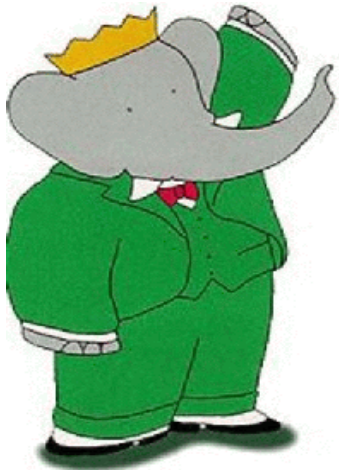


# Study of three-body decays of the $\eta_c$ meson at the BABAR experiment

Racha Cheaib  
Lake Louise Winter Institute  
Sat., Feb. 13<sup>th</sup>, 2016

McGill University  
Montreal, Canada



*On behalf of the BaBar Collaboration*



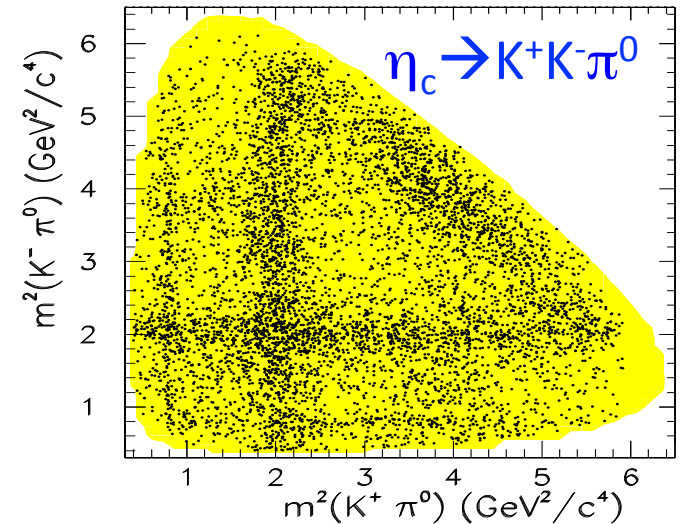
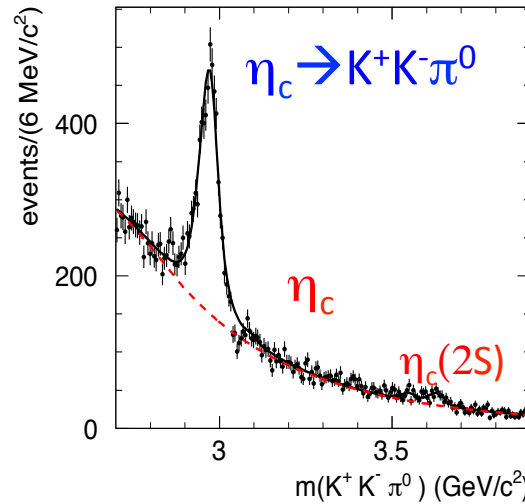
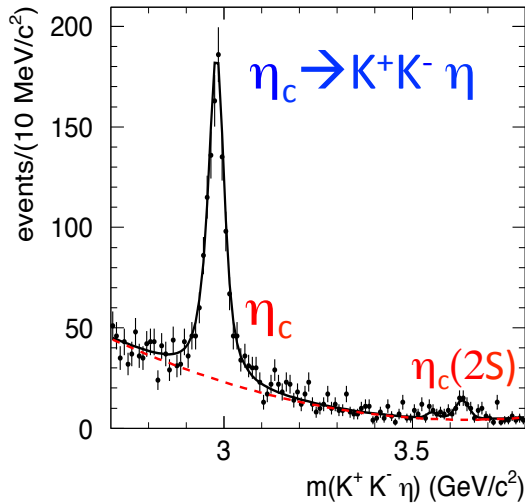
# Theoretical Motivation

- Charmonium decays,  $\eta_c$ , are an excellent probe for light meson spectroscopy.
- Scalar mesons ( $J^{PC}=0^{++}$ ) are still a puzzle.
  - Too many states: inconsistent with quark model
  - Large decay widths: overlap between resonance and background.
- Structure of  $l=1/2$   $K\pi$  S-wave is poorly known
  - Introduces large systematic uncertainties in analyses with 3 or 4-body decays of heavy flavour mesons
- Previous measurements of  $l=1/2$   $K\pi$  S-wave, limited to below  $D^+$  mass and contamination from  $l=3/2$ 
  - LASS experiment: Nucl. Phys. B **296**,493 (1988)
  - E791 experiment: Phys. Rev. D **73**, 032004 (2006)



# Dalitz plot analysis of $\eta_c$

BaBar performed first Dalitz plot analysis of  $\eta_c \rightarrow K^+ K^- \pi^0$  and  $\eta_c \rightarrow K^+ K^- \eta$ .

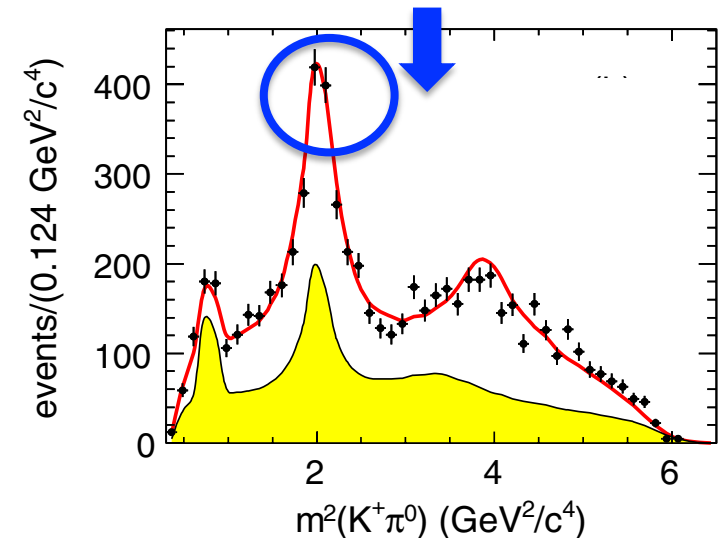


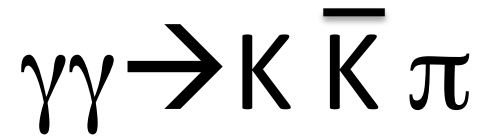
First observation of  $K^*(1430)$  as a Breit-Wigner peak:

$$\frac{\mathcal{B}(K_0^*(1430) \rightarrow \eta K)}{\mathcal{B}(K_0^*(1430) \rightarrow \pi K)} = \mathcal{R}(\eta_c) \frac{f_{\eta K}}{f_{\pi K}} = 0.092 \pm 0.025^{+0.010}_{-0.025}$$

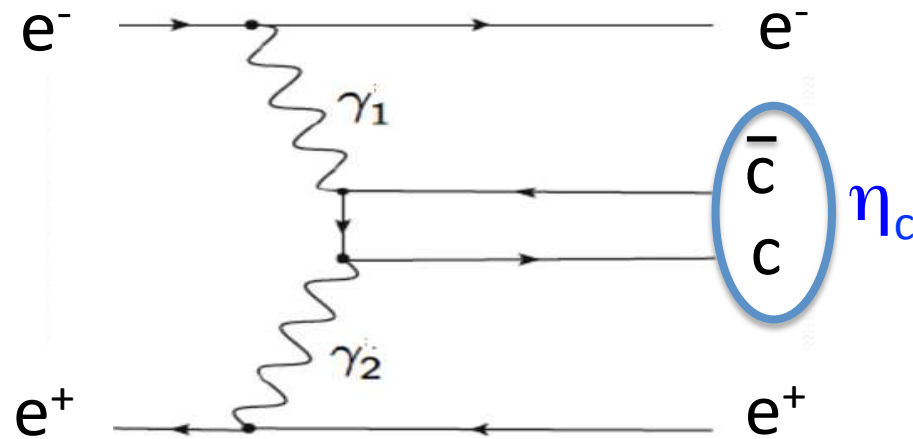
Dominance of scalar meson amplitudes:

$\eta_c \longrightarrow$  pseudoscalar + scalar

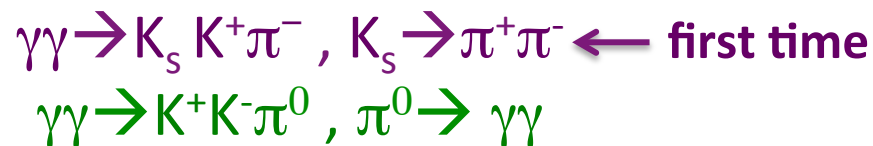




- Two photon interactions are used to produce charmonium resonances.
- $e^+ e^-$  beam particles are scattered at small angles and undetected in the final state.



- Produced resonances can have  $J^{PC}=0^{\pm+}, 2^{\pm+}, 3^{\pm+}, 4^{\pm+}$ , etc ... with the exception that  $K\bar{K}\pi$  cannot be in a  $J^P=0^+$  state.
- The following two photon interactions are considered using an integrated luminosity of  $519 \text{ fb}^{-1}$ :







# $\gamma\gamma \rightarrow K \bar{K} \pi$

$$\gamma\gamma \rightarrow K_s K^+ \pi^-, K_s \rightarrow \pi^+ \pi^-$$

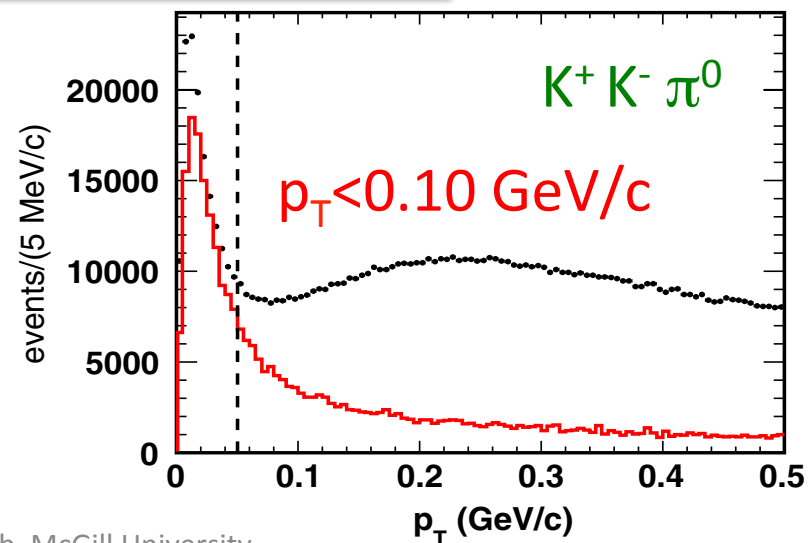
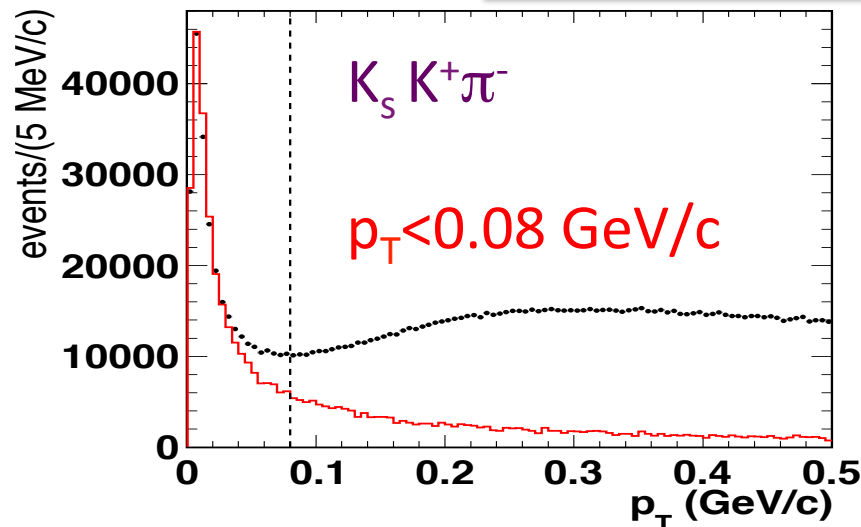
- Exactly 4 charged tracks with loose Kaon and Pion PID.
- Vertex fit of oppositely charged tracks to select  $K_s$  candidates.

$$\gamma\gamma \rightarrow K^+ K^- \pi^0$$

- Exactly 2 charged tracks with loose Kaon PID.
- $\gamma\gamma$  pairs fit to a  $\pi^0$  hypothesis, with  $E_\gamma > 50$  MeV.

Background from ISR events with  $J^{PC} = 1^{-}$  - resonance production suppressed:

$$M_{\text{rec}}^2 = (p_{e^+e^-} - p_{\text{rec}})^2 > 10 \text{ GeV}^2/c^4$$

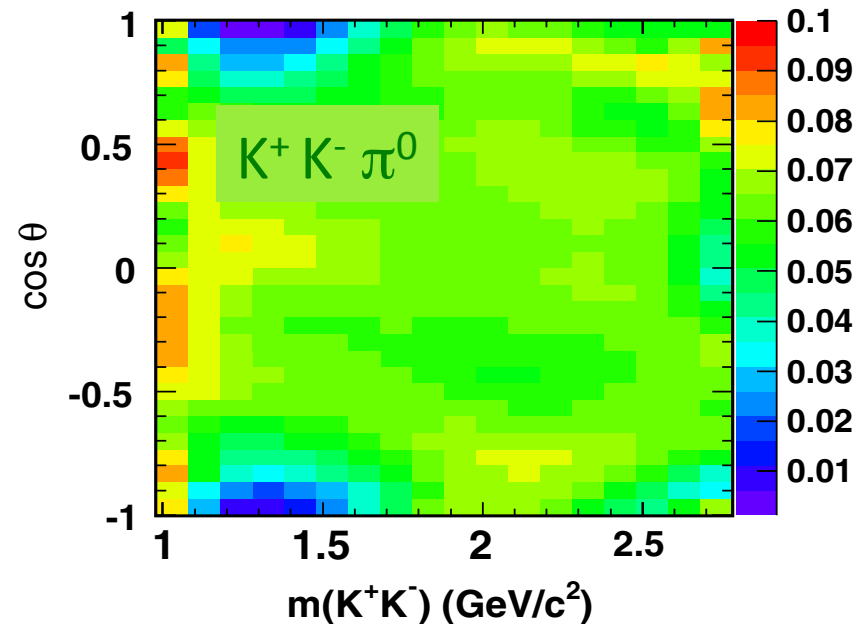
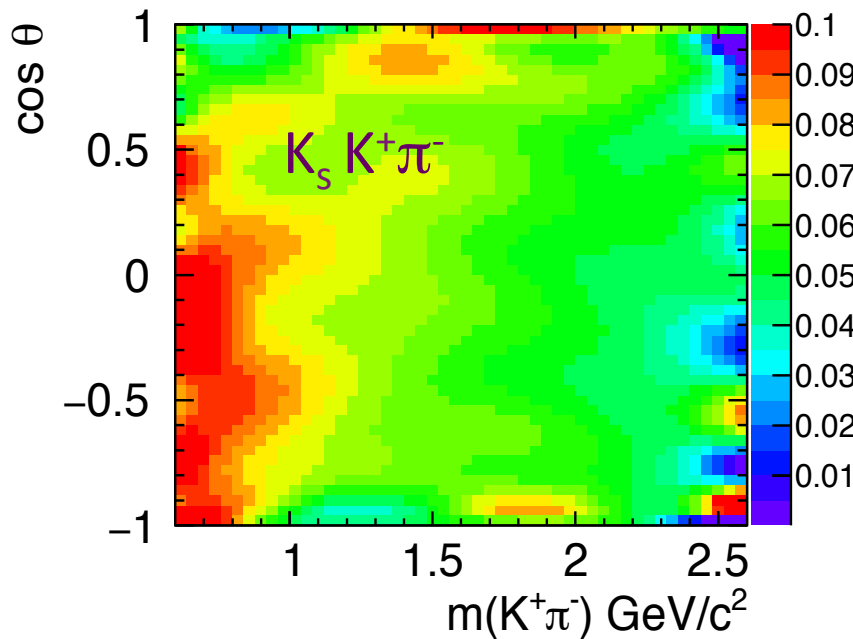




# Signal Efficiency

- MC signal events in which the  $\eta_c$  decays uniformly in phase space.
- Restrict mass regions to  $\eta_c$  signal region.
- Efficiency:  $N_{\text{reconstructed}}/N_{\text{generated}}$
- Express as a function of  $m(K^+K^-)$  and  $\cos\theta_{K^+}$
- $\cos\theta_{K^+}$ : angle, in  $K^+K^-$  rest frame, between the direction of the  $K^+$  and the boost from  $K_s K^- \eta$  or  $K^+ K^- \pi^0$  rest frame.

$$\epsilon(\cos\theta) = \sum_{L=0}^{12} a_L(m) Y_L^0(\cos\theta)$$



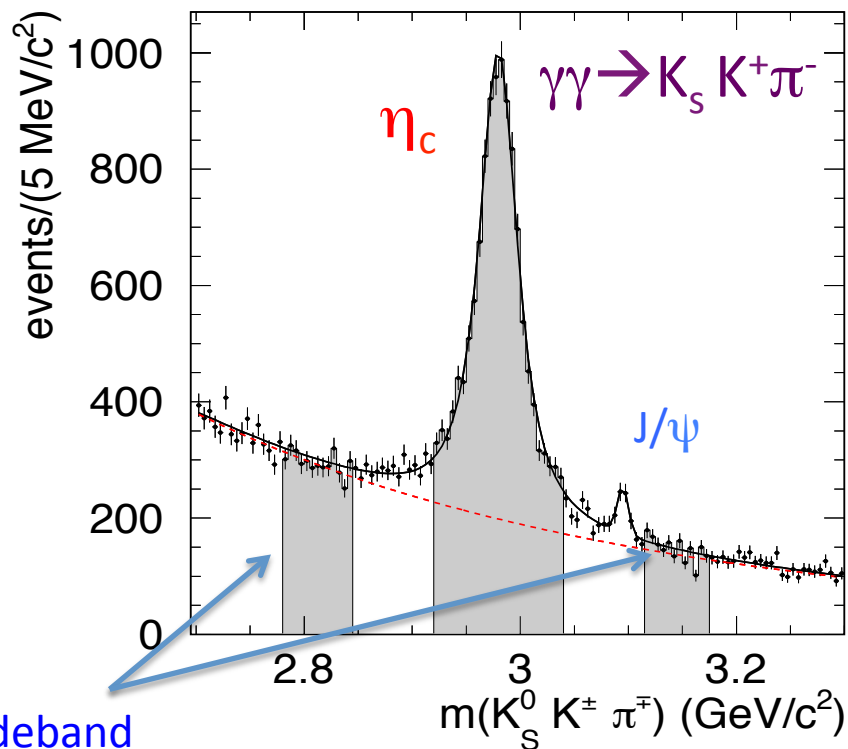
Efficiency loss due to low momentum kaons (<200 MeV/c)  
 $K_s$  and  $\pi^\pm$  (<100 MeV/c)



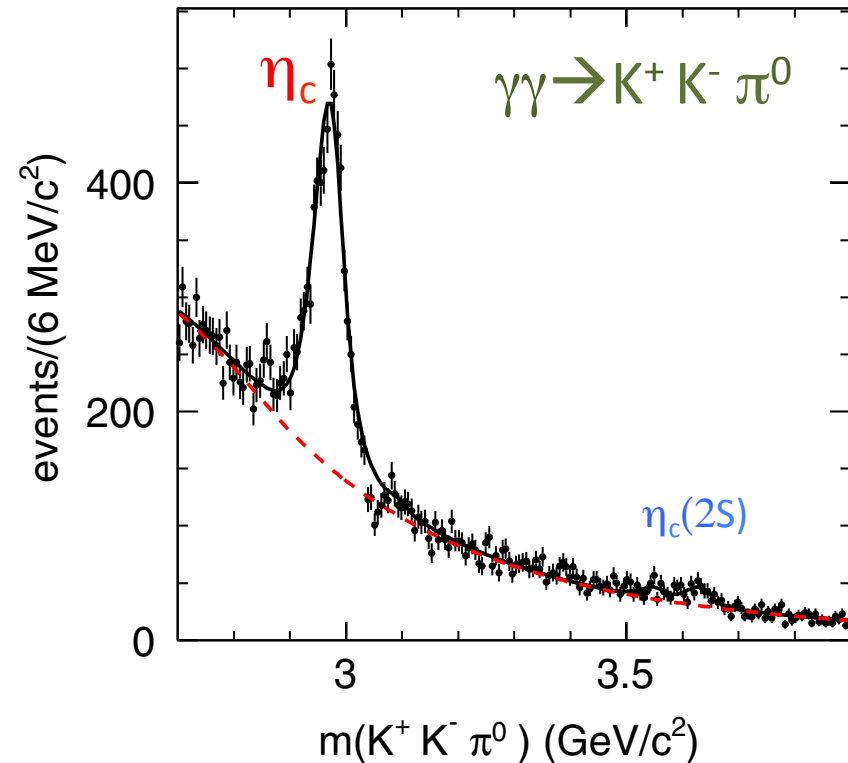
# Mass Spectra

Clear  $\eta_c$  signal fit with a Breit-Wigner function convolved with the mass resolution function.

Background described with 2<sup>nd</sup> order polynomial.



12849 signal events  
with 64.3% purity

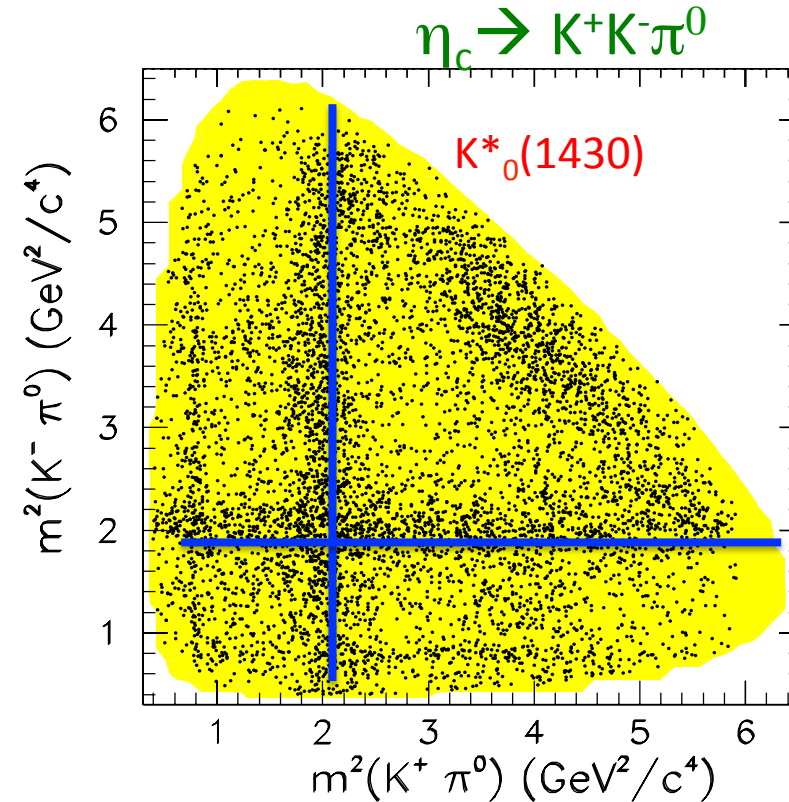
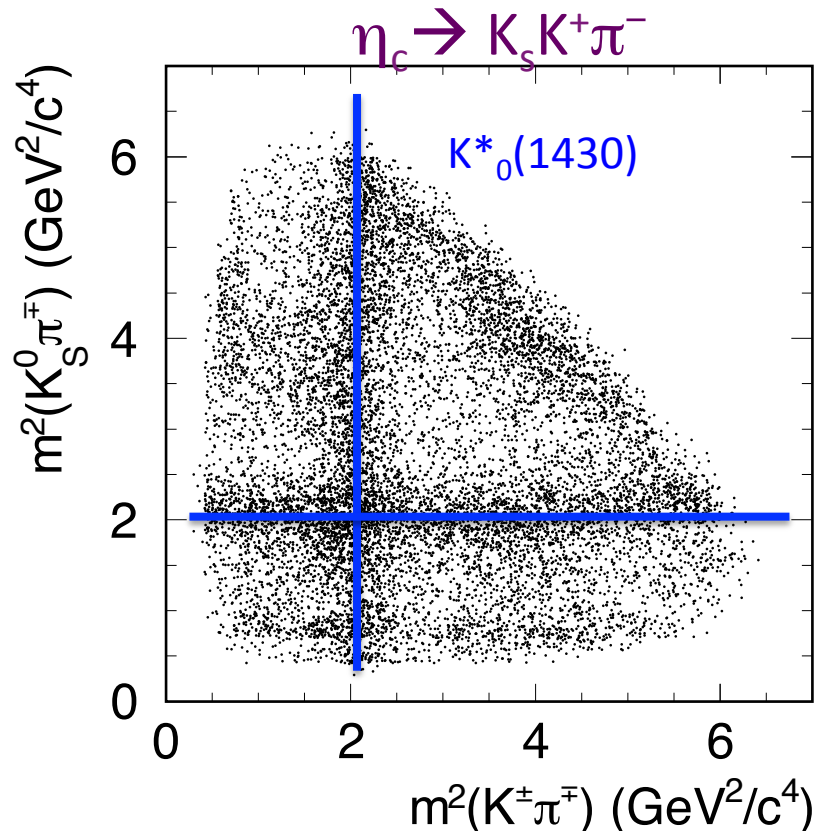


6710 signal events  
with 55.2% purity



# Dalitz Plot Analysis of $\eta_c \rightarrow K \bar{K} \pi$

- Unbinned maximum likelihood fit in the  $\eta_c$  mass region
- Full interference allowed among amplitudes.
- Fits performed using:
  - Isobar model: Phys. Lett. B **592**, 1 (2004)
  - Model Independent Partial Wave Analysis (MIPWA): Phys. Rev. D **73**, 032004 (2006)





# Model Independent Partial Wave Analysis of $\eta_c \rightarrow K \bar{K} \pi$

Amplitude is written as a sum of partial waves with  $A_i$  and  $\phi_i$

$$A = |A_1 + c_2 A_2 e^{i\phi_2} + c_3 A_3 e^{i\phi_3} + \dots|$$

- $\eta_c = 0^+ \rightarrow$  Interference between ( $K\pi$ )  $K$  systems is constructive.
- $K\pi$  amplitude is symmetrized

$$\eta_c = K_S K^+ \pi^-$$

$$A_{S\text{-wave}} = \frac{1}{\sqrt{2}} (a_j^{K^+ \pi^-} e^{i\phi_j^{K^+ \pi^-}} + a_j^{\bar{K}^0 \pi^-} e^{i\phi_j^{\bar{K}^0 \pi^-}})$$

where  $a^{K^+ \pi^-}(m) = a^{K_S^0 \pi^-}(m)$  and  $\phi^{K^+ \pi^-}(m) = \phi^{K_S^0 \pi^-}(m)$

$$\eta_c = K^+ K^- \pi^0$$

$$A_{S\text{-wave}} = \frac{1}{\sqrt{2}} (a_j^{K^+ \pi^0} e^{i\phi_j^{K^+ \pi^0}} + a_j^{K^- \pi^0} e^{i\phi_j^{K^- \pi^0}})$$

where  $a^{K^+ \pi^0}(m) = a^{K^- \pi^0}(m)$  and  $\phi^{K^+ \pi^0}(m) = \phi^{K^- \pi^0}(m)$

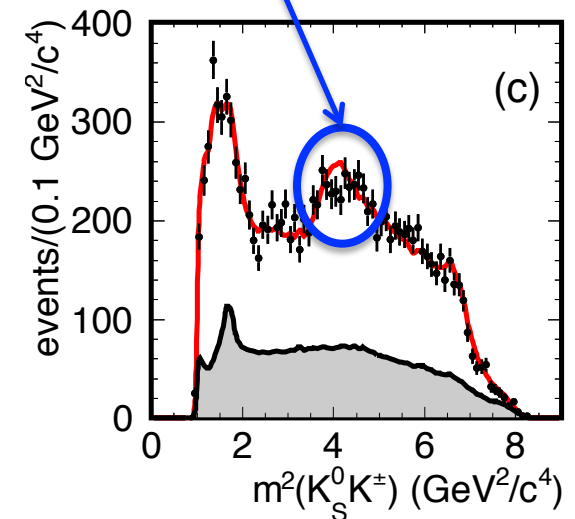
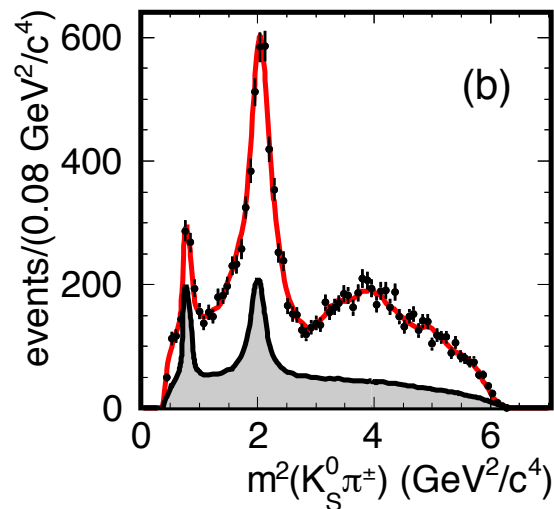
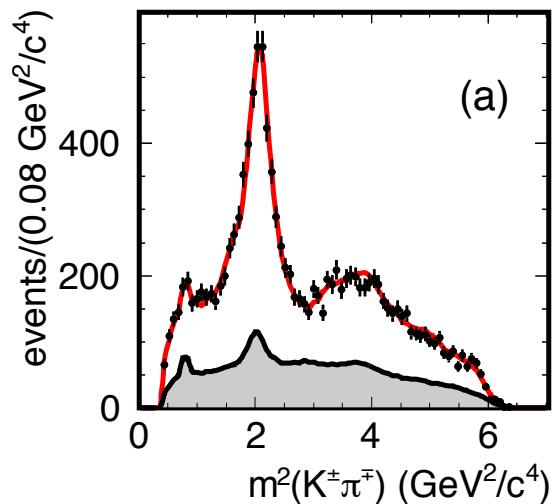
- Mass spectrum is divided into 30 equally spaced mass intervals 60 MeV wide.
- Each interval has 2 new free parameters,  $A_i$  and  $\phi_i$ , with a total of 58
- Fix  $A=1$  and  $\phi=\pi/2$  in one arbitrary bin (bin 11 with mass=1.45 GeV/c<sup>2</sup>)
- Other resonance contributions,  $A_2, A_3$ , etc..., are parameterized as Breit-Wigner amplitudes multiplied by their corresponding angular functions.
- Backgrounds are interpolated from sideband regions:  $K^*(892)$  entirely background.



# MIPWA of $\eta_c \rightarrow K_S K^+ \pi^-$

- Dominance of  $K\pi$  S-wave with contributions from  $a_0\pi$  and  $K^*(1430)K$ .
  - New resonance  $a_0(1950)\pi$  with  $2.5\sigma$  significance.
  - Good description of the data!

	$\eta_c \rightarrow K_S^0 K^\pm \pi^\mp$	
Amplitude	Fraction (%)	Phase (rad)
$(K\pi \text{ S-wave}) \bar{K}$	$107.3 \pm 2.6 \pm 17.9$	fixed
$a_0(980)\pi$	$0.8 \pm 0.5 \pm 0.8$	$1.08 \pm 0.18 \pm 0.18$
$a_0(1450)\pi$	$0.7 \pm 0.2 \pm 1.4$	$2.63 \pm 0.13 \pm 0.17$
$a_0(1950)\pi$	$3.1 \pm 0.4 \pm 1.2$	$-1.04 \pm 0.08 \pm 0.77$
$a_2(1320)\pi$	$0.2 \pm 0.1 \pm 0.1$	$1.85 \pm 0.20 \pm 0.20$
$K_2^*(1430)\bar{K}$	$4.7 \pm 0.9 \pm 1.4$	$4.92 \pm 0.05 \pm 0.10$
Total	$116.8 \pm 2.8 \pm 18.1$	→ Interference effects
$-2 \log \mathcal{L}$	$-4314.2$	
$\chi^2/N_{\text{cells}}$	$301/254$	$\approx 1.17$

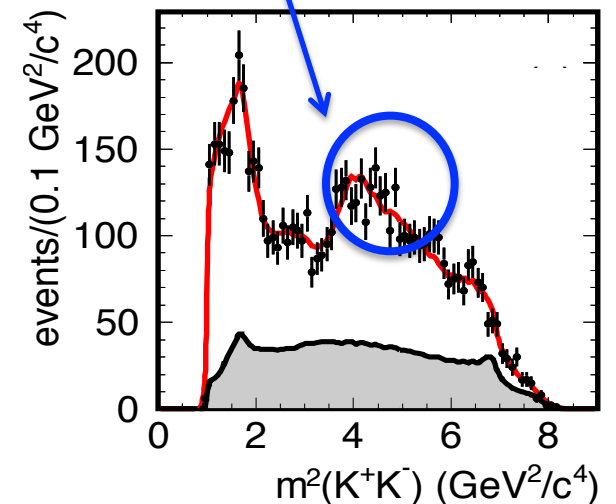
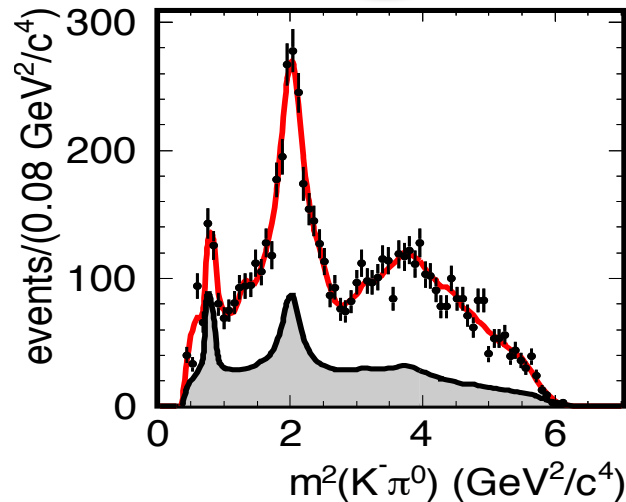
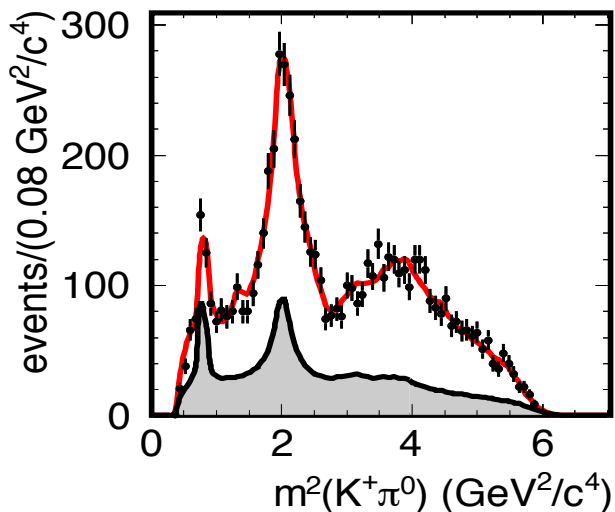




# MIPWA of $\eta_c \rightarrow K^+ K^- \pi^0$

- Dominance of  $K\pi$  S-wave with contributions from  $a_0\pi$  and  $K^*(1430)K$ .
  - New resonance  $a_0(1950)\pi$  with  $4.2\sigma$  significance
  - Good description of the data!

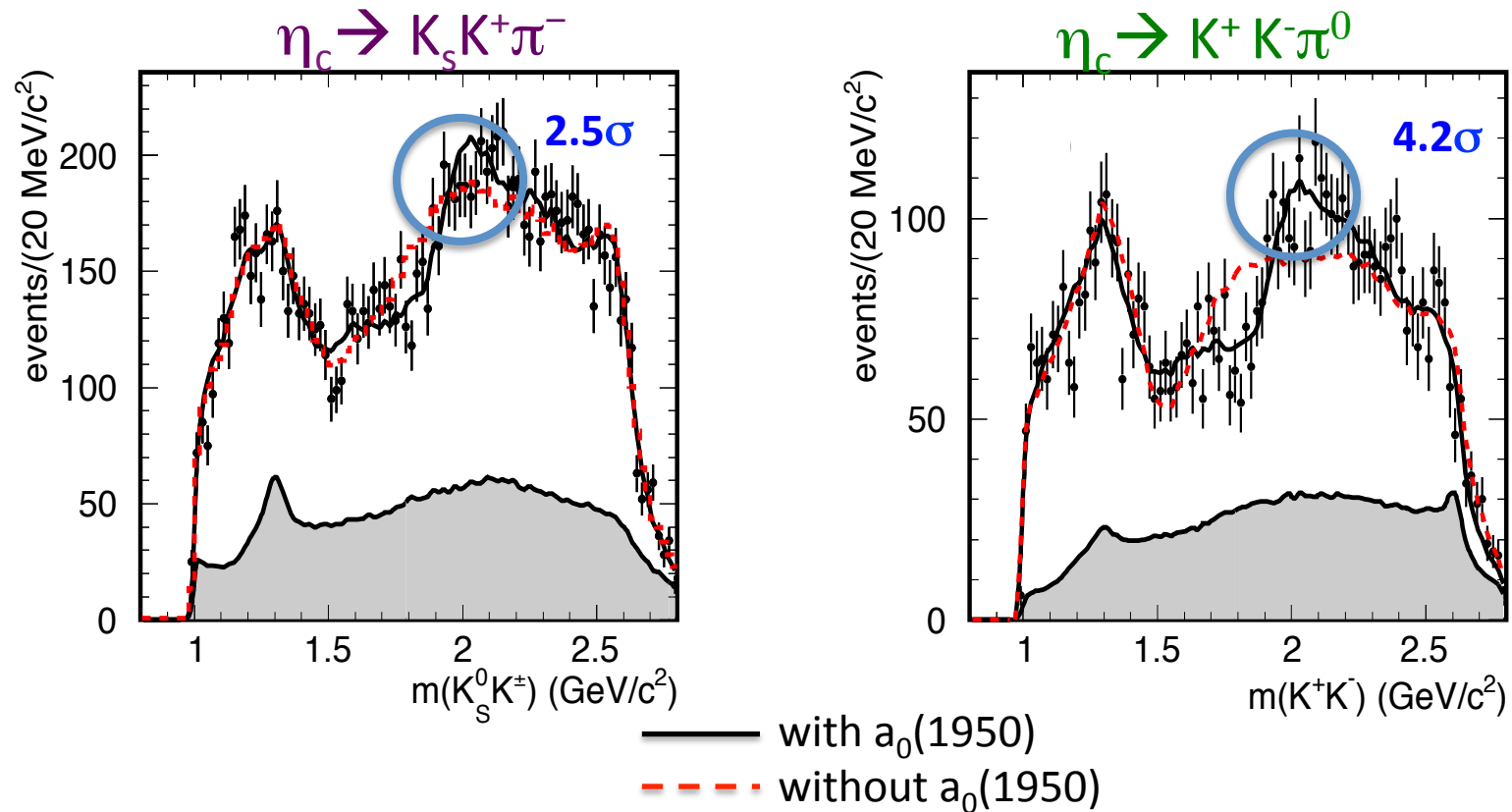
$\eta_c \rightarrow K^+ K^- \pi^0$		
Amplitude	Fraction (%)	Phase (rad)
$(K\pi \text{ S-wave}) \bar{K}$	$125.5 \pm 2.4 \pm 4.2$	fixed
$a_0(980)\pi$	$0.0 \pm 0.1 \pm 1.7$	-
$a_0(1450)\pi$	$1.2 \pm 0.4 \pm 0.7$	$2.90 \pm 0.12 \pm 0.25$
$a_0(1950)\pi$	$4.4 \pm 0.8 \pm 0.8$	$-1.45 \pm 0.08 \pm 0.27$
$a_2(1320)\pi$	$0.6 \pm 0.2 \pm 0.3$	$1.75 \pm 0.23 \pm 0.42$
$K_2^*(1430)\bar{K}$	$3.0 \pm 0.8 \pm 4.4$	$5.07 \pm 0.09 \pm 0.30$
Total	$134.8 \pm 2.7 \pm 6.4$	→ Interference effects
$-2 \log \mathcal{L}$	-2339	
$\chi^2 / N_{\text{cells}}$	$283.2/233 = 1.22$	





# $a_0(1950)$ resonance

MIPWA fit improved with the  $a_0(1950)$  resonance in both  $\eta_c$  channels.



Final state	Mass ( $\text{MeV}/c^2$ )	Width (MeV)
$\eta_c \rightarrow K_S^0 K^\pm \pi^\mp$	$1949 \pm 32 \pm 76$	$265 \pm 36 \pm 110$
$\eta_c \rightarrow K^+ K^- \pi^0$	$1927 \pm 15 \pm 23$	$274 \pm 28 \pm 30$
Weighted mean	$1931 \pm 14 \pm 22$	$271 \pm 22 \pm 29$

large systematics





# Isobar Model of $\eta_c \rightarrow K_S K^+ \pi^-$

- Resonances are modeled as Breit-Wigner multiplied by their angular functions.
- $K\pi$  S-wave is parametrized as a superposition of interfering  $K^*_0(1430)$ ,  $K^*_0(1950)$ , and non-resonant amplitude (NR).

Amplitude	Fraction %	Phase (rad)
$K^*_0(1430)\bar{K}$	$40.8 \pm 2.2$	0.
$K^*_0(1950)\bar{K}$	$14.8 \pm 1.7$	$-1.00 \pm 0.07$
NR	$18.0 \pm 2.5$	$1.94 \pm 0.09$
$a_0(980)\pi$	$10.5 \pm 1.2$	$0.94 \pm 0.12$
$a_0(1450)\pi$	$1.7 \pm 0.5$	$2.94 \pm 0.13$
$a_0(1950)\pi$	$0.7 \pm 0.2$	$-1.76 \pm 0.24$
$a_2(1320)\pi$	$0.2 \pm 0.2$	$-0.53 \pm 0.42$
$K^*_2(1430)\bar{K}$	$2.3 \pm 0.7$	$-1.55 \pm 0.11$
Total	$88.8 \pm 4.3$	
$-2 \log \mathcal{L}$	-4290.7	
$\chi^2 / N_{\text{cells}}$	467/256 = 1.82	

}  $K\pi$  S-wave

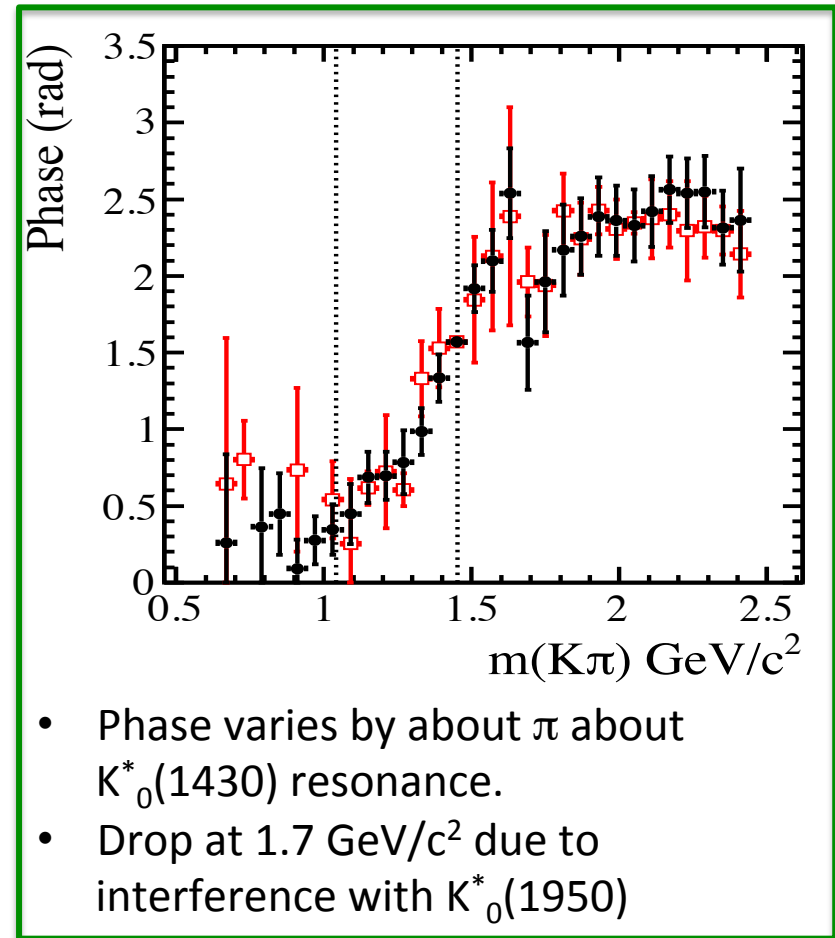
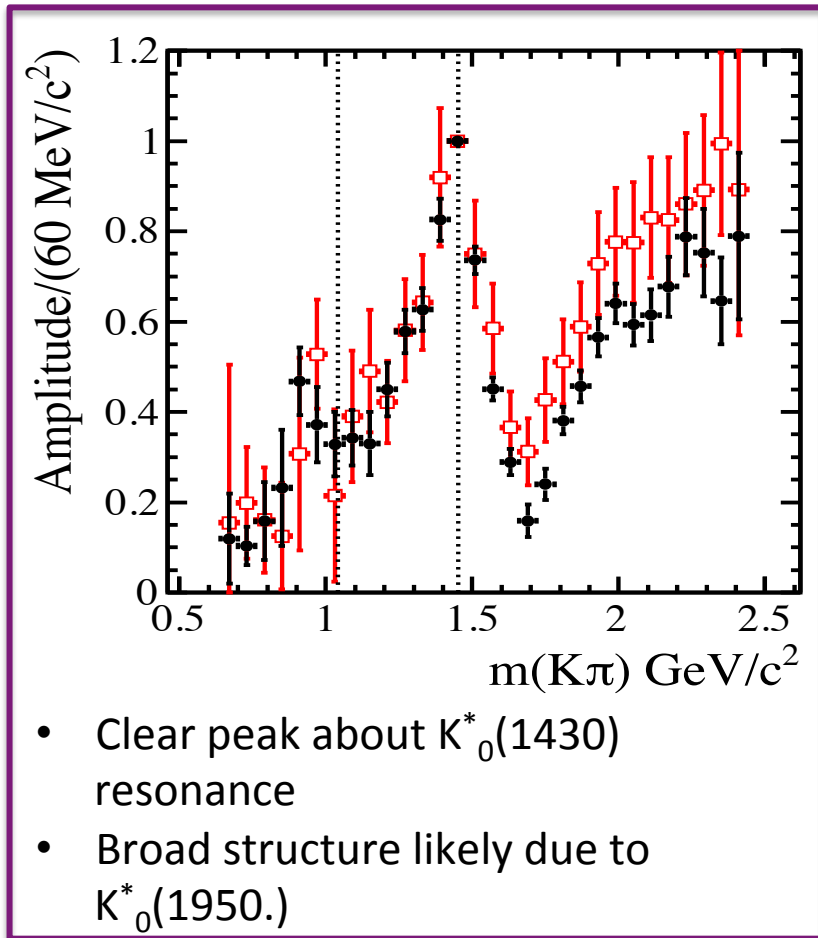
→ Interference effects

Poor description of data!  
 Compare with **1.17** with MIWPA fit.  
 Similar result for  $K^+K^-\pi^0$



# $I=1/2$ $K\pi$ S-wave Amplitude

- Amplitude extending up to a mass of 2.5  $\text{GeV}/c^2$ .
  - Due to isospin conservation, no contribution from  $I=3/2$ .
- $\eta_c \rightarrow K_s K^+ \pi^-$   
 $\eta_c \rightarrow K^+ K^- \pi^0$





# $I=1/2$ $K\pi$ S-wave Amplitude

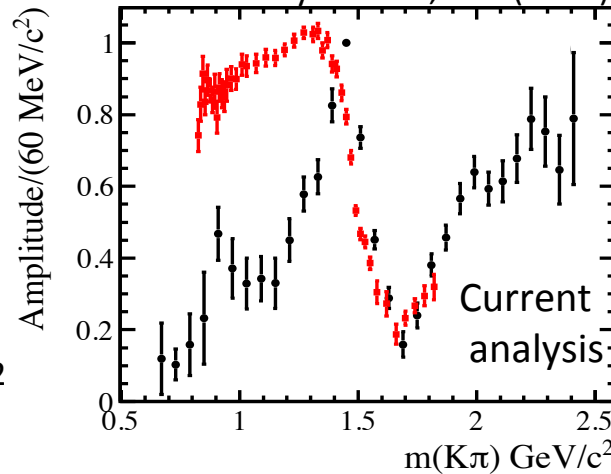
Difficulty separating  $I=1/2$  and  $I=3/2$  contributions in **LASS** and **E791** experiments.

LASS measurement has two-fold ambiguity above  $1.82 \text{ GeV}/c^2$ .

E791 measurements are limited to below  $1.6 \text{ GeV}/c^2$

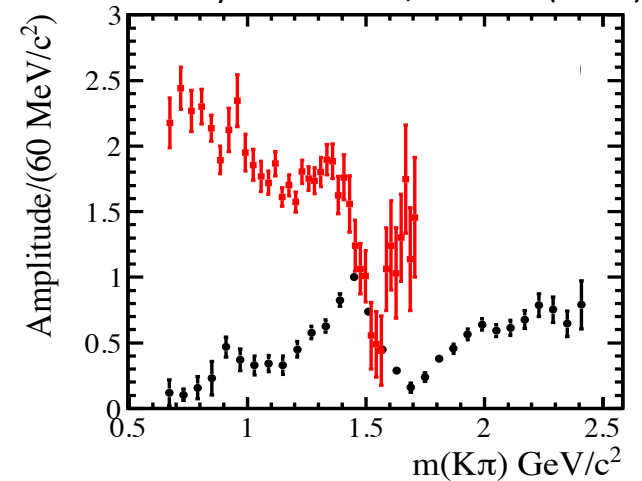
LASS:  $K^-p^- \rightarrow K^+\pi^+n$

Nucl. Phys. B **296**, 493 (1988)



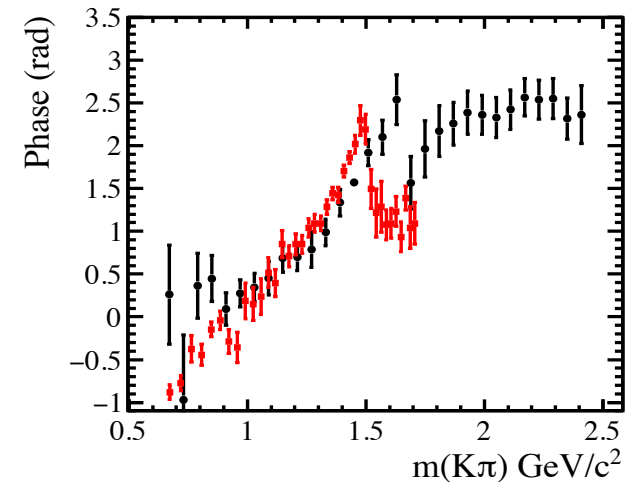
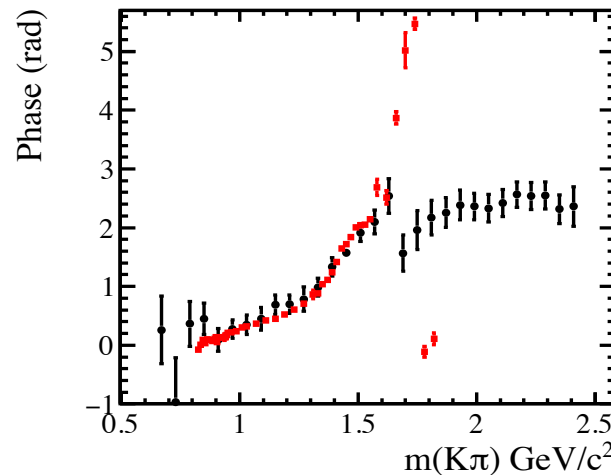
E791:  $D^- \rightarrow K^+\pi^+\pi^-$

Phys. Rev. D **73**, 032004 (2006)



Striking difference in the mass dependence of amplitudes!

Phase behaviour is similar as expected by Watson's theorem!

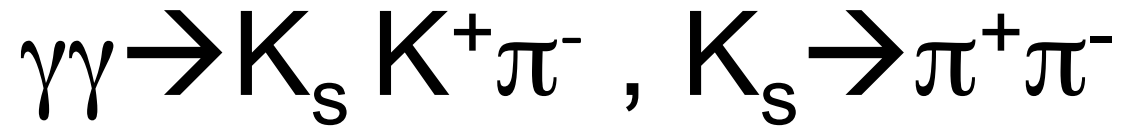




# Conclusion

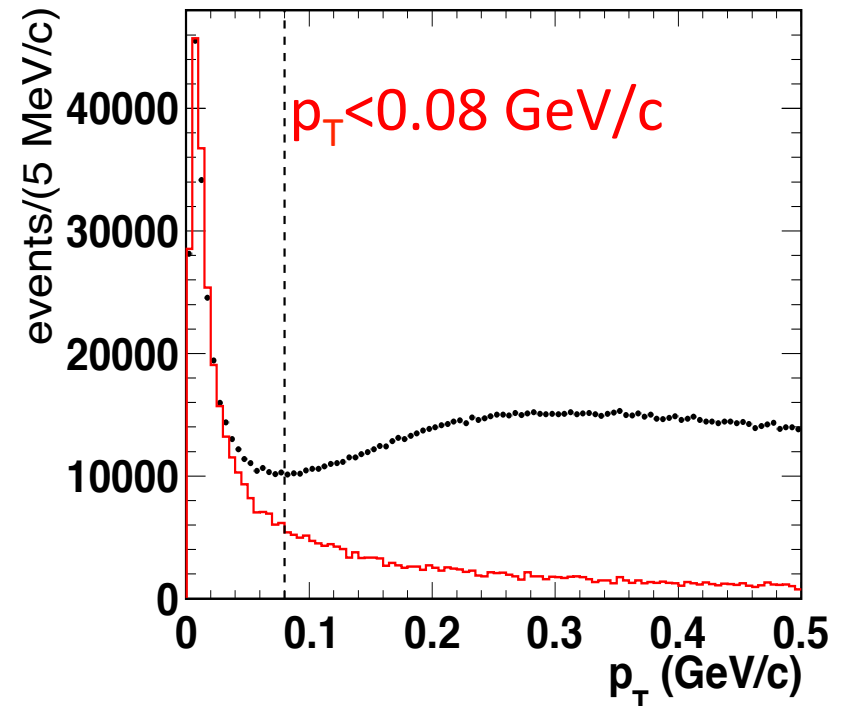
- Dalitz plot analysis of  $\eta_c \rightarrow K \bar{K} \pi$  using MIPWA and isobar model.
  - **Improved** description with MIPWA.
  - $a_0(1950)$  resonance with  $m = 1931 \pm 14 \pm 22$  MeV/ $c^2$  and  $\Gamma = 271 \pm 22 \pm 29$  MeV with significance of  **$2.5\sigma$**  and  **$4.2\sigma$** .
- Extraction of  $I=1/2$   $K\pi$  S-wave using MIPWA
  - Amplitudes show **distinct differences** with previous measurements by E791 and LASS.

# **BACK UP SLIDES**



- Consider only events with 4 charged tracks and no more than 5 photon candidates with energy  $>100$  MeV.
- Vertex fit of oppositely charged tracks to select  $K_S$  candidates.
  - No PID on daughter  $\pi$ , decay length  $>0.2$  cm.
  - Mass  $\pm 2\sigma$  from Gaussian-fitted  $\pi^+\pi^-$  mass spectrum with  $m=497.24$  MeV/ $c^2$ ,  $\sigma=2.9$  MeV/ $c^2$ .
- Loose Kaon and Pion PID for remaining two tracks.
- Background from ISR events with  $J^{PC} = 1^{--}$  resonance production suppressed:

$$M_{rec}^2 = (p_{e^+e^-} - p_{rec})^2 > 10 \text{ GeV}/c^2$$



Selection optimized as a function of  $p_T$ .

**Maximize P•S:**

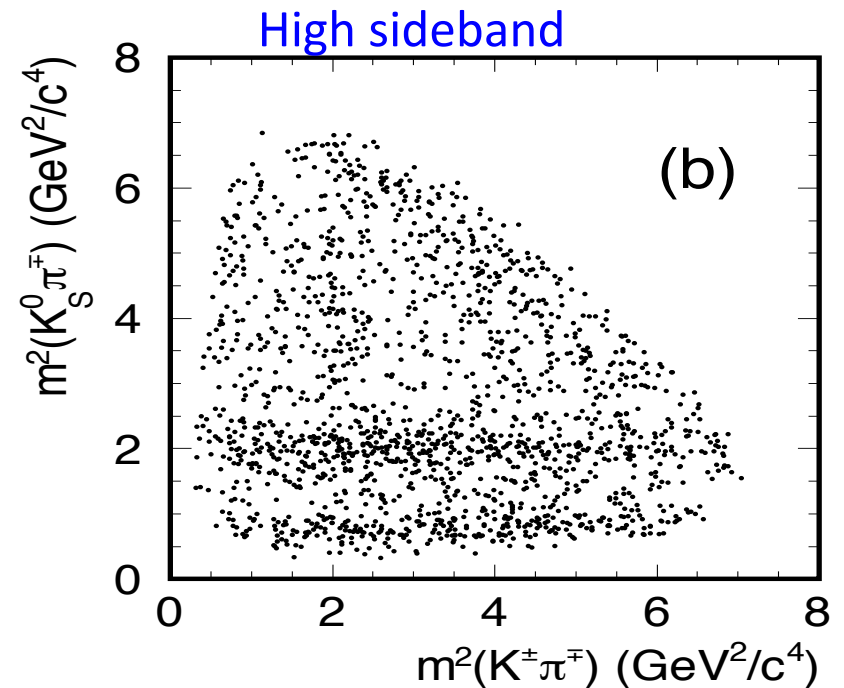
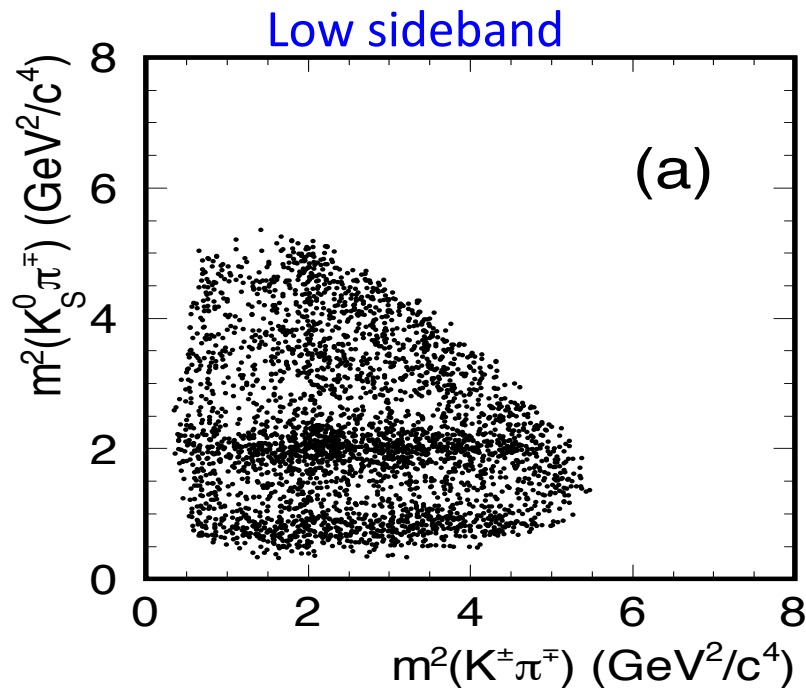
$$P = N_s / (N_s + N_b)$$

$$S = N_s / \sqrt{N_s + N_b}$$



# $\eta_c$ sideband regions

- Uniformly distributed resonant structures in multi-channel likelihood.
- Weaker in  $K^+\pi^-$  than in  $K^0_s\pi^-$  due to interference between  $I=0$  and  $I=1$  configurations.



- Backgrounds from  $a_0(980)$ ,  $a_0(1450)$ ,  $a_2(1320)$ ,  $K^*(892)$ ,  $K^*_0(1430)$ ,  $K^*_2(1430)$ .
- Spin-1 resonance contributions are entirely from background.



# MIPWA of $\eta_c \rightarrow K K \pi$

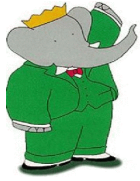
## Systematic uncertainties:

1. Fit bias.
  - Generate signal MC starting from fit.
2. Replace  $K\pi$  S-wave representation with cubic spline
3. Remove low significance contributions
4. Vary signal purity
5. Account for efficiency variation as a function of  $\eta_c$  mass.

N	$K\pi$ mass	$\eta_c \rightarrow K_S^0 K^\pm \pi^\mp$		$\eta_c \rightarrow K^+ K^- \pi^0$	
		Amplitude	Phase (rad)	Amplitude	Phase (rad)
1	0.67	0.119 ± 0.100 ± 0.215	0.259 ± 0.577 ± 1.290	0.154 ± 0.350 ± 0.337	3.786 ± 1.199 ± 0.857
2	0.73	0.103 ± 0.043 ± 0.113	-0.969 ± 0.757 ± 1.600	0.198 ± 0.124 ± 0.216	3.944 ± 0.321 ± 0.448
3	0.79	0.158 ± 0.086 ± 0.180	0.363 ± 0.381 ± 1.500	0.161 ± 0.116 ± 0.098	1.634 ± 0.584 ± 0.448
4	0.85	0.232 ± 0.128 ± 0.214	0.448 ± 0.266 ± 1.500	0.125 ± 0.118 ± 0.031	3.094 ± 0.725 ± 0.448
5	0.91	0.468 ± 0.075 ± 0.194	0.091 ± 0.191 ± 0.237	0.307 ± 0.213 ± 0.162	0.735 ± 0.326 ± 0.255
6	0.97	0.371 ± 0.083 ± 0.129	0.276 ± 0.156 ± 0.190	0.528 ± 0.121 ± 0.055	-0.083 ± 0.178 ± 0.303
7	1.03	0.329 ± 0.071 ± 0.102	0.345 ± 0.164 ± 0.273	0.215 ± 0.191 ± 0.053	0.541 ± 0.320 ± 0.638
8	1.09	0.343 ± 0.062 ± 0.062	0.449 ± 0.196 ± 0.213	0.390 ± 0.146 ± 0.046	0.254 ± 0.167 ± 0.144
9	1.15	0.330 ± 0.070 ± 0.081	0.687 ± 0.167 ± 0.221	0.490 ± 0.135 ± 0.089	0.618 ± 0.155 ± 0.099
10	1.21	0.450 ± 0.059 ± 0.042	0.696 ± 0.156 ± 0.226	0.422 ± 0.092 ± 0.102	0.723 ± 0.242 ± 0.267
11	1.27	0.578 ± 0.048 ± 0.112	0.785 ± 0.208 ± 0.358	0.581 ± 0.113 ± 0.084	0.605 ± 0.186 ± 0.166
12	1.33	0.627 ± 0.047 ± 0.053	0.986 ± 0.153 ± 0.166	0.643 ± 0.106 ± 0.039	1.330 ± 0.264 ± 0.130
13	1.39	0.826 ± 0.047 ± 0.105	1.334 ± 0.155 ± 0.288	0.920 ± 0.153 ± 0.056	1.528 ± 0.161 ± 0.160
14	1.45	1.000	1.570	1.000	1.570
15	1.51	0.736 ± 0.031 ± 0.059	1.918 ± 0.153 ± 0.132	0.750 ± 0.118 ± 0.076	1.844 ± 0.149 ± 0.048
16	1.57	0.451 ± 0.025 ± 0.053	2.098 ± 0.202 ± 0.277	0.585 ± 0.099 ± 0.047	2.128 ± 0.182 ± 0.110
17	1.63	0.289 ± 0.029 ± 0.065	2.539 ± 0.292 ± 0.180	0.366 ± 0.079 ± 0.052	2.389 ± 0.230 ± 0.213
18	1.69	0.159 ± 0.036 ± 0.089	1.566 ± 0.308 ± 0.619	0.312 ± 0.074 ± 0.043	1.962 ± 0.195 ± 0.150
19	1.75	0.240 ± 0.034 ± 0.067	1.962 ± 0.331 ± 0.655	0.427 ± 0.093 ± 0.063	1.939 ± 0.150 ± 0.182
20	1.81	0.381 ± 0.031 ± 0.059	2.170 ± 0.297 ± 0.251	0.511 ± 0.094 ± 0.063	2.426 ± 0.156 ± 0.277
21	1.87	0.457 ± 0.035 ± 0.085	2.258 ± 0.251 ± 0.284	0.588 ± 0.098 ± 0.080	2.242 ± 0.084 ± 0.210
22	1.93	0.565 ± 0.042 ± 0.067	2.386 ± 0.255 ± 0.207	0.729 ± 0.114 ± 0.095	2.427 ± 0.098 ± 0.254
23	1.99	0.640 ± 0.044 ± 0.055	2.361 ± 0.228 ± 0.092	0.777 ± 0.119 ± 0.075	2.306 ± 0.102 ± 0.325
24	2.05	0.593 ± 0.046 ± 0.065	2.329 ± 0.235 ± 0.268	0.775 ± 0.134 ± 0.075	2.347 ± 0.107 ± 0.299
25	2.11	0.614 ± 0.057 ± 0.083	2.421 ± 0.230 ± 0.169	0.830 ± 0.134 ± 0.078	2.374 ± 0.105 ± 0.199
26	2.17	0.677 ± 0.067 ± 0.117	2.563 ± 0.218 ± 0.137	0.825 ± 0.140 ± 0.070	2.401 ± 0.127 ± 0.189
27	2.23	0.788 ± 0.085 ± 0.104	2.539 ± 0.228 ± 0.241	0.860 ± 0.158 ± 0.123	2.296 ± 0.131 ± 0.297
28	2.29	0.753 ± 0.097 ± 0.125	2.550 ± 0.234 ± 0.168	0.891 ± 0.167 ± 0.133	2.320 ± 0.131 ± 0.273
29	2.35	0.646 ± 0.096 ± 0.118	2.315 ± 0.241 ± 0.321	0.994 ± 0.202 ± 0.076	2.297 ± 0.153 ± 0.197
30	2.41	0.789 ± 0.184 ± 0.187	2.364 ± 0.336 ± 0.199	0.892 ± 0.322 ± 0.098	2.143 ± 0.292 ± 0.393

Effect of systematic uncertainties is an average of 16 %.

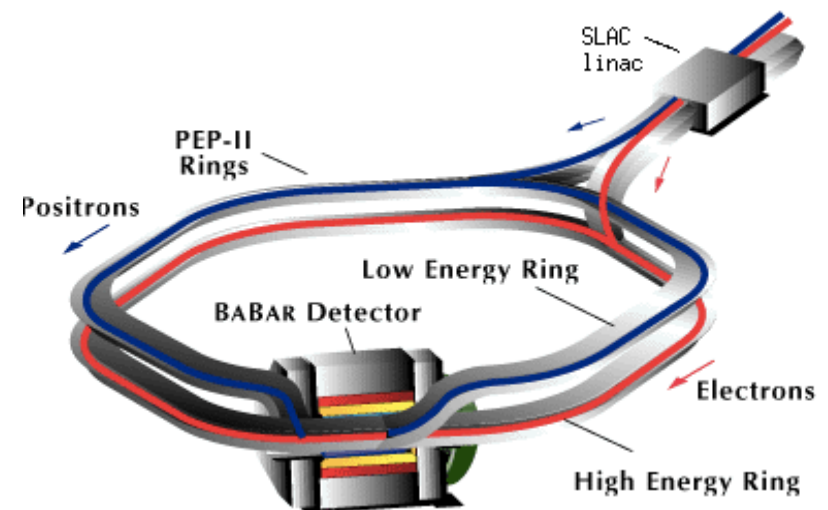
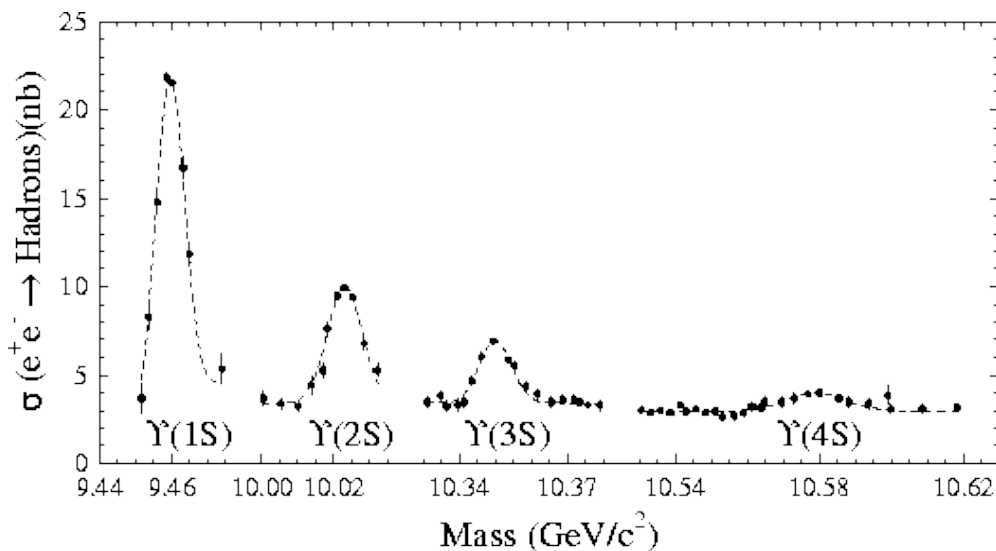
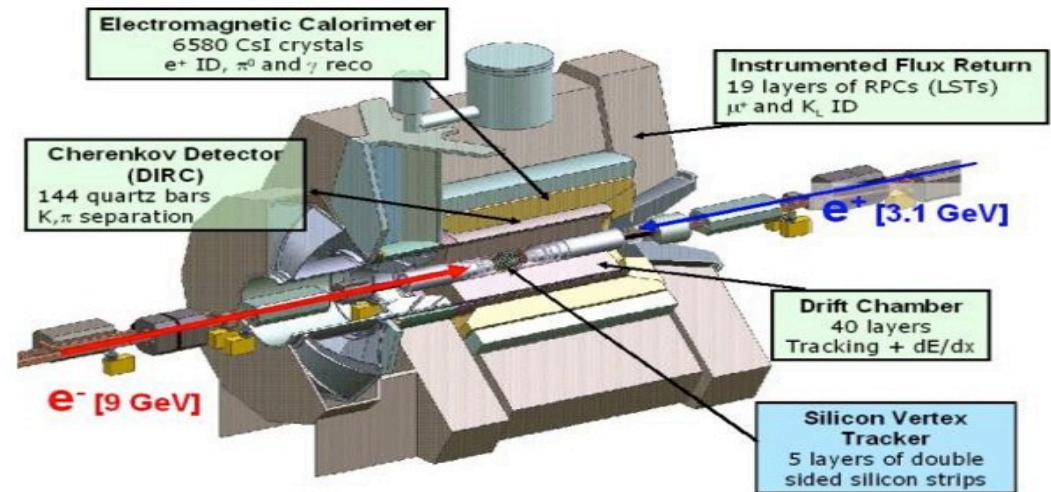


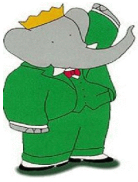


# BaBar Experiment:

*Phys. Rev. D 89, 112004 (2014)*

- Located at SLAC National Accelerator Laboratory
- Asymmetric  $e^+e^-$  collisions at CM energy of 10.58 GeV .
- Data collection 1999 to 2008.





# Resonance Parameterization

- Each amplitude is parameterized as the product of a complex Breit-Wigner and a real angular term T:

$$A(x, y) = BW(m) \times T(\Omega).$$

- Relativistic BW is written as:  $D \rightarrow rc, r \rightarrow ab$

$$BW(M_{AB}) = \frac{F_r F_D}{M_r^2 - M_{AB}^2 - i\Gamma_{AB} M_r} \quad \Gamma_{AB} = \Gamma_r \left( \frac{p_{AB}}{p_r} \right)^{2J+1} \left( \frac{M_r}{M_{AB}} \right) F_r^2$$

- $F_r$  and  $F_D$  are form factors

- $f_0(980)$  amplitude parameterized as:

$$A_{f_0(980)} = \frac{1}{m_0^2 - m^2 - im_0 \Gamma_0 \rho_{KK}},$$

$$\rho_{KK} = 2p/m$$

$$m_0 = (0.922 \pm 0.003_{\text{stat}}) \text{ GeV}/c^2,$$

$$\Gamma_0 = (0.24 \pm 0.08_{\text{stat}}) \text{ GeV}.$$

- Coupled channel BW (Flatte) formalism does not take into account coupling to the  $\pi\pi$  channel:

- $a_0(980)$  amplitude parameterized using Flatte formalism, because of its coupling to KK and  $\pi\eta$ :

$$F_0 = \beta'_0 \frac{\begin{pmatrix} g_1 \\ g_2 \end{pmatrix}}{m_0^2 - m^2 - i(\rho_1 g_1^2 + \rho_2 g_2^2)}.$$



# Model Independent Partial Wave Analysis

- $k\pi$  S-wave is taken as reference amplitude,

$$A = |A_1 + c_2 A_2 e^{\phi^2} + c_3 A_3 e^{\phi^3} + \dots|$$

- Other resonance contributions,  $A_2$ ,  $A_3$ , etc..., are parametrized as Breit-Wigner amplitudes multiplied by their corresponding angular functions.
- Mass spectrum is divided into 30 equally spaced mass intervals 60 MeV wide.
- In each interval, 2 new free parameters are added:  $A_i$  and  $\phi_i$



# Outline

- Theoretical Motivation
- $\gamma\gamma \rightarrow K_S K^+ \pi^-$  and  $\gamma\gamma \rightarrow K^+ K^- \pi^0$ 
  - PHYSICAL REVIEW D **89**, 112004 (2014)
- Model Independent Partial Wave Analysis
- $K\pi$  S-wave amplitude