Supernova Neutrino Theory

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Neutrino evolution: quantum kinetic equations (QKEs)

$$\frac{d\mathcal{F}}{d\lambda} + \mathrm{force} + \mathrm{drift} = -p^{\mu}u_{\mu}\left(\mathcal{C} - \frac{i}{\hbar c}[\mathcal{H}, \mathcal{F}]\right)$$

This equation represents a nine by nine matrix and in principle a similar equation is needed for every neutrino, or at least every position, direction and energy.

Neutrino evolution: separating left and right handed neutrinos

$$p^\mu \frac{\partial f}{\partial x^\mu} - \Gamma^\mu_{\alpha\beta} p^\alpha p^\beta \frac{\partial f}{\partial p^\mu} = -p^\mu u_\mu \left(C - \frac{i}{\hbar c}[H, f] \right)$$

$$p^\mu \frac{\partial \bar{f}}{\partial x^\mu} - \Gamma^\mu_{\alpha\beta} p^\alpha p^\beta \frac{\partial \bar{f}}{\partial p^\mu} = -p^\mu u_\mu \left(\bar{C} - \frac{i}{\hbar c}[\bar{H}, \bar{f}] \right)$$

Removing “small” terms, particularly those suppressed by neutrino mass allows the right and left handed neutrinos to decouple. So now, each equation is “only” a 3 x 3 matrix, but you still need one for every angle, position energy

Neutrino evolution: flat spacetime, homogeneous

$$\frac{\partial f}{\partial t} = C - \frac{i}{\hbar c}\left[H, f\right]$$

Now, the spatial dependence is no long independent of the time, dependence.
Still, need an equation for every energy, direction of neutrino.

If you also assume isotropy, however, you only need one equation for each energy..

Neutrino evolution, cont

$$\frac{\partial f}{\partial t} = C - \frac{i}{\hbar c}\,[H, f]$$

$$H = H_{\mathrm{vacuum}} + H_{\mathrm{matter}} + H_{\mathrm{neutrino}}$$

C is the collision term

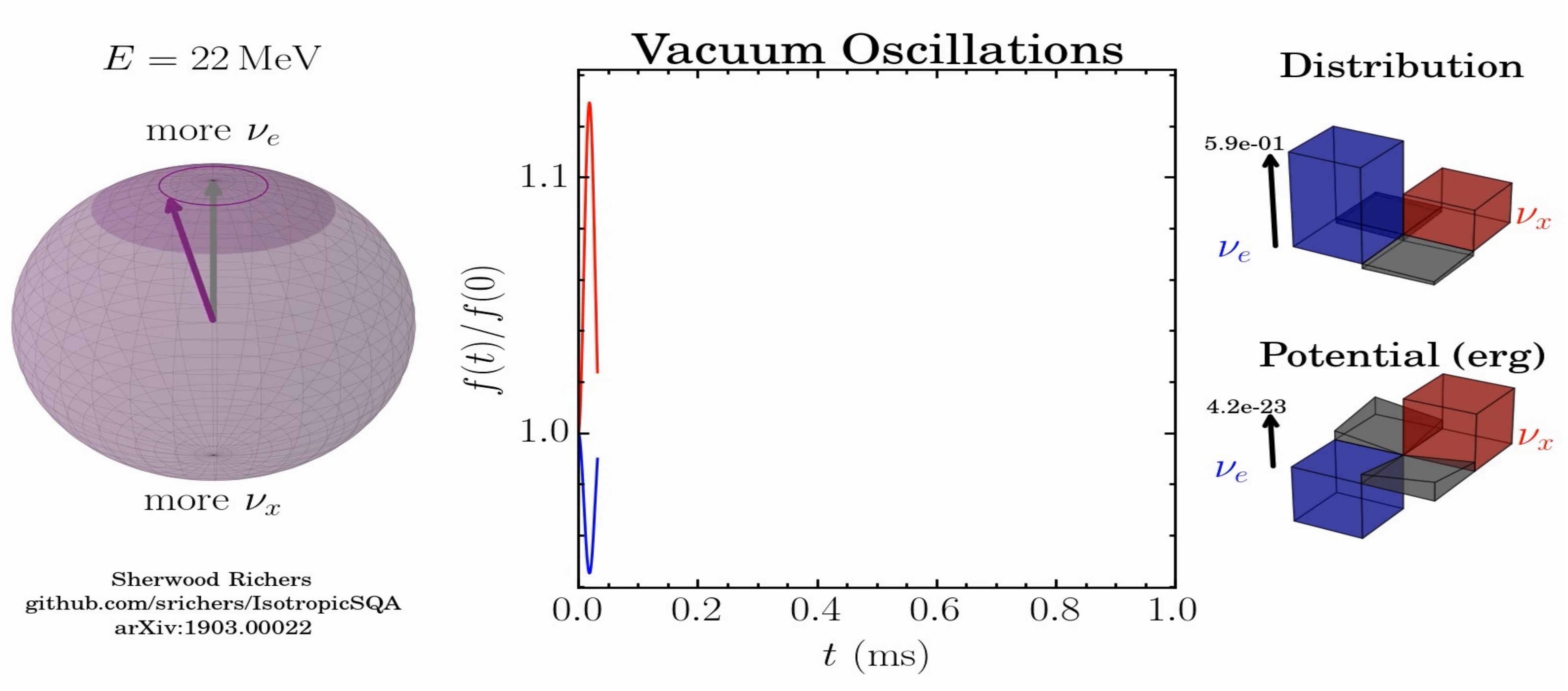
Vacuum oscillations
(not a main effect in SN, but a useful example)

$$\frac{\partial f}{\partial t} = -\frac{i}{\hbar c}[H, f]$$

$$H = H_{\text{vacuum}}$$

Since all neutrinos are independent of each other, we can make the homogeneous isotropic approximation, and solve for all neutrinos Independently.

Vacuum Oscillations



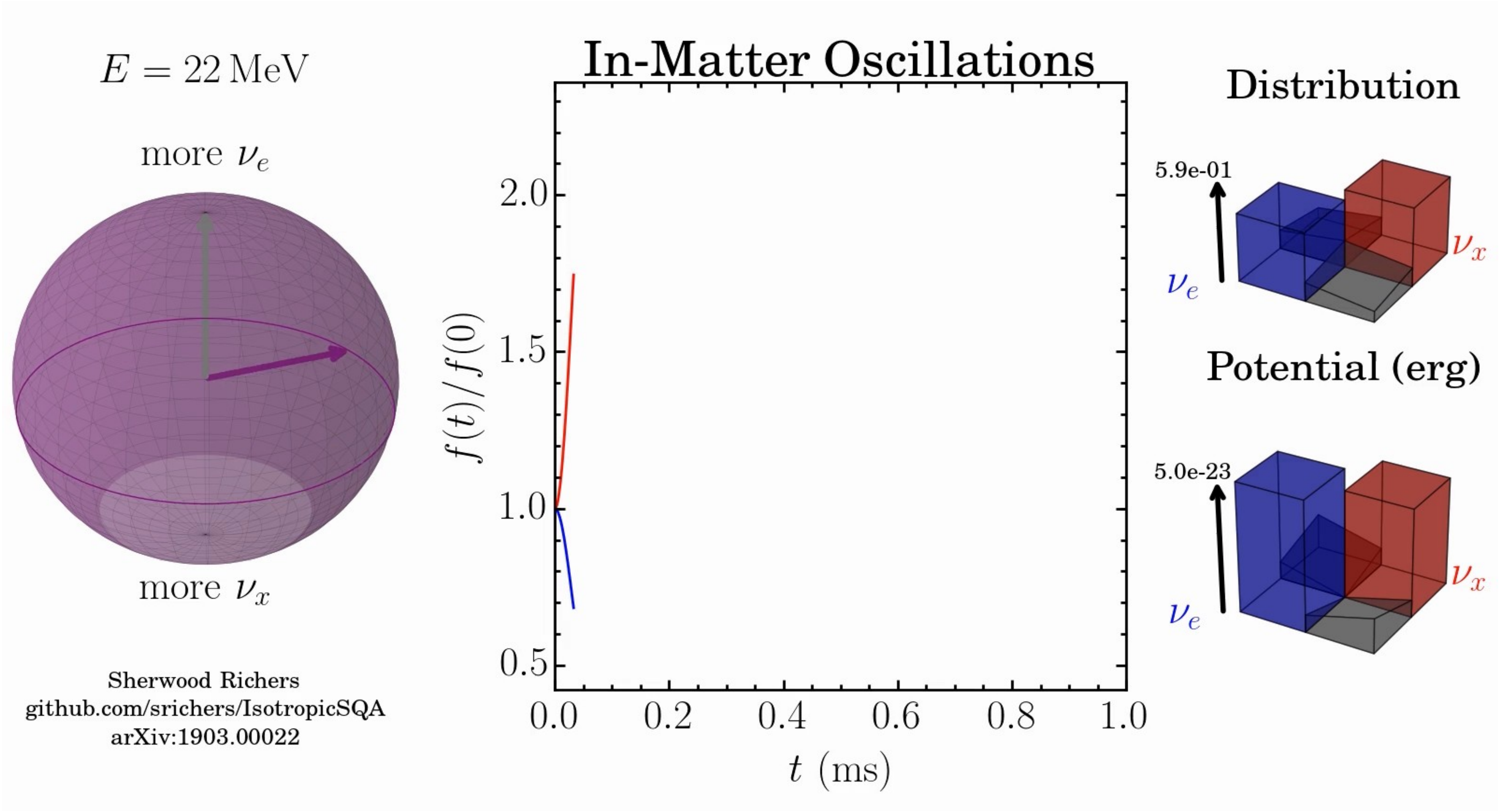
Matter Hamiltonian

$$\frac{\partial f}{\partial t} = \quad -\frac{i}{\hbar c}[H, f]$$

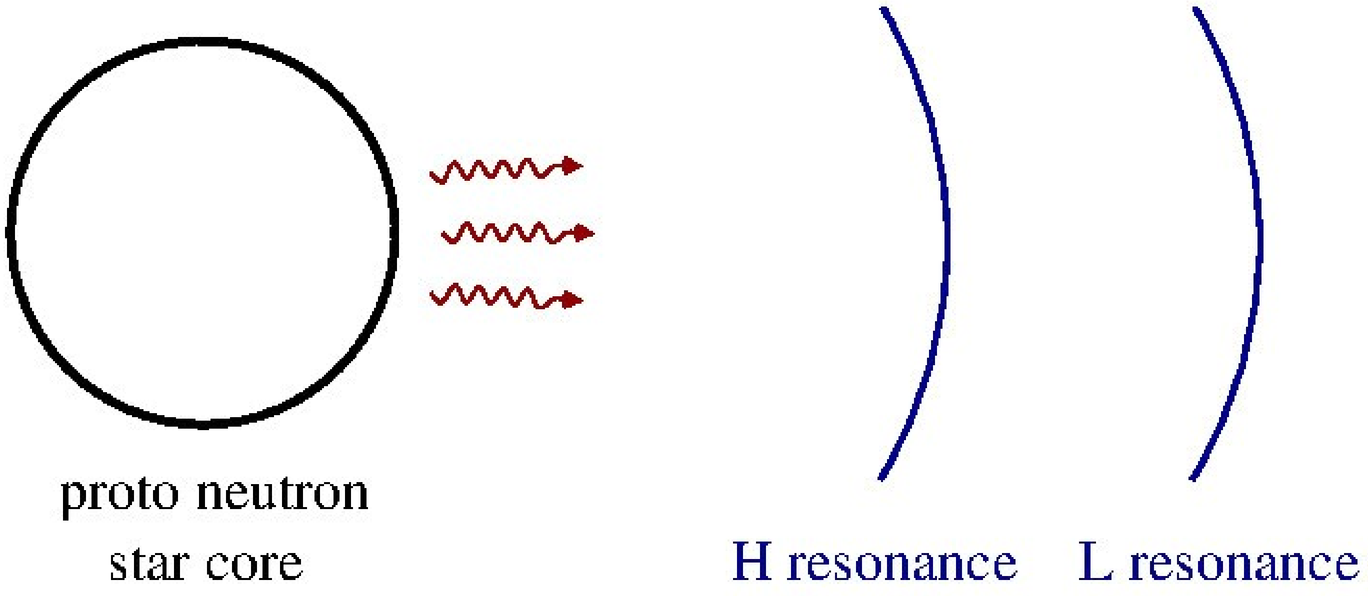
$$H = H_{\text{vacuum}} + H_{\text{matter}}$$

For many applications, homogeneous, isotropic, approximation is still okay, since the evolution of one neutrino doesn't depend on the others. In the absence of other terms each neutrino energy and direction can be solve independently

MSW Oscillations



MSW in Supernovae



Resonances can be adiabatic or non-adiabatic

Effect of the shock on the time signal

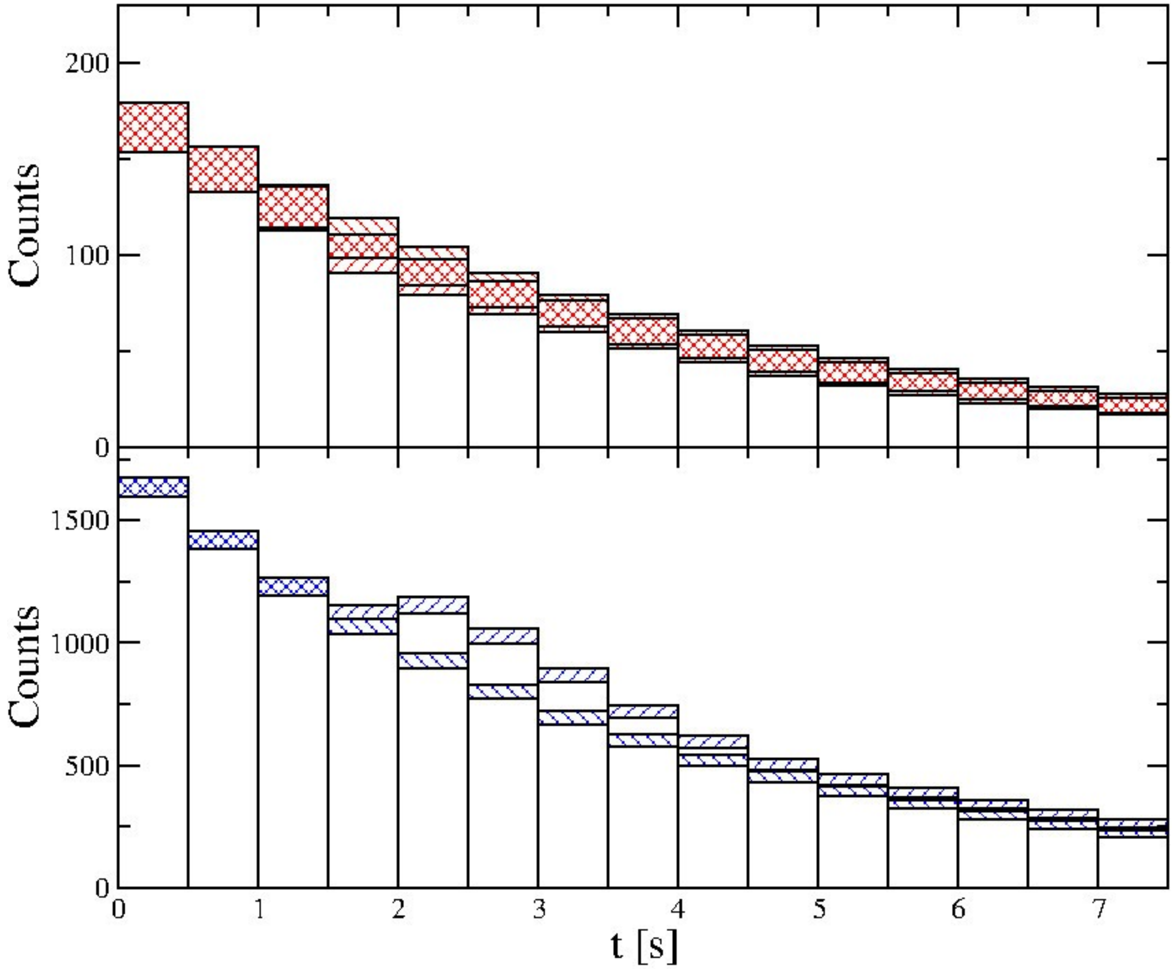


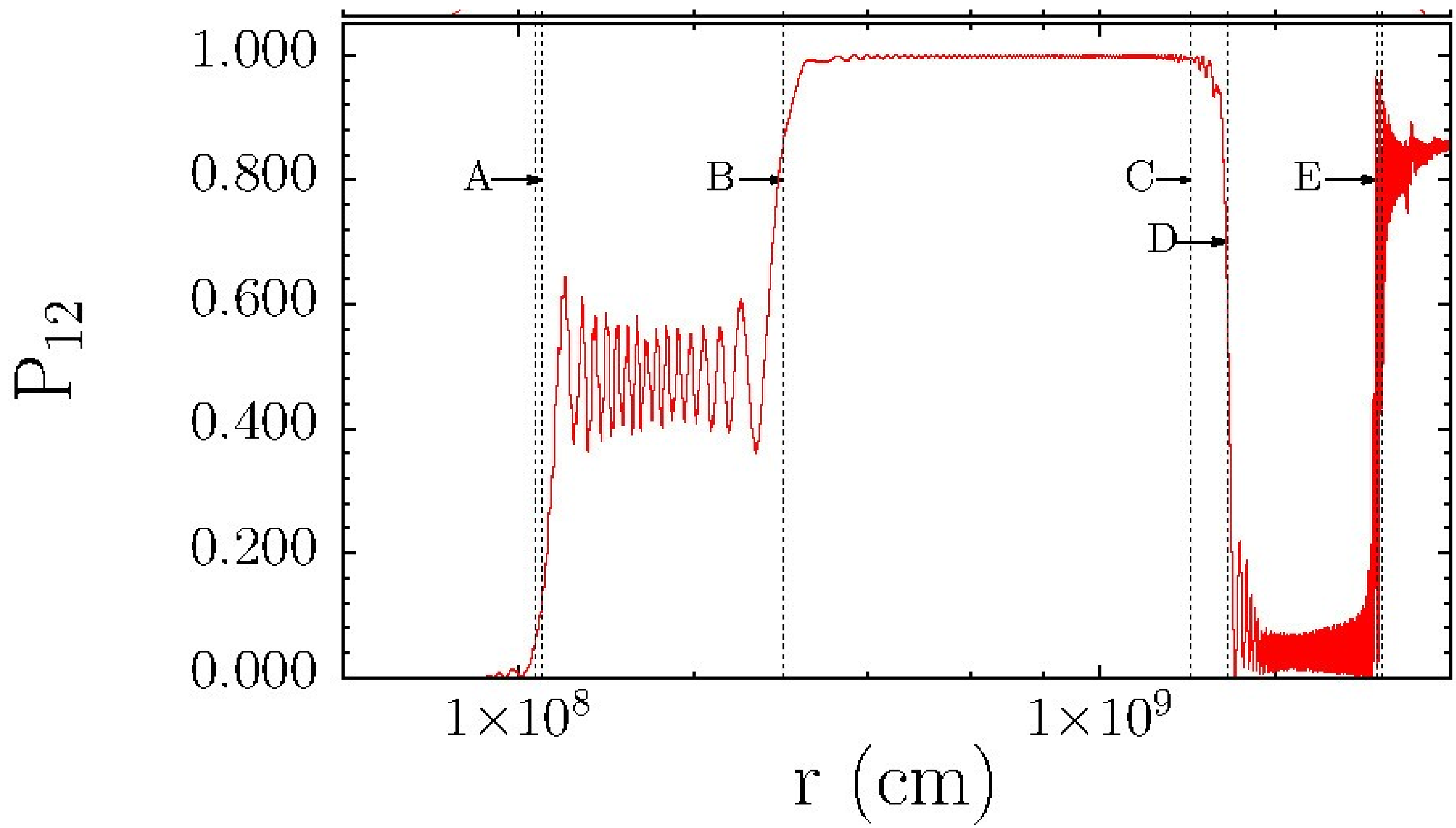
Fig. From Gava et al 2009

Density perturbations

- Any density profile can be Fourier decomposed to a series of sinusoidal fluctuations with inverse wavelengths k, and amplitudes C.
- A parametric resonance occurs when a the oscillation wavelength is approximately propotional to a multiple of the inverse wavelength
- Transitions occur if the neutrino spends enough time in the parametric resonance

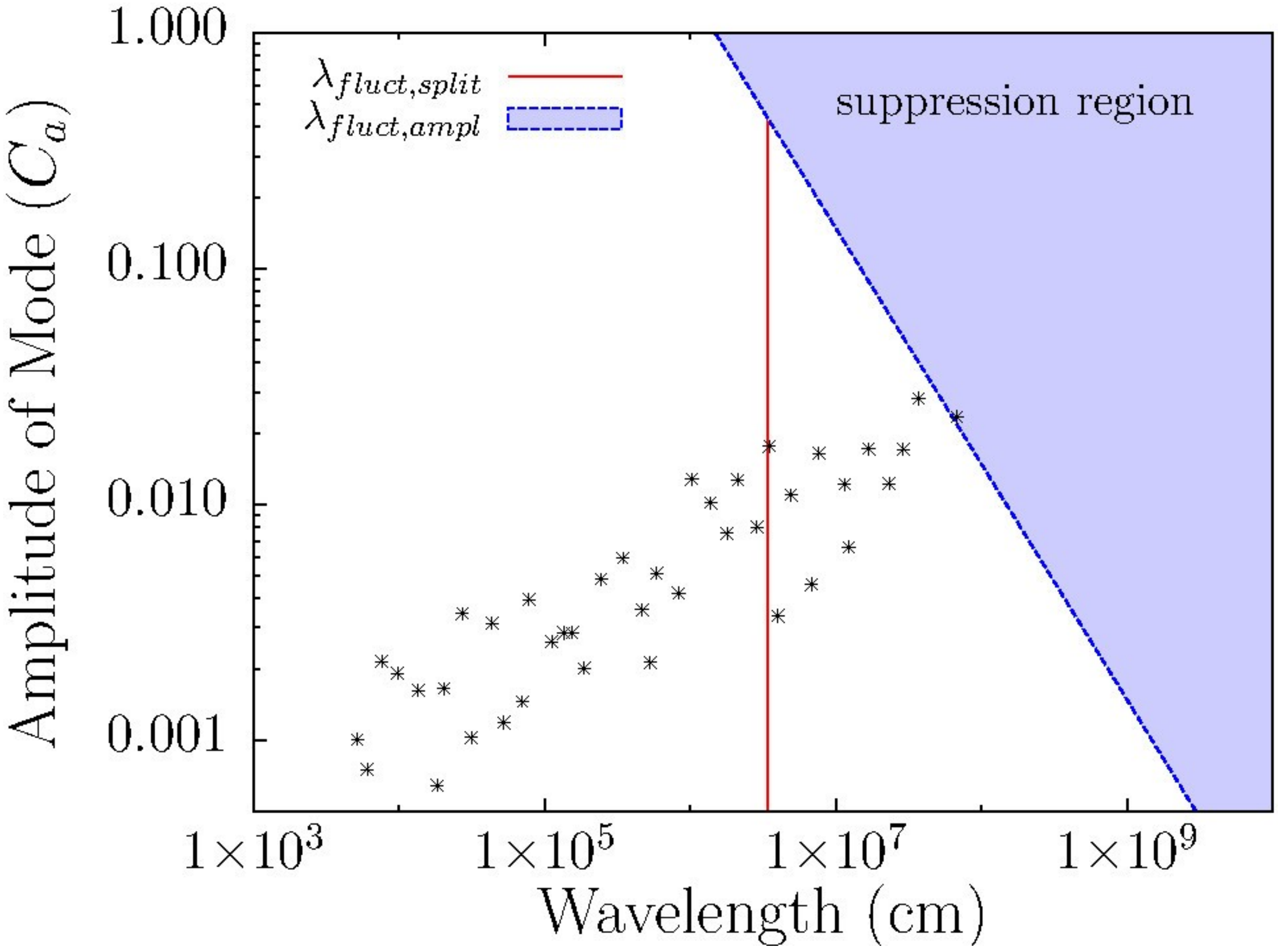
(Theory developed in a series of papers by J. Kneller, Patton, Yang)

Effect of density perturbations



Dramatic effects occur when the oscillation frequency is similar to the perturbation wave number. Fig from Patton et al

Relevant phase space for density fluctuations



Patton et al 2015

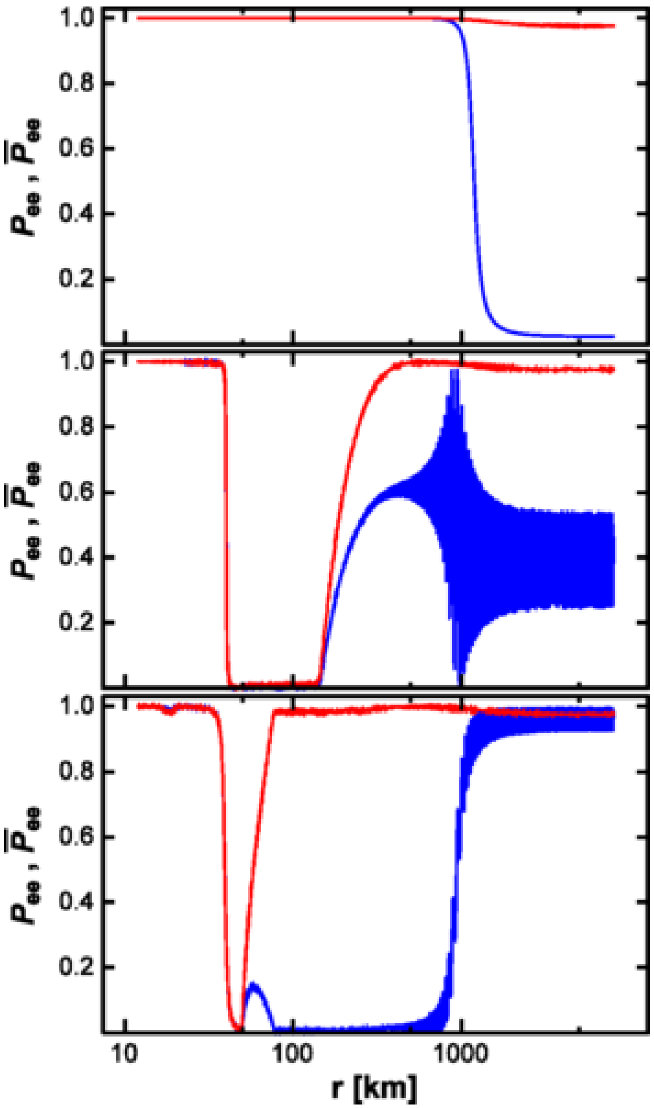
Beyond the Standard Model

Since the matter Hamilton represents an interaction of neutrinos with matter, any other interaction of neutrinos with matter will change Hamiltonian.

$$\begin{pmatrix} |\epsilon_{ee}| < 4.2 & |\epsilon_{e\mu}| < 0.33 & |\epsilon_{e\tau}| < 3.0 \\ & |\epsilon_{\mu\mu}| < 0.068 & |\epsilon_{\mu\tau}| < 0.33 \\ & & |\epsilon_{\tau\tau}| < 21 \end{pmatrix}$$

Limits on couplings of neutrinos to matter through non-standard interactions remarkably weak. Limits above are from Biggo, Blenno And Fernandez-Martin as reported in Stapleford et al.

Non standard interactions (NSI)



Top panel shows survival probability without NSI.

Middle panel shows the I resonance and subsequent Collective oscillations in the presence of NSI

Bottom panel: shows the I resonance and subsequent Matter Neutrino Resonance with slightly different NSI parameters

Figure from Stapleford et al 2018

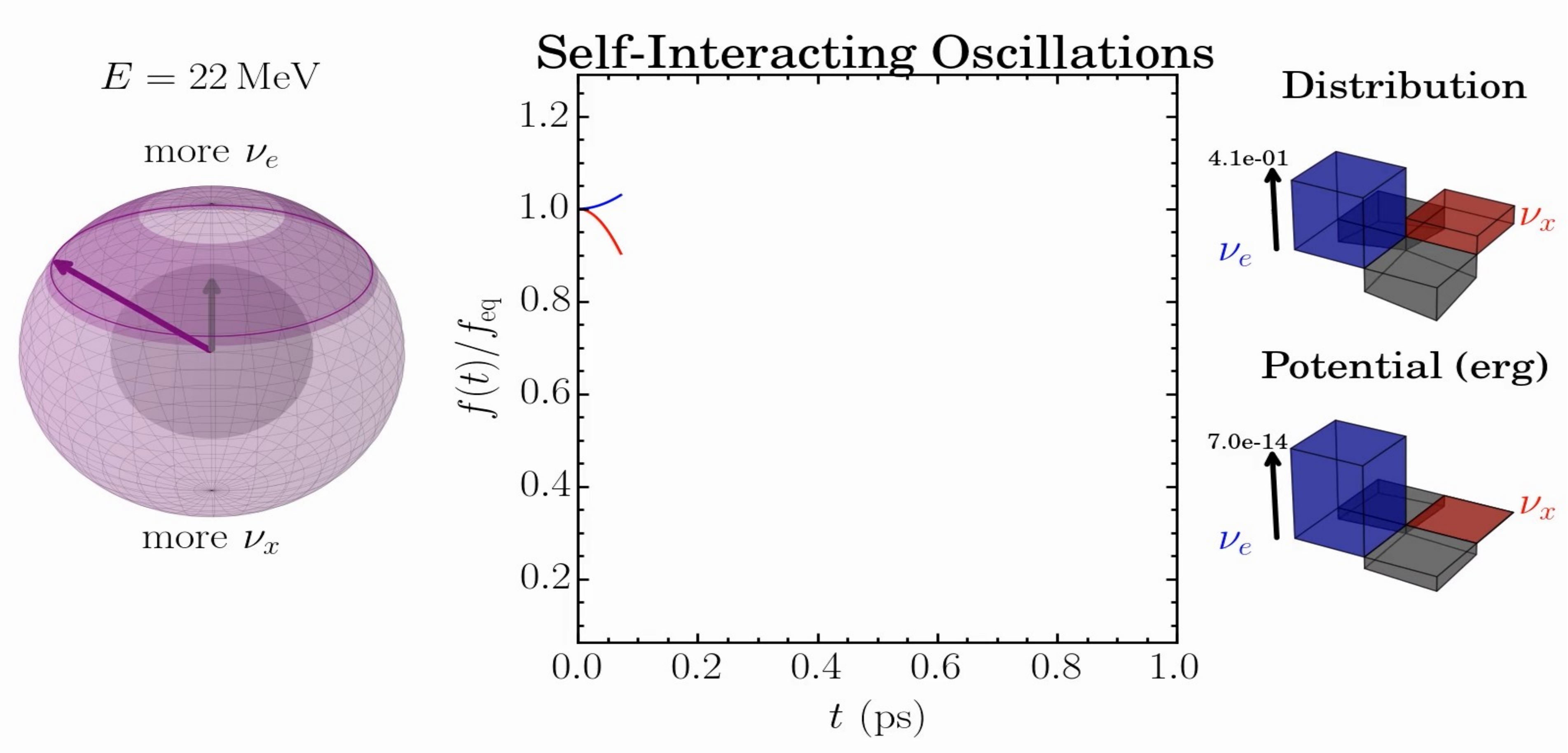
Neutrino "self interaction" Hamiltonian

$$\frac{\partial f}{\partial t} = - \frac{i}{\hbar c}[H, f]$$

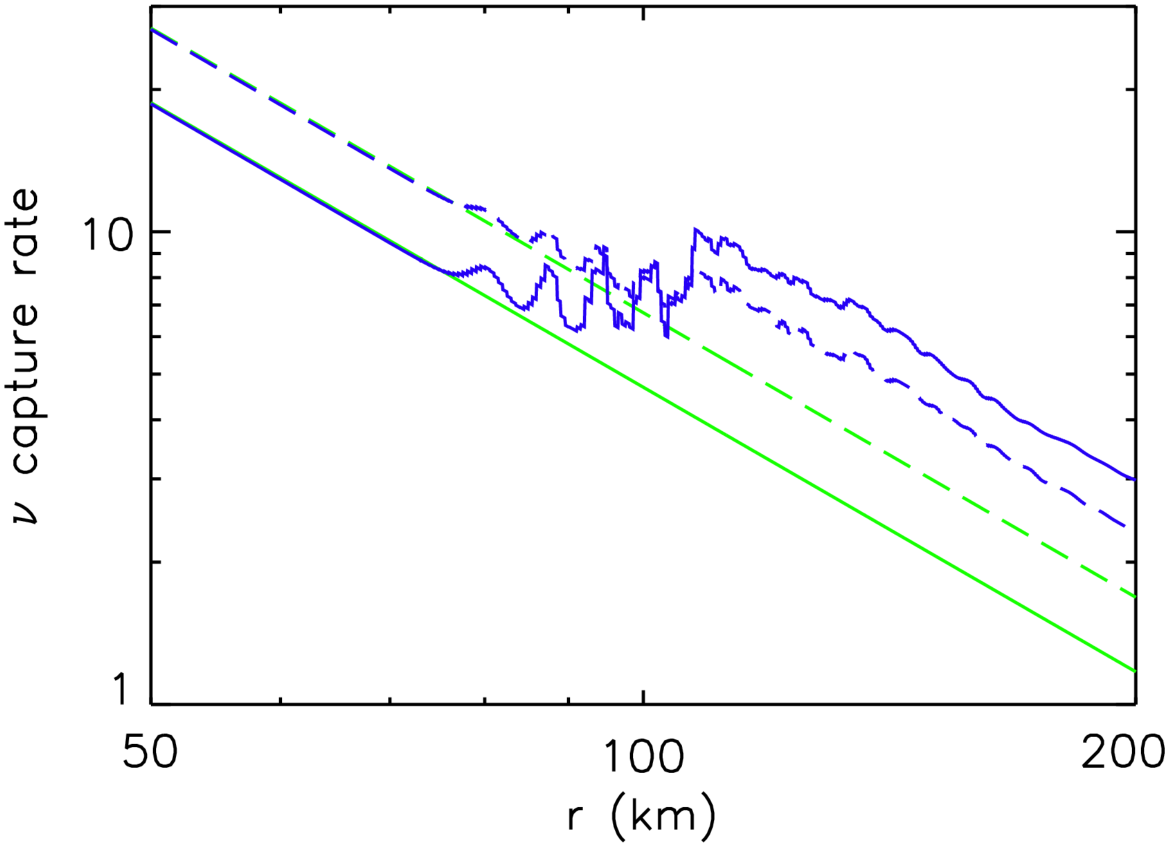
$$H = H_{\rm vacuum} + H_{\rm matter} + H_{\rm neutrino}$$

Collective effects come about from include the neutrino interaction with other neutrinos in the Hamiltonian. Effect is non-linear but sometimes the homogeneous, isotropic approximation is okay – also known as "single angle" and one equation per energy is okay. However, usually "multi-angle" Is needed, i.e., one equation for each angle and energy

Bipolar (collective) oscillations



Multi-angle oscillations in SN wind



From Duan et al, 2012, dotted lines are antineutrino capture rate
Solid lines are neutrino capture rate.

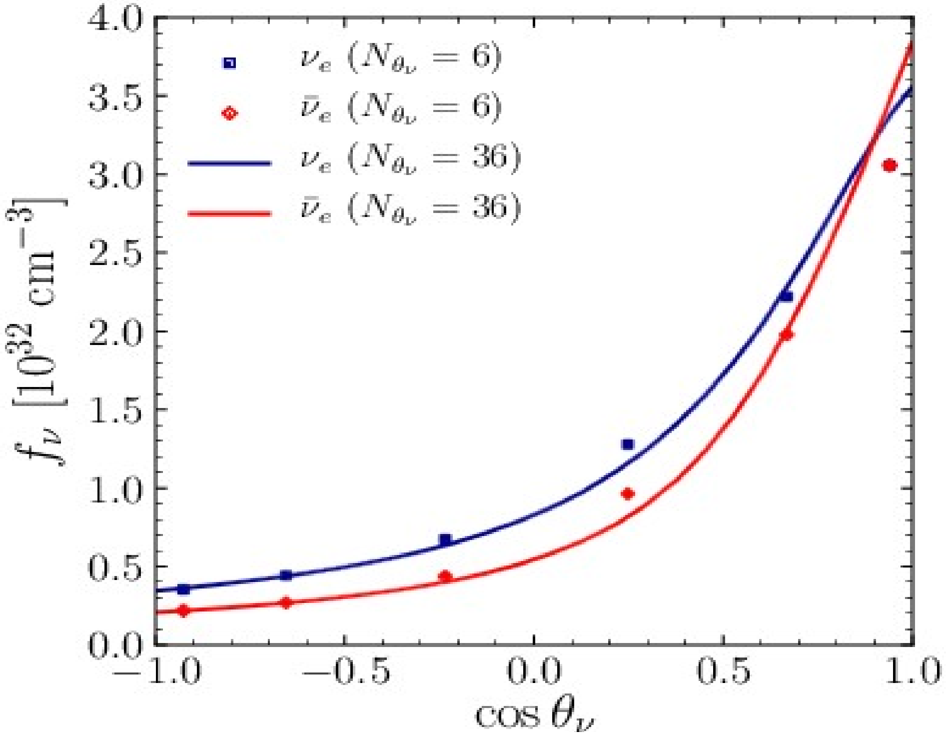
Fast oscillations: a multi-angle effect

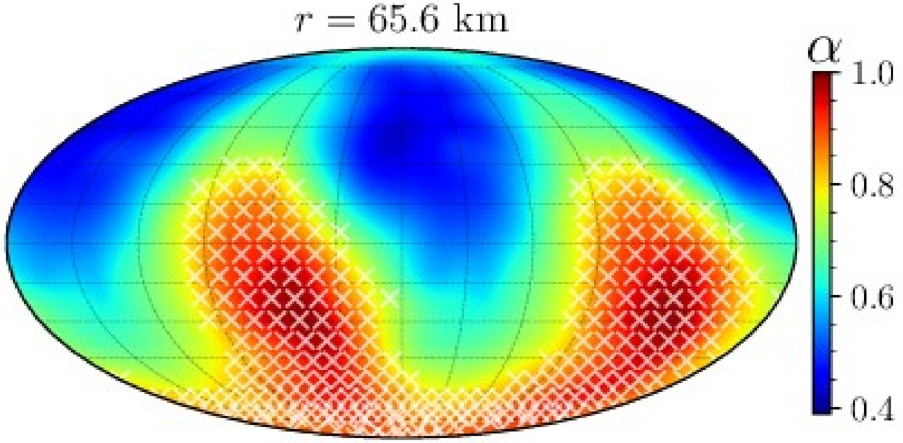
Previous oscillation analysis, even multi-angle, assumed no perturbations in the neutrino field.

If you allow perturbations to develop, stability analysis shows that these perturbations are unstable, e.g. growing.

We only know about this instability, not how the flavor evolution proceeds.

Fast oscillation instability instability correlated with "crossings"





Figures from Abbar et al 2018,
supernova model from Sumiyoshi et al

When collisions and oscillations are both important

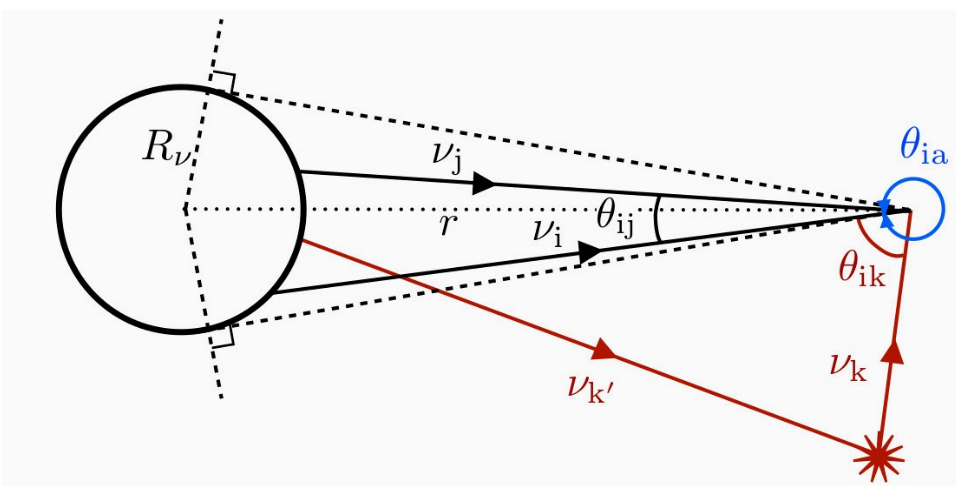
$$\frac{\partial f}{\partial t} = C - \frac{i}{\hbar c}[H, f]$$

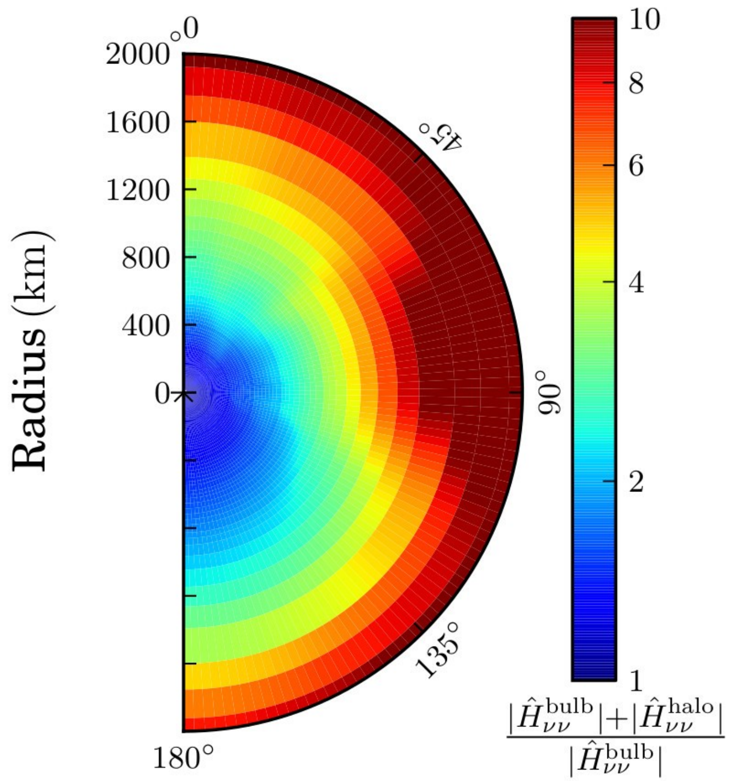
$$H = H_{\mathrm{vacuum}} + H_{\mathrm{matter}} + H_{\mathrm{neutrino}}$$

C is the collision term which includes scattering (change of direction), absorption and emission

Halo effect

Significant numbers of neutrinos can scatter "backward"

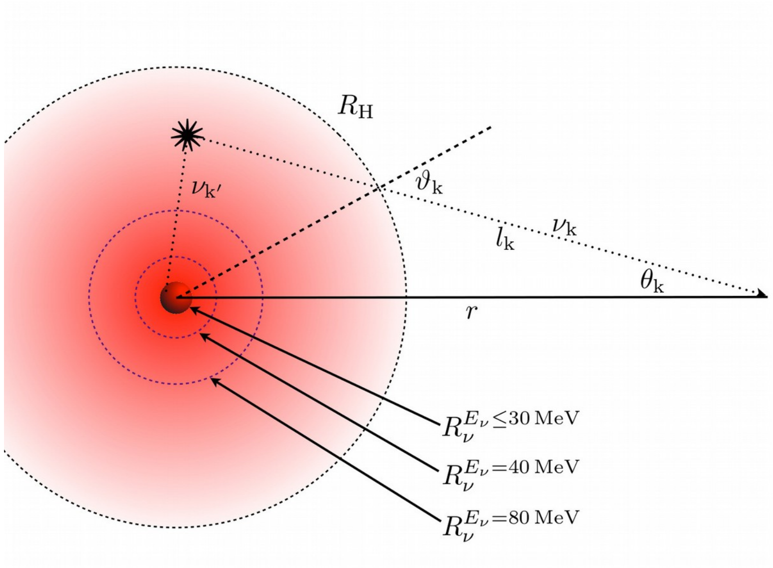




Cherry et al 2012

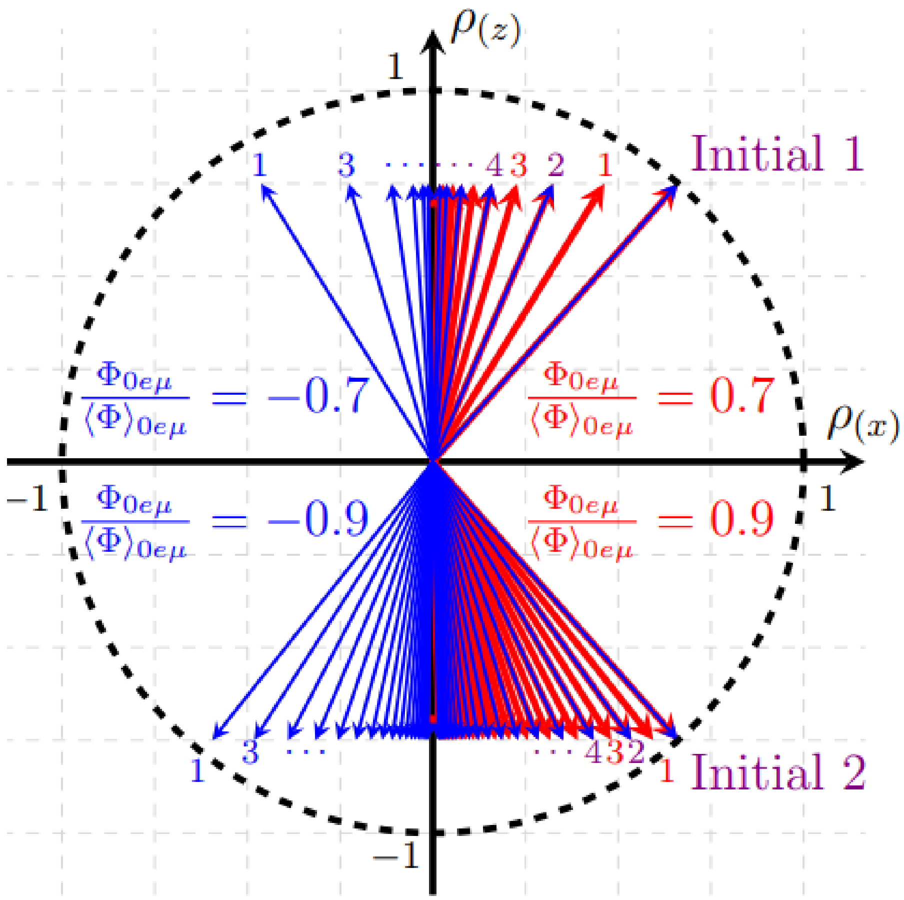
Halo effect: computation

In principle you need both space and time derivatives from many energies and angles



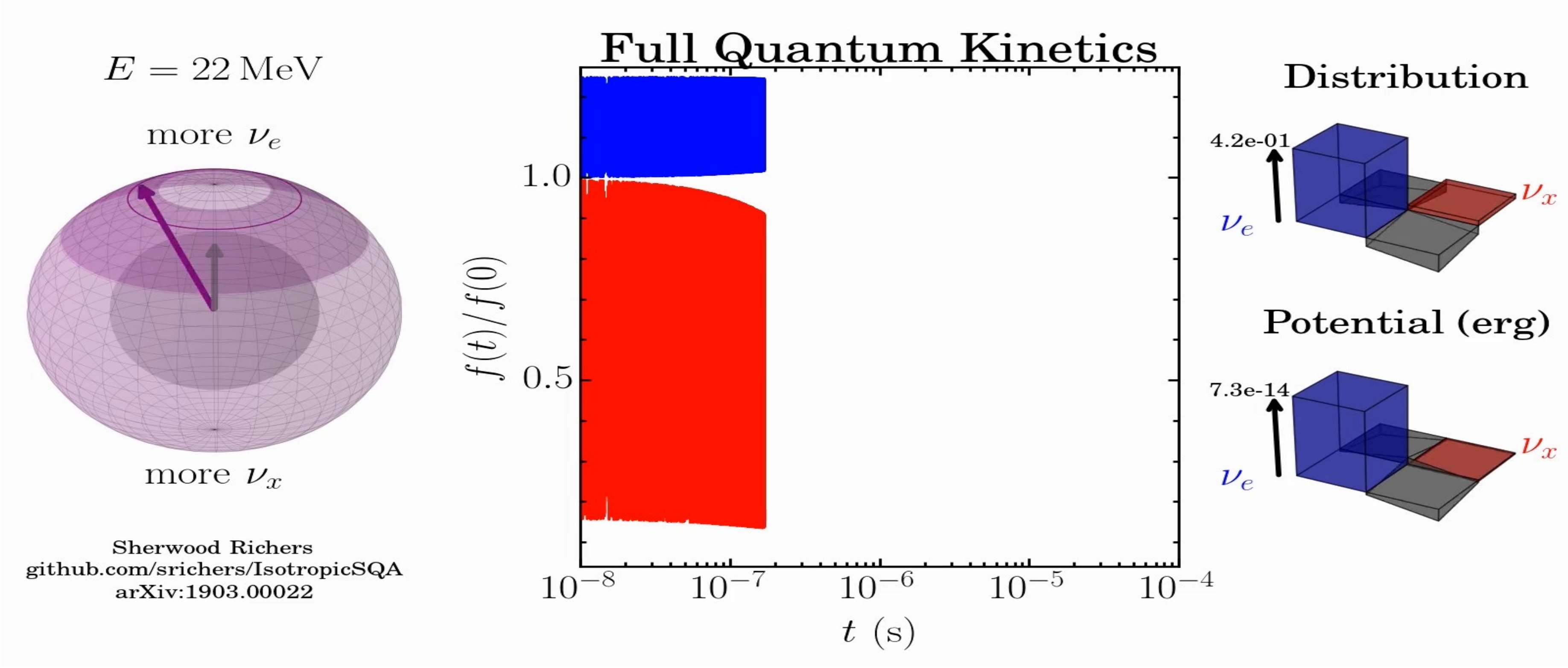
but watch for new clever ideas for the steady state problem from Cherry et al 2013, Shalgar et al 2019, Valsenko et al, in prep

Collisions and oscillations are also important near decoupling region



Evolution of flavor vector due to collisions, Fig. From Richers et al, 2019

Isotropic QKE calculation



Conclusions

- The neutrino quantum kinetic equations produce many effects that need to be better understood before we can “invert” a neutrino signal
- However, this is a very active field, and there is a lot of progress
- In the near future look for: 1)results from steady state halo calculations, 2) improved resolution from simulation feeding into density perturbation analysis 3) first attempts to calculate fast flavor evolution, 4) first attempts to study the QKEs in supernova-like environments