

Loop corrections to supergravity

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based on:

1706.02388 w/Alday

1612.03891 w/Aharony, Alday and Perlmutter

Boundary and defect CFT:

open problems and applications

AdS/CFT correspondence:

Gravitational theory in
D+1 dimensions

CFT in D dimensions

G_N

Newton Constant

Loops in AdS



N

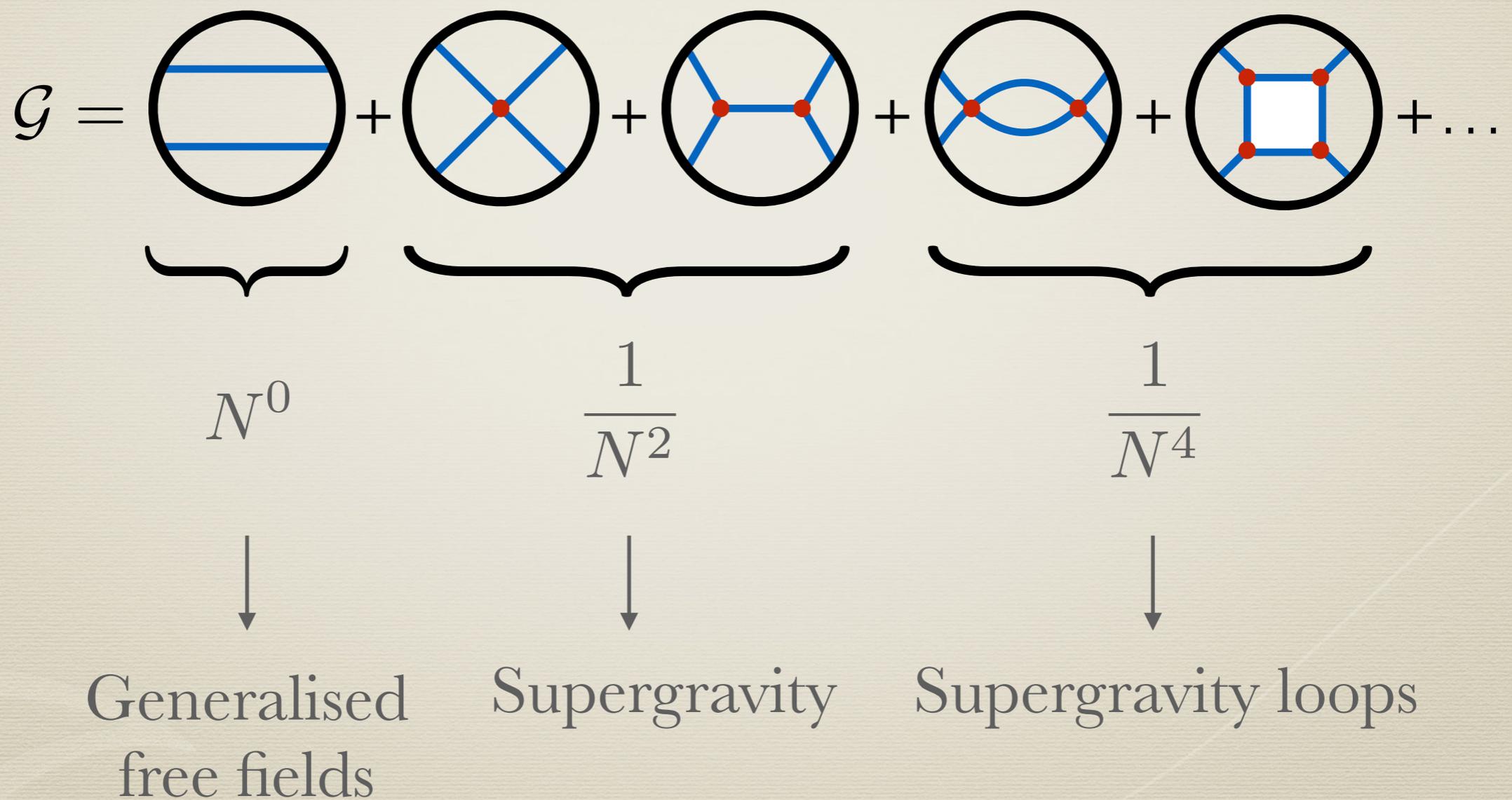
rank of the gauge group

$\frac{1}{N^2}$ expansion

Use structure and symmetries (superconformal symmetry)
of the CFT to compute loop amplitudes in AdS

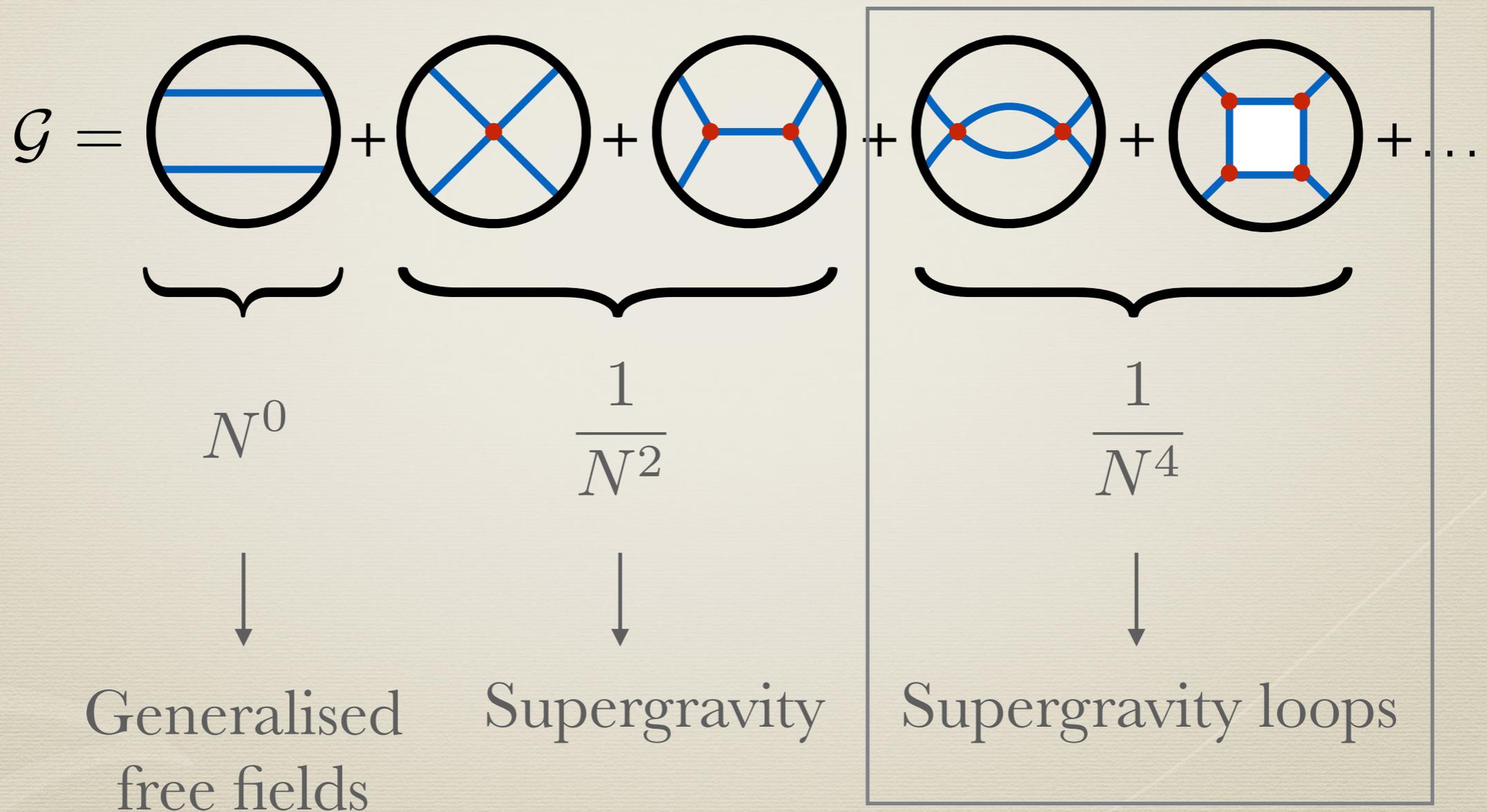
General idea:

$\mathcal{N} = 4$ SYM with $SU(N)$ gauge group at large $\lambda = g^2 N$



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$\mathcal{N} = 4$ SYM with $SU(N)$ gauge group at large $\lambda = g^2 N$



- Consider the superconformal primary \mathcal{O}_2 with protected dimension $\Delta = 2$, transforming in the $[0, 2, 0]$ of $SU(4)$ R-symmetry $\longrightarrow \frac{1}{2}$ BPS operator

$$\langle \mathcal{O}_2(x_1) \mathcal{O}_2(x_2) \mathcal{O}_2(x_3) \mathcal{O}_2(x_4) \rangle = \sum_R \frac{\mathcal{G}^R(u, v)}{x_{12}^4 x_{34}^4}$$

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all the reps in $[0, 2, 0] \times [0, 2, 0]$
cross ratios

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- Superconformal Ward identities imply relations among \mathcal{G}^R allowing the entire four point function to be expressed in terms of a non trivial function $\mathcal{G}(u, v)$

OPE content

$$\mathcal{O}_2 \times \mathcal{O}_2 \sim \mathcal{O}_{long} + \mathcal{O}_{short}$$

\mathcal{O}_{long}

acquire anomalous dimension

$$\Delta_L = \Delta(g_{YM}, N)$$

$$c_L = c_{\mathcal{O}_2 \mathcal{O}_2 \mathcal{O}_{long}}(g_{YM}, N)$$

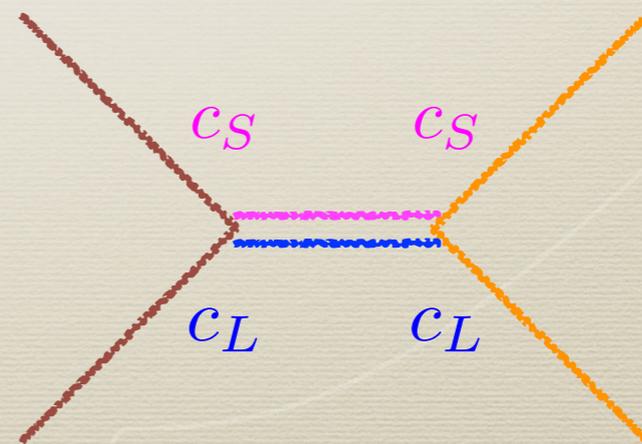
\mathcal{O}_{short}

protected ops (1/2 and 1/4 BPS)

$$\Delta_S = \Delta(N)$$

$$c_S = c_{\mathcal{O}_2 \mathcal{O}_2 \mathcal{O}_{short}}(N)$$

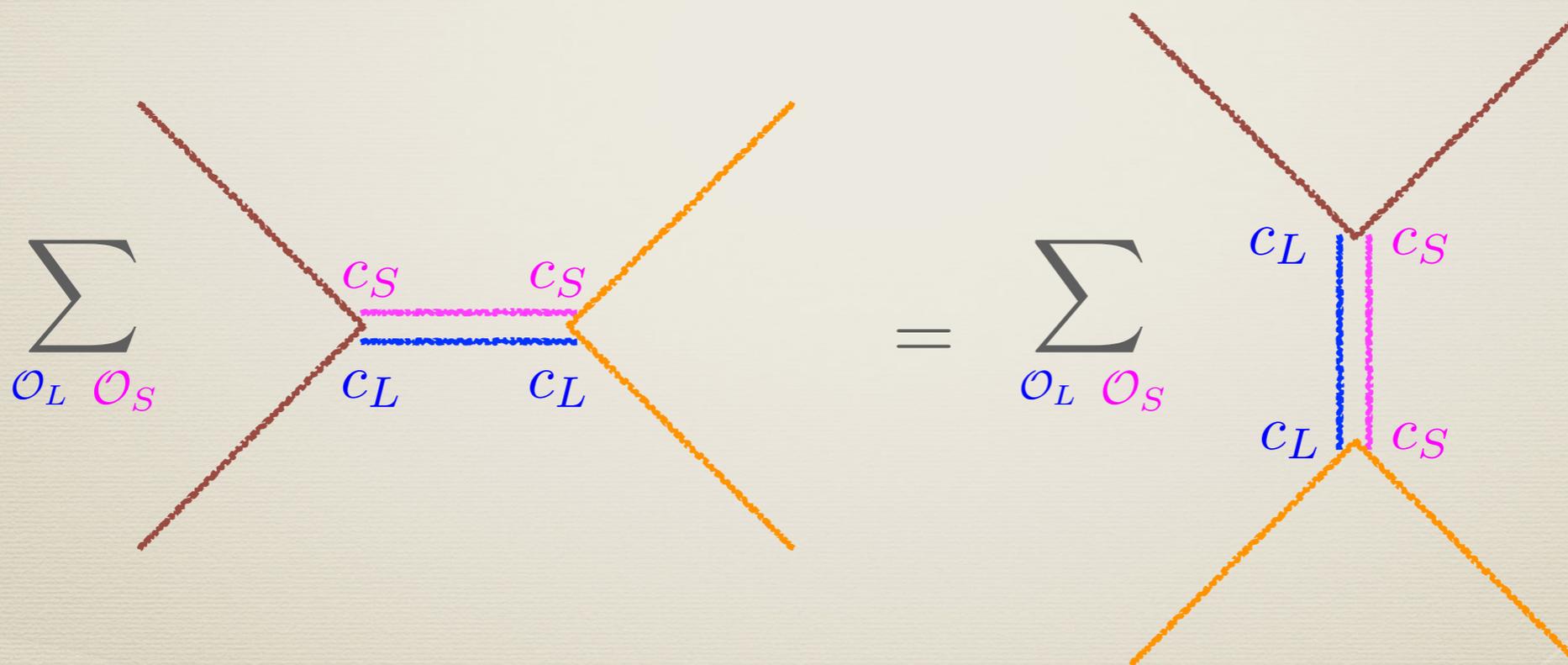
$$\mathcal{G}(u, v) = \sum_{\mathcal{O}_L \mathcal{O}_S}$$

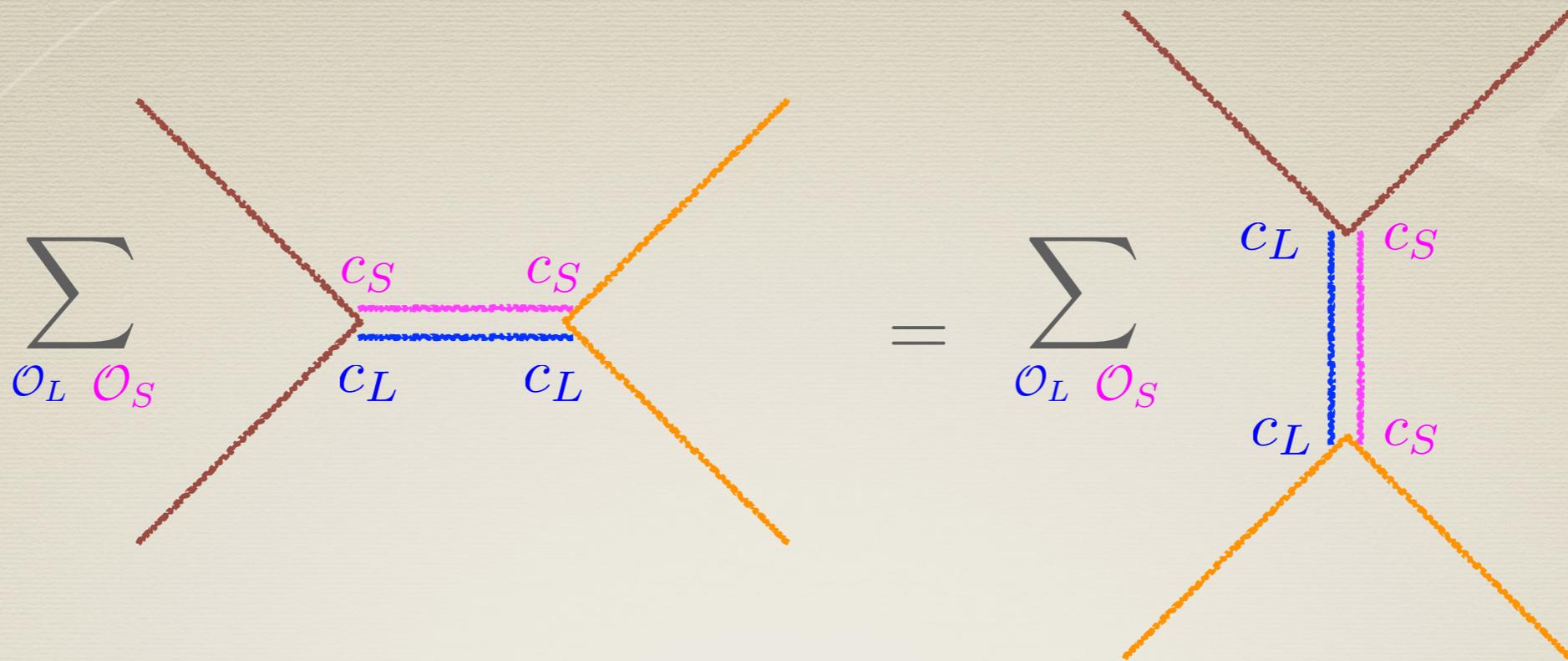


$$\langle \mathcal{O}_{20}(x_1) \mathcal{O}_{20}(x_2) \mathcal{O}_{20}(x_3) \mathcal{O}_{20}(x_4) \rangle = \langle \mathcal{O}_{20}(x_1) \mathcal{O}_{20}(x_2) \mathcal{O}_{20}(x_3) \mathcal{O}_{20}(x_4) \rangle$$



$$v^2 \mathcal{G}(u, v) - u^2 \mathcal{G}(v, u) + 4(u^2 - v^2) + \frac{16(u - v)}{N^2 - 1} = 0$$





$$\mathcal{G}(u, v) = \mathcal{G}^{short}(u, v) + \mathcal{H}(u, v)$$

Since \mathcal{O}_S are protected,
this function is completely determined!

$$\sum_{\mathcal{O}_L} c_L^2 g_{\mathcal{O}_L}$$

The sum runs over superconformal primaries, which are singlet of $SU(4)_R$ and $g_{\mathcal{O}_L}$ are superconformal blocks.

Properties of conformal blocks

- Conformal blocks are fixed by conformal symmetry and encode the contribution of a primary and all its descendants. In 4d they are known in a closed form, for scalar external operators.
- They are eigenfunctions of the quadratic Casimir operator with eigenvalue $J^2 \sim \ell^2$

$$\mathcal{C}G_{\Delta,\ell}(u, v) = J^2 G_{\Delta,\ell}(u, v)$$

- Note that (Δ, ℓ) label intermediate operators and denote the conformal dimension and spin₈, respectively.

Properties of conformal blocks II

$$G_{\Delta,\ell}(u, v) = u^{\frac{\tau}{2}} \tilde{g}_{\Delta,\ell}(u, v) \quad \text{where } \tau = \Delta - \ell$$

- Small u limit

$$G_{\Delta,\ell}(u, v) \rightarrow u^{\frac{\tau}{2}} \tilde{g}_{\Delta,\ell}^{coll}(v) + \dots$$

- Small v limit

$$\tilde{g}_{\Delta,\ell}(u, v) \rightarrow \log(v)$$

-
- Superconformal blocks $g_{\Delta,\ell}(u, v) = u^{\frac{\tau}{2}} \tilde{g}_{\Delta+4,\ell}(u, v)$

Large N expansion

$$\mathcal{H}(u, v) = \mathcal{H}^{(0)}(u, v) + \frac{1}{N^2} \mathcal{H}^{(1)}(u, v) + \frac{1}{N^4} \mathcal{H}^{(2)}(u, v)$$

$$\Delta = \Delta^{(0)} + \frac{1}{N^2} \gamma^{(1)} + \frac{1}{N^4} \gamma^{(2)}$$

$$c_{\Delta, \ell}^2 = a_{\Delta, \ell} = a_{\Delta, \ell}^{(0)} + \frac{1}{N^2} a_{\Delta, \ell}^{(1)} + \frac{1}{N^4} a_{\Delta, \ell}^{(2)}(u, v)$$

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Order N^0

For large λ and large N the result reduces to the one of generalised free fields:

$$\mathcal{G}^{(0)}(u, v) = 4 \left(1 + \frac{1}{v^2} \right)$$

From the OPE content of protected operators

$$\mathcal{G}^{(0)}(u, v) = G_{short}^{(0)}(u, v) + \sum_{n, \ell} a_{n, \ell}^{(0)} u^{n+2} g_{n, \ell}(u, v)$$

$\mathcal{H}^{(0)}(u, v)$
↓

Double trace operators!

$$\mathcal{O}_2 \square^n \partial^\ell \mathcal{O}_2$$



$$\Delta^{(0)} = 2n + \ell + 4$$

$$a_{n, \ell}^{(0)}$$

Comments on large spin limit

$$\mathcal{H}^{(0)}(u, v) = \sum_{\Delta, \ell} a_{\Delta, \ell} u^{\frac{\Delta - \ell}{2}} g_{\Delta, \ell}(u, v)$$

For small u and v :

LHS

RHS

Comments on large spin limit

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$$\mathcal{H}^{(0)}(u, v) \rightarrow \frac{2}{3} \frac{u^2}{v^2} + \dots$$

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Power law divergence!

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RHS

- Each conformal block diverges as $\log(v)$
- Need of an infinite sum on the spin, of operators whose twist approaches $\Delta - \ell = 4$

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LHS

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Power law divergence!

Fix dimensions and
OPE coefficients to all
orders in $1/\ell$

RHS

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Large N expansion

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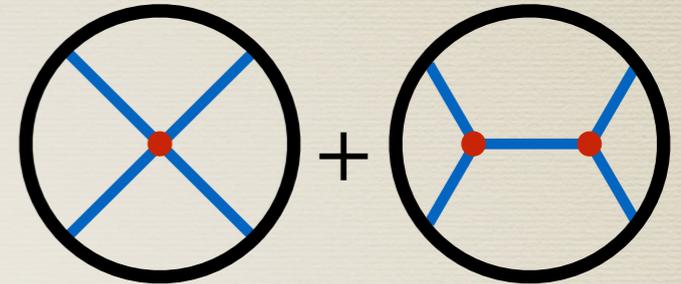
$$a_{\Delta, \ell} = a_{\Delta, \ell}^{(0)} + \frac{1}{N^2} a_{\Delta, \ell}^{(1)} + \frac{1}{N^4} a_{\Delta, \ell}^{(2)}$$



Order N^{-2}

Supergravity result:

$$\begin{aligned} \mathcal{G}^{(1)}(u, v) &= -16u^2 \bar{D}_{2422}(u, v) \\ &= G_{short}^{(1)}(u, v) + \mathcal{H}^{(1)}(u, v) \end{aligned}$$



Absence of new operators \rightarrow only corrections to the dimensions and the OPE coefficients of double trace operators

$$\mathcal{H}^{(1)}(u, v) = \sum_{n, l} u^{2+n} \left(a_{n, l}^{(1)} + \frac{1}{2} a_{n, l}^{(0)} \gamma_{n, l}^{(1)} \left(\log u + \frac{\partial}{\partial n} \right) \right) g_{4+2n+l, l}(u, v)$$

$$\gamma_{n, l}^{(1)}$$

$$a_{n, l}^{(1)} = \frac{1}{2} \frac{\partial}{\partial n} \left(a_{n, l}^{(0)} \gamma_{n, l}^{(1)} \right)$$

Alternative method

Crossing equation:

$$v^2 \mathcal{G}(u, v) - u^2 \mathcal{G}(v, u) + 4(u^2 - v^2) + \frac{16(u - v)}{N^2 - 1} = 0$$



$$\mathcal{G}^{(1)}(u, v) = \mathcal{G}_{short}^{(1)}(u, v) + \mathcal{H}^{(1)}(u, v)$$



- protected operators
- no $\log u$



- double trace operators
 $\tau = 4 + 2n \geq 0$

Equation for $\mathcal{H}^{(1)}(u, v)$

$$\mathcal{H}^{(1)}(u, v) = \frac{u^2}{v^2} \mathcal{H}^{(1)}(v, u) - \frac{4(u^2 - v^2)}{v^2} - \frac{16}{v^2(N^2 - 1)}$$
$$+ \frac{u^2}{v^2} \mathcal{G}_{short}^{(1)}(v, u) - \mathcal{G}_{short}^{(1)}(u, v)$$

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$$\frac{u^2}{v^2} \mathcal{H}^{(1)}(v, u) = \frac{u^2}{v^2} \sum_{n, \ell} a_{n, \ell}^{(0)} v^{2+n} \gamma_{n, \ell}^{(1)} g_{4+2n+\ell, \ell}(v, u) \log v + \dots$$

- no divergence!
- all the divergences in v come from the protected part

Equation for $\mathcal{H}^{(1)}(u, v)$

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$$\mathcal{H}(u, v)|_{\log u} = \sum_{n, \ell} a_{n, \ell}^{(0)} u^{2+n} \gamma_{n, \ell}^{(1)} g_{4+2n+\ell, \ell}(u, v)$$

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$$= \frac{u^2}{v^2} \mathcal{G}_{short}^{(1)}(v, u)|_{\log u}$$

- match the divergences on both sides $\rightarrow \gamma_{n, \ell}^{(1)}$ to all orders in $1/\ell$

Large N expansion

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Order N^{-4}

Idea: extract information about $\gamma_{n,l}^{(2)}$ using

- CFT data from previous orders
- crossing symmetry

General picture:

$$\mathcal{H}^{(2)}(u, v) = \sum_{n,l} \left(a_{n,l}^{(2)} + \frac{1}{2} a_{n,l}^{(0)} \gamma_{n,l}^{(2)} \partial_n + \frac{1}{2} a_{n,l}^{(1)} \gamma_{n,l}^{(1)} \partial_n + \frac{1}{8} a_{n,l}^{(0)} (\gamma_{n,l}^{(1)})^2 \partial_n^2 \right) u^{2+n} g_{n,l}(u, v)$$

Order N^{-4}

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$\log^2 u$ with coefficient fixed by known CFT data

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**CROSSING
SYMMETRY**

$$\left. + \frac{1}{8} a_{n,l}^{(0)} (\gamma_{n,l}^{(1)})^2 \partial_n^2 \right) u^{2+n} g_{n,l}(u, v)$$

$\log^2 u$ with coefficient fixed by known CFT data

Order N^{-4}

General picture:

$\log^2 v \longrightarrow$ extract $\gamma_{n,l}^{(2)}$ to all orders
in $1/l$

$$\mathcal{H}^{(2)}(u, v) = \sum_{n,l} \left(a_{n,l}^{(2)} + \frac{1}{2} a_{n,l}^{(0)} \gamma_{n,l}^{(2)} \partial_n + \frac{1}{2} a_{n,l}^{(1)} \gamma_{n,l}^{(1)} \partial_n \right.$$

**CROSSING
SYMMETRY**

$$\left. + \frac{1}{8} a_{n,l}^{(0)} (\gamma_{n,l}^{(1)})^2 \partial_n^2 \right) u^{2+n} g_{n,l}(u, v)$$

$\log^2 u$ with coefficient fixed by known CFT data

It is not so “simple”...

More than one operator with the same $\tau^{(1)}$, ℓ and R-symmetry

$$\mathcal{H}^2(u, v)|_{\log^2 u} = \frac{1}{8} \sum_{n, \ell, I} u^{n+2} a_{n, \ell, I}^{(0)} (\gamma_{n, \ell, I}^{(1)})^2 g_{4+n+\ell, \ell}$$



does NOT follow from

$$\sum a_{n, \ell}^{(0)}$$

$$\sum a_{n, \ell}^{(0)} \gamma_{n, \ell}^{(1)}$$

$$\{\mathcal{O}_2 \square^n \partial^\ell \mathcal{O}_2, \mathcal{O}_3 \square^{n-1} \partial^\ell \mathcal{O}_3, \mathcal{O}_4 \square^{n-2} \partial^\ell \mathcal{O}_4, \dots\}$$

Solve the mixing problem!!

Mixing problem

$$\{\mathcal{O}_2 \square^n \partial^\ell \mathcal{O}_2, \mathcal{O}_3 \square^{n-1} \partial^\ell \mathcal{O}_3, \mathcal{O}_4 \square^{n-2} \partial^\ell \mathcal{O}_4, \dots\}$$



intermediate operators in the four point functions $\langle \mathcal{O}_p \mathcal{O}_p \mathcal{O}_q \mathcal{O}_q \rangle$

Data (Dolan Osborn, Uruchurtu, Rastelli Zhou):

$$\sum_I c_{ppI}^{(0)} c_{qqI}^{(0)}$$



$$\langle \mathcal{O}_p \mathcal{O}_p \mathcal{O}_q \mathcal{O}_q \rangle^{(0)}$$

$$\sum_I c_{ppI}^{(0)} c_{qqI}^{(0)} \gamma_I^{(1)}$$



$$\langle \mathcal{O}_p \mathcal{O}_p \mathcal{O}_q \mathcal{O}_q \rangle^{(1)}$$

$$\sum_I c_{ppI}^{(0)} c_{qqI}^{(0)} (\gamma_I^{(1)})^2$$

Example

$$\Delta^{(0)} = 8 + \ell \quad n = 2$$

$$\{\mathcal{O}_2 \square^2 \partial^\ell \mathcal{O}_2, \mathcal{O}_3 \square \partial^\ell \mathcal{O}_3, \mathcal{O}_4 \partial^\ell \mathcal{O}_4\}$$



$$\langle \mathcal{O}_2 \mathcal{O}_2 \mathcal{O}_2 \mathcal{O}_2 \rangle \quad \langle \mathcal{O}_3 \mathcal{O}_3 \mathcal{O}_4 \mathcal{O}_4 \rangle$$

$$\langle \mathcal{O}_2 \mathcal{O}_2 \mathcal{O}_3 \mathcal{O}_3 \rangle \quad \langle \mathcal{O}_4 \mathcal{O}_4 \mathcal{O}_4 \mathcal{O}_4 \rangle$$

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6 x 2



$$\sum_I c_{ppI}^{(0)} c_{qqI}^{(0)}$$

$$\sum_I c_{ppI}^{(0)} c_{qqI}^{(0)} \gamma_I^{(1)}$$

Example

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$$\{\mathcal{O}_2 \square^2 \partial^\ell \mathcal{O}_2, \mathcal{O}_3 \square \partial^\ell \mathcal{O}_3, \mathcal{O}_4 \partial^\ell \mathcal{O}_4\}$$



$$\langle \mathcal{O}_2 \mathcal{O}_2 \mathcal{O}_2 \mathcal{O}_2 \rangle \quad \langle \mathcal{O}_3 \mathcal{O}_3 \mathcal{O}_4 \mathcal{O}_4 \rangle$$

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6 x 2



N^0 and N^{-2}

$$\sum_I c_{ppI}^{(0)} c_{qqI}^{(0)} \quad 3+3+3$$

$$\sum_I c_{ppI}^{(0)} c_{qqI}^{(0)} \gamma_I^{(1)} \quad 3$$

Example

$$\Delta^{(0)} = 8 + \ell \quad \{ \mathcal{O}_2 \square^2 \partial^\ell \mathcal{O}_2, \mathcal{O}_3 \square \partial^\ell \mathcal{O}_3, \mathcal{O}_4 \partial^\ell \mathcal{O}_4 \}$$



$$\begin{array}{cc} \langle \mathcal{O}_2 \mathcal{O}_2 \mathcal{O}_2 \mathcal{O}_2 \rangle & \langle \mathcal{O}_3 \mathcal{O}_3 \mathcal{O}_4 \mathcal{O}_4 \rangle \\ \langle \mathcal{O}_2 \mathcal{O}_2 \mathcal{O}_3 \mathcal{O}_3 \rangle & \langle \mathcal{O}_4 \mathcal{O}_4 \mathcal{O}_4 \mathcal{O}_4 \rangle \\ \langle \mathcal{O}_3 \mathcal{O}_3 \mathcal{O}_3 \mathcal{O}_3 \rangle & \langle \mathcal{O}_2 \mathcal{O}_2 \mathcal{O}_4 \mathcal{O}_4 \rangle \end{array}$$

6 x 2



N^0 and N^{-2}

$$\sum_I c_{ppI}^{(0)} c_{qqI}^{(0)} \quad 3+3+3$$

$$\sum_I c_{ppI}^{(0)} c_{qqI}^{(0)} \gamma_I^{(1)} \quad 3$$

- # of operators: 3
- # of 4 point functions = eqs : 12
- # of unknown: 12

MIXING PROBLEM CAN BE SOLVED!



Solving the mixing problem

$$\frac{\sum_I a_{n,\ell,I}^{(0)} (\gamma_{n,\ell,I}^{(1)})^2}{\sum_I a_{n,\ell,I}^{(0)}}$$

1. From the explicit results we get the structure:

$$\frac{(n+1)^3 (n+2)^3 (n+3)^3 (n+4)^3 (5+2n)}{120(J^2 - (n+2)(n+3))^2} + \sum_{j=2}^{n+2} \frac{\beta_{n,j}}{J^2 - j(j+1)}$$

2. Looking at the term $\log^2 u \log^2 v$ fix all the coefficients!

$$\mathcal{H}^{(2)}(u, v) \Big|_{\log^2 u \log^2 v} \longrightarrow \text{crossing symmetric!}$$

Fix the average to all orders in
 $1/\ell$

Solving the mixing problem

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Nicer way

$$\langle (\gamma_{n,\ell}^{(1)})^2 \rangle = \sum_{p=2}^{\infty} \prod_{k=2}^{p-1} \frac{\alpha_p (n+1)^2 (n+2)^2 (n+3)^2 (n+4)^2 (n-k+2)(n+k+3)}{(J^2 - (n+2)(n+3))^2 (J^2 - k(k+1))}$$

Each term is the contribution of a Kaluza Klein mode!

$$\mathcal{O}_p \square^n \partial^\ell \mathcal{O}_p$$

Next step: contribution of each KK mode to $\gamma_{0,\ell}^{(2)}$

Full expansion

$$\mathcal{H}^{(2)}(u, v) = \sum_{n,l} \left(a_{n,l}^{(2)} + \frac{1}{2} a_{n,l}^{(0)} \gamma_{n,l}^{(2)} \partial_n + \frac{1}{2} a_{n,l}^{(1)} \gamma_{n,l}^{(1)} \partial_n + \frac{1}{8} a_{n,l}^{(0)} (\gamma_{n,l}^{(1)})^2 \partial_n^2 \right) u^{2+n} g_{n,l}(u, v)$$

- Small u, v limit
- Focus on the terms proportional to $\log u \log^2 v$

$$\sum_l \frac{1}{4} a_{0,l}^{(0)} (\gamma_{0,l}^{(1)})^2 \partial_n g_{n,l}(u, v) \Big|_{n=0} + \frac{1}{2} \left(a_{0,l}^{(0)} \gamma_{0,l}^{(2)} + \frac{1}{2} a_{0,l}^{(1)} \gamma_{0,l}^{(1)} \right) g_{0,l}^{coll}(v) \Big|_{\log^2 v}$$

$$= \sum_{n,l} \frac{1}{8} a_{n,l}^{(0)} (\gamma_{n,l}^{(1)})^2 v^n g_{n,l}(v, u) \Big|_{u^0 \log u}$$

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$$(u, v) \rightarrow (v, u)$$

- Small u, v limit
- Focus on the terms proportional to $\log u \log^2 v$

$$\sum_l \frac{1}{4} a_{0,l}^{(0)} (\gamma_{0,l}^{(1)})^2 \partial_n g_{n,l}(u, v) \Big|_{n=0} + \frac{1}{2} \left(a_{0,l}^{(0)} \gamma_{0,l}^{(2)} + \frac{1}{2} a_{0,l}^{(1)} \gamma_{0,l}^{(1)} \right) g_{0,l}^{coll}(v) \Big|_{\log^2 v}$$

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$$\sum_{\ell} \frac{1}{4} a_{0,\ell}^{(0)} (\gamma_{0,\ell}^{(1)})^2 \partial_n g_{n,\ell}(u, v) \Big|_{n=0} + \frac{1}{2} \left(a_{0,\ell}^{(0)} \gamma_{0,\ell}^{(2)} + \frac{1}{2} a_{0,\ell}^{(1)} \gamma_{0,\ell}^{(1)} \right) g_{0,\ell}^{coll}(v) \Big|_{\log^2 v}$$

$$= \sum_{n,\ell} \frac{1}{8} a_{n,\ell}^{(0)} (\gamma_{n,\ell}^{(1)})^2 v^n g_{n,\ell}(v, u) \Big|_{u^0 \log u}$$

Technical complications:

- deal with derivatives of the blocks
- infinite sum with insertion of $(\gamma_{n,\ell}^{(1)})^2$

Next: Plug the expression for $(\gamma_{n,\ell}^{(1)})^2$ in terms of KK modes and

see the contribution of each KK mode to $\gamma_{0,\ell}^{(2)}$

$$\gamma_{0,\ell}^{(2)} - \frac{1}{2} \gamma_{0,\ell}^{(1)} \partial_n \gamma_{n,\ell}^{(1)} \Big|_p = \alpha_p \frac{P^{(14+2\ell)}(p)}{(p^2-4)(p^2-1)p} + \alpha_p (p^2-4)(p^2-1)p^3 Q^{(4+2\ell)}(p) \psi^{(2)}(p)$$

polynomials

$$\sim p^{1-2\ell} \quad \text{for large } p$$

sum over p diverges for spin 0!

$$\Delta_{0,2} = 6 - \frac{4}{N^2} - \frac{45}{N^4} + \dots$$

$$\Delta_{0,4} = 8 - \frac{48}{25} \frac{1}{N^2} - \frac{12768}{3125} \frac{1}{N^4} + \dots$$

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first loop correction to the \mathfrak{e}_{26} graviton superamplitude!

Comments and outlook

- Divergence at spin 0:

quadratic divergence in 10d sugra, at large λ with only spin zero support

- Going beyond $n=0$
- Extend to other 4 point functions, e.g. external operator with
 $\Delta = 3$
- Extend to other susy theory