Diamagnetic levitation – a platform for quantum sensing

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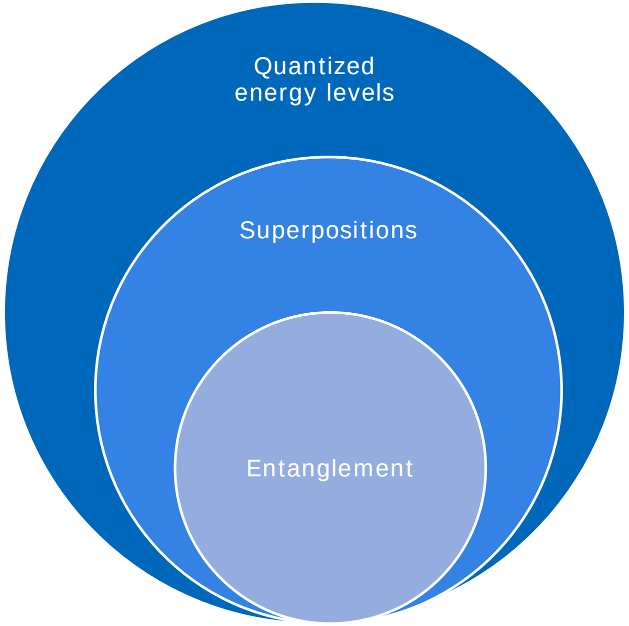
What is quantum sensing

- The measurement of any property of a physical system is ultimately described by quantum mechanics.
- Quantum sensors use quantization, superpositions, or entanglement to enhance physical measurements.
- Advanced quantum sensors leverage quantum error correction and phase transitions.
- Advantages of quantum sensors
  - Potential to increase sensitivity and resolution (including improved scaling)
  - Accuracy based on fundamental constants for quantum sensors based on microscopic systems ("absolute sensors")

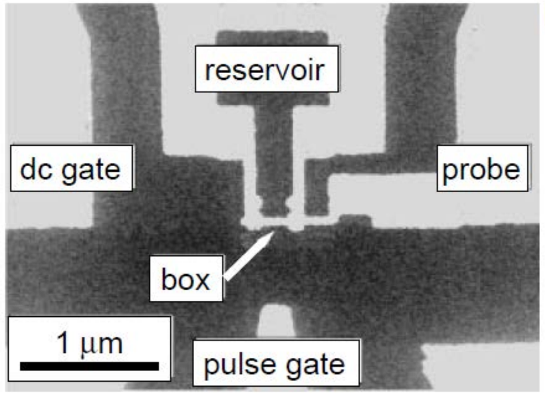
Quantum sensing hierarchy



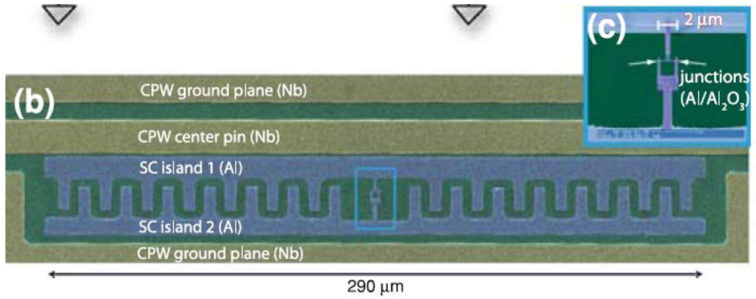
Degen et al., Reviews of Modern Physics 89, 035002 (2017).
Arrad et al., Physical Review Letters 112, 150801 (2014).
Macieszczak et al., Physical Review A 93, 022103 (2016).

Superconducting qubits

Charge qubits/transmon

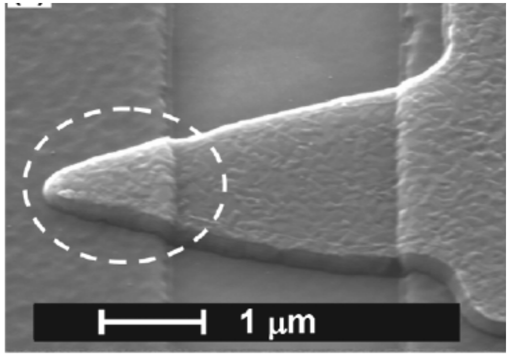


Nakamura et al. (1999)



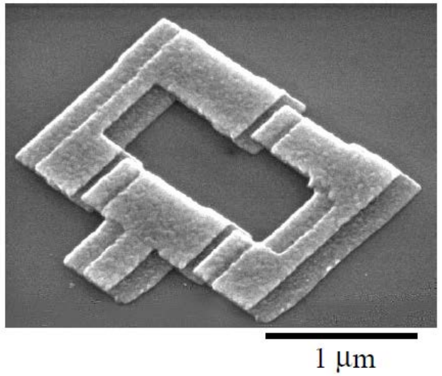
Houck et al. (2009)

Phase qubits



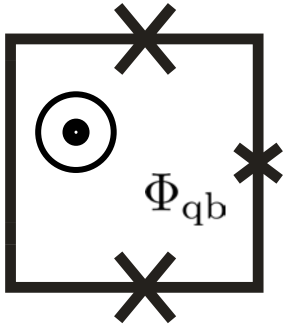
Steffen et al. (2006)

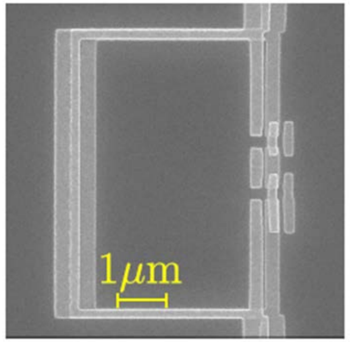
Flux qubits

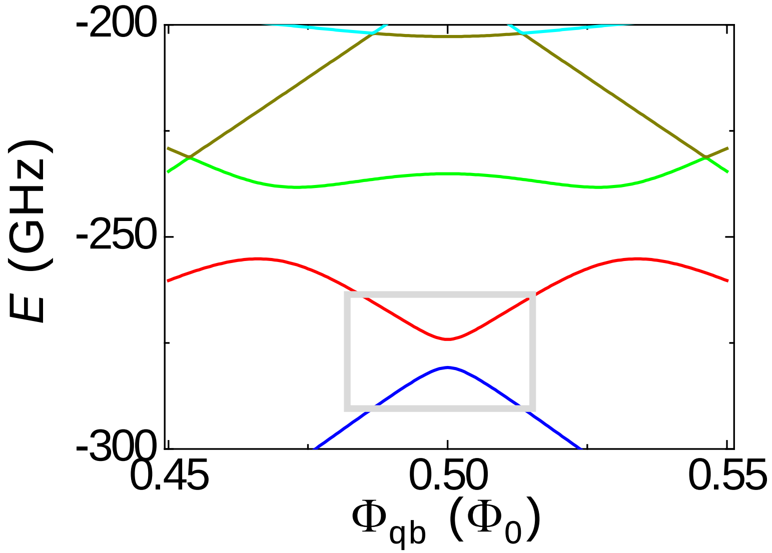


van der Wal et al. (2001)

The flux qubit







Large anharmonicity – two-level system

Two-level system (qubit) model

$$H = -\frac{h\epsilon}{2}\sigma_z - \frac{h\Delta}{2}\sigma_x$$

In basis: $\{|\circlearrowright\rangle, |\circlearrowleft\rangle\}$

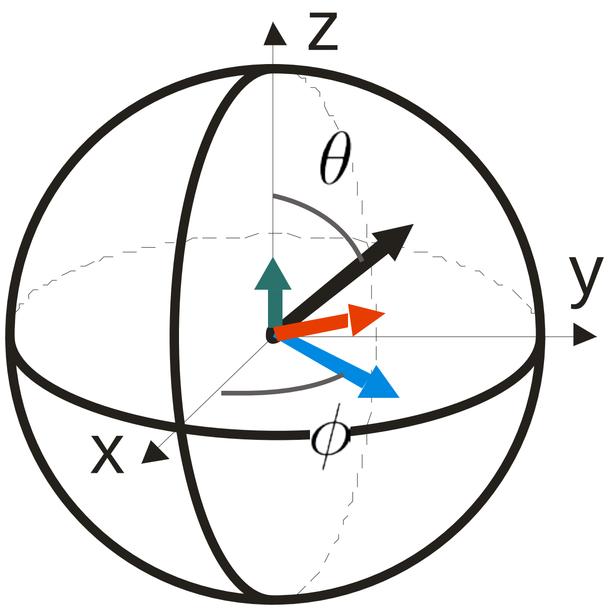
$$\epsilon = 2I_p\left(\Phi_{qb} - \frac{\Phi_0}{2}\right)$$

$$\hat{\sigma}_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \hat{\sigma}_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

I_p , Δ: ***design*** parameters
Φ_{qb} : ***control*** parameter

Mooij et al., Science 285 1036 (1999).

Single qubit control



State representation

$$|\psi\rangle = \cos\left(\frac{\theta}{2}\right)|0\rangle + \sin\left(\frac{\theta}{2}\right)e^{i\phi}|1\rangle$$

Control

$$H = -\frac{\hbar\Delta}{2}\sigma_x - \frac{\hbar\epsilon}{2}\sigma_z$$

$$\epsilon = \epsilon_0 + A\cos(\omega t + \phi)$$

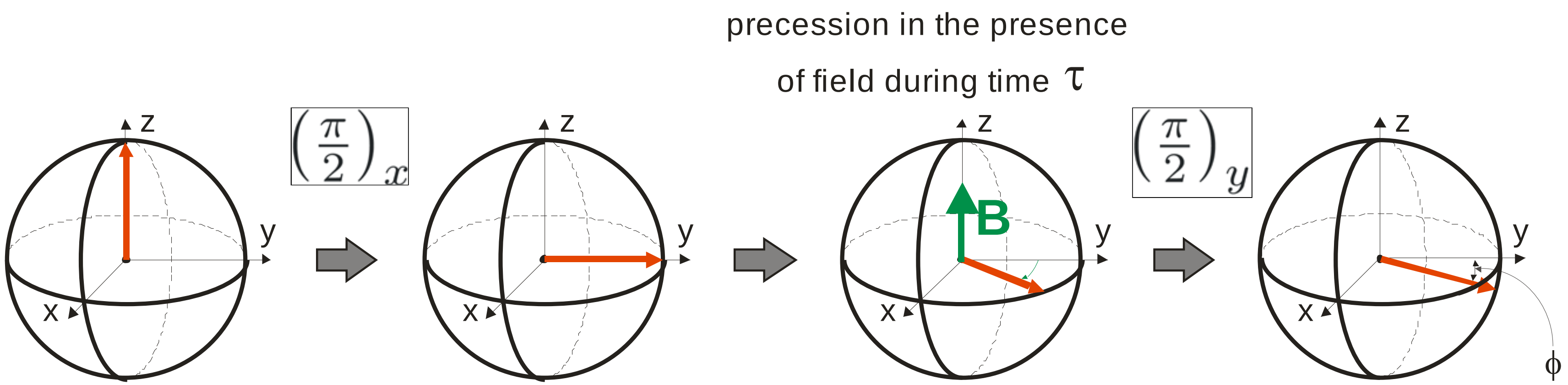
$$\omega_{01} = \sqrt{\Delta^2 + \epsilon_0^2}$$

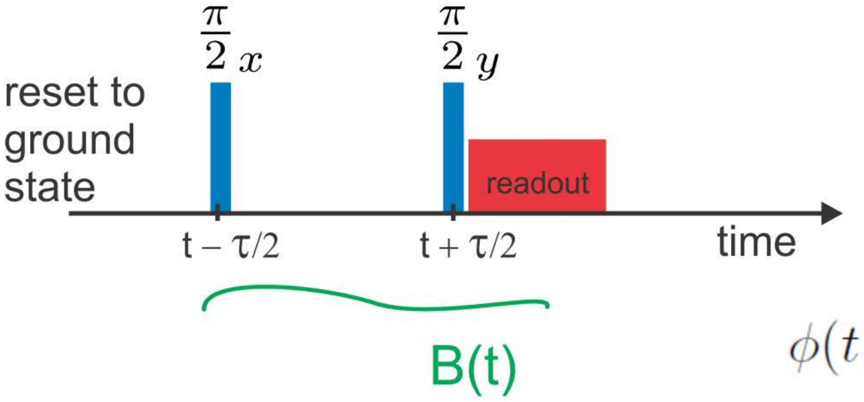
Evolution in the rotating frame

Polarization vector precesses around $\boldsymbol{\omega} = (\omega_x, \omega_y, \omega_z)$

$$\omega_x = -A\frac{\Delta}{\sqrt{\Delta^2+\epsilon_0^2}}\cos(\phi) \quad \omega_y = A\frac{\Delta}{\sqrt{\Delta^2+\epsilon_0^2}}\sin(\phi) \quad \omega_z = \omega - \omega_{01}$$

Magnetic field detection based on Ramsey interferometry using atoms





$$\phi(t,\tau) = \frac{1}{\hbar}\int_{t-\tau/2}^{t+\tau/2} \mu B(t')dt'$$

Budkert and Romalis, Nature Physics 3, 227 (2007).

Signal and noise

- Phase precession

$$\phi = -\frac{2\mu B}{\hbar}\tau$$

- Statistics of an average of N measurements

$$r_N = \frac{1}{N}\sum_{i=1}^{N} r_i \qquad \langle r_N \rangle = \sin\phi \approx -\frac{2\mu B}{\hbar}\tau$$

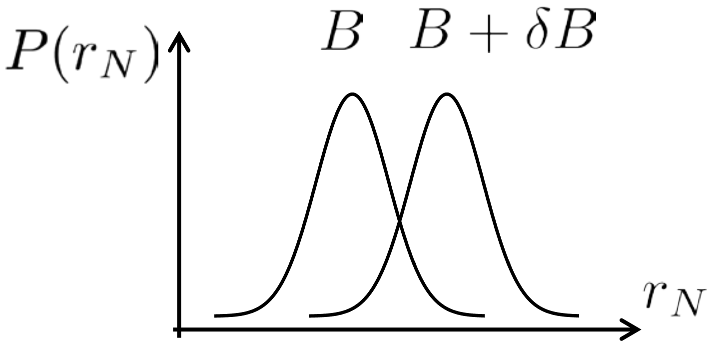
$$r_i = -1 \text{or} 1 \qquad \sigma_N = \frac{1}{\sqrt{N}} = \frac{\sqrt{\tau}}{\sqrt{T}}$$

- Signal to noise ratio

$$SNR = \frac{2\mu B}{\hbar}\sqrt{\tau}\sqrt{T}$$

Sensitivity

- Condition for distinguishability



Signal difference δB

$$1 = SNR = \frac{2\mu\delta B}{\hbar}\sqrt{\tau}\sqrt{T}$$

We can tell any field difference δB if T is long enough. The minimum detectable difference

$$\delta B_{\min}\sqrt{T} = \frac{\hbar}{2\mu\sqrt{\tau}}$$

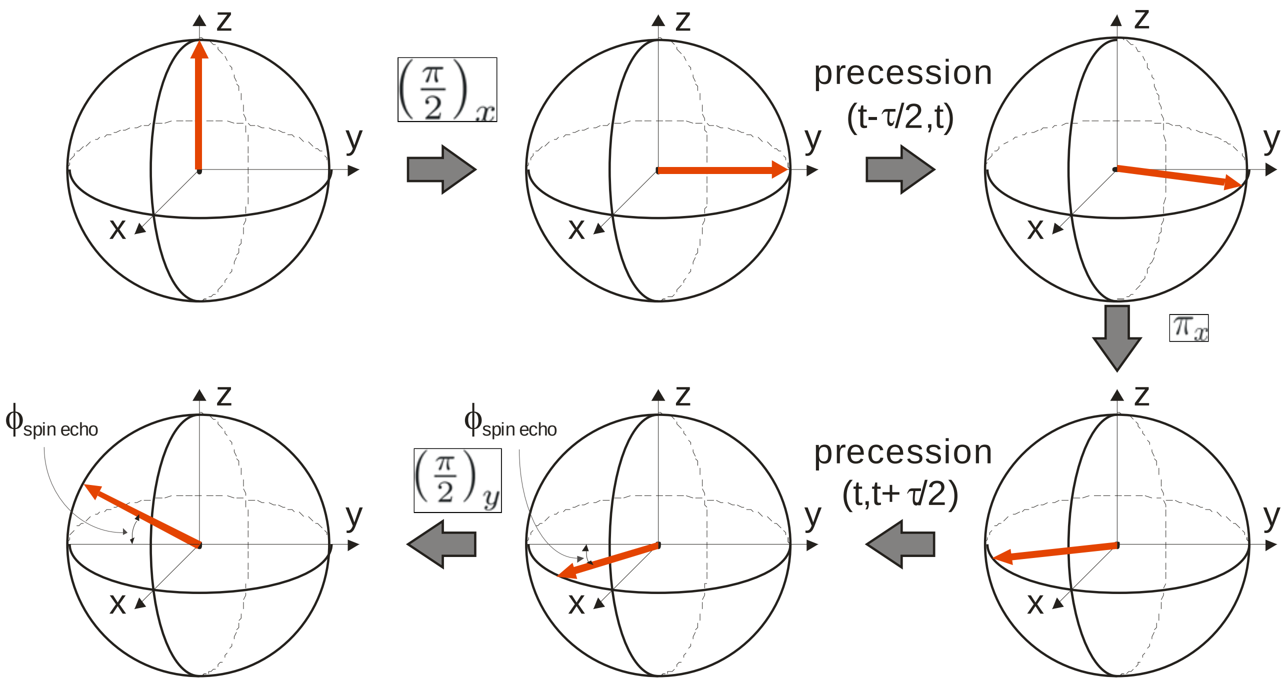
This is the *sensitivity:*

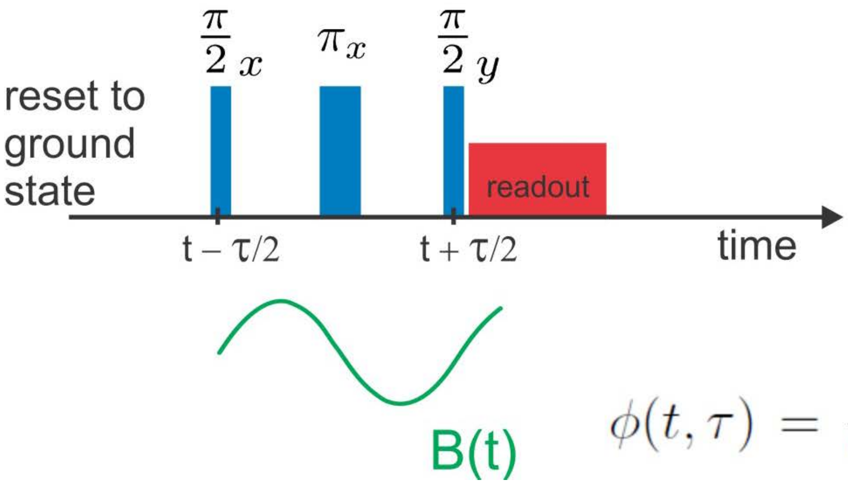
$$\text{Sensitivity} = \frac{\hbar}{2\mu\sqrt{\tau}}$$

- Optimal time: $\tau = T_{\text{coh}}$ (coherence time)

$$\text{Sensitivity} = \frac{\hbar}{2\mu\sqrt{T_{\text{coh}}}}$$

AC magnetic field detection, spin-echo





$$\phi(t,\tau) = \frac{1}{\hbar}\int_{t-\tau/2}^{t} \mu B(t')dt' - \frac{1}{\hbar}\int_{t}^{t+\tau/2} \mu B(t')dt'$$

Spin magnetometry with natural vs artificial atoms

- Single spin magnetometers sensitivity $\sim \frac{1}{\mu\sqrt{T_{\text{coh}}}}$
- Comparison

	Magnetic moment	Dephasing time
Potassium	μ_B	0.1 s
Flux qubit	$3.8 \times 10^5\ \mu_B$	126 ns

  - Flux qubit size is comparable with atomic density (which impacts atomic coherence time)
  - The coherence time for the flux qubit
    - Coherence depends on frequency (assumed ~MHz)
    - Short coherence is due to coupling to magnetic field, which activates coupling to intrinsic flux noise
- Tradeoff between coupling and coherence does not have a simple fundamental scaling interpretation

Comparison with other sensors

Comparison with DC-SQUIDs

Relevant figure of merit for comparison: $S_\epsilon = S_\Phi / 2L$

	${S_\Phi}^{1/2}$ $(\Phi_0/\sqrt{\mathrm{Hz}})$	$S_\epsilon = S_\Phi/2L$
This Work	3.9×10^{-8}	$1.1\hbar$
D. D. Awschalom *et al.*, APL **53**, 2108 (1988)	8.4×10^{-8}	$1.4\hbar$
F. C. Wellstood *et al.*, IEEE Tran. Magn. **25**, 1001 (1989)	3.5×10^{-7}	$5\hbar$

Comparison with atomic magnetometers

Relevant figure of merit for comparison: $\delta B_{\mathrm{min}}\sqrt{T}\sqrt{V}$

$\delta B_{\mathrm{min}}\sqrt{T} \sim 0.1 - 1\ \mathrm{fT}/\sqrt{\mathrm{Hz}}$ for $V \sim \mathrm{cm}^3$

- I. Kominis *et al.*, Nature **422**, 596 (2003)
- H. B. Dang *et al.*, APL **97**, 151110 (2010)

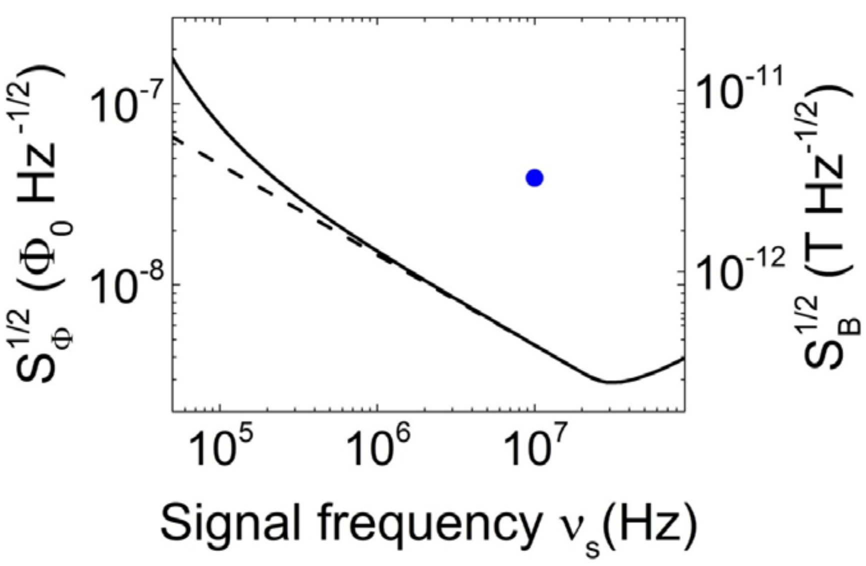
Theoretical limit to sensitivity:
$\delta B_{\mathrm{min}}\sqrt{T} \sim 1\ \mathrm{pT}/\sqrt{\mathrm{Hz}}$ for $V \sim \mu\mathrm{m}^3$

- V. Shah *et al.*, Nature Photonics **1**, 649 (2007)

This work: demonstrated 3.3 pT/rt(Hz), one order of magnitude improvement possible.

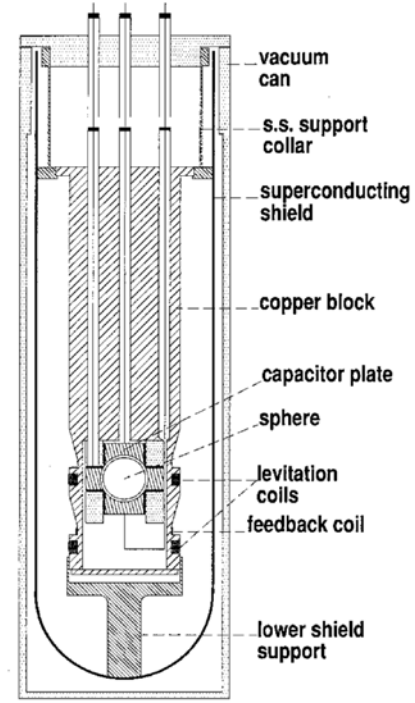
Fundamental limits

- Theoretical model
  - Perfect fidelity
  - State preparation and measurement short compared to evolution time
  - 1/f noise spectrum
- Qubits attain the detection limit set by 1/f noise (in contrast with DC-SQUIDs)
- Any future improvements in 1/f noise will lead to further improvements of sensitivity



Superconducting levitation

https://en.wikipedia.org/wiki/SCMaglev

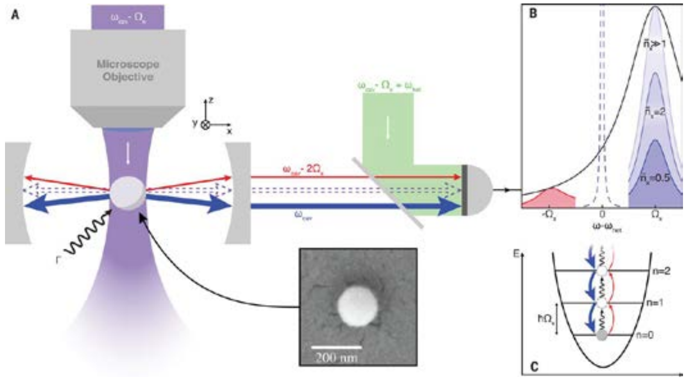


Goodkind, Rev. Sci. Instr. 70, 4131 (1999).

Levitation of nano/micro particles

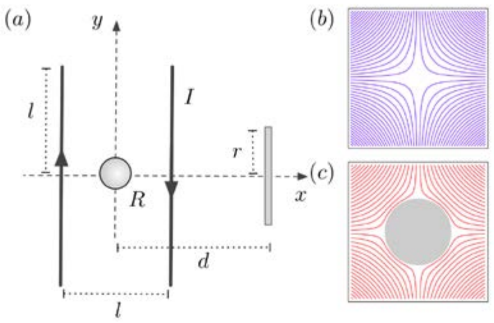
Superconducting diamagnetic levitation

Levitation using radiation pressure (optical tweezers)



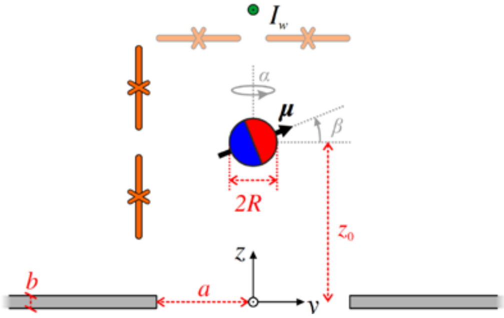
Delic et al., Science 367, 892 (2020).

Superconducting spheres



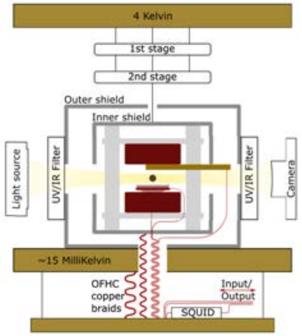
Romero-Isart et al., Phys. Rev. Lett. 109, 147205 (2012).

Ferromagnetic spheres

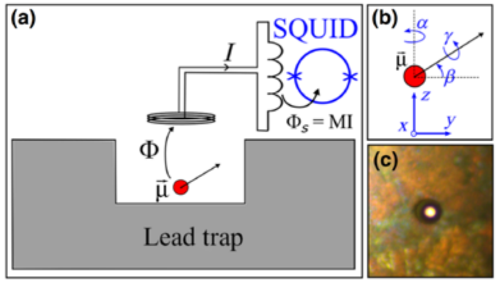


Prat-Camps et al., Phys. Rev. Appl. 8, 034002 (2017).

Recent experiments: mechanical quality factors of the order 10^7 !



Hofer et al., Phys. Rev. Lett. 131, 043603 (2023).



Vinante et al., Phys. Rev. Appl. 13, 064027 (2020).

Prospects for low temperature diamagnetic levitation

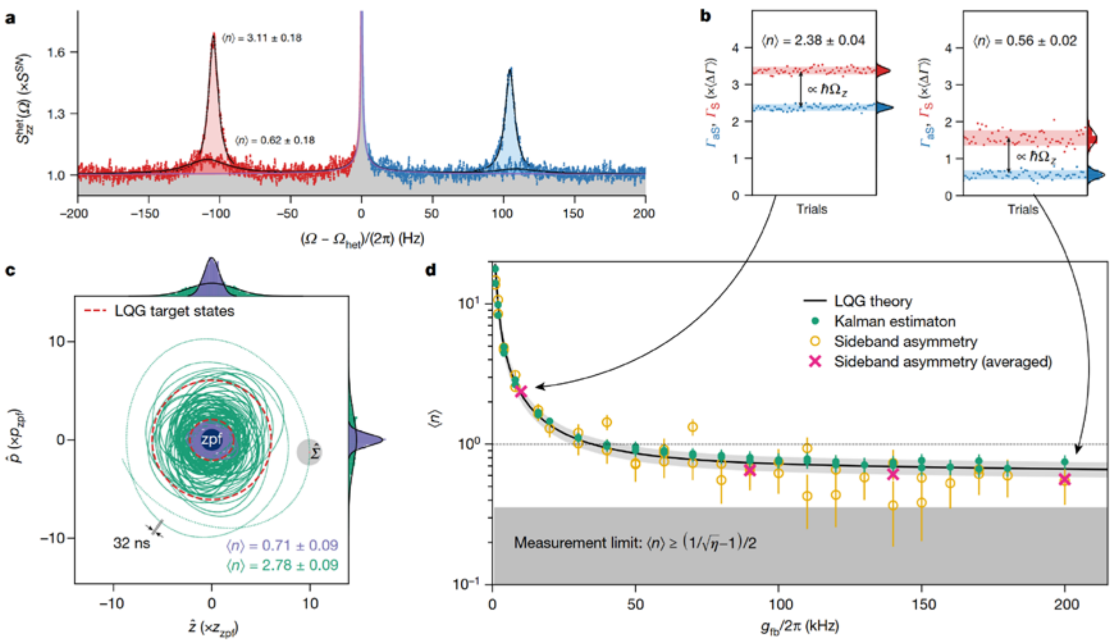
- The long coherence time is an asset for both quantum sensing and quantum computing
- Quantum sensing
  - Versatility: coupling to acceleration/gravity and electric/magnetic field
  - Applications: geophysics, navigation, dark matter, etc
- Quantum computing
  - Quantum memory is a potential intriguing application
- Challenges
  - Isolation from the environment
  - State preparation
  - State measurement

Decoupling from the environment

- Trapping potential noise
  - It is expected that operation of trapping coils in persistent current mode is required
- Residual gas collisions
  - Analyzed theoretically and in recent experiments with levitated superconducting spheres and diamagnetically levitated ferromagnets. This source of dissipation is strongly reduced in a cryogenic environment.
  - Levitation in helium (Arrayás et al. JLTP 212, 363, 2023) https://doi.org/10.1007/s10909-022-02925-3.)
- Decoherence due to surfaces
  - Known to be a limiting factor in trapped ion and atom chip experiments
- Mechanical vibrations

State preparation

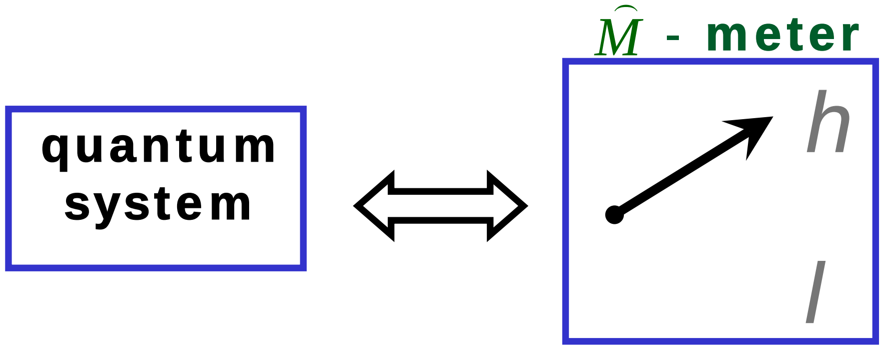
- Feedback cooling
  - Requires quantum limited measurements of position
  - Demonstrated for optical trapping of microsphere



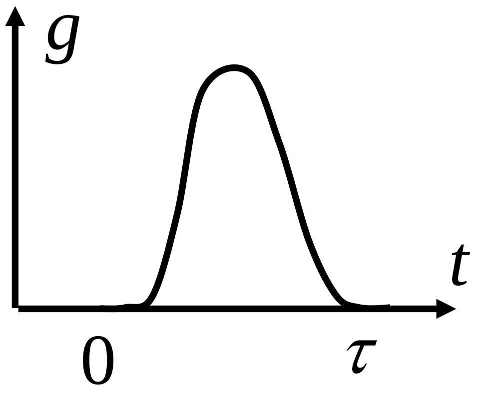
Magrini et al., Nature 595, 373 (2021).

- Cooling based on measurement

Quantum non-demolition measurements



$$\boxed{\hat{H}_s + g(t)\hat{H}_{int}} + \cancel{\hat{H}_{meter}}$$



- *Consideration of the* ***measured system – detector interaction*** *only can give useful insight.*

- ***Detector dynamics*** does not need to be taken into account.

***Quantum non-demolition (QND) measurements* – particular choice of $\hat{M}$, $\hat{H}_{int}$, and $g(t)$ which guarantees that the eigenstates $|i\rangle$ of $\hat{M}$ are preserved.**

Quantum non-demolition measurements for harmonic oscillators

- The number of quanta
  - The operator $n = a^{\dagger}a$ trivially commutes with the Hamiltonian
- Quadrature operators
  - $X_1 + i\,X_2 = \left(x + \frac{ip}{(m\,\omega)p}\right)e^{i\,\omega t}$
  - The quadrature operators and their linear combinations are constants of motion in the rotating frame
- Quadrature vs energy measurements
  - Rely on linear vs quadratic interactions, which are more common and stronger
  - Require suitable modulation of the interaction strength

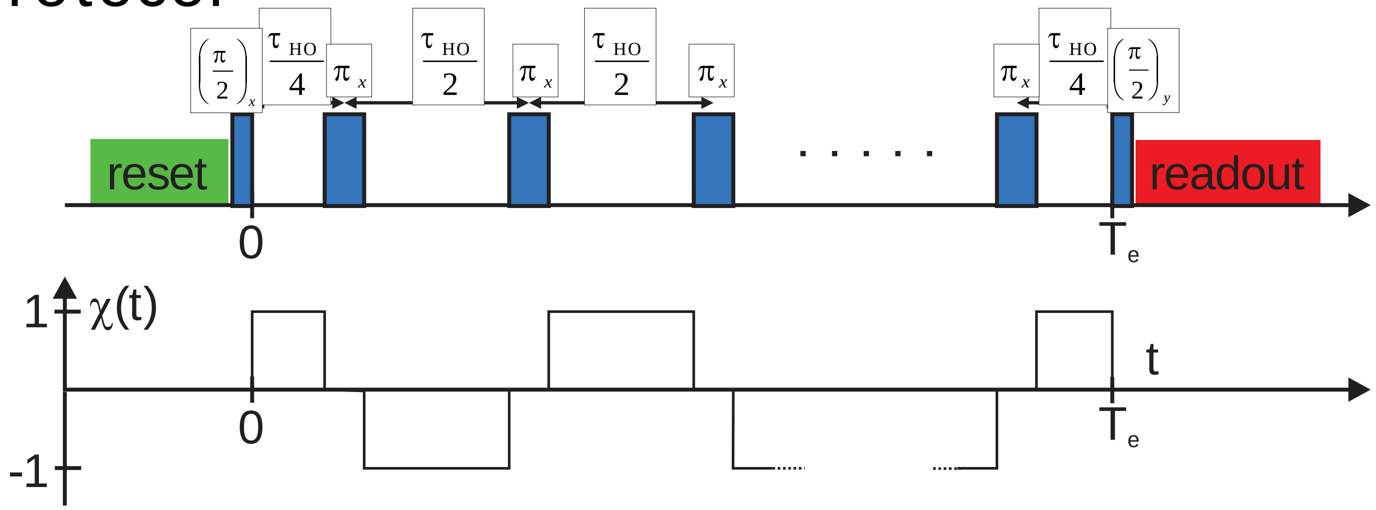
Thorne et al., Phys. Rev. Lett. 40, 667 (1978).

Quadratures measurements with a qubit

- Interaction (in the lab frame)

$$H = \omega_r a^\dagger a - \frac{\omega_{ge}}{2}\sigma_z + g(a + a^\dagger)\sigma_z + f(t)\sigma_x$$

- Control protocol



$$H_{\text{eff}}(t) = g\chi(t)(ae^{-i\omega_r t} + a^\dagger e^{i\omega_r t})\sigma_z$$

$$H_{\text{avg}} = \frac{2}{\pi} g\sigma_z(a + a^\dagger)$$

See also modulated interactions,
Caves et al. RMP (1980)

Canturk and Lupascu, arXiv:1704.04533 (2017).

Measurement protocol

$$H_{\mathrm{avg}} = \frac{2}{\pi} g \sigma_z I$$

Detect quadrature I by measuring qubit phase (Ramsey interferometry)

Formal treatment

$$|g\rangle \otimes |\alpha_{i-1}\rangle$$

interaction

$$|g\rangle \otimes D_{\mathrm{g}}|\alpha_{i-1}\rangle + |e\rangle \otimes D_{\mathrm{e}}|\alpha_{i-1}\rangle$$

Qubit readout

$r = -1$ $r = 1$

$$|\alpha_i\rangle = D_{\mathrm{g}}|\alpha_{i-1}\rangle / \sqrt{P_g}$$

$$P_g = ||D_{\mathrm{g}}|\alpha_{i-1}\rangle||^2$$

$$|\alpha_i\rangle = D_{\mathrm{e}}|\alpha_{i-1}\rangle / \sqrt{P_e}$$

$$P_e = ||D_{\mathrm{e}}|\alpha_{i-1}\rangle||^2$$

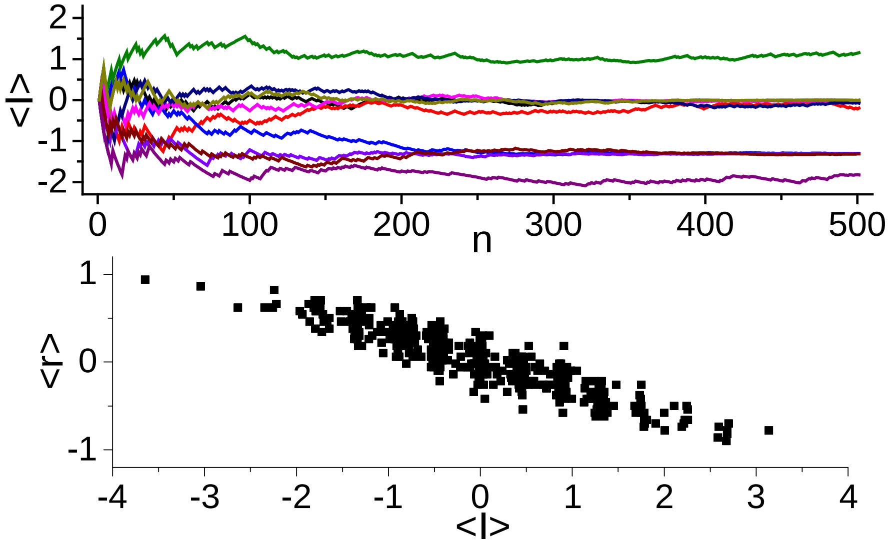
$$D_{\mathrm{g}} = -\frac{1}{2}(D - iD^\dagger)$$

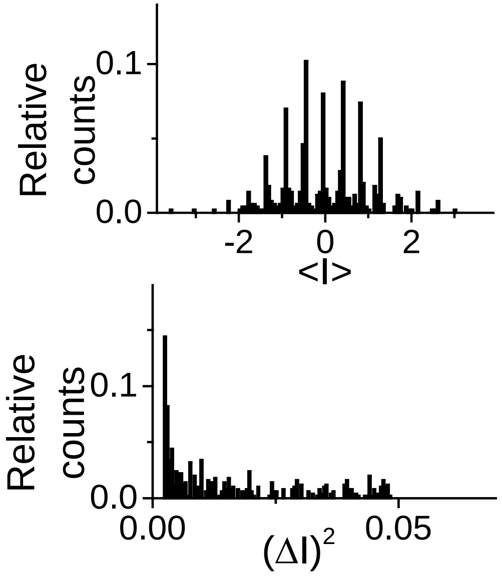
$$D_{\mathrm{e}} = -\frac{1}{2}(D + iD^\dagger)$$

$$D = \mathcal{D}(i\phi) \quad \phi = \frac{2}{\pi} g T_e$$

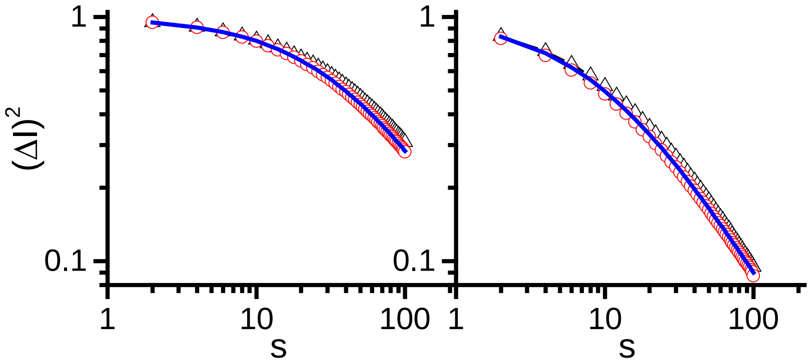
Simulation of trajectories

- Stochastic simulation confirm squeezing of quadratures





- Analytical approximation for squeezing



n=s/2

$$(\Delta I)^2_{s/2,s/2} = \frac{1}{1+4\phi^2 s}$$

average over n

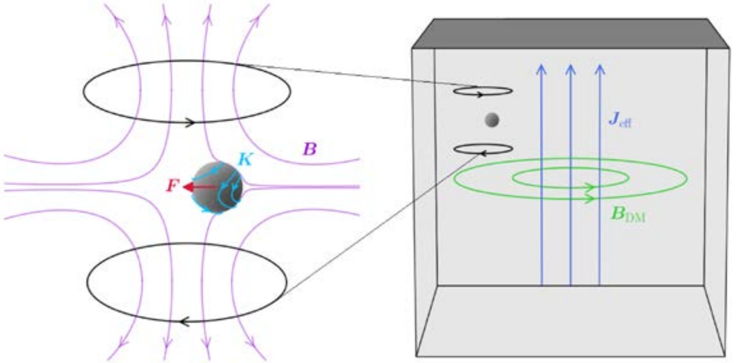
Implementation of high efficiency quadrature measurements

- The protocol discussed requires long qubit coherence: $T_2 \gtrsim 2\,\pi\omega_r$.
  - Longest qubit coherence realized for fluxonium $T_2 \sim 1$ ms in optimized geometries
  - Levitated object resonance frequency $\frac{\omega_r}{2\pi} \sim 100$ Hz
- An alternative to the protocol above is implementation of stroboscopic measurements, where the HO – qubit coupling is turned on for $T_i \ll \frac{2\pi}{\omega_r}$.
  - Measurements can exceed the SQL by a factor $\frac{1}{\sqrt{\omega_r T_i}}$
- Efficient quadrature measurements can be used both for
  - State preparation
  - High efficiency force detection

Braginsky and Khalili, Quantum Measurement (1995).
Brunelli et al., Phys. Rev. Res. 2, 023241 (2020).

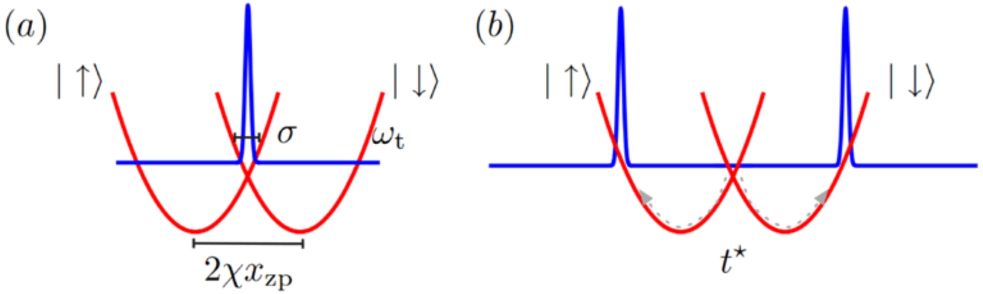
Fundamental physics applications

- Dark matter detection



Higgins et al., https://arxiv.org/abs/2310.18398v1 (2023).

- Decoherence at macroscopic scales

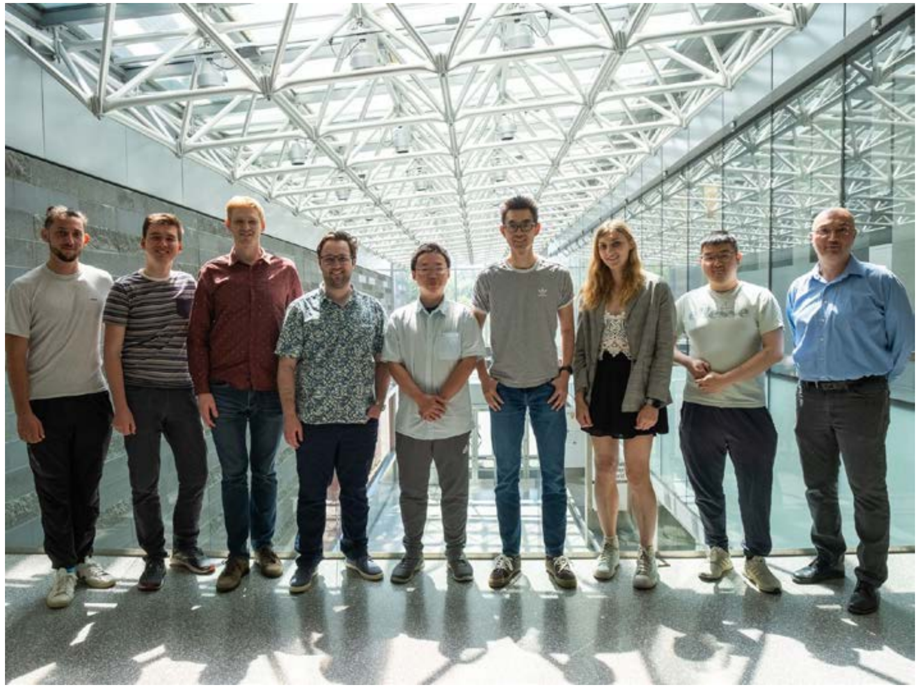


Romero-Isart et al., Physical Review Letters 109, 147205 (2012).

Summary and outlook

- Superconducting qubits have properties that enable unique advantages in quantum sensing
- A hybrid platform formed of superconducting qubits and diamagnetically levitated microspheres has potential for new quantum sensing and computing applications

Acknowledgements



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