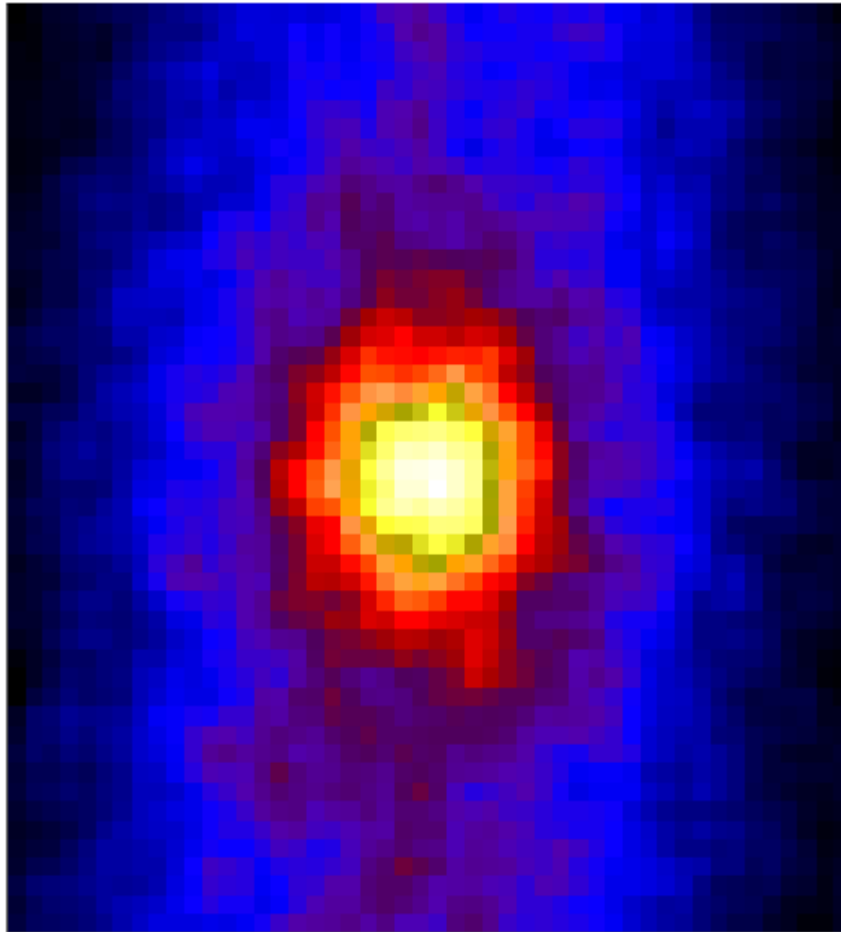


# Lecture 3

## *The neutrino oscillation industry*

# Solar Neutrinos

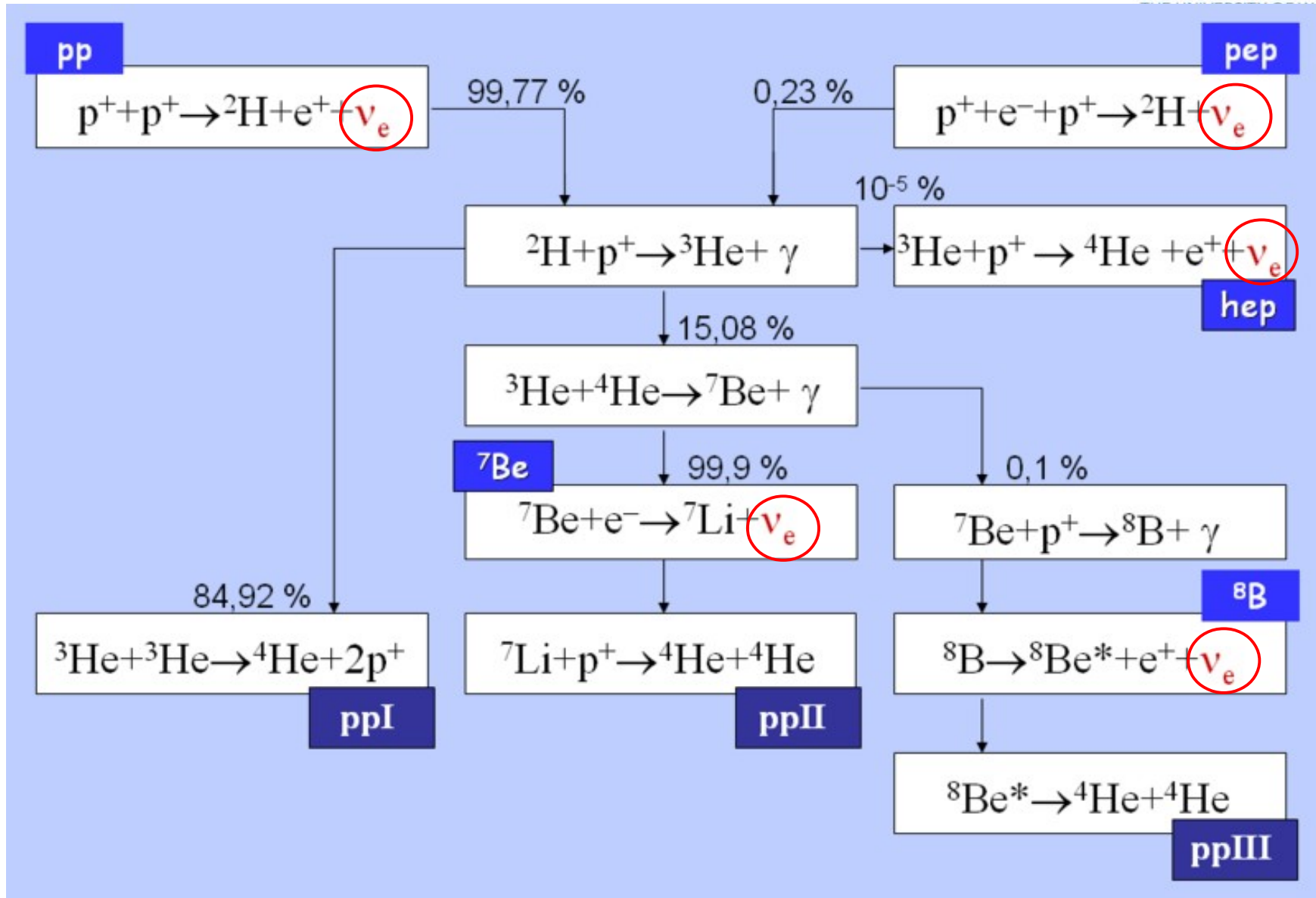
SuperK : Solar neutrino-gram



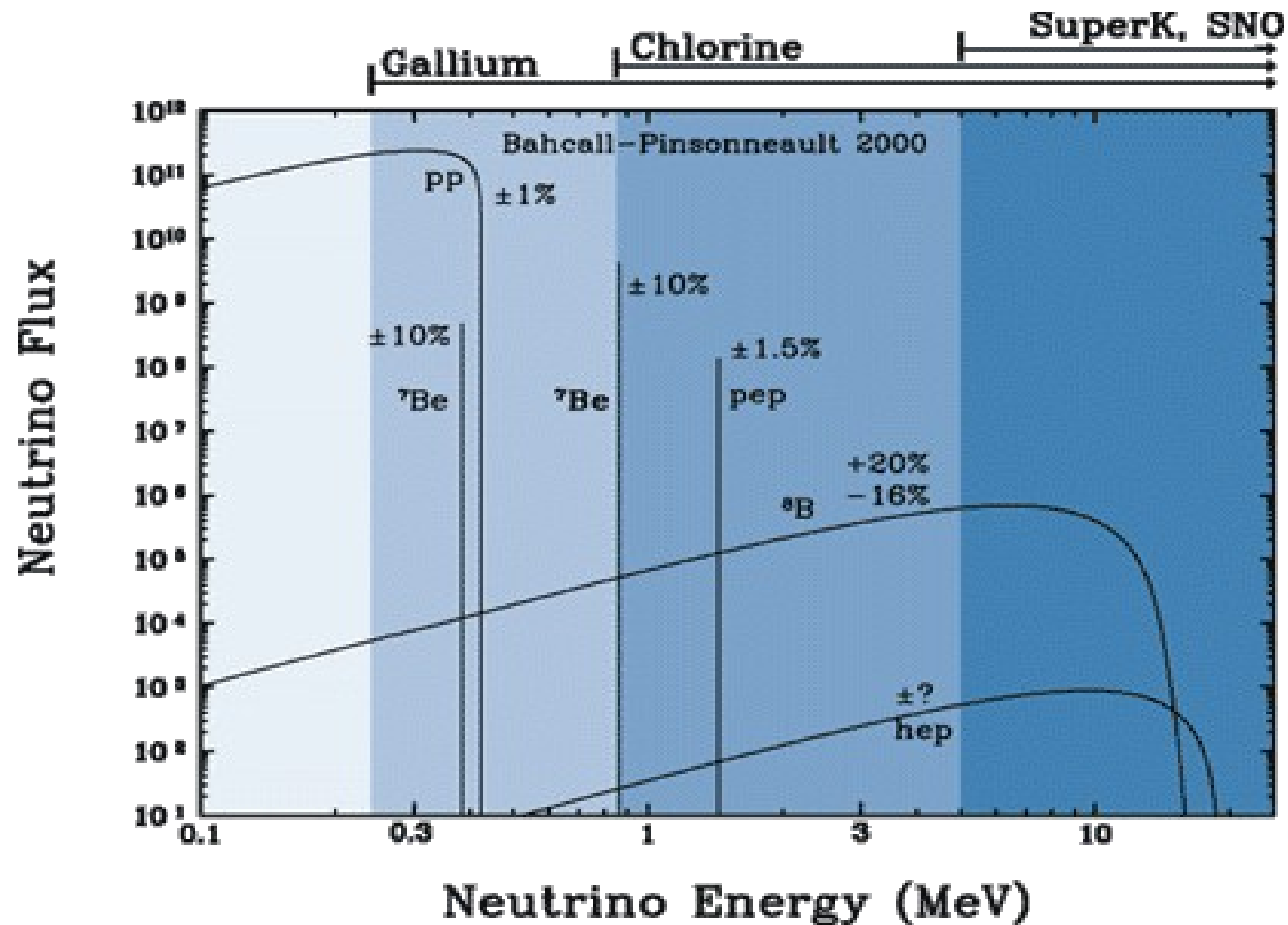
- Light from the solar core takes a million years to reach the surface
- Fusion processes generate electron neutrinos which take 2s to leave
- Solar neutrinos are a direct probe of the solar core
- Roughly  $4.0 \times 10^{10}$  solar  $n_e$  per  $\text{cm}^2$  per second on earth



# Solar neutrino - pp Cycle

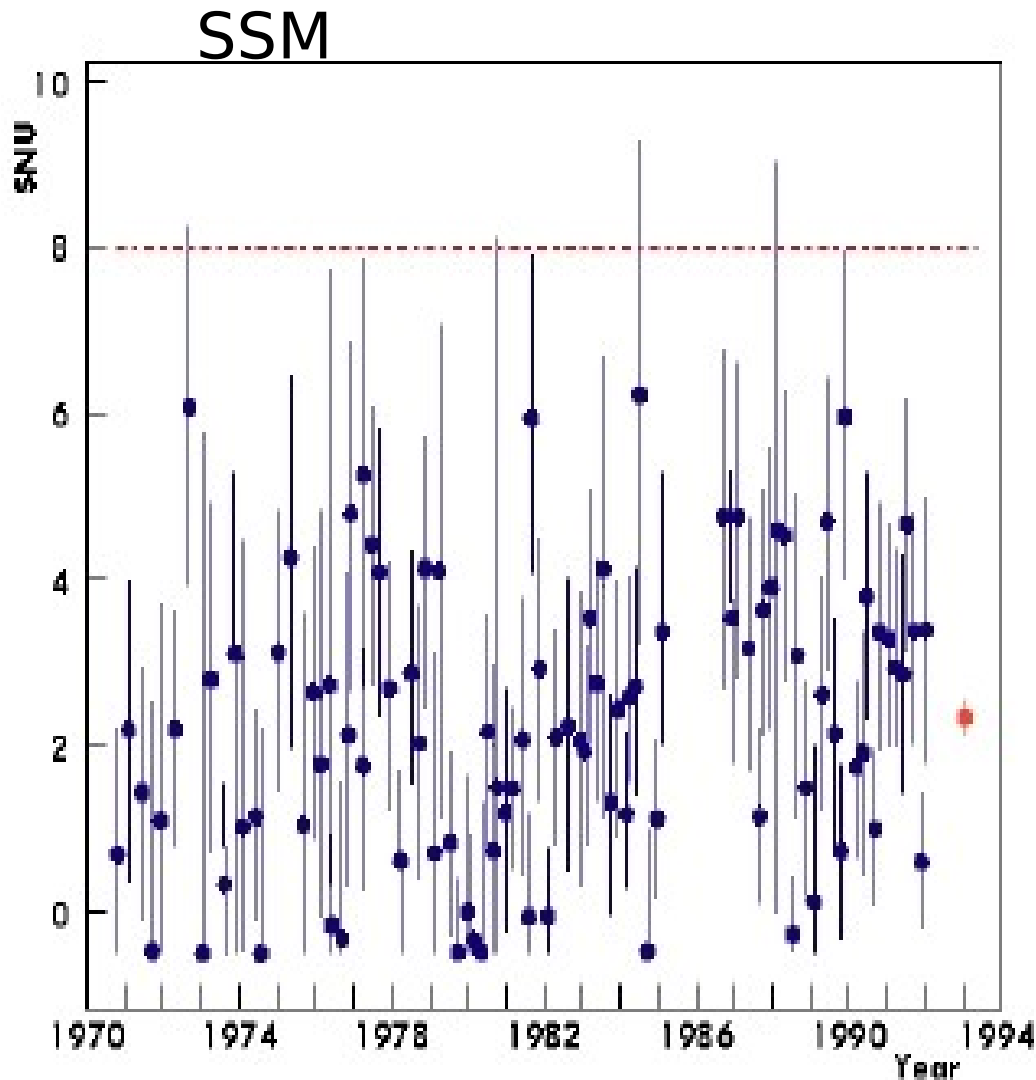


# Solar Neutrino Flux



As predicted by Bahcall's Solar model

# The Solar Neutrino Problem - Homestake



Homestake sensitive to  ${}^8\text{B}$  and  ${}^7\text{Be}$  *electron neutrinos*

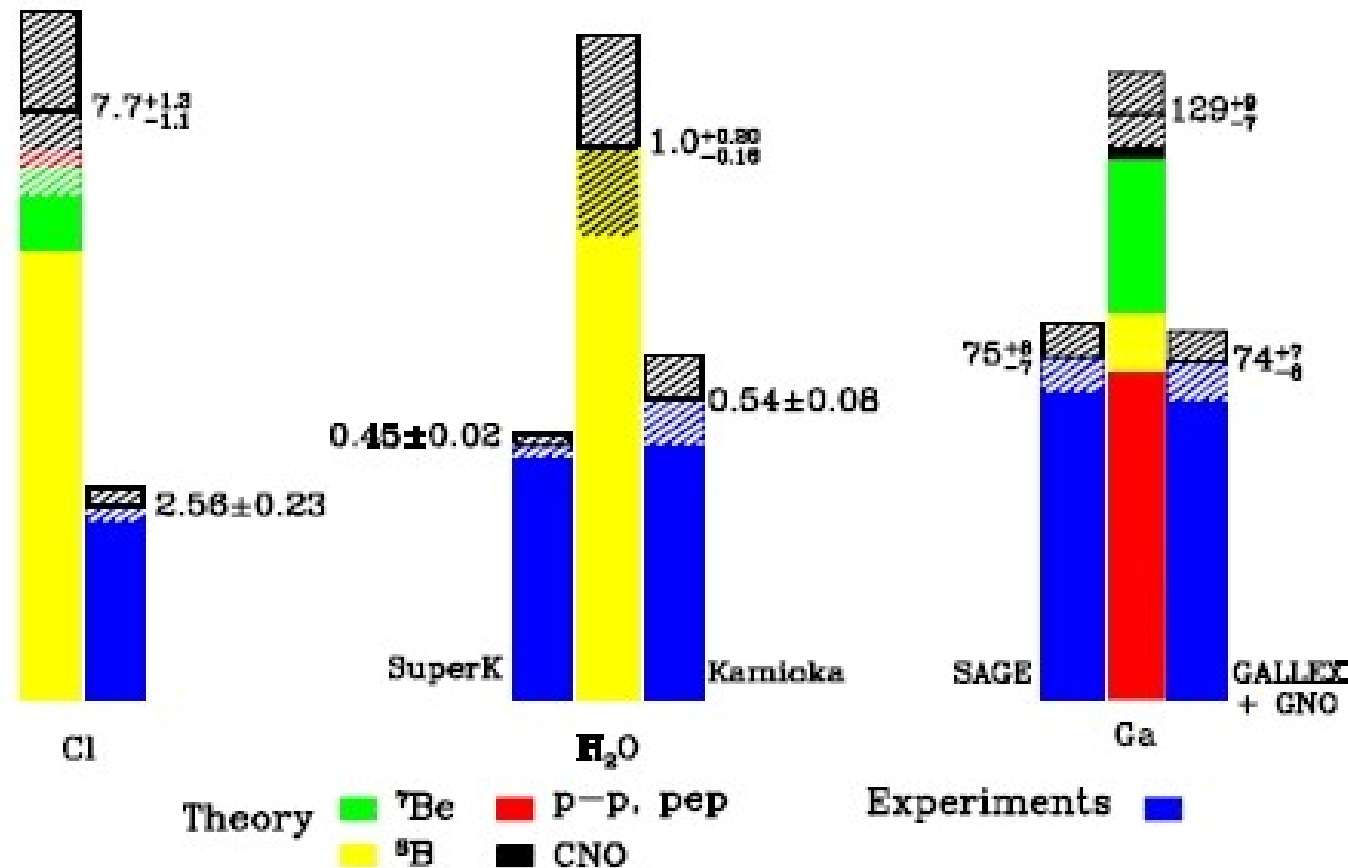
$$E_{\nu} > 800 \text{ keV}$$

Observe 1/3 of the expected number of solar neutrinos

1 SNU = 1 interaction per  $10^{36}$  atoms per second

# Experimental summary

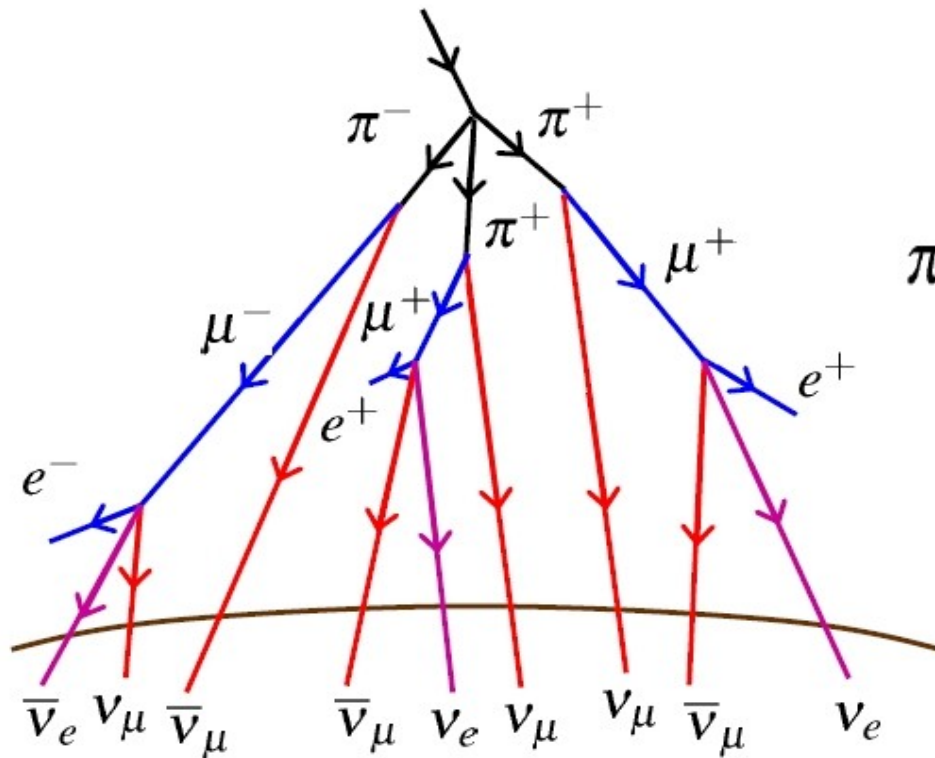
Total Rates: Standard Model vs. Experiment  
Bahcall–Pinsonneault 2000



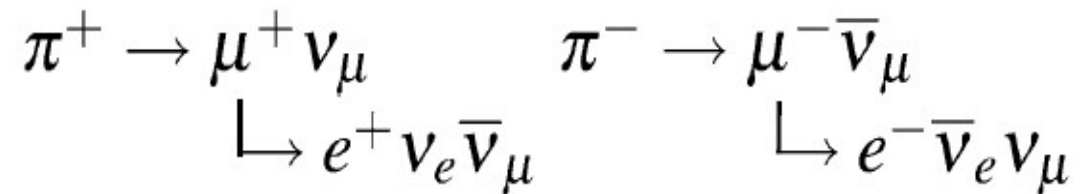


# Atmospheric neutrinos

High energy cosmic rays interact in the upper atmosphere producing showers of mesons (mostly pions)

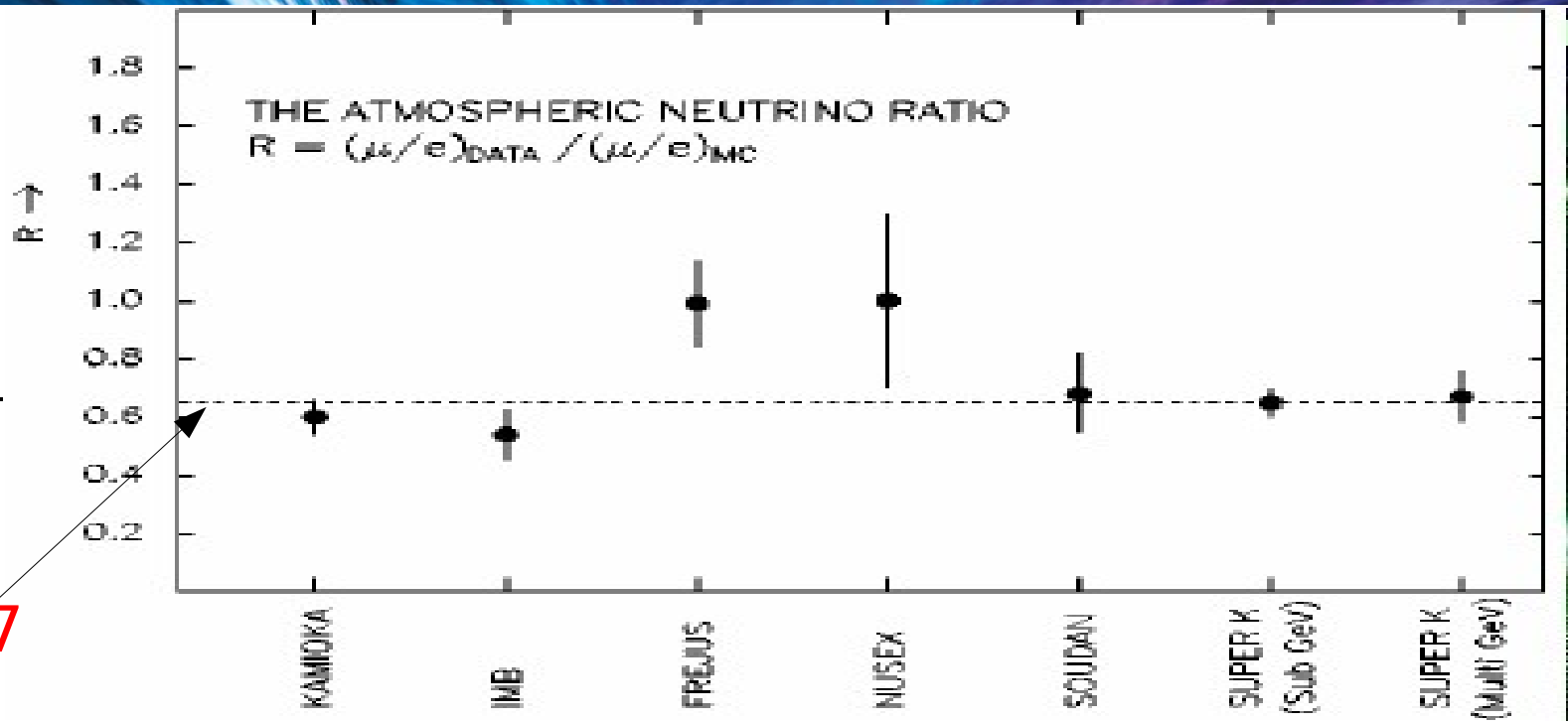


Neutrinos produced by



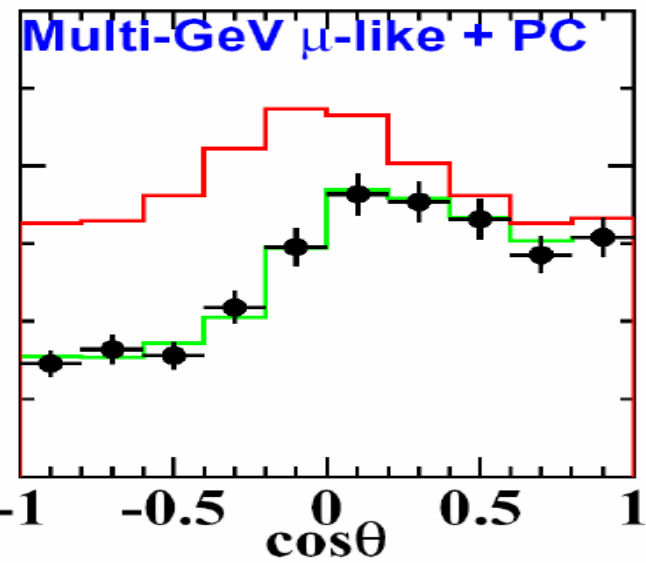
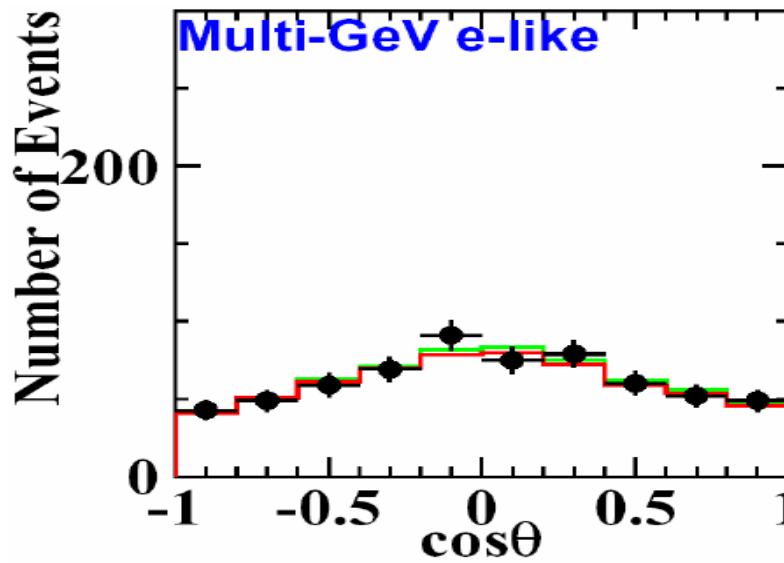
Expect 
$$\frac{N(\nu_\mu + \bar{\nu}_\mu)}{N(\nu_e + \bar{\nu}_e)} \approx 2$$

At higher energies, the muons can reach the ground before decaying so ratio increases



$$R = \frac{(\mu/e)_{Data}}{(\mu/e)_{MC}}$$

$R \sim 0.6 - 0.7$



The Atmospheric Neutrino Anomaly



# *Neutrino Flavour Oscillations*

# Mixing

CKM  
Mechanism

$$\begin{pmatrix} u \\ d' \end{pmatrix}_L, \begin{pmatrix} c \\ s' \end{pmatrix}_L$$
$$d' = d \cos \theta_c + s \sin \theta_c$$
$$s' = -d \sin \theta_c + s \cos \theta_c$$

In the quark sector, the flavour eigenstates (those states which couple to the W/Z) are not identical to the mass eigenstates (those states which are solutions of the Dirac equation)

**Weak states**  $\longrightarrow$   $\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} 0.97 & 0.23 & 0.003 \\ 0.23 & 0.97 & 0.04 \\ 0.008 & 0.04 & 0.99 \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix} \longleftarrow$  **Mass states**

# Mixing

CKM  
Mechanism

$$\begin{pmatrix} u \\ d' \end{pmatrix}_L, \begin{pmatrix} c \\ s' \end{pmatrix}_L$$
$$d' = d \cos \theta_c + s \sin \theta_c$$
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In the quark sector, the flavour eigenstates (those states which couple to the W/Z) are not identical to the mass eigenstates (those states which are solutions of the Dirac equation)

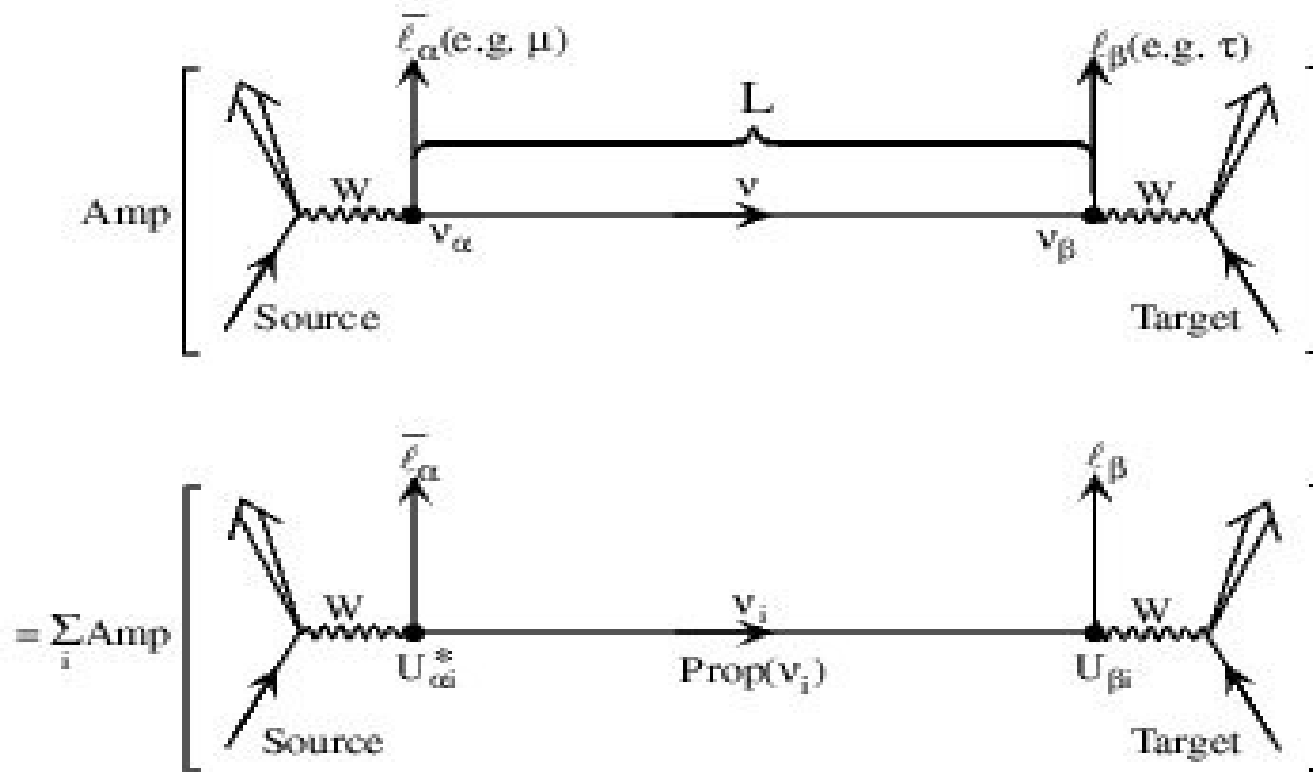
Weak states  $\longrightarrow$

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

← Mass states

Unitary mixing matrix  $\longleftarrow$

# Neutrino Oscillations



$$Prob(\nu_\alpha \rightarrow \nu_\beta) \propto \left| \sum_i U_{\alpha i}^* \text{Prop}(\nu_i) U_{\beta i} \right|^2$$

If we don't know which mass state was created then the the amplitude involves a coherent superposition of  $\nu_i$  states

$$\begin{aligned}
 \text{Prob}(\nu_\alpha \rightarrow \nu_\beta) = & \delta_{\alpha\beta} - 4 \sum_{i>j} \Re(U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^*) \sin^2\left(\Delta m_{ij}^2 \frac{L}{4E}\right) \\
 & + 2 \sum_{i>j} \Im(U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^*) \sin\left(\Delta m_{ij}^2 \frac{L}{2E}\right)
 \end{aligned}$$

- ▶ If  $\Delta m_{ij}^2 = 0$  then neutrinos don't oscillate
- ▶ Oscillation depends on  $|\Delta m^2|$  - absolute masses cannot be determined
- ▶ If there is no mixing (If  $U_{ai} = 0$ ) neutrinos don't oscillate
- ▶ One can detect flavour change in 2 ways : start with  $\nu_\alpha$  and look for  $\nu_\beta$  (appearance) or start with  $\nu_\alpha$  and see if any disappears (disappearance)
- ▶ Flavour change oscillates with  $L/E$ .  $L$  and  $E$  are chosen by the experimenter to maximise sensitivity to a given  $\Delta m^2$
- ▶ Flavour change doesn't alter total neutrino flux – it just redistributes it amongst different flavours (unitarity)

# Two flavour oscillations

$$\begin{pmatrix} \nu_\alpha \\ \nu_\beta \end{pmatrix} = U \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} \Rightarrow U = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}$$

$$P(\nu_\alpha \rightarrow \nu_\beta) = \delta_{\alpha\beta} - 4 \sum_{i>j} U_{\alpha i} U_{\beta i} U_{\alpha j} U_{\beta j} \sin^2 \left( \Delta m_{ij}^2 \frac{L}{4E} \right)$$

$P(\nu_\alpha \rightarrow \nu_\beta)$  : Appearance Probability

$P(\nu_\alpha \rightarrow \nu_\alpha)$  : Survival Probability

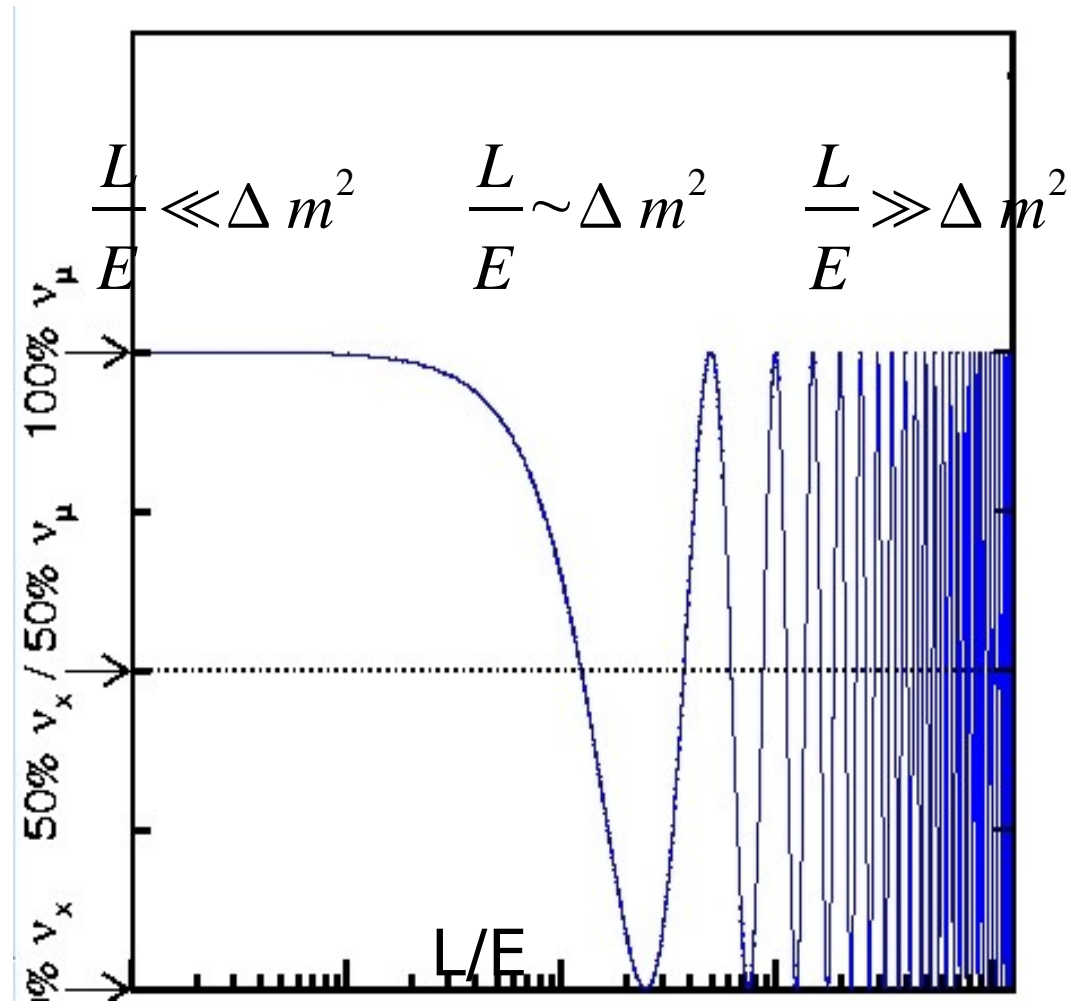
$$P(\nu_\alpha \rightarrow \nu_\beta) = -4 (U_{\alpha 1} U_{\beta 1} U_{\alpha 2} U_{\beta 2}) \sin^2 \left( \Delta m_{ij}^2 \frac{L}{4E} \right)$$

$$= \sin^2(2\theta) \sin^2 \left( 1.27 \Delta m^2 (\text{eV}^2) \frac{L(\text{km})}{E(\text{GeV})} \right)$$

(changing to useful units)



Survival Probability



$$P(\nu_\alpha(0) \rightarrow \nu_\alpha(x)) = 1 - \sin^2(2\theta) \sin^2\left(1.27 \Delta m^2 \frac{(L/\text{km})}{(E/\text{GeV})}\right)$$

Question : Would you see any effect if you were able to know what mass state propagated from source to detector?

# Three Flavour Oscillation

The three flavour case is more complicated, but no different

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} \Leftrightarrow U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}$$

U is the Pontecorvo-Maskawa-Nakayama-Sakata (PMNS) matrix

$$\begin{aligned} \text{Prob}(\nu_\alpha \rightarrow \nu_\beta) = & \delta_{\alpha\beta} - 4 \sum_{i>j} \Re(U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^*) \sin^2\left(\Delta m_{ij}^2 \frac{L}{4E}\right) \\ & + 2 \sum_{i>j} \Im(U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^*) \sin\left(\Delta m_{ij}^2 \frac{L}{2E}\right) \end{aligned}$$

# Oscillation parameters

$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha} & 0 \\ 0 & 0 & e^{i\beta} \end{pmatrix}$$

2 independent  $\Delta m^2$

$$\begin{aligned} \text{Prob}(v_\alpha \rightarrow v_\beta) = & \delta_{\alpha\beta} - 4 \sum_{i>j} \Re(U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^*) \sin^2\left(\Delta m_{ij}^2 \frac{L}{4E}\right) \\ & + 2 \sum_{i>j} \Im(U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^*) \sin\left(\Delta m_{ij}^2 \frac{L}{2E}\right) \end{aligned}$$

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$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} \\ U_{\tau1} & U_{\tau2} & U_{\tau3} \end{pmatrix} = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha} & 0 \\ 0 & 0 & e^{i\beta} \end{pmatrix}$$

Three angles

$$\begin{aligned} \text{Prob}(v_\alpha \rightarrow v_\beta) = & \delta_{\alpha\beta} - 4 \sum_{i>j} \Re(U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^*) \sin^2\left(\Delta m_{ij}^2 \frac{L}{4E}\right) \\ & + 2 \sum_{i>j} \Im(U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^*) \sin\left(\Delta m_{ij}^2 \frac{L}{2E}\right) \end{aligned}$$

# Oscillation parameters

$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} \\ U_{\tau1} & U_{\tau2} & U_{\tau3} \end{pmatrix} = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} e^{i\delta} \\ 0 & 1 & 0 \\ -s_{13} e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha} & 0 \\ 0 & 0 & e^{i\beta} \end{pmatrix}$$

CP violating phase

$$\begin{aligned} \text{Prob}(\nu_{\alpha} \rightarrow \nu_{\beta}) = & \delta_{\alpha\beta} - 4 \sum_{i>j} \Re(U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^*) \sin^2(\Delta m_{ij}^2 \frac{L}{4E}) \\ & + 2 \sum_{i>j} \Im(U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^*) \sin(\Delta m_{ij}^2 \frac{L}{2E}) \end{aligned}$$



# Oscillation parameters

$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} \\ U_{\tau1} & U_{\tau2} & U_{\tau3} \end{pmatrix} = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha} & 0 \\ 0 & 0 & e^{i\beta} \end{pmatrix}$$

Extra Majorana phases

The extra Majorana matrix does not affect flavour oscillation processes....so is usually dropped. However it will affect the interpretation of neutrinoless double beta decay results

# Explaining the solar data

# Testing the oscillation hypothesis

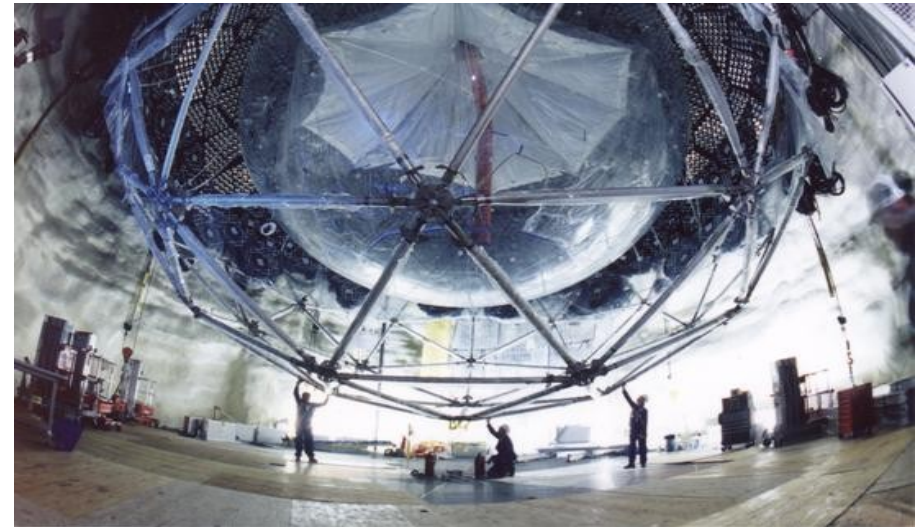
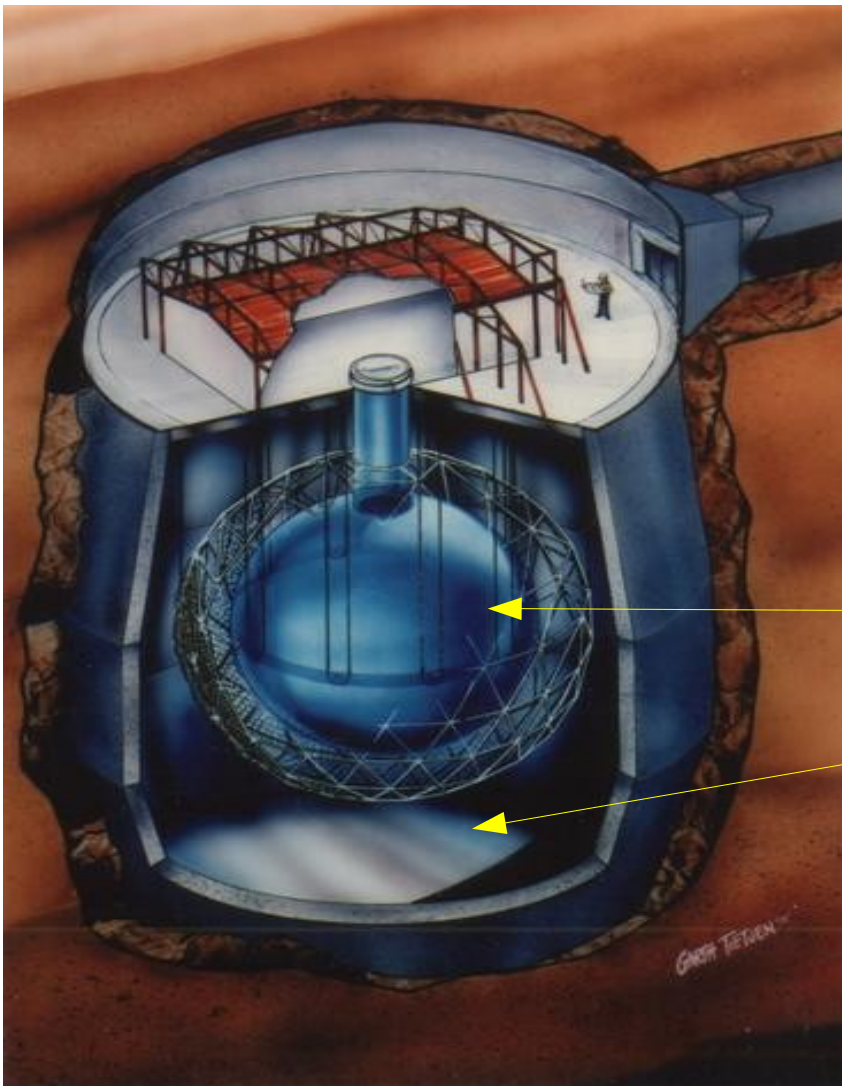
## Solar neutrino problem

$\nu_e$  from sun would change to  $\nu_\mu$  or  $\nu_\tau$ . However these have too little energy to interact via the charged current, and all the detectors are only sensitive to charge current interactions.

Non- $\nu_e$  component would effectively disappear, reducing the apparent  $\nu_e$  flux.

**Proof : Neutral current event rate shouldn't change.**

# Sudbury Neutrino Observatory



1000 tonnes of  $D_2O$

6500 tons of  $H_2O$

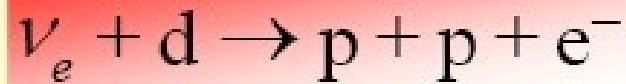
Viewed by 10,000 PMTS

In a salt mine 2km underground  
in Sudbury, Canada



# SNO

CC

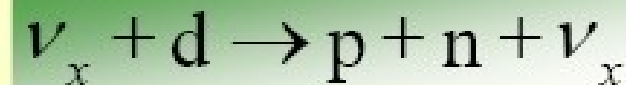


- $Q = 1.445 \text{ MeV}$
- good measurement of  $\nu_e$  energy spectrum
- some directional info  $\propto (1 - 1/3 \cos\theta)$
- $\nu_e$  only

Produces Cherenkov  
Light Cone in  $D_2O$

$$n_e$$

NC



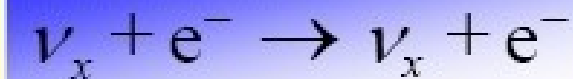
- $Q = 2.22 \text{ MeV}$
- measures total  $^8B \nu$  flux from the Sun
- equal cross section for all  $\nu$  types

n captures on deuteron  
 $^2H(n, \gamma)^3H$

Observe  $6.25 \text{ MeV } \gamma$

$$n_e + n_m + n_t$$

ES

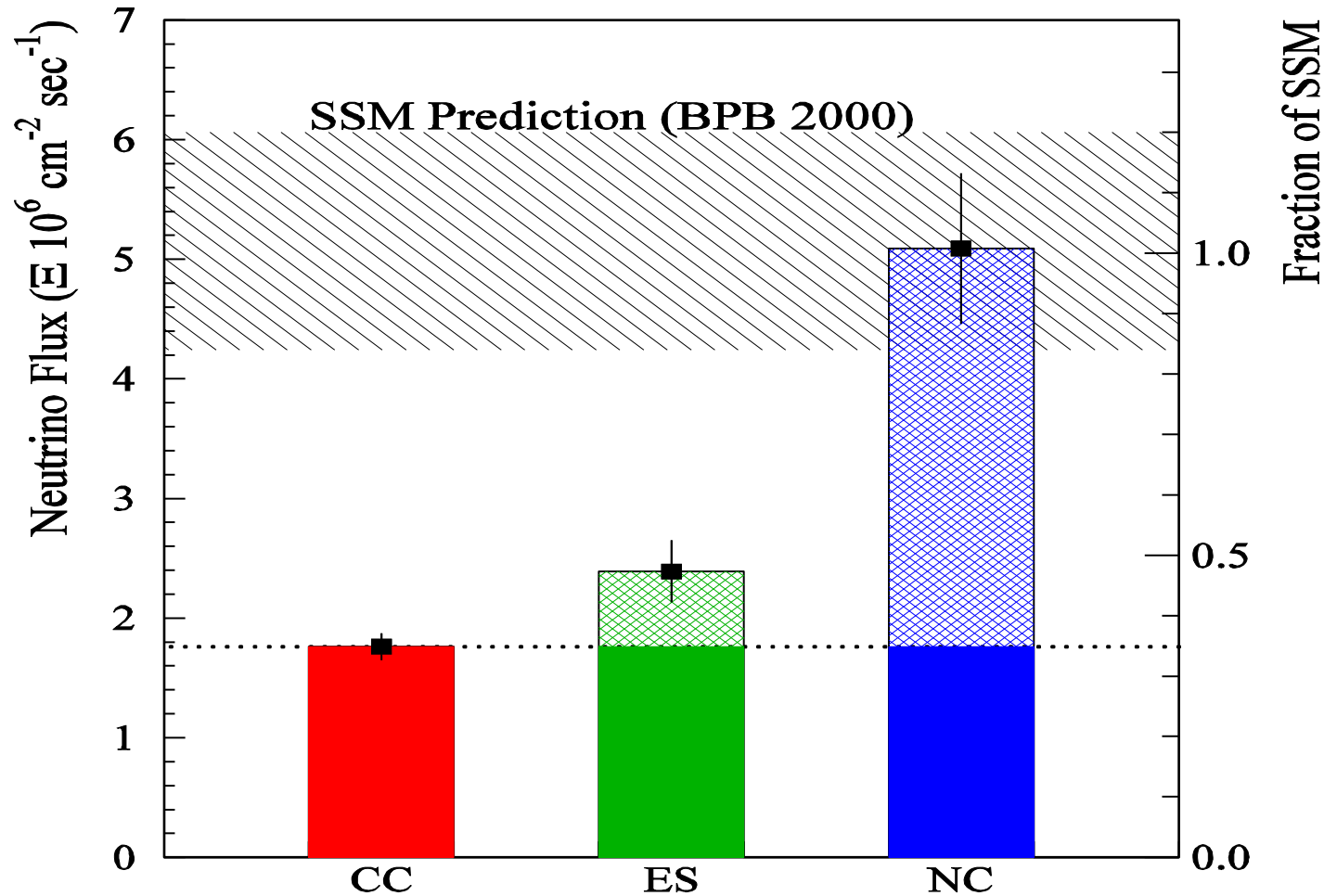


- low statistics
- mainly sensitive to  $\nu_e$ , some  $\nu_\mu$  and  $\nu_\tau$
- strong directional sensitivity

Produces Cherenkov  
Light Cone in  $D_2O$

$$n_e + 0.15*(n_m + n_t)$$

# SNO Results



5.3  $\sigma$  appearance of  $\nu_{\mu\tau}$  in a  $\nu_e$  beam  
Roughly 70% of  $\nu_e$  oscillates away



# Naively...

First instinct is to assume that neutrinos leave the sun as  $\nu_e$  and oscillate on their way to the earth. Assuming this

$$L \sim 10^8 \text{ km}, E_\nu < 10 \text{ MeV} \rightarrow \Delta m^2 \sim 3 \times 10^{-10} \text{ eV}^2$$

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$$L \sim 10^8 \text{ km}, E_\nu < 10 \text{ MeV} \rightarrow \Delta m^2 \sim 7 \times 10^{-5} \text{ eV}^2$$

Oscillations come from phase difference between mass states. In a vacuum the phase diff comes from free particle Hamiltonian. In a material there are interaction potentials as well

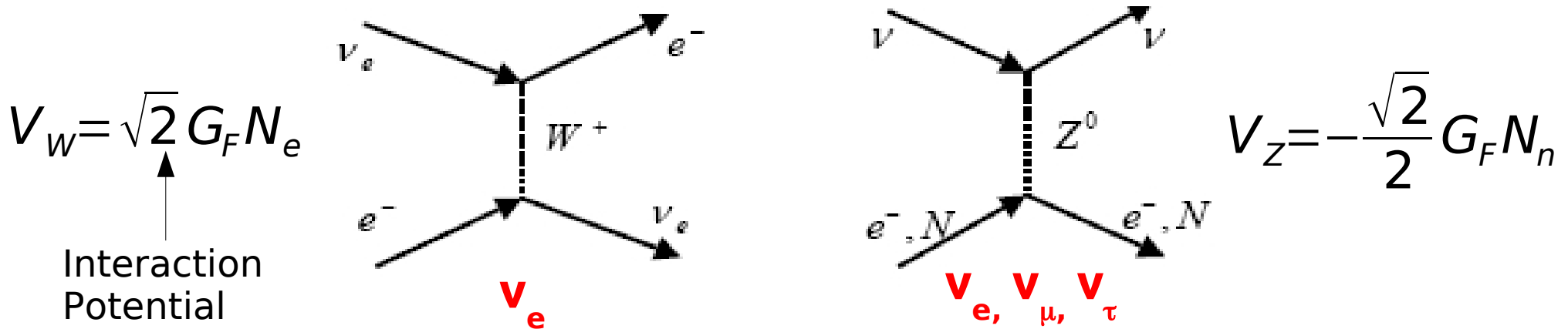
$$-i\hbar \frac{\partial \psi}{\partial t} = E \psi = \frac{-\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} \rightarrow -i\hbar \frac{\partial \psi}{\partial t} = (E + V) \psi = \frac{-\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2}$$

$$E^2 - p^2 = m_{\text{vac}}^2 \rightarrow (E + V)^2 - p^2 = m_{\text{mat}}^2 \rightarrow m_{\text{mat}} \approx \sqrt{m_{\text{vac}}^2 + 2EV}$$

c.f. effective mass of an electron in a semiconductor or light in glass

# Oscillations in Matter

Electrons exist in standard matter –  $\mu/\tau$  do not. Electron neutrinos travelling in matter can experience an extra charged current interaction that other flavours cannot.



$$P(\nu_e \rightarrow \nu_e) = 1 - \sin^2(2\theta_M) \sin^2\left(\frac{\Delta m_M^2 L}{4E}\right)$$

Oscillation probability modified by matter effects

$$\Delta m_M^2 = \Delta m_V^2 \sqrt{\sin^2(2\theta) + (\cos 2\theta - \zeta)^2}$$

$$\sin^2 2\theta_M = \frac{\sin^2 2\theta}{\sin^2 2\theta + (\cos 2\theta - \zeta)^2}$$

$$\zeta = \frac{2\sqrt{2} G_F N_e E}{\Delta m_V^2}$$

# Implications

$$\sin^2 2\theta_M = \frac{\sin^2 2\theta}{\sin^2 2\theta + (\cos 2\theta - \zeta)^2} \quad \zeta = \frac{2\sqrt{2}G_F N_e E}{\Delta m_{Vac}^2}$$

- If  $\Delta m_{Vac}^2 = 0$  or matter is very dense,  $\zeta = \infty$  and  $\theta_m = 0$
- Similarly, if  $\theta_{vac} = 0$ , then  $\theta_M = 0 \Rightarrow$  need mixing in vacuum
- If there is no matter, then  $\zeta = 0$  and we have vacuum mixing
- At a particular electron density, dependent on  $\Delta m^2$ ,

$$\zeta = \frac{2\sqrt{2}G_F N_e E}{\Delta m^2} = \cos 2\theta \Rightarrow \sin^2 2\theta_M = 1$$

Even if the vacuum mixing angle is tiny, there is a density for which the matter mixing angle is maximal

# Mass hierarchy

$$\sin^2 2\theta_M = \frac{\sin^2 2\theta}{\sin^2 2\theta + (\cos 2\theta - \zeta)^2} \quad \zeta = \frac{2\sqrt{2}G_F N_e E}{\Delta m_V^2}$$

If mass of  $\nu_1 <$  mass of  $\nu_2$ ,  $\Delta m_V^2 = m_1^2 - m_2^2 < 0$

$$\zeta = -\frac{2\sqrt{2}G_F N_e E}{|\Delta m^2|} \rightarrow \sin^2 2\theta_M = \frac{\sin^2 2\theta}{\sin^2 2\theta + (\cos 2\theta + |\zeta|)^2}$$

Positive definite - no resonance

If mass of  $\nu_1 >$  mass of  $\nu_2$ ,  $\Delta m^2 = m_1^2 - m_2^2 > 0$

$$\zeta = \frac{2\sqrt{2}G_F N_e E}{|\Delta m^2|} \rightarrow \sin^2 2\theta_M = \frac{\sin^2 2\theta}{\sin^2 2\theta + (\cos 2\theta - |\zeta|)^2}$$



# Mass hierarchy

$$\sin^2 2\theta_M = \frac{\sin^2 2\theta}{\sin^2 2\theta + (\cos 2\theta - \xi)^2} \quad \xi = \pm \frac{2\sqrt{2} G_F N_e E}{|\Delta m_\nu^2|}$$

The effect of matter on neutrino oscillations can be used to measure the mass hierarchy.

This is about the only way we know how to do this.

# Mixing matrix

$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} \\ U_{\tau1} & U_{\tau2} & U_{\tau3} \end{pmatrix} = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}$$

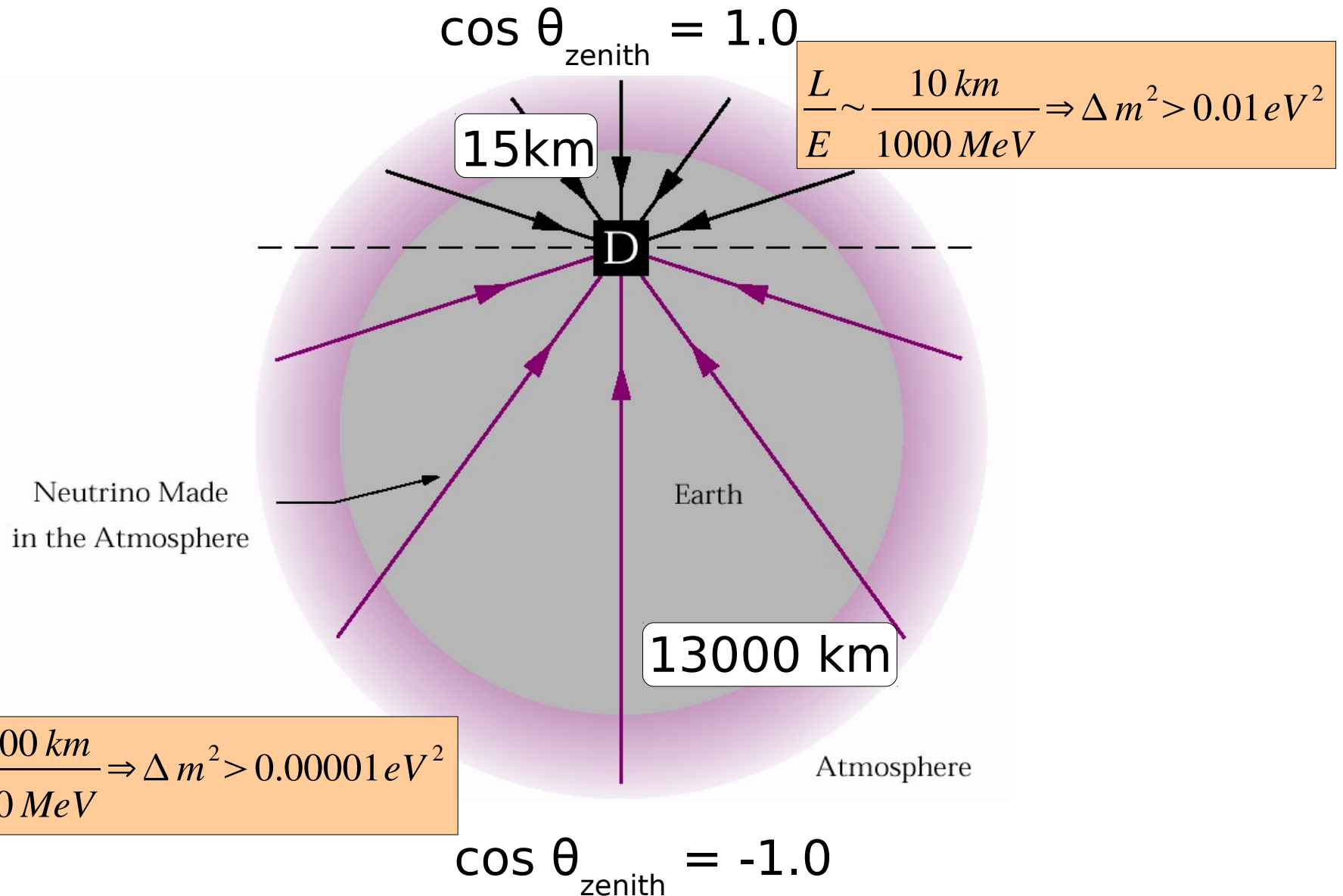
Solar sector

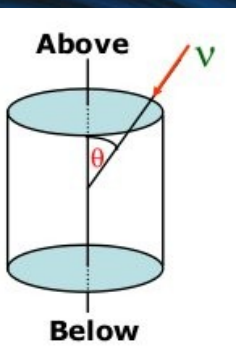
$$\theta_{e\mu} = 32.5^\circ \pm 2.4^\circ$$

$$\Delta m_{12}^2 = +7.9 \times 10^{-5} eV^2$$

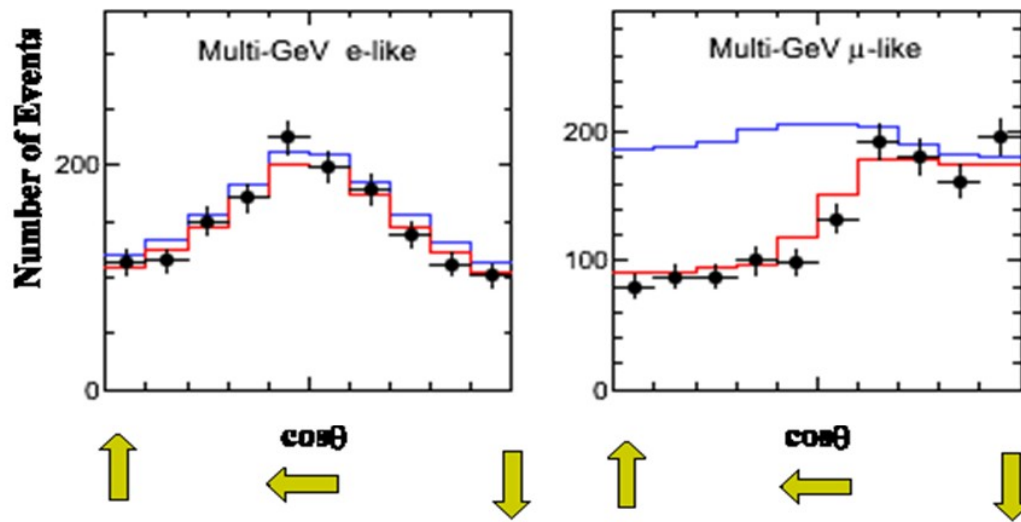
# Explaining the atmospheric data

# Cosmic Labs

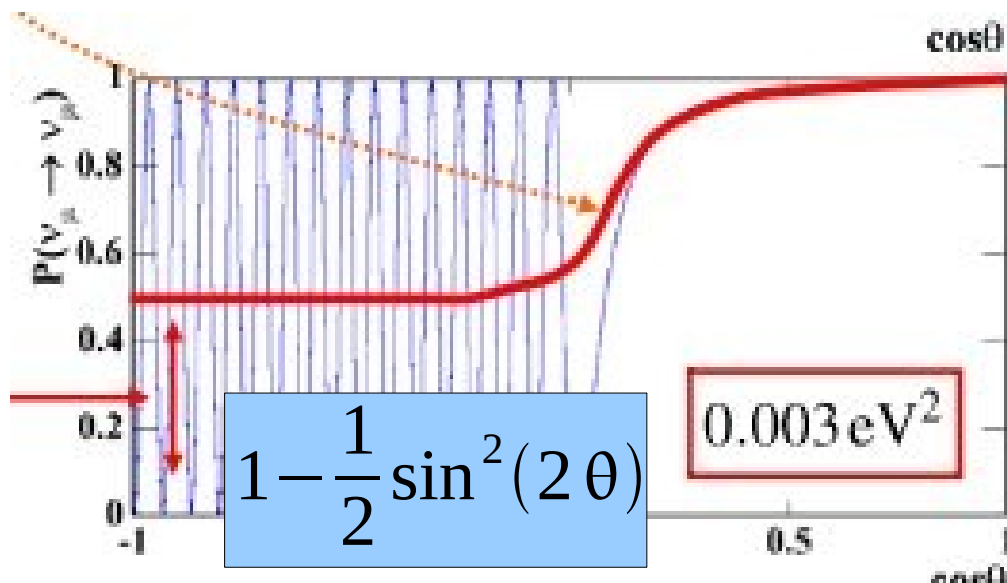




# Atmospheric results



- Prediction for  $\nu_e$  rate agrees with data.
- $\nu_\mu$  disappear at large baseline consistent with  $\nu_\mu \rightarrow \nu_\tau$
- Don't detect  $\nu_\tau$  as
  - below  $\tau$  mass threshold
  - SuperK is awful at  $\tau$  detection



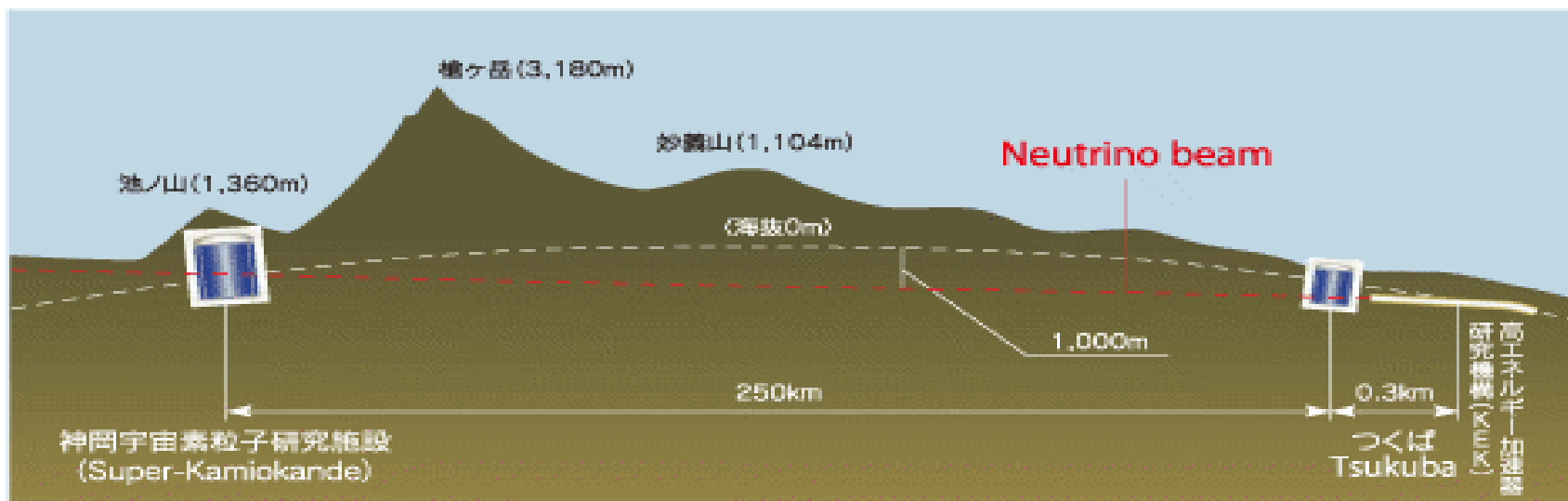
$$|\Delta m_{atmos}^2| \approx 0.0025 eV^2$$

$$\sin^2(2\theta_{atmos}) \approx 1.0$$

# Accelerator Cross-check

$$\Delta m_{atmos}^2 \approx 3 \times 10^{-3} eV^2 \rightarrow L/E \approx 400 km GeV^{-1}$$

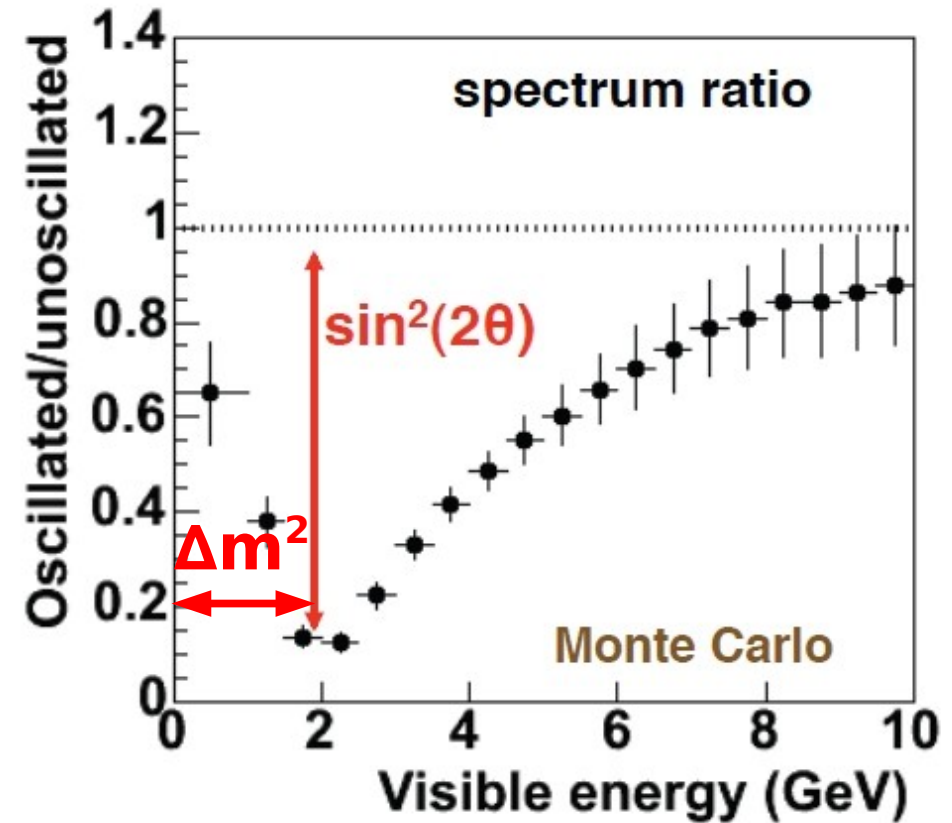
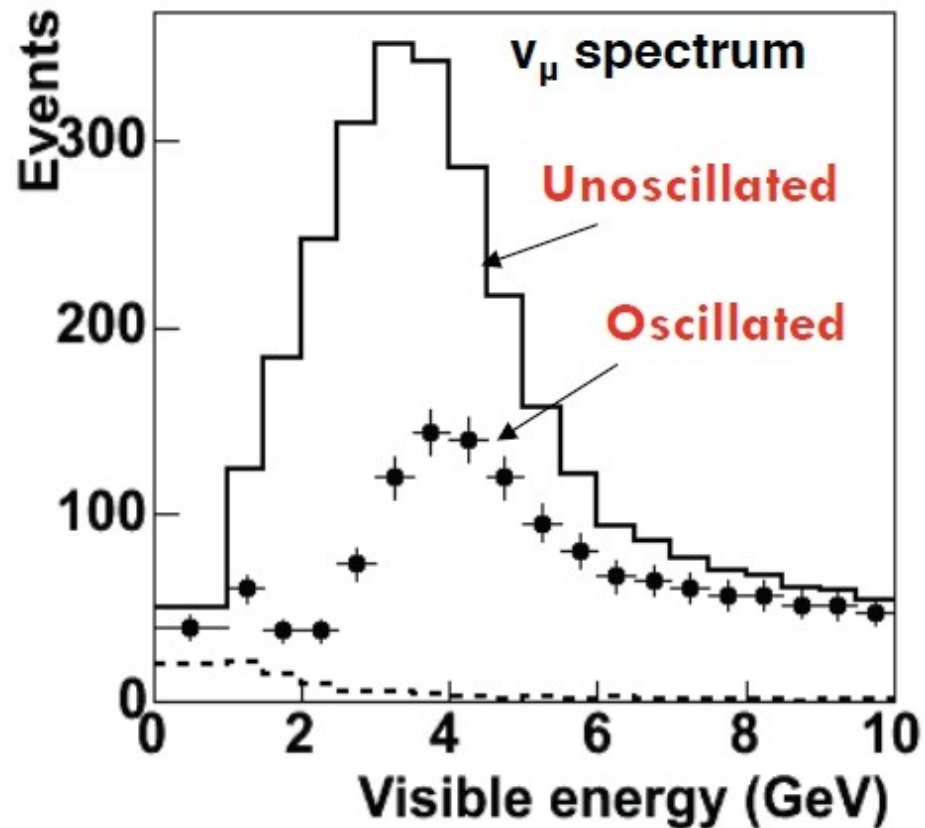
$$L = 250 km \rightarrow E_\nu \approx 0.6 GeV$$



Beam events tagged using GPS at both near and far detector sites



# Disappearance Experiments



$$P(\nu_\alpha \rightarrow \nu_\alpha) \rightarrow \frac{\Phi_\nu(@FD)}{\Phi_\nu(@ND)}$$

$F_n$  : Neutrino Flux

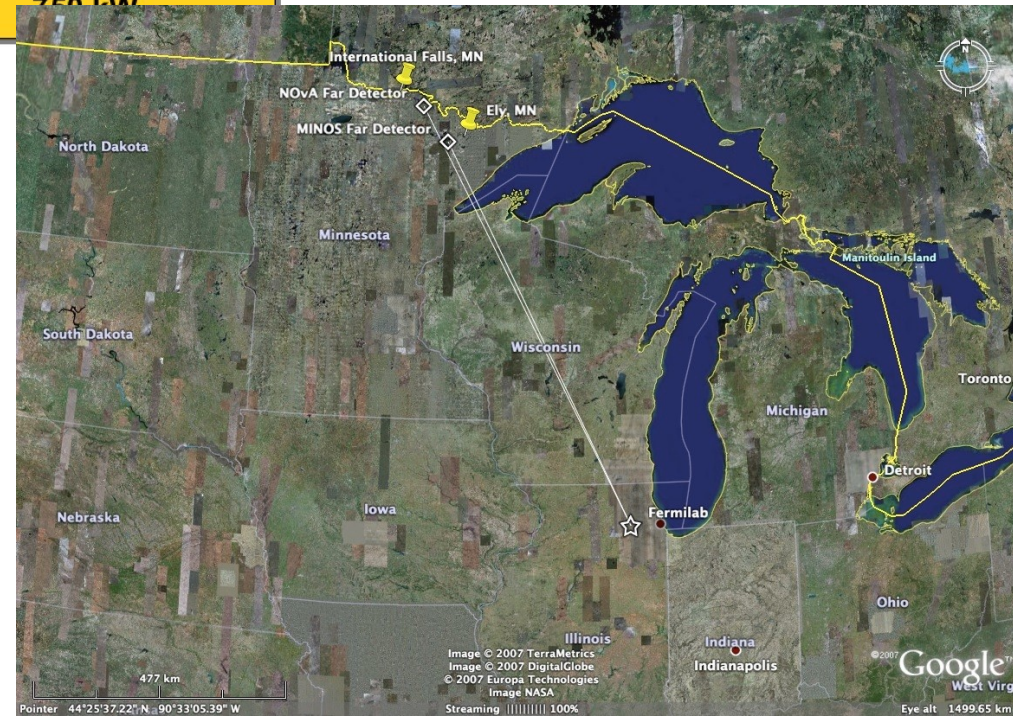
Use Near Detector to measure  $F_n(@ND)$

# T2K and NOVA



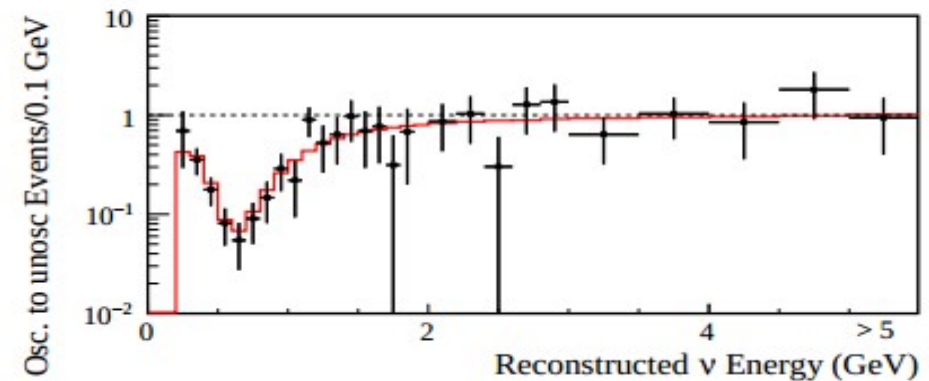
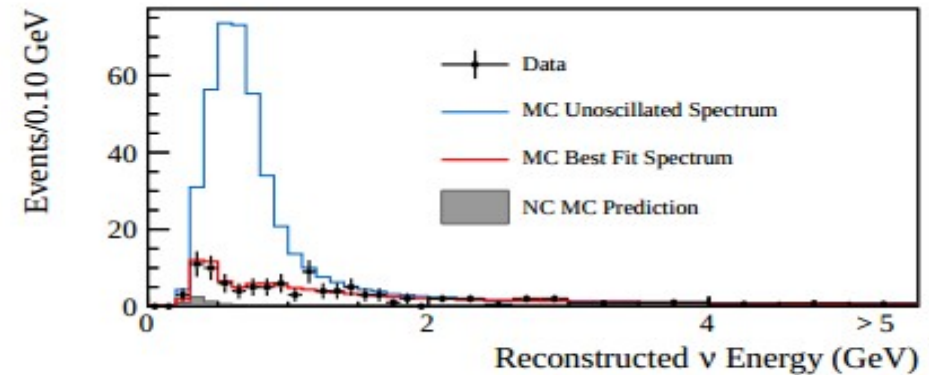
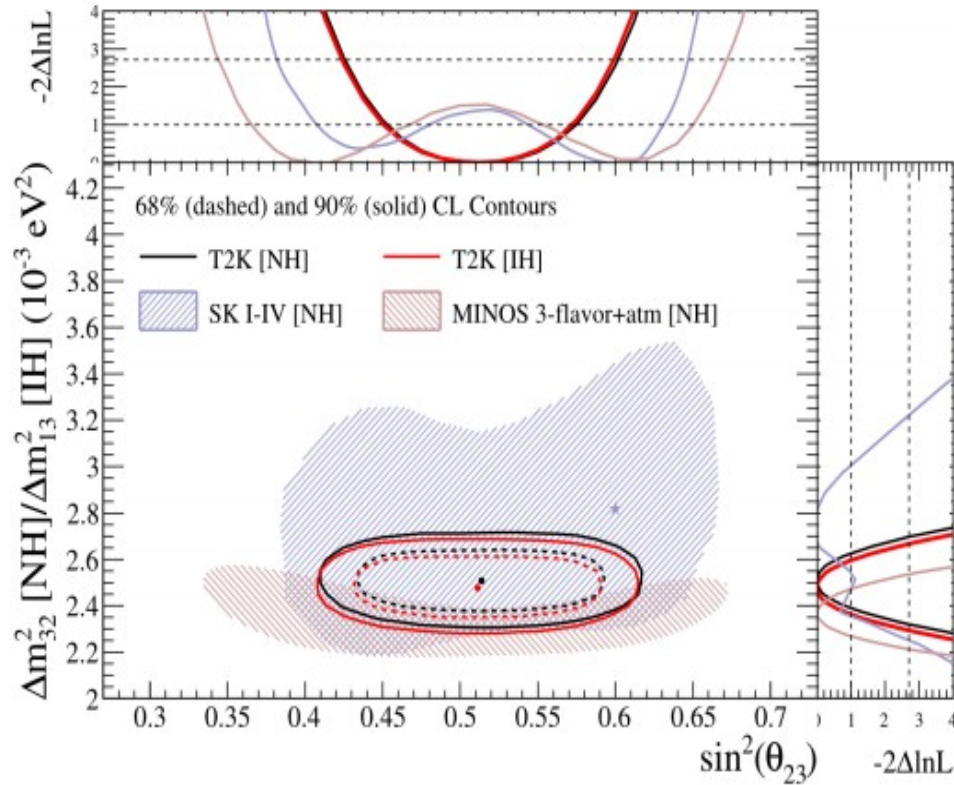
- ▶ JPARC to Kamioka
- ▶  $L = 295$  km
- ▶  $E_{\nu} \sim 0.6$  GeV
- ▶ Far Det : 22.6 kton water Cerenkov detector

- ▶ Fermilab to Ash River, MN
- ▶  $L = 810$  km
- ▶  $E_{\nu} \sim 2.0$  GeV
- ▶ Far Det : 14 kton of liquid scintillator (in bars)





# T2K Disappearance



$$\frac{\# \text{ events observed}}{\# \text{ events expected}} = P(\nu_\mu \rightarrow \nu_\mu) = 1 - \sin^2(2\theta) \sin^2\left(\frac{\Delta m^2 L}{4E}\right)$$

$$|\Delta m_{23}^2| = (2.51 \pm 0.1) \times 10^{-3} \text{ eV}^2$$

$$\sin^2(\theta_{23}) = 0.514_{-0.056}^{+0.055} \rightarrow \theta_{23} = 45.8 \pm 3.2$$

(best fit)

# Mixing matrix

$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} e^{i\delta} \\ 0 & 1 & 0 \\ -s_{13} e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}$$

Solar sector :  $\nu_{\mu} \rightarrow \nu_e$

$$\theta_{e\mu} = 33.7^\circ \pm 1.1^\circ$$

$$\Delta m_{12}^2 = +(7.54 \pm 0.24) \times 10^{-5} eV^2$$

Atmospheric sector

$$\nu_{\mu} \rightarrow \nu_{\tau}$$

$$\theta_{\mu\tau} = 42^\circ \pm 3.0^\circ$$

$$\Delta m_{23}^2 = |(2.43 \pm 0.06) \times 10^{-3}| eV^2$$

# How do we measure $\theta_{13}$ ?

$\nu_{\mu} \rightarrow \nu_e$  oscillations with atmospheric L/E

$$P(\nu_{\mu} \rightarrow \nu_e) = \sin^2 2\theta_{13} \sin^2 \theta_{23} \sin^2 \left( 1.27 \Delta m_{23}^2 \frac{L}{E} \right)$$

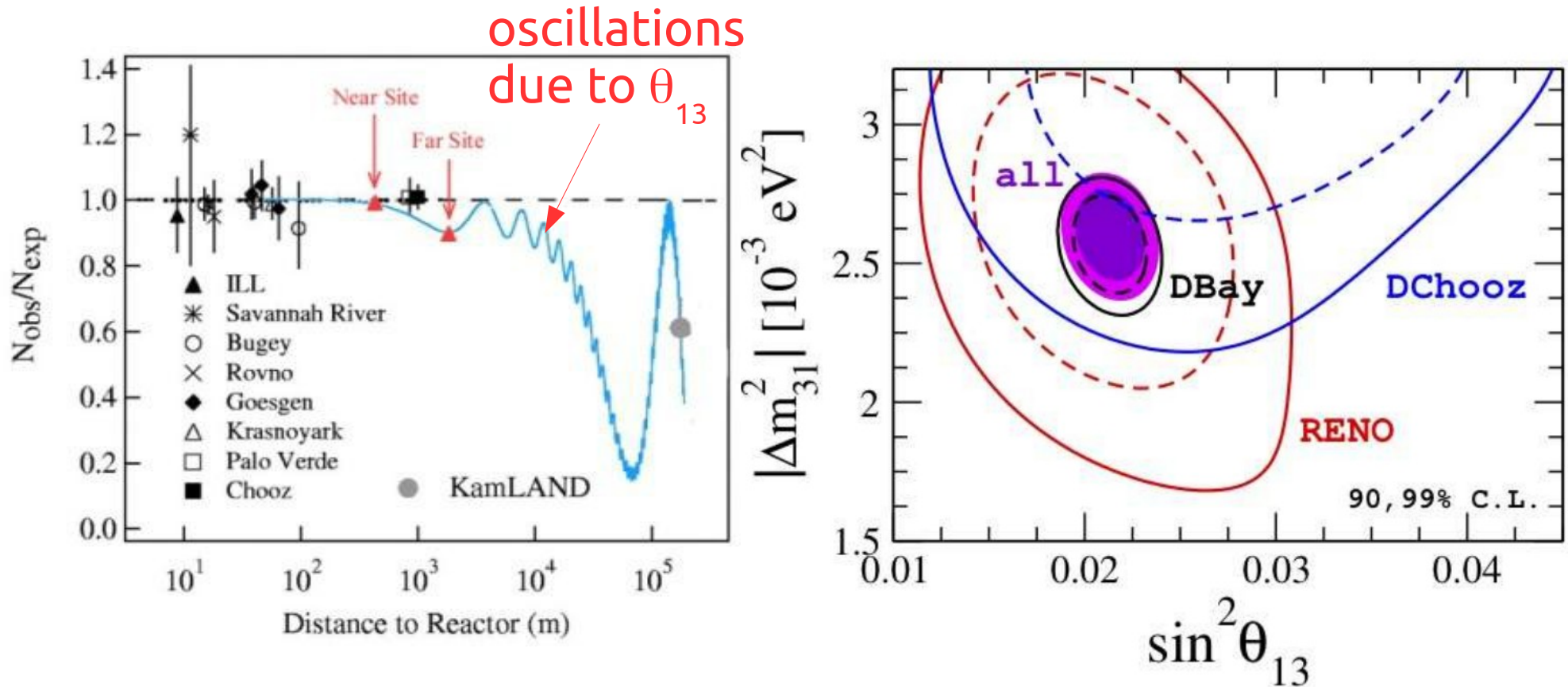
$\nu_e$  appearance in a  $\nu_{\mu}$  beam – ideal for *accelerator experiments*

$\bar{\nu}_e \rightarrow \bar{\nu}_x$  disappearance oscillations with atmospheric L/E

$$p(\bar{\nu}_e \rightarrow \bar{\nu}_x) = 1 - \sin^2(2\theta_{13}) \sin^2 \left( 1.27 \Delta m_{23}^2 \frac{L}{E} \right)$$

$\bar{\nu}_e$  disappearance – ideal for *reactor experiments*

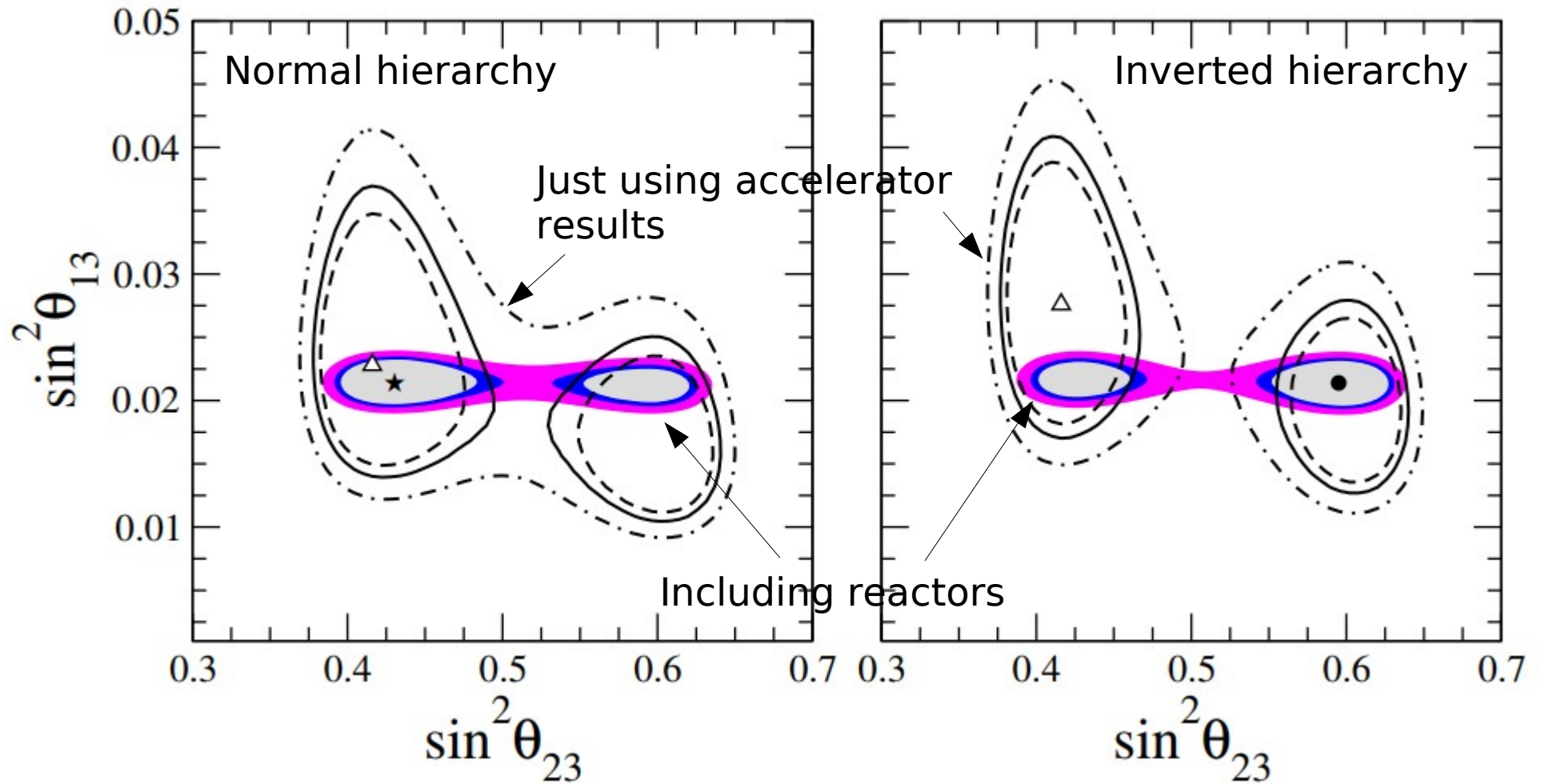
# $\theta_{13}$ from reactors



$$\theta_{13} = (8.44(41) \pm 0.16)^\circ \text{ (NO(IO))}$$



# Global results



# 3-Neutrino Mixing

$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} e^{i\delta} \\ 0 & 1 & 0 \\ -s_{13} e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}$$

## Solar sector

$$\nu_e \rightarrow \nu_\mu$$

$$\theta_{12} = 34.5^\circ \pm 1.1^\circ$$

$$\Delta m_{12}^2 = +7.56 \times 10^{-5} eV^2$$

## 13 Sector

$$\nu_\mu \rightarrow \nu_e$$

$$\theta_{13} = 8.44^\circ \pm 0.16^\circ$$

$$\Delta m_{23}^2 = |2.52 \times 10^{-3}| eV^2$$

## Atmospheric sector

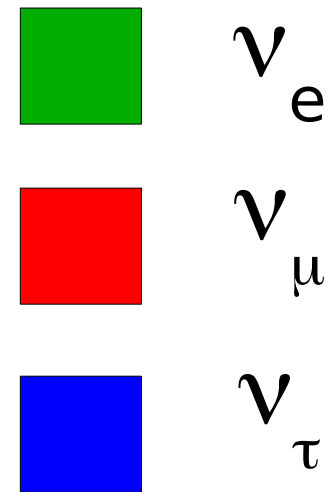
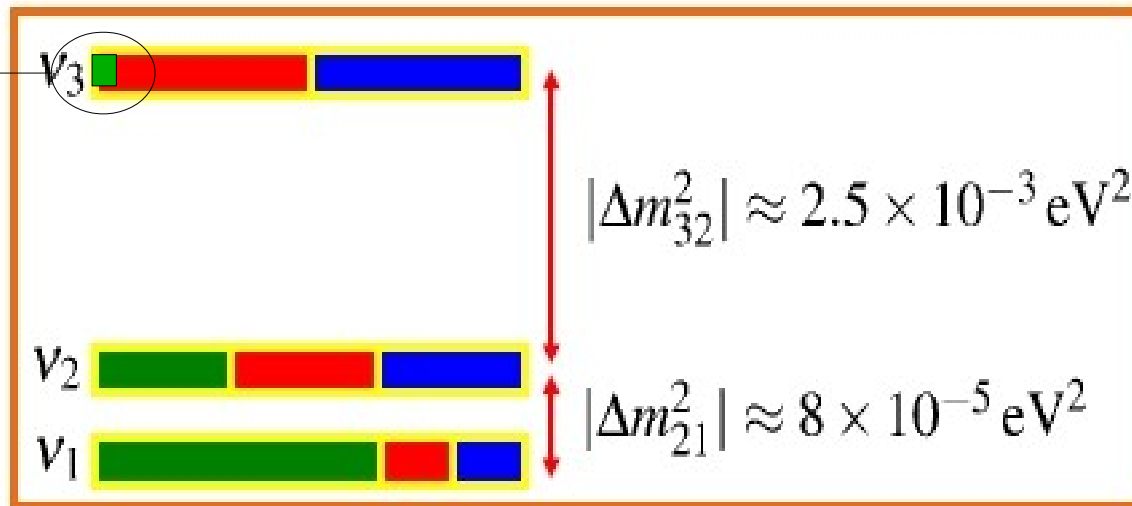
$$\nu_\mu \rightarrow \nu_\tau$$

$$\theta_{23} = 41.0(50.5)^\circ \pm 1.1^\circ$$

$$\Delta m_{23}^2 = |2.52 \times 10^{-3}| eV^2$$

# Summary of Current Knowledge

$\theta_{13}$  : how much  $\nu_e$  is in  $\nu_3$



$$U_{MNSP} \approx \begin{pmatrix} 0.8 & 0.5 & 0.15 \\ 0.4 & 0.5 & 0.6 \\ 0.4 & 0.5 & 0.7 \end{pmatrix}$$

Some elements only known to 10-30%

Very very different from the quark CKM matrix

# Lecture 4

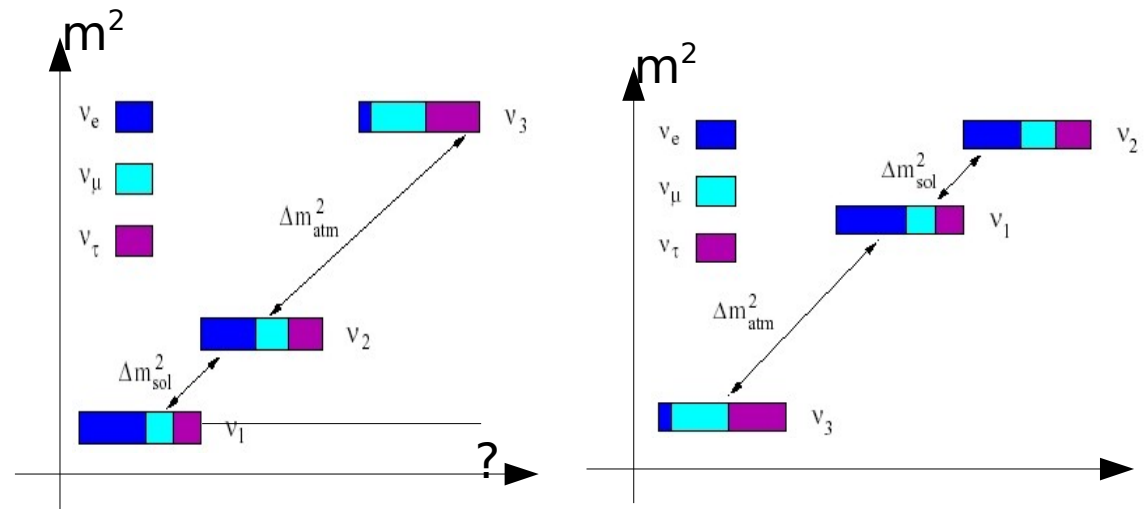
## *The Future*

# The Quest

$$\begin{pmatrix} c_{13} & 0 & s_{13} e^{i\delta} \\ 0 & 1 & 0 \\ -s_{13} e^{i\delta} & 0 & c_{13} \end{pmatrix}$$

Value of  $\delta$ ?

## Normal or Inverted mass hierarchy?



- Better estimates of the oscillation parameters using accelerators
- Is  $\theta_{23}$  maximal?
- Is the neutrino Majorana?
- What is the absolute mass?

$$U_{PMNS} = \begin{pmatrix} 0.8 & 0.5 & -0.15 \\ -0.4 & 0.7 & 0.6 \\ 0.4 & -0.5 & 0.7 \end{pmatrix} \quad ?$$

$$U_{CKM} = \begin{pmatrix} 0.975 & 0.222 & 0.004 \\ 0.221 & 0.97 & 0.04 \\ 0.01 & 0.04 & 0.999 \end{pmatrix}$$