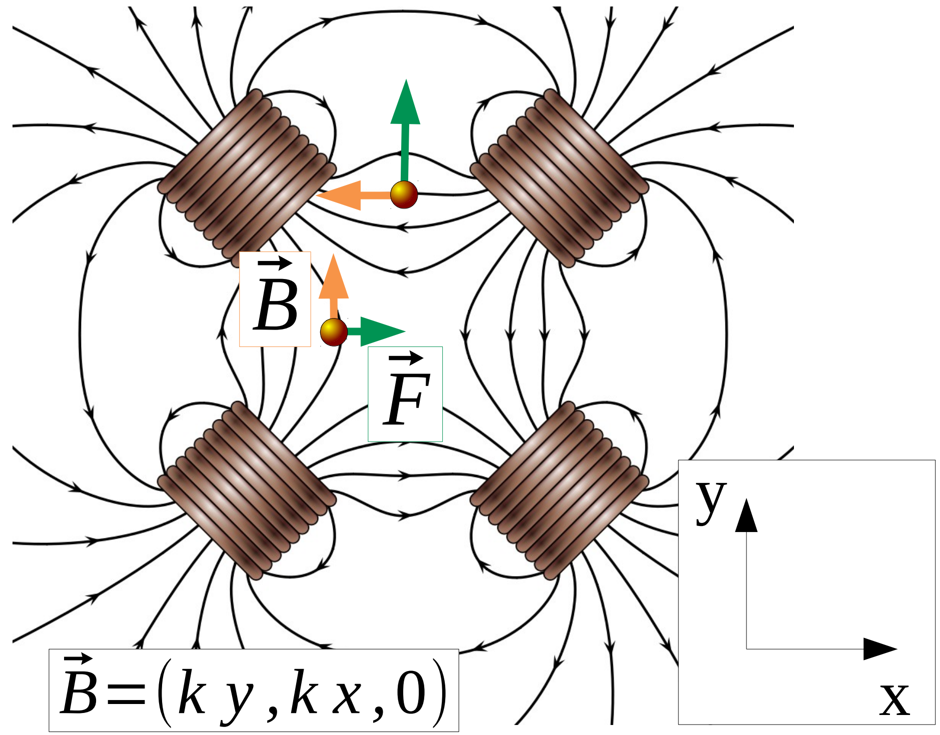
$$\rho = \frac{p}{qB} = \frac{7}{0.3 \times 8.3} = 2.8\,km$$

- Nb: LHC radius ~ 4.1 km
  - Need space for detectors, etc

Science and Technology Facilities Council

Quadrupole magnets

- If we only had bending magnets, particles would soon be lost from the accelerator
- Need to keep the particles in the accelerator using focussing elements
  - Usually use quadrupoles
- Field stronger away from beam centre
  - Like a spring or pendulum
  - Simple harmonic motion
- Overall focussing by alternating the gradient



Quadrupole field – horizontal (1)

- For a particle moving near to the z-axis

$$\vec{F}=q\vec{v}\times\vec{B}+q\vec{E}$$

$$\vec{B}=\left(b_0 y, b_0 x, 0\right)$$

- Considering only p_x for now

$$\frac{dp_x}{dt}=q\frac{dz}{dt}B_y$$

- Use the chain rule

$$\frac{dp_x}{dt}=\frac{dp_x}{dz}\frac{dz}{dt}$$

- Combining these equations:

$$\frac{dp_x}{dz}=q b_0 x$$

Science and Technology Facilities Council

Quadrupole field – horizontal (2)

$$\frac{dp_x}{dz}=q\,b_0\,x$$

- Definition of x-component of momentum

$$p_x=m\,\gamma\,v_x=m\,\gamma\frac{dz}{dt}\frac{dx}{dz}=p_z\frac{dx}{dz}$$

- Substitute this definition into gives

$$p_z\frac{d^2x}{dz^2}=q\,b_0\,x$$

- Rearrange and wrap up constant terms in focussing strength *k*

$$\frac{d^2x}{dz^2}-k\,x=0$$

Quadrupole field – vertical

- Lorentz force law with quadrupole field definition

$$\frac{dp_y}{dt} = -q\, b_0\, v_z\, y$$

- Use chain rule and eliminate v_z

$$p_z \frac{d^2 y}{dz^2} = -q\, b_0\, y$$

- Rearrange and wrap up constant terms in defocussing strength *k*

$$\frac{d^2 y}{dz^2} + k\, y = 0$$

Solutions

- Motion is governed by

$$\frac{d^2x}{dz^2} - k\,x = 0 \qquad \frac{d^2y}{dz^2} + k\,y = 0$$

- This is simple harmonic motion – solutions are of form

$$x = x_0 \cos\left(\sqrt{k}\,z\right) + \frac{dx_0}{dz}\frac{1}{\sqrt{k}}\sin\left(\sqrt{k}\,z\right)$$

- Taking derivative

$$\frac{dx}{dz} = -x_0\sqrt{k}\sin\left(\sqrt{k}\,z\right) + \frac{dx_0}{dz}\cos\left(\sqrt{k}\,z\right)$$

For y

$$y = y_0 \cosh\left(\sqrt{k}\,z\right) + \frac{dy_0}{dz}\frac{1}{\sqrt{k}}\sinh\left(\sqrt{k}\,z\right)$$

$$\frac{dy}{dz} = y_0\sqrt{k}\sinh\left(\sqrt{k}\,z\right) + \frac{dy_0}{dz}\cosh\left(z\right)$$

Transfer Matrix

- Just thinking about *x*, the particles move according to

$$x = x_0 \cos\left(\sqrt{k}\, z\right) + \frac{dx_0}{dz} \sin\left(\sqrt{k}\, z\right)$$

$$\frac{dx}{dz} = -x_0 \sqrt{k} \sin\left(\sqrt{k}\, z\right) + \frac{dx_0}{dz} \sqrt{k} \cos\left(\sqrt{k}\, z\right)$$

- We can rewrite this as a matrix

$$\begin{pmatrix} x \\ \frac{dx}{dz} \end{pmatrix} = \begin{pmatrix} \cos\left(\sqrt{k}\, z\right) & \frac{1}{\sqrt{k}} \sin\left(\sqrt{k}\, z\right) \\ \sqrt{k} \sin\left(\sqrt{k}\, z\right) & \cos\left(\sqrt{k}\, z\right) \end{pmatrix} \begin{pmatrix} x_0 \\ \frac{dx_0}{dz} \end{pmatrix}$$

- This matrix is known as the quadrupole's **transfer matrix**

$$\underline{u}_1 = \boldsymbol{M}_{\mathbf{01}}\, \underline{u}_0$$

Questions

Questions

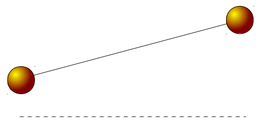
- Exercise – what is the transfer matrix for a drift space, that is a region with no fields at all?
  - What is the force acting on the particle?
  - What is x(z) in terms of dx_0/dz and x_0
  - What is dx/dz in terms of dx_0/dz
  - Now write that as a matrix

Science and Technology Facilities Council

Questions

- Exercise – what is the transfer matrix for a drift space?
  - What is the force acting on the particle?
    - No force
  - What is x(z) in terms of dx_0/dz and x_0

$$x = x_0 + \frac{dx_0}{dz} z$$



  - What is dx/dz in terms of dx_0/dz

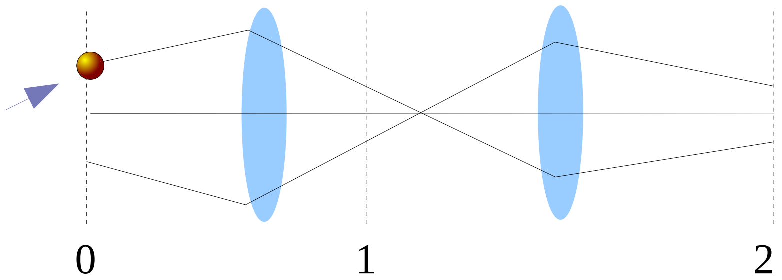
$$\frac{dx}{dz} = \frac{dx_0}{dz}$$

  - Now write that as a matrix

$$\begin{pmatrix} x \\ \frac{dx}{dz} \end{pmatrix} = \begin{pmatrix} 1 & z \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x_0 \\ \frac{dx_0}{dz} \end{pmatrix}$$

Transfer Lines

$$\underline{u}_0 = \begin{pmatrix} x_0 \\ \dfrac{dx_0}{dz} \end{pmatrix}$$



- Transfer matrix defines transport through a region
- Transfer matrices can be combined by multiplication
- Say we have transfer matrices like:

$$\underline{u}_1 = \boldsymbol{M}_{01}\,\underline{u}_0$$

$$\underline{u}_2 = \boldsymbol{M}_{12}\,\underline{u}_1$$

- Then

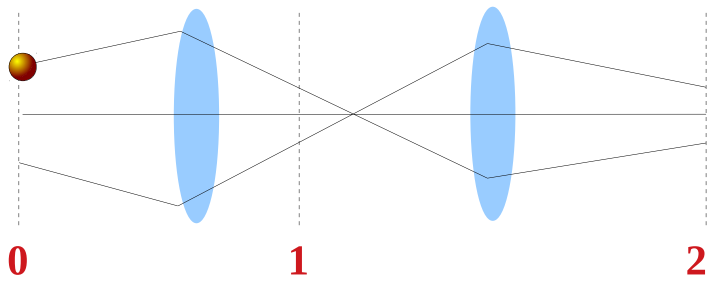
$$\underline{u}_2 = \boldsymbol{M}_{12}\,\boldsymbol{M}_{01}\,\underline{u}_0$$

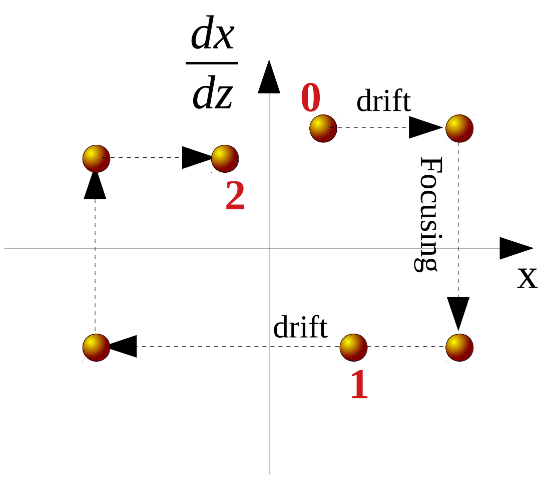
- i.e. we can define a combined transfer matrix like

$$\boldsymbol{M}_{02} = \boldsymbol{M}_{12}\,\boldsymbol{M}_{01}$$

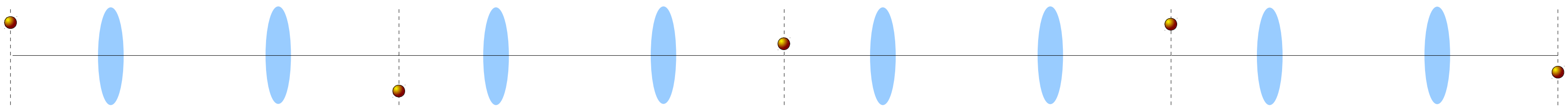
Phase space

- Another instructive way to look at beam optics is by considering the phase space





Periodic Lattices



- Following *n* identical cells or turns in a ring with one-turn matrix $\boldsymbol{M}$

$$\underline{u}_n = \boldsymbol{M}^n \underline{u}_0$$

- Rewrite

$$\boldsymbol{M} = \boldsymbol{I}\cos\mu + \boldsymbol{J}\sin\mu$$

$$\boldsymbol{J} = \begin{pmatrix} \alpha & \beta \\ -\gamma & -\alpha \end{pmatrix} \quad \text{with} \quad \gamma\beta - \alpha^2 = 1 \quad \text{and} \quad \boldsymbol{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

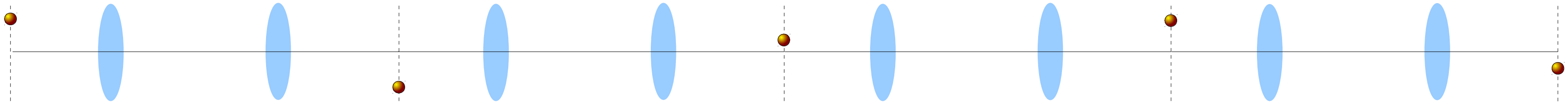
- So

$$\boldsymbol{J}^2 = -\boldsymbol{I}$$

- And

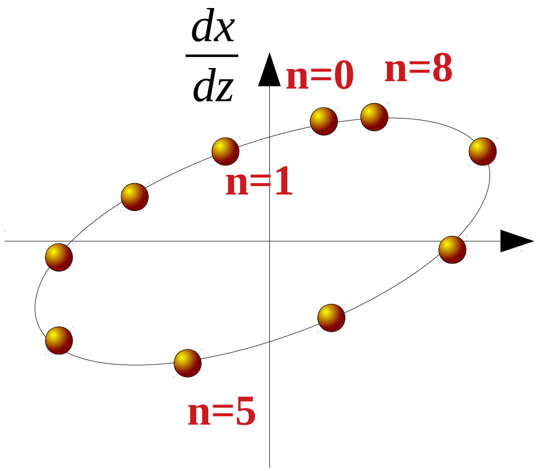
$$\boldsymbol{M}^n = \boldsymbol{I}\cos(n\mu) + \boldsymbol{J}\sin(n\mu)$$

Periodic Lattices



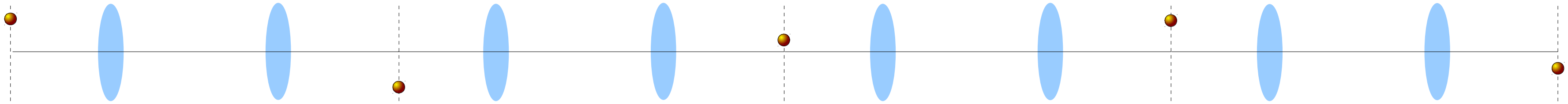
- What does this mean?

$$\boldsymbol{M}^n = \boldsymbol{I}\cos(n\mu) + \boldsymbol{J}\sin(n\mu)$$

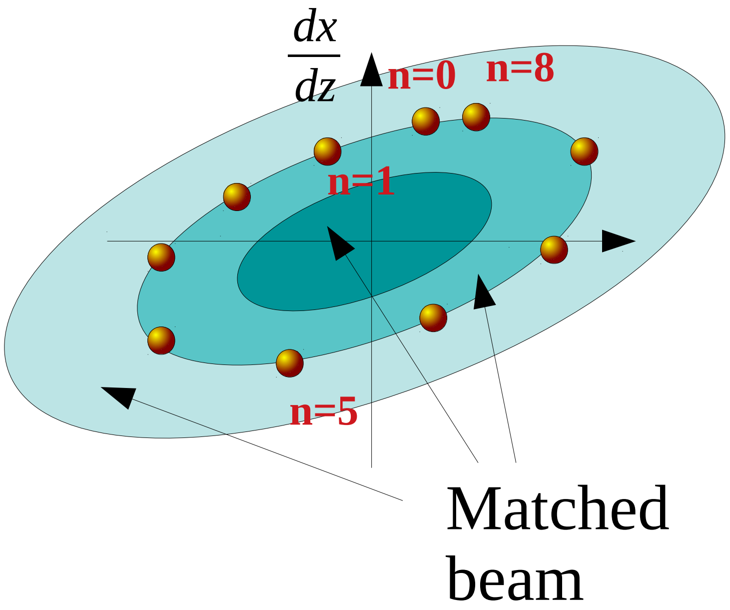


- Particles move around an ellipse in phase space if Trace(**M**) < 2
- μ is the "phase advance"
- α, β and γ are "Twiss parameters"
  - Tell us the alignment of the ellipse
- Each particle sits on ellipse area ε - the particle's amplitude

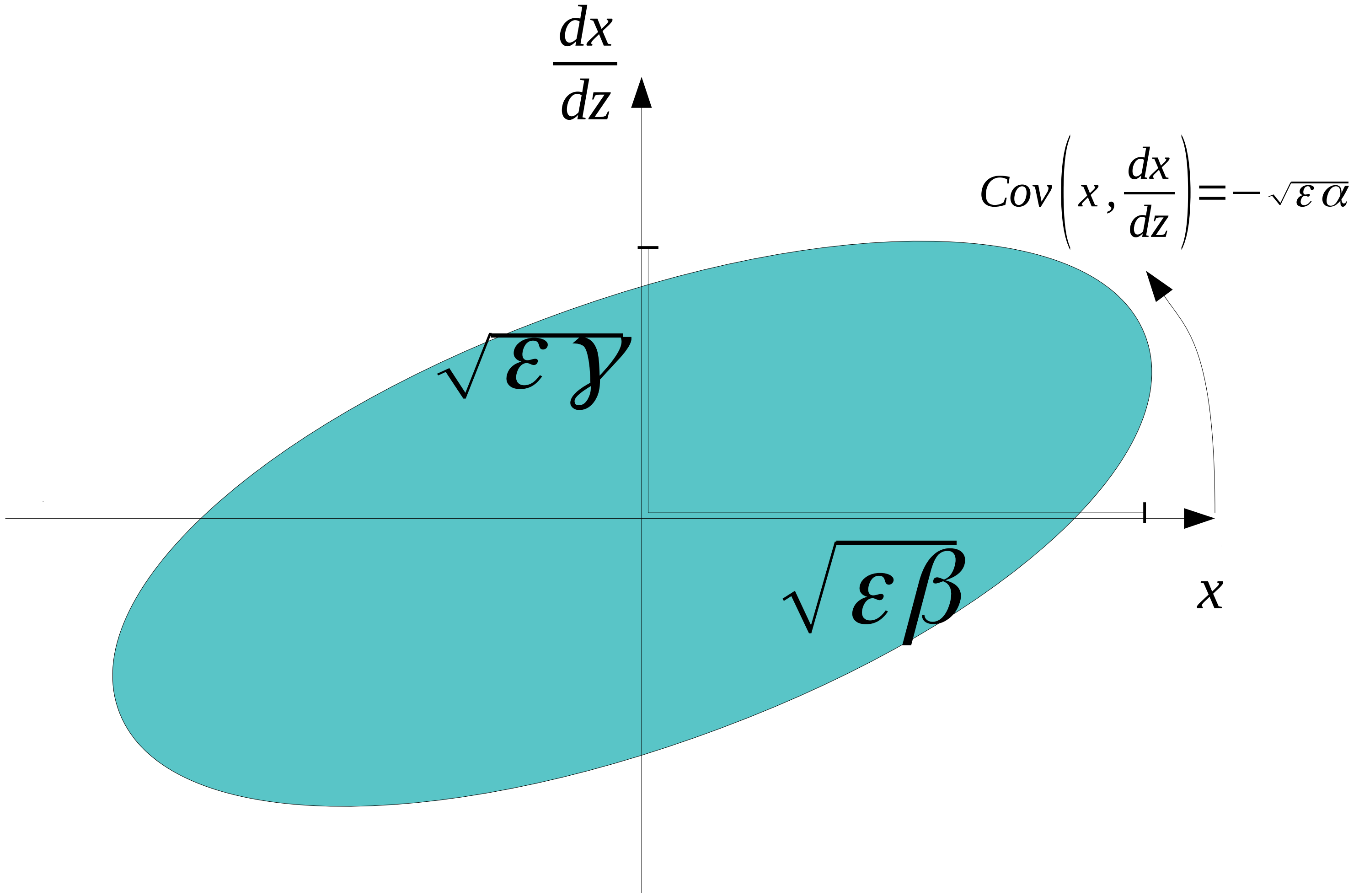
Periodic Lattices and beams



- Beam is composed of many particles
  - Particles occupy a region in phase space
- "Emittance" is area occupied by the entire beam
- Sometimes classify "RMS emittance"
  - Area occupied by ellipse 1 RMS distance from beam centre
- Low emittance is crucial for
  - High luminosity
  - Low losses



Beam ellipse



Science and Technology Facilities Council

Emittance Growth

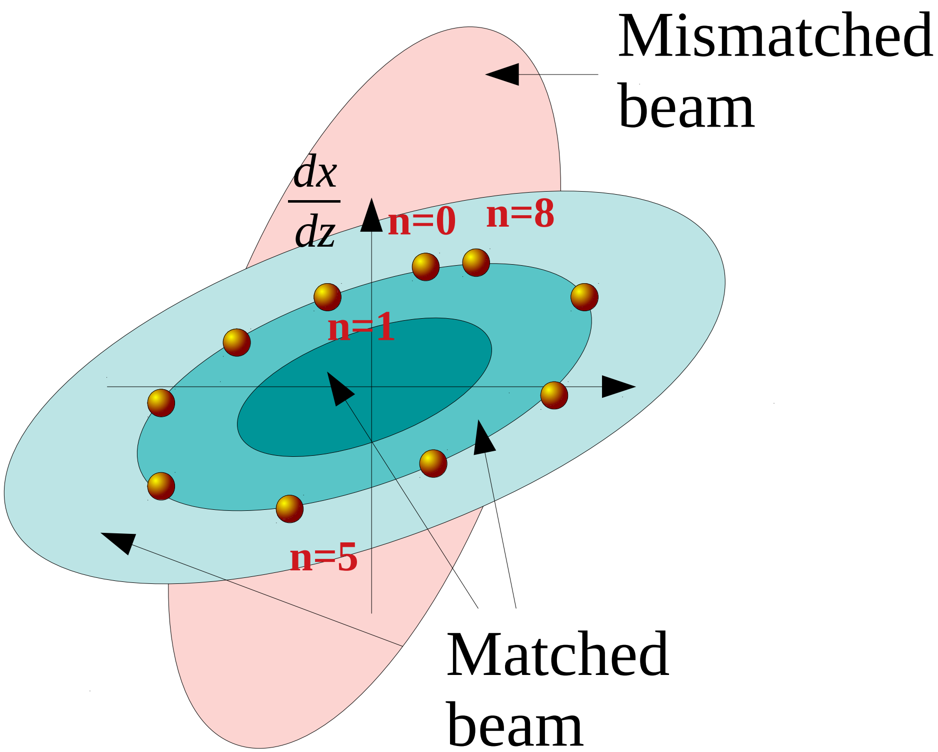
- Ideally emittance is conserved, but this is not always the case
- Long list of effects that can cause emittance growth
  - Beam mismatch
  - Scattering off residual gas
  - Scattering off particles in the same beam
  - Scattering off particles in other beams (e.g. in collider)
  - Space charge
  - Resonances:

$$\nu_x = \frac{\mu_x}{2\pi}$$

$$j\,\nu_x + k\,\nu_y = N$$

Science and Technology Facilities Council

Mismatch



Emittance Reduction (Cooling)

- Several techniques to reduce emittance
  - Synchrotron radiation cooling
  - Stochastic cooling
  - Laser cooling
  - Electron cooling
  - Ionisation cooling
- Fundamental principle is to remove “heat” from the beam using a neighbouring heat sink
  - Comoving electron beam → electron cooling
  - Comoving laser → laser cooling
  - Emission of synchrotron radiation
    - Photon emission caused by (principally) electrons bending in magnetic field

Questions

Science and Technology Facilities Council

Questions

- What is behaviour of particles in phase space if
  - Trace(M) < 2
  - Trace(M) = 2
  - Trace(M) > 2

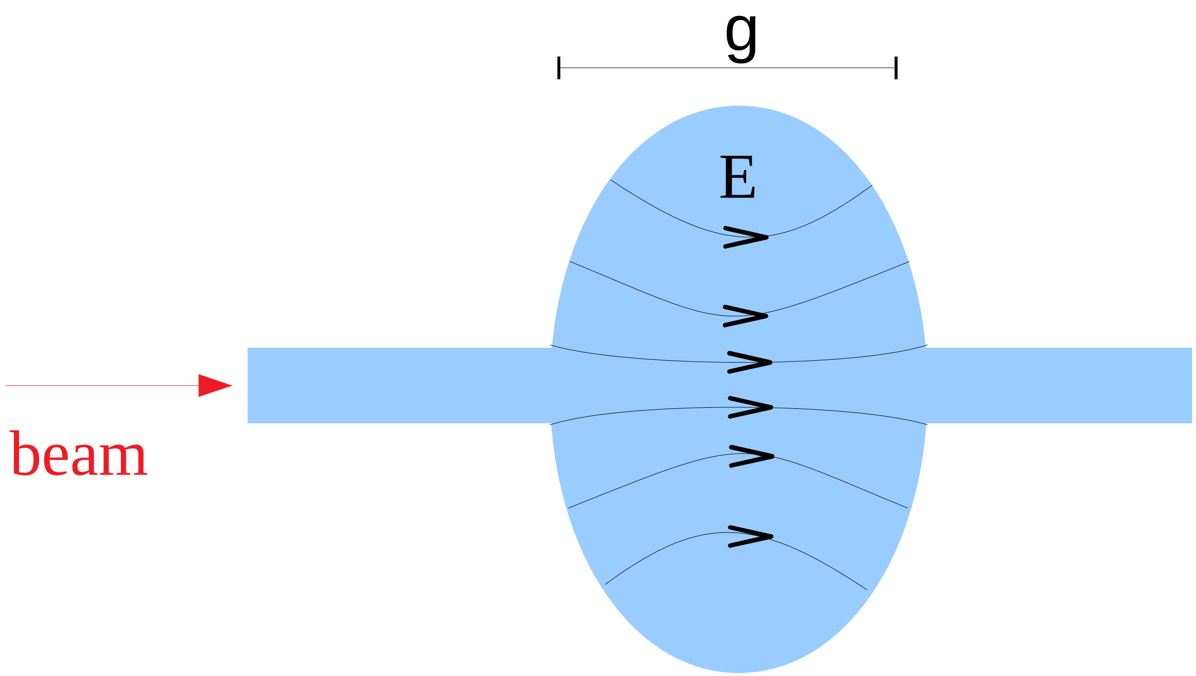
Questions

- What is behaviour of particles in phase space if
  - Trace(M) < 2
    - Motion is an ellipse
  - Trace(M) = 2
    - x → +/- x
  - Trace(M) > 2
    - Motion is a hyperbola

Longitudinal Dynamics

- So much for transverse motion (i.e. x and y planes)
- What about energy and acceleration?
- Electrostatic acceleration limited by breakdown potential
  - Change in energy is given by voltage differential
  - High voltage differentials cause breakdown (sparks)
  - Practically limits electrostatic acceleration to few MeV
- To accelerate beyond MeV require oscillating electric field
- RF Cavities

RF cavity field

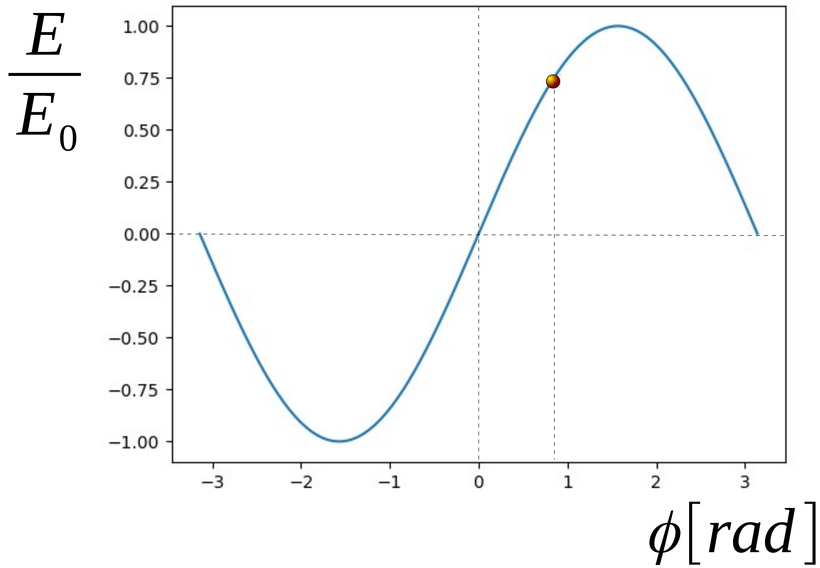


- RF cavity holds a resonating EM wave
- Recall Lorentz force law

$$\vec{F} = q\vec{v} \times \vec{B} + q\vec{E}$$

- Force is in direction of motion - energy changes!

RF cavity field



- In RF cavity

$$\vec{E} = E_0 \sin(\omega t + \phi)$$

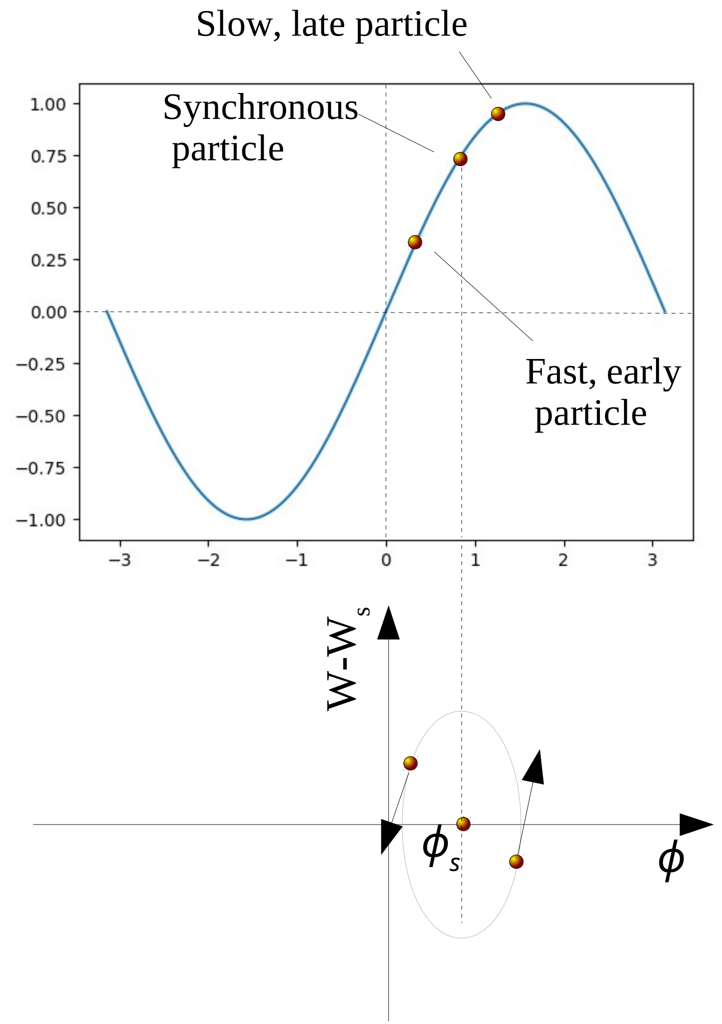
- Energy change of synchronous particle crossing at ϕ_s

$$\delta W = qTg\, E_0 \sin(\phi_s)$$

  - T is factor to allow for phase to vary a bit during crossing
  - g is the gap length

Phase stability

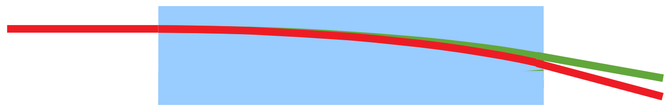
- Particle crossing at phase ϕ relative to synchronous particle
$$\delta W = q\,T\,g\,E_0 \sin(\phi + \phi_s)$$
- Particle arriving early
  - Fast
  - t negative
  - Gets smaller energy kick
  - Ends up relatively slower
- Particle arriving late
  - Slow
  - t positive
  - Gets bigger energy kick
  - Ends up relatively faster
- Phase stability!



Dealing with momentum spread

- Momentum spread introduces a few effects
  - Dispersion
  - Chromaticity
  - Momentum compaction
- Dispersion:
  - Off-momentum particles follow a different trajectory
- Momentum compaction (rings):
  - Different path length yields different time of flight
- Chromaticity:
  - Off-momentum particles get a different focussing strength

Dispersion



- Recall the definition of magnetic rigidity

$$B\rho = \frac{p}{q}$$

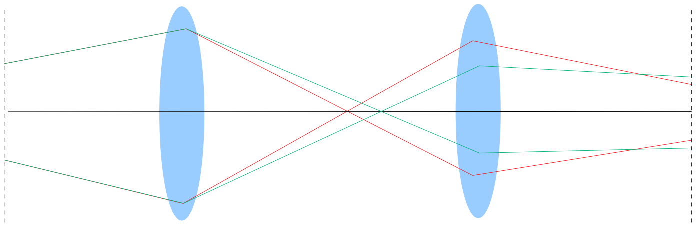
- Particles having different momentum (p) get different radius of curvature
  - Introduce dispersion *D*

$$D = p\frac{dx}{dp}$$

  - Which is another optical function that we must make periodic

Chromaticity

- Chromaticity arises because quadrupoles focus differently for different momenta



$$k = q\frac{b_0}{p}$$

- This often limits the degree of focussing at a collision point
  - Limits luminosity
- Can deliberately enhance/reduce chromaticity by
  - Introduce a dispersion
  - Using a magnet with variable focussing strength across the aperture - “sextupole”

Questions

Review

- Dipoles are used to bend a beam – rigidity is $B\rho = \frac{p}{q}$
- Quadrupoles are used to focus a beam: $k = q\frac{b_0}{p}$
- Beam in each of *x* and y can be characterised by 3 Twiss parameters and an emittance
- Lattices can be characterised by a phase advance
- RF cavities are used to accelerate the beam
- Introducing momentum spread, one can also define a dispersion (and its derivative with respect to z)

Finally… luminosity

- Luminosity defines the number of interactions in a collider per unit time for a given cross section
- Luminosity will increase if
  - Beam is narrower
  - Current is higher

Number of particles in each bunch

Revolution frequency

Number of bunches

$$\tilde{L} = \frac{\boxed{N_1 N_2} \boxed{f} \boxed{N_b}}{4\pi \boxed{\sigma_x \sigma_y}}$$

Width of Each bunch

Science and Technology Facilities Council

What dictates luminosity?

$$\tilde{L} = \frac{N_1 N_2 f N_b}{4\pi\sigma_x \sigma_y}$$

- Typically
  - Number of particles → space charge
  - Revolution frequency → ring circumference
  - Number of bunches → RF frequency
  - Beam width → $\sqrt{\varepsilon\beta}$
    - Emittance (cooling?)
    - Twiss beta (final focus and chromaticity)

Science and Technology Facilities Council

Next lecture...

- Accelerator equipment
- Types of accelerator
- Current facilities
- Future facilities