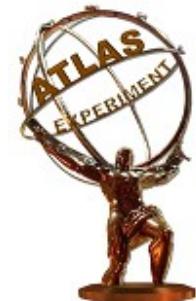
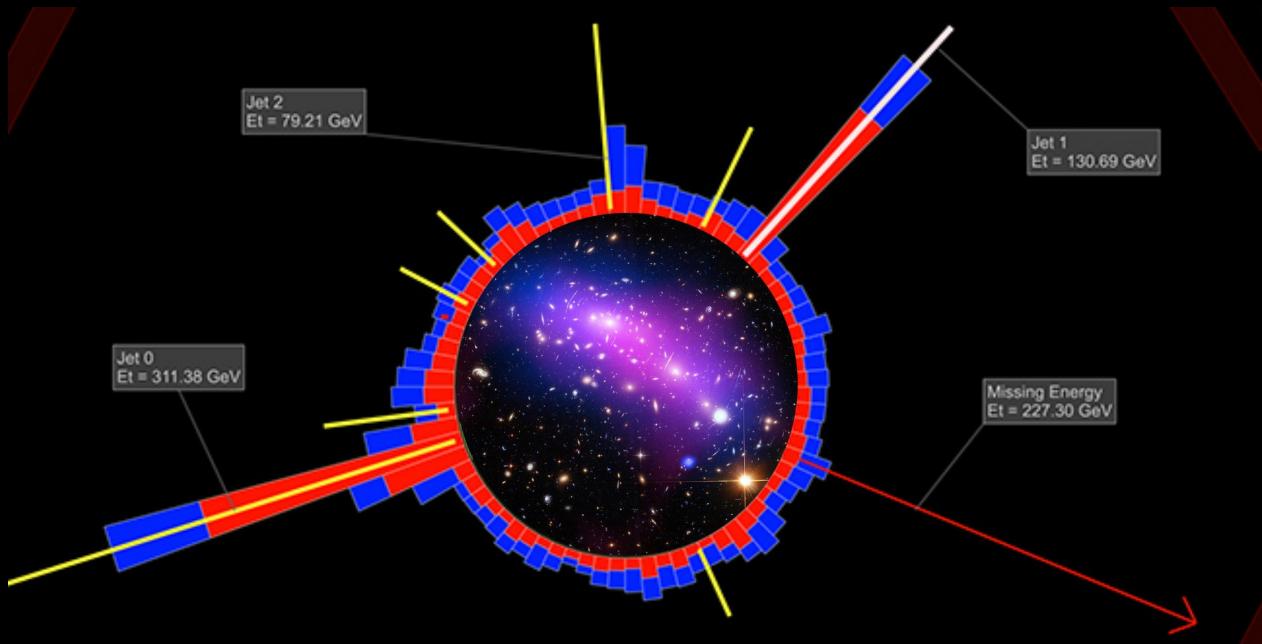
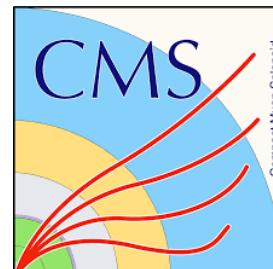


Dark Matter Searches at the LHC



Kristian Hahn
Northwestern University
on behalf of the ATLAS & CMS Collaborations



Topics in Astroparticle and Underground Physics
TAUP 2017, Sudbury ON, Canada
July 25, 2017

Dark Matter Detection

Complementary capabilities for observing DM interactions

Collider Production



Direct Detection (DD)
DM-Nucleon Scattering



Indirect Detection (ID)
DM annihilation



WIMP DM falls under the LHC “lamppost” ...

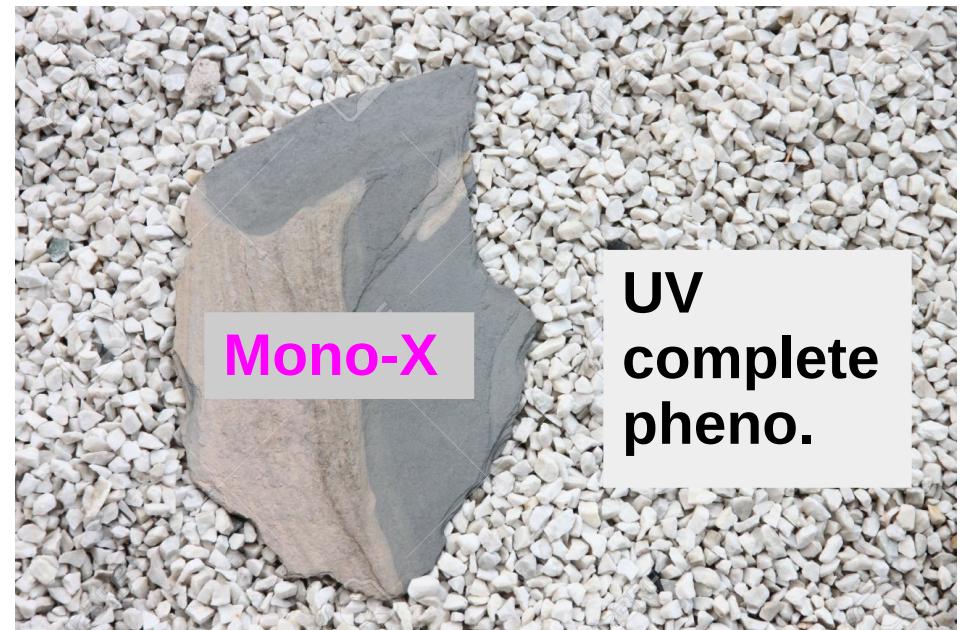
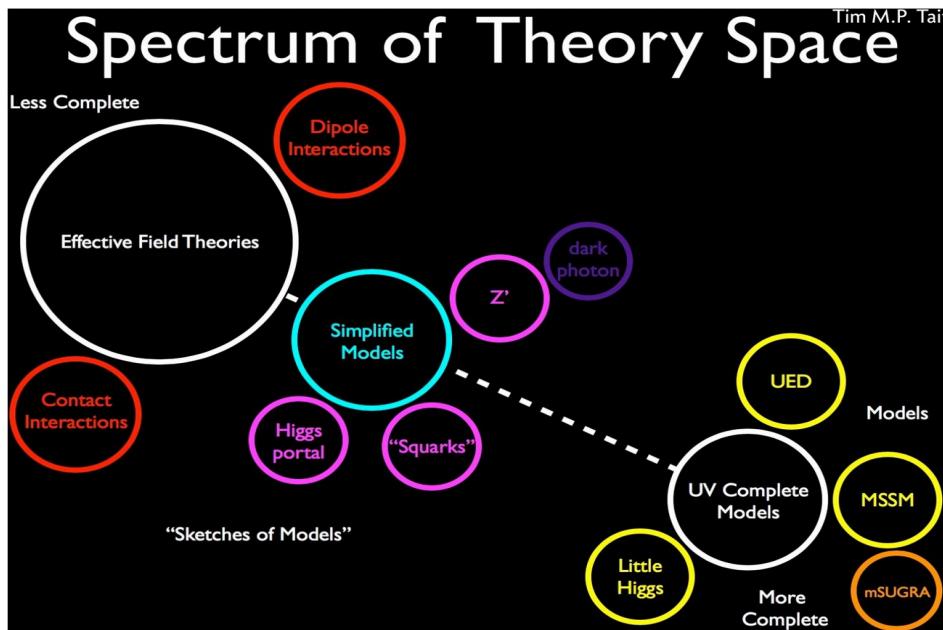
- DM as a thermal relic implies weak-ish mass/interaction scales

Some of the benefits of collider searches:

- Excellent control of systematic uncertainties
- No velocity / p-wave suppression
- Possible resonant enhancement
- Possibility to characterize DM particle properties

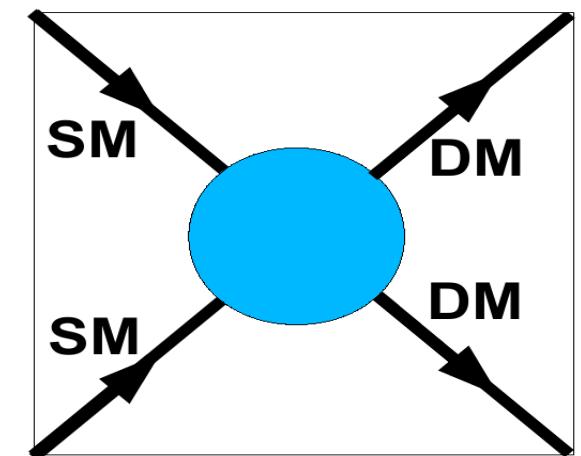
Many BSM models provide DM candidates

- Eg: SUSY (nMSSM, cMSSM, ...), UED/ADD, Little Higgs ...
- Large numbers of parameters, wide range of phenomenology

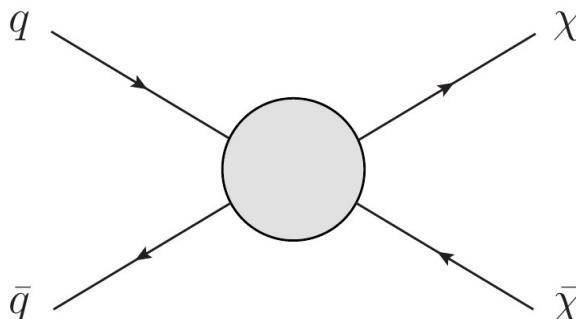


Can we take a more general approach?

- Perform broad searches based on general DM phenomenology
- Use “models” that are simple as possible, even if they are incomplete (eg: EFT →)
- Turn as large a stone as possible ... mono-X!



Simplest mode of DM production *unobservable* @ LHC



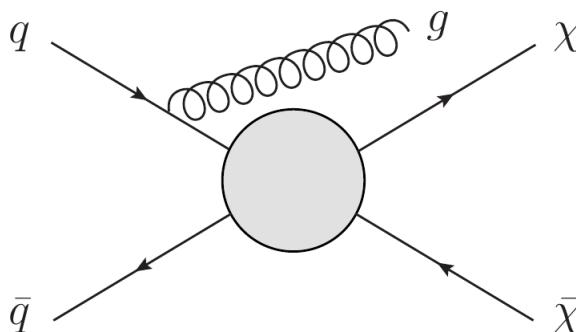
Dark Matter is DARK

- Leaves no activity in the detector
- Nothing to trigger on / reconstruct above

DM must instead recoil against *something* to become “visible”

“Mono-X” (or “MET+X”) includes “X” for viable detection

- X: quarks/gluons, photons, W/Z ...

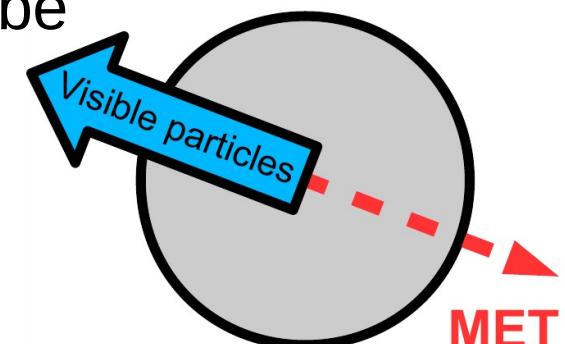
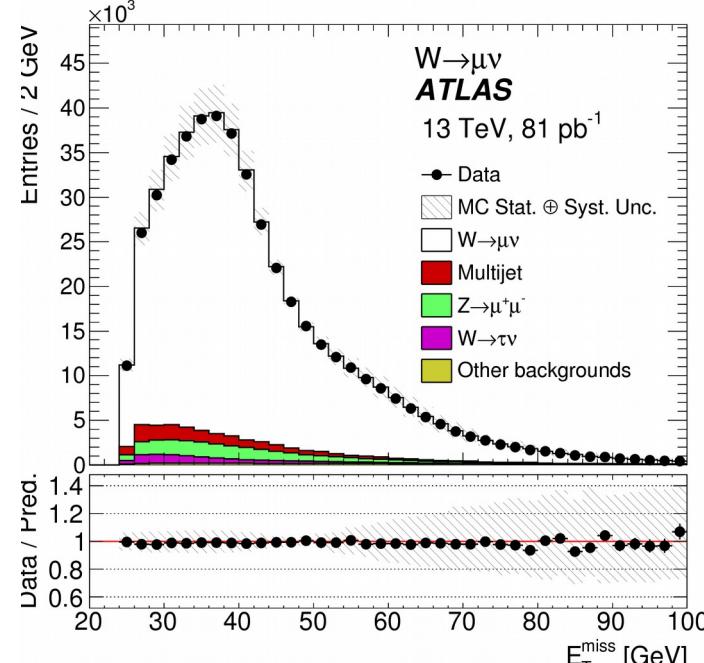
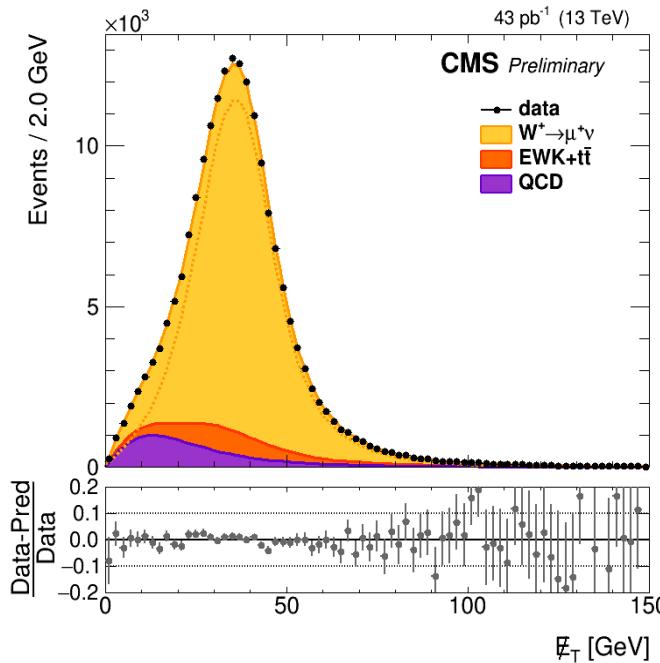


Non-interacting particles escape the detector

- Their presence inferred from energy/momentum imbalance

(Transverse) analog of nuclear recoil in DD ...

- Transverse → because final state particles can be lost in the beampipe
- E_T^{miss} = Negative vector sum of all visible pT



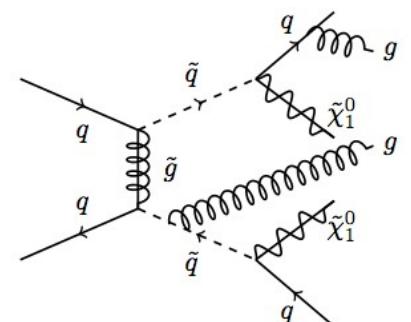
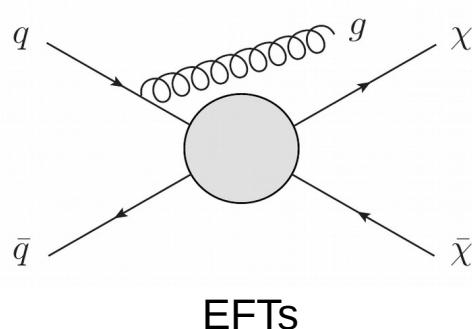
$$\vec{E}_T \equiv - \sum_i E_T^i \hat{n}_i = - \sum_{\text{all visible}} \vec{E}_T$$

A well understood collider observable

- Wide use in SM measurements

Modeling DM Collider Production

- Models used in the design and interpretation of DM searches
- Need to balance model complexity with predictive accuracy ...

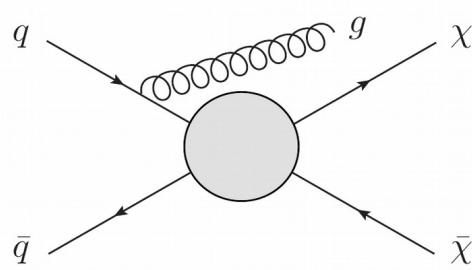


Validity issues @ LHC ..
cf: 1307.2253, 1308.6799

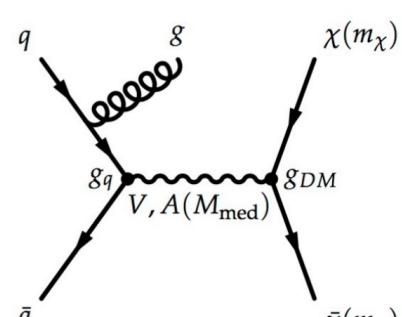
Too specific?
Theory baggage?

Modeling DM Collider Production

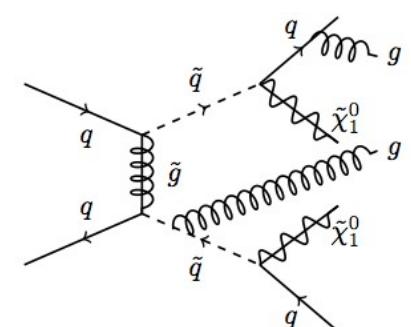
- Models used in the design and interpretation of DM searches
- Need to balance model complexity with predictive accuracy ...



EFTs



Simplified Models



UV-complete Models

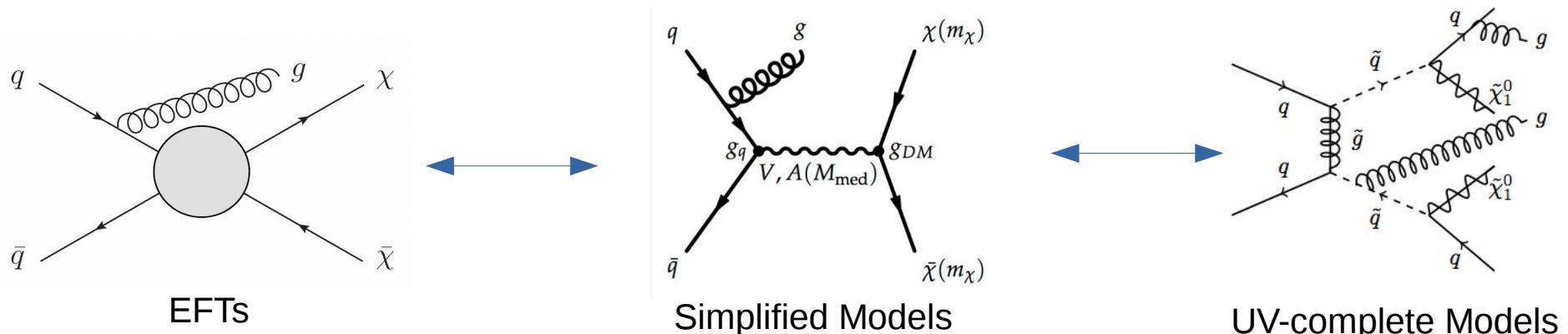
Validity issues @ LHC ..
cf: 1307.2253, 1308.6799

Just right?

Too specific?
Theory baggage?

Modeling DM Collider Production

- Models used in the design and interpretation of DM searches
- Need to balance model complexity with predictive accuracy ...



Simplified models: capture kinematics, lack completion

- Pair-produced DM Dirac fermions, χ
- Massive DM \leftrightarrow SM mediator, on/off-shell production
- Couplings: vector/axial/scalar/pseudo
- Minimal flavor violation
- Minimal mediator width: couples only to SM and χ

Only four parameters:
 $g_q, g_{\text{DM}}, m_\chi, M_{\text{med}}$

LHC DM searches using simplified models/benchmarks from
the LHC Dark Matter Forum: 1507.00966

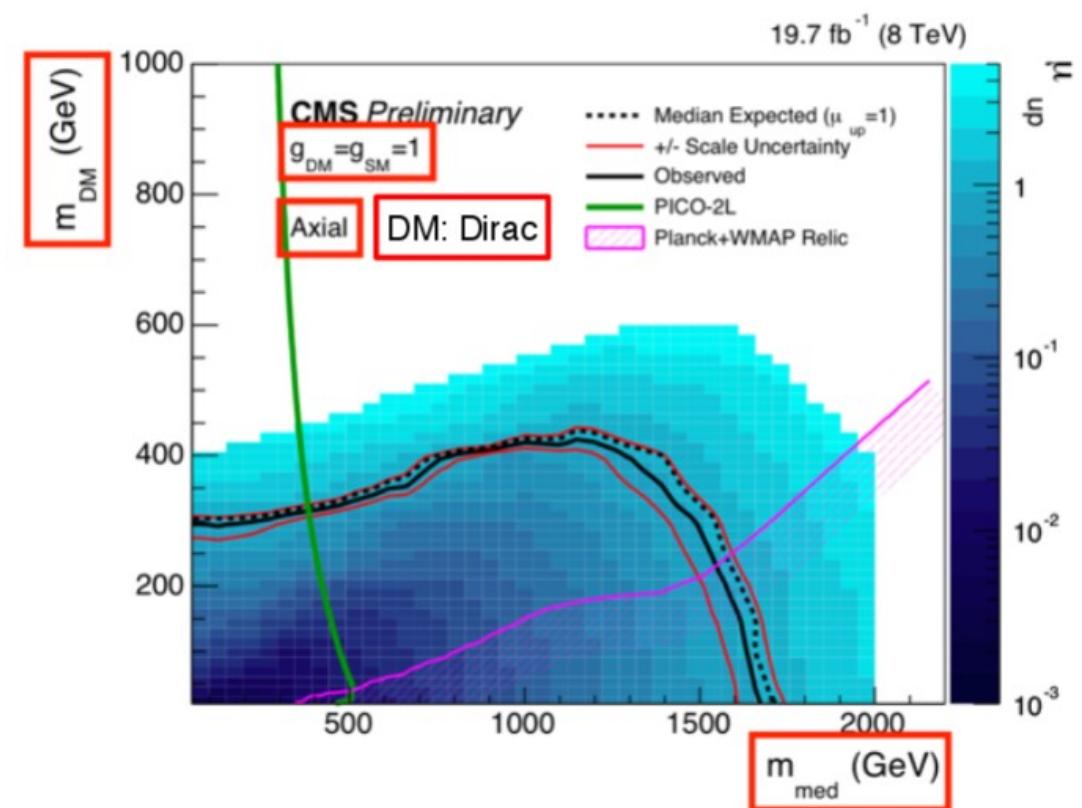
Extraction of potential DM signals ...

In absence of excess: limit setting, model constraints

- NB: 95% CLs limits are standard in collider world

$m(\text{Med})$ - $m(\text{DM})$ plane: provides natural representation of collider results

- Results shown as limit on signal cross section or on signal strength ($\mu = \sigma_{\text{obs}}/\sigma_{\text{th}}$)
- Fixed g_{DM} & g_{SM}
- All model assumptions (eg: mediator & DM type) specified



Interpretation

Comparison of collider results with (in)direct detection

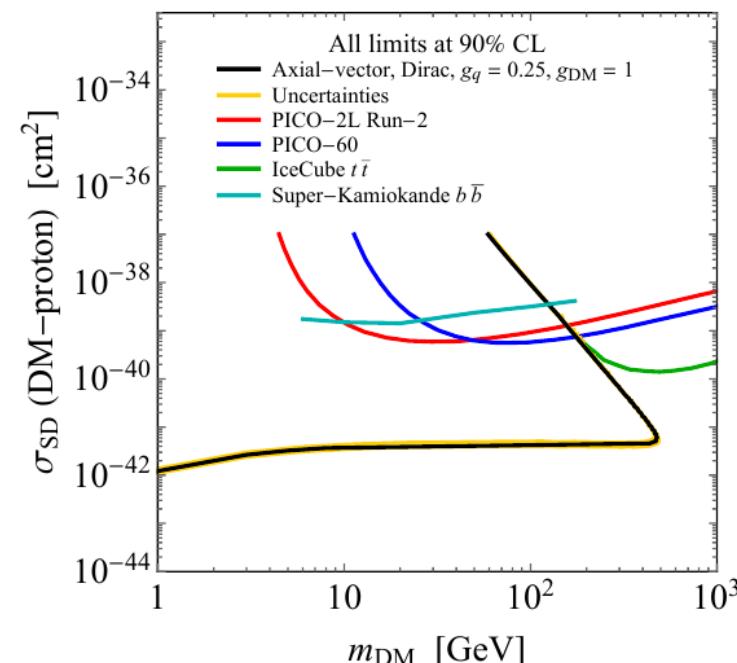
- Recent focus of LHC Dark Matter Working Group (DMWG)
- Developed recommendations for collider/non-collider comparison

Translate collider limits to $\sigma_{\text{DM-N}}$ & σv_{rel} , rather than reverse

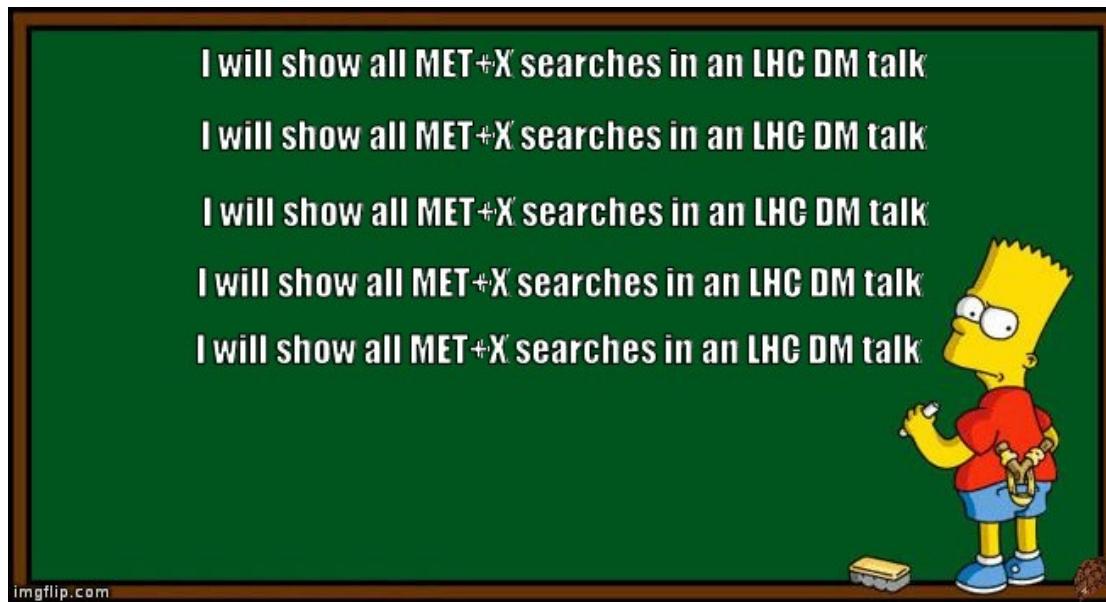
- Avoid subtleties and assumptions involved in mapping DD/ID to collider
- DD: vector/scalar (SI) axial (SD) mediators
- ID: pseudoscalar mediators

Recommendations on presenting LHC searches for missing transverse energy signals using simplified s -channel models of dark matter

Antonio Boveia,^{1,*} Oliver Buchmueller,^{2,*} Giorgio Busoni,³ Francesco D'Eramo,⁴ Albert De Roeck,^{1,5} Andrea De Simone,⁶ Caterina Doglioni,^{7,*} Matthew J. Dolan,³ Marie-Helene Genest,⁸ Kristian Hahn,^{9,*} Ulrich Haisch,^{10,11,*} Philip C. Harris,¹ Jan Heisig,¹² Valerio Ippolito,¹³ Felix Kahlhoefer,^{14,*} Valentin V. Khoze,¹⁵ Suchita Kulkarni,¹⁶ Greg Landsberg,¹⁷ Steven Lowette,¹⁸ Sarah Malik,² Michelangelo Mangano,^{11,*} Christopher McCabe,^{19,*} Stephen Mrenna,²⁰ Priscilla Pani,²¹ Tristan du Pree,¹ Antonio Riotto,¹¹ David Salek,^{19,22} Kai Schmidt-Hoberg,¹⁴ William Shepherd,²³ Tim M.P. Tait,^{24,*} Lian-Tao Wang,²⁵ Steven Worm²⁶ and Kathryn Zurek²⁷



Recent ATLAS & CMS DM Results



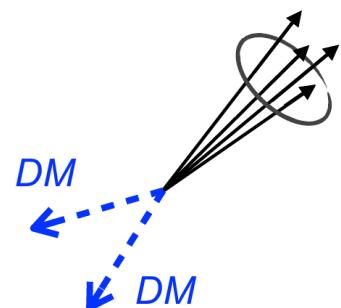
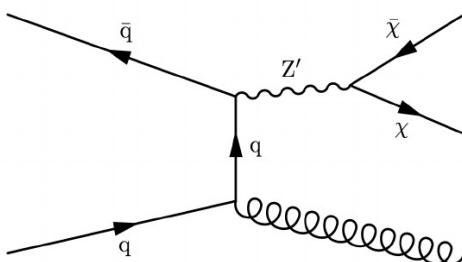
Focusing on the hadronic search channels

- Monojet, tt/bb + DM, dijet
- In the simplified model framework, these provide most of the DM reach at the LHC

Complete list of recent results in the backups

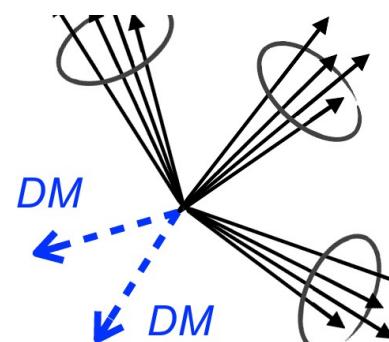
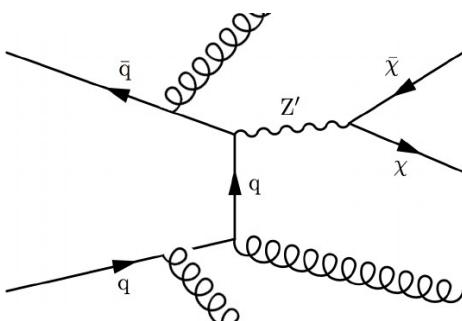
A generic & powerful DM search strategy at the LHC

- Assumes only that DM couples in someway to incoming quarks
- Require energetic recoiling jet to trigger detector



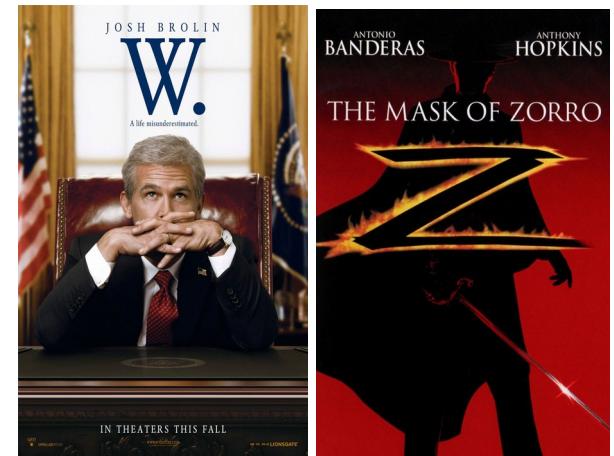
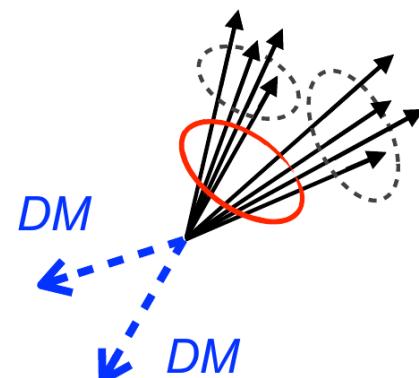
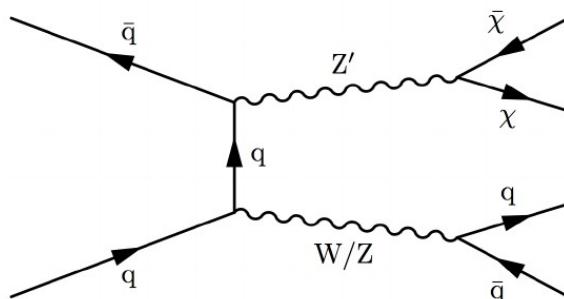
But no need to limit to a single recoiling jet ...

- The “monojet” search actually targets multijet + E_T^{miss} !

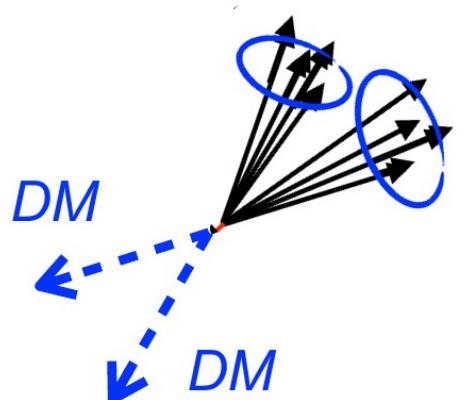


Monojet / Mono-V

DM + hadronic decays of EWK bosons can also produce a multijet + E_T^{miss} signature ... mono-V



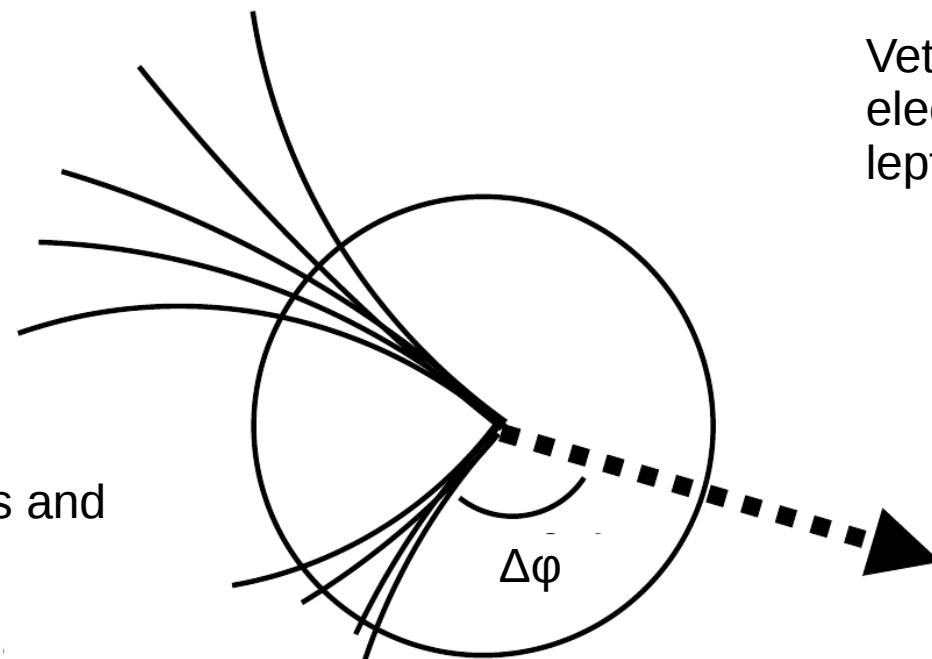
- W/Z decay products will be **boosted** when DM recoil is significant
- Reconstruction algorithms can merge these into a ~small radius jet
- But can use jet grooming / substructure techniques to identify the underlying 2-prong nature



Monojet : general strategy

At least one central ($|n| < 2.4$), good-quality, high- pT (eg >250 GeV) jet

Require minimum $\Delta\phi$ separation between jets and E_T^{miss} to suppress misreconstruction BGs

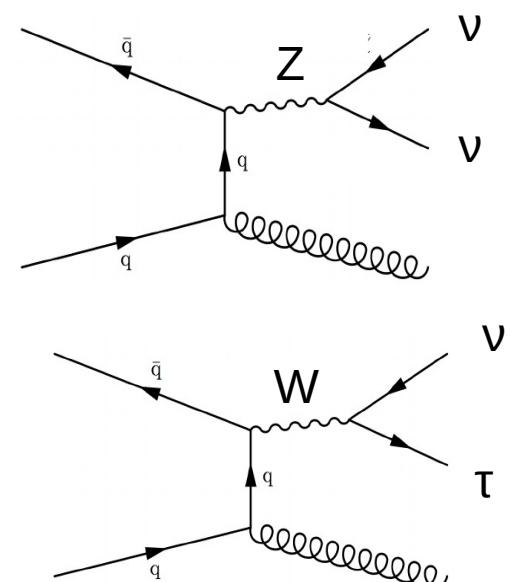


Veto additional objects: electrons, muons, tau leptons, photons, bjets ...

Significant E_T^{miss} (eg >200 GeV)

Dominant backgrounds from SM processes with real E_T^{miss} and/or leptons out of detector acceptance

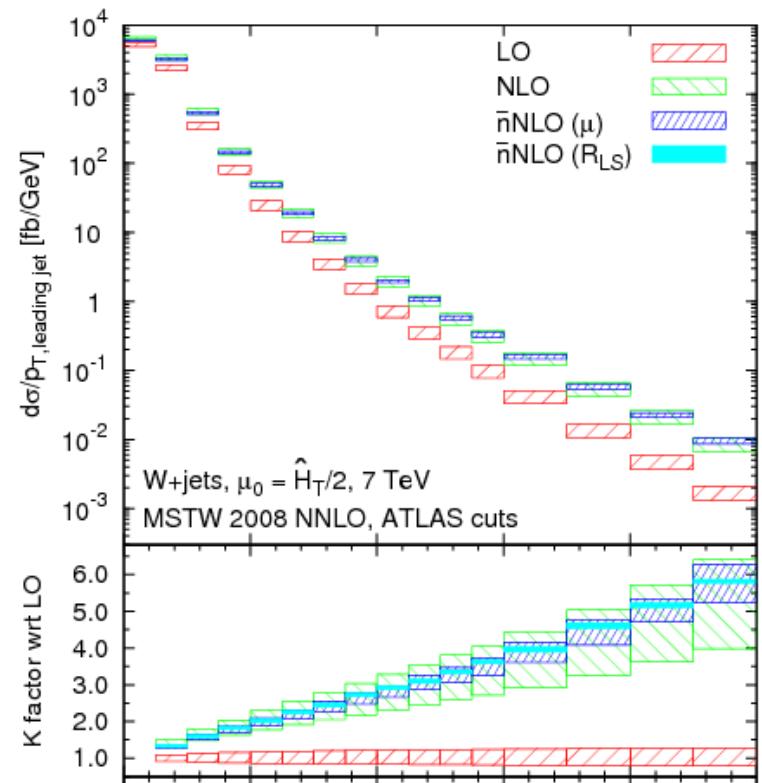
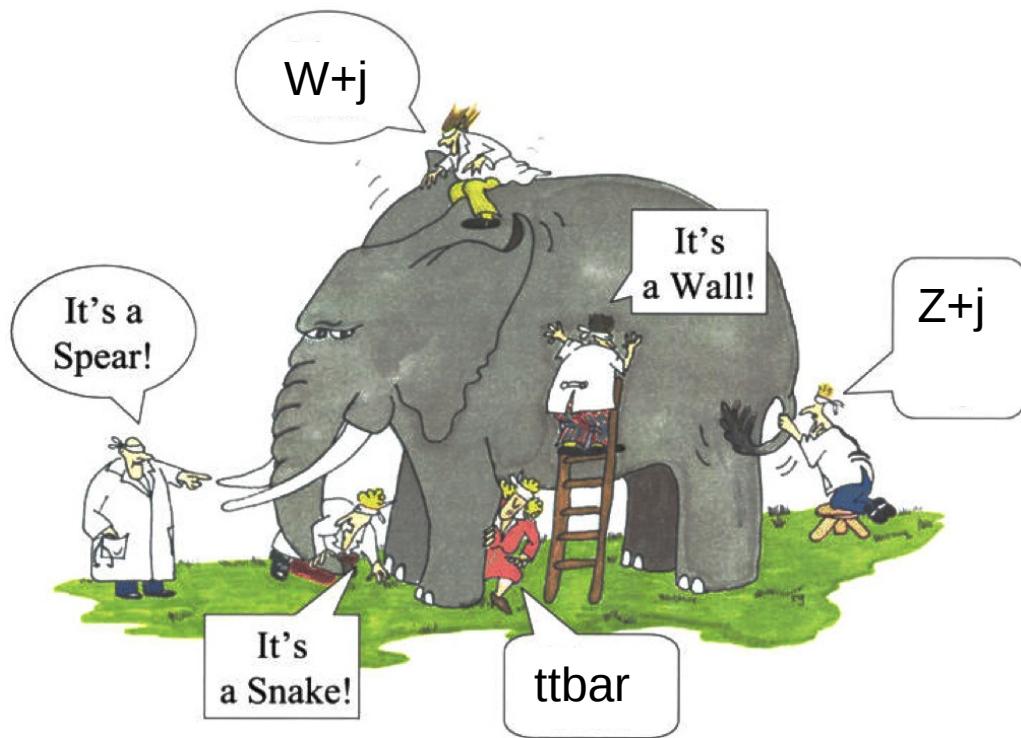
- $Z(vv) + \text{jets}$, $W(\tau[qq']vv) + \text{jets}$, $W(lv) + \text{jets}$
- Bread & butter EWK processes @ the LHC
- Wealth of precise calculations & simulation tools available



Monojet : general strategy

Selections define signal enriched regions (SR) in data

- Residual backgrounds in these regions from events in tails of E_T^{miss} kinematic distributions
- Associated SM theory uncertainties are typically large here ...



BG dominated control regions (CR) help constrain SM rates & kinematics in the SRs

- *Augment* precise calculations of EW processes with measurements!

35.9 fb⁻¹ : CMS-PAS-EXO-16-048, 12.9 fb⁻¹: JHEP 07 (2017) 014, 1703.01651

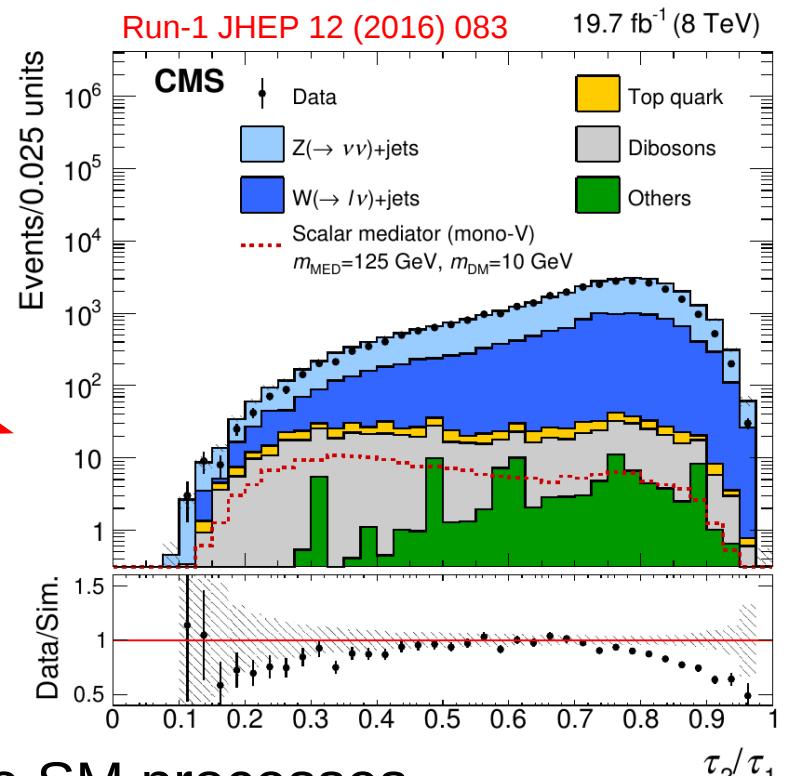
SR selection : large E_T^{miss} ,
 ≥ 1 high- p_T jet, $\Delta\phi > 0.5$ radian

- Mono-V : $p_T^{\text{AK8}}, E_T^{\text{miss}} > 250 \text{ GeV}$,
 $m_{jj} 65-105 \text{ GeV}, \tau_{12} < 0.6$ ("n-subjettiness")
- Mono-jet : remaining events,
 $p_T^{\text{AK4}} > 100 \text{ GeV}, E_T^{\text{miss}} > 250 \text{ GeV}$

5 (categorized) SM control regions
to constrain high- E_T^{miss} BGs

- Use observable analogues of the invisible SM processes
 - $Z(\mu\mu), Z(ee), W(\mu\nu), W(e\nu)$ + jets, **high-stat γ +jet**
- Subtract visible signatures → hadronic recoil, a proxy for E_T^{miss}
- Use NLO QCD + NLO EWK calculations to translate rates + distributions in CRs into SR predictions!

Extract signal from combined likelihood fit to E_T^{miss} distributions

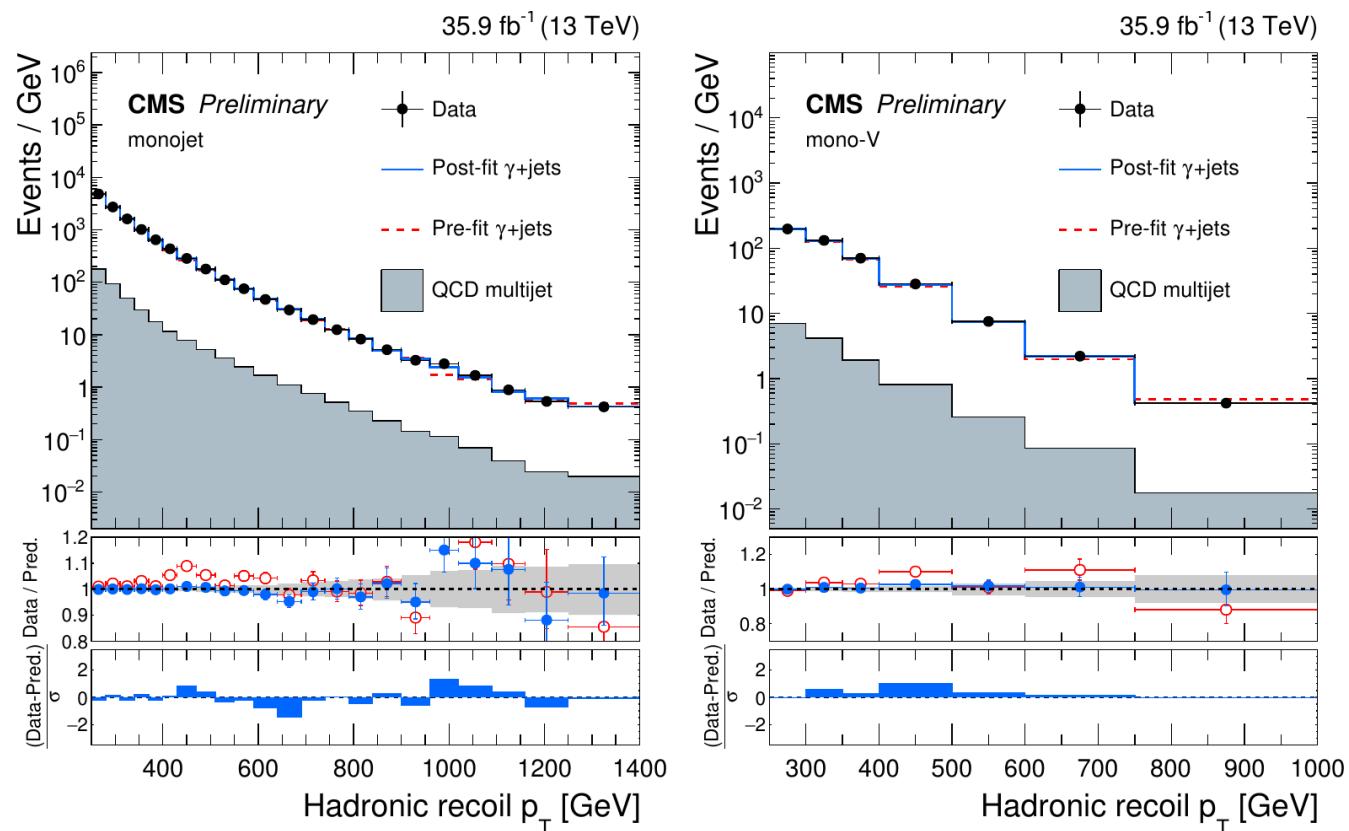


Uncertainties & correlations on transfer factors (see 1705.04664)

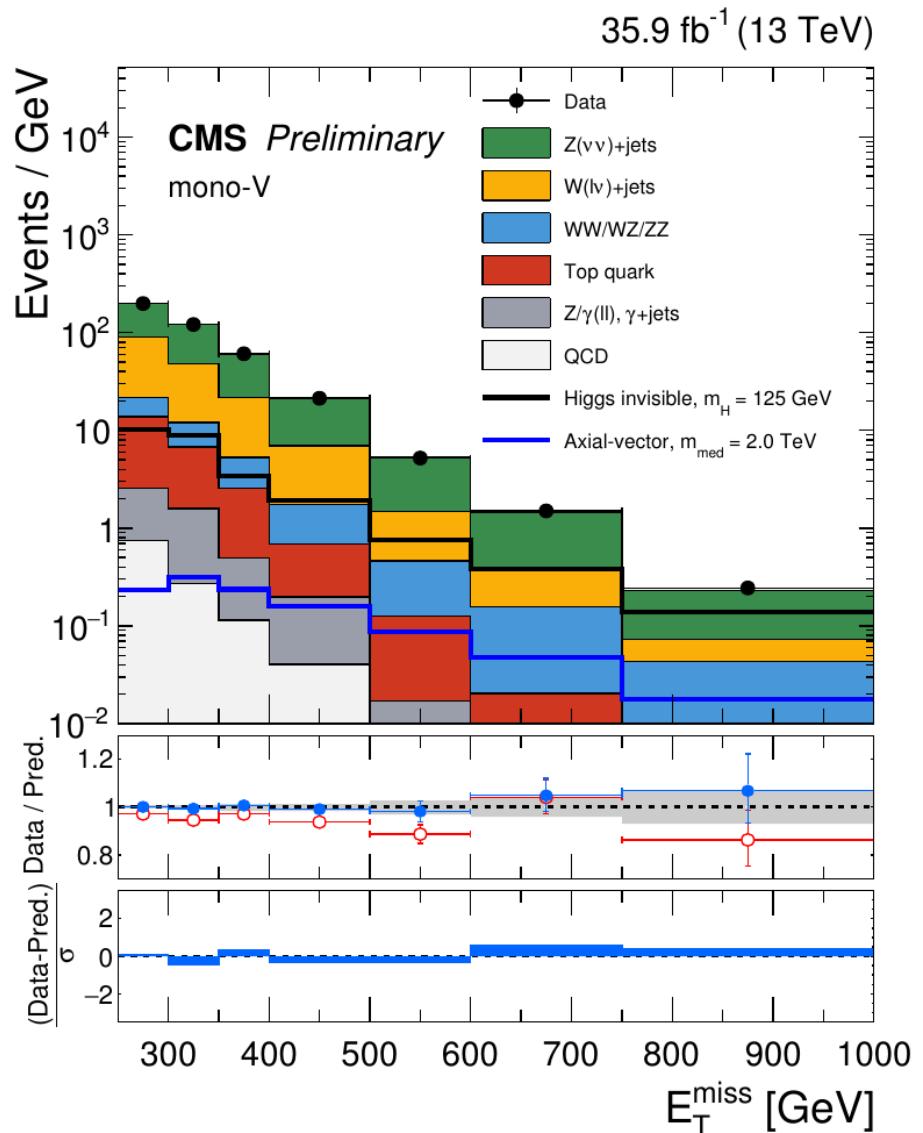
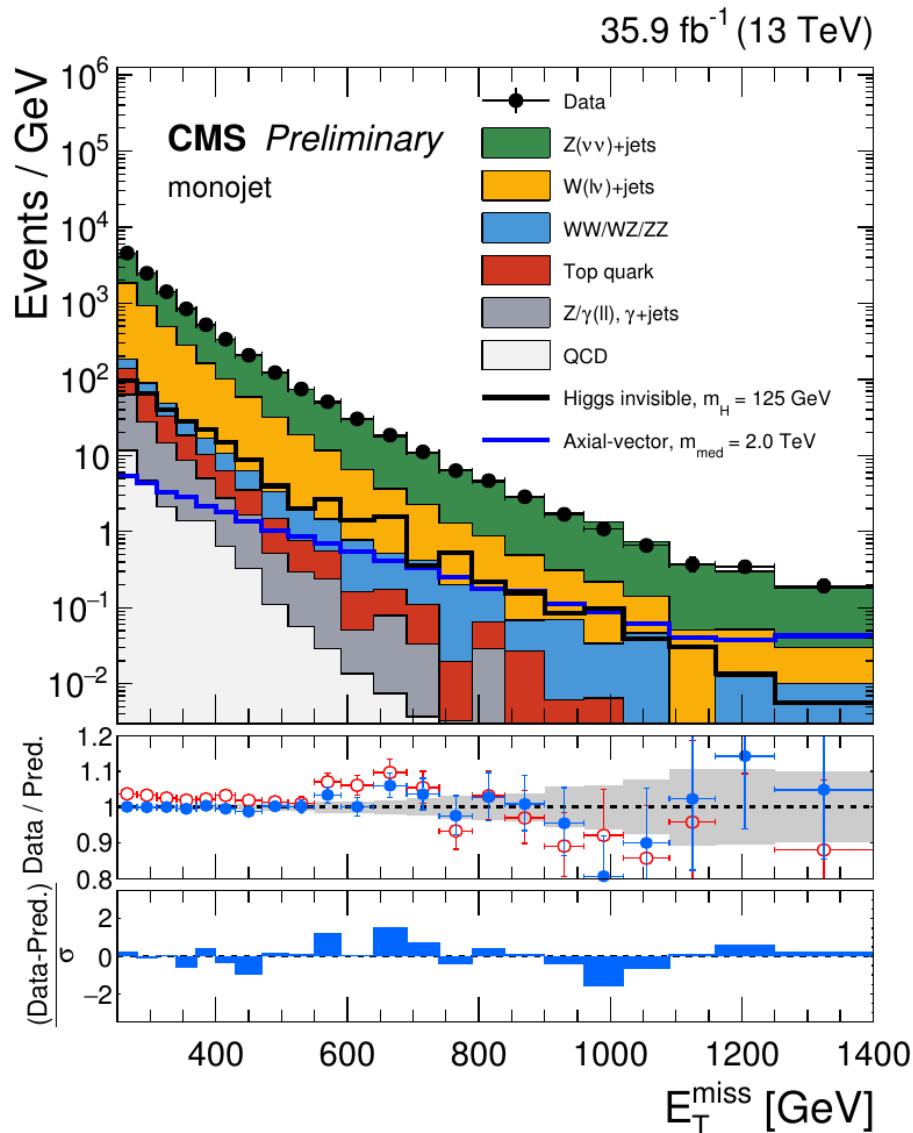
- Incorporated as nuisance parameters in the fit
- Pure QCD effects: scale/normalization, recoil shape pT dependence, cross section ratios
- Pure EWK effects: missing NNLO, unknown Sudakov logs, NLL Sudakov approximation
- Combined multiplicatively, nuisance added for possible non-factorization

Control regions
fit simultaneously
with the signal
regions

- Excellent post-fit agreement in CRs

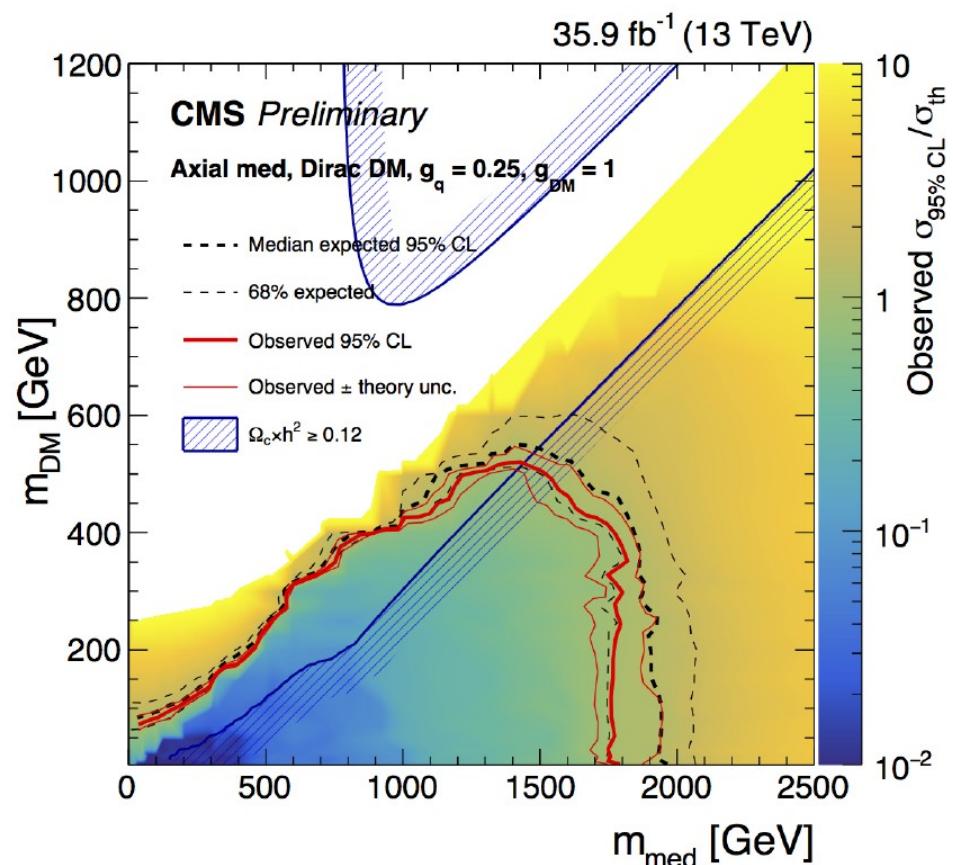
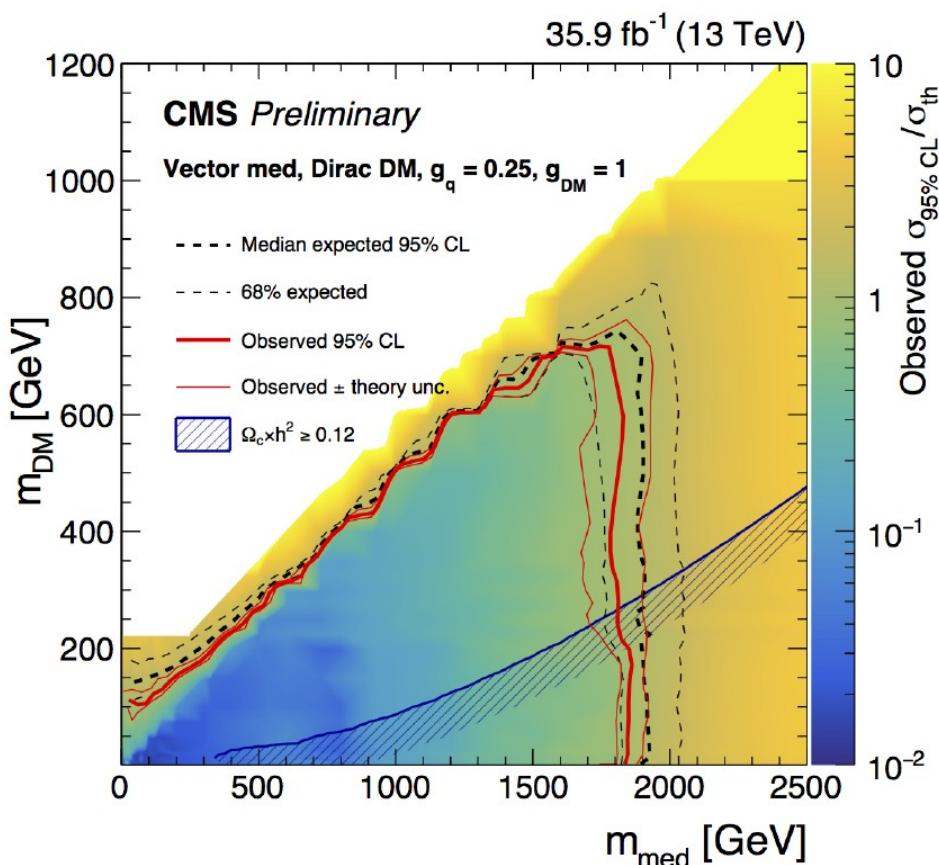
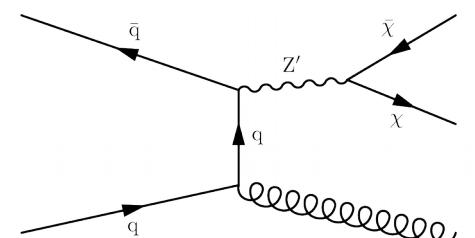


Data in signal region consistent w/ post-fit SM expectations ...



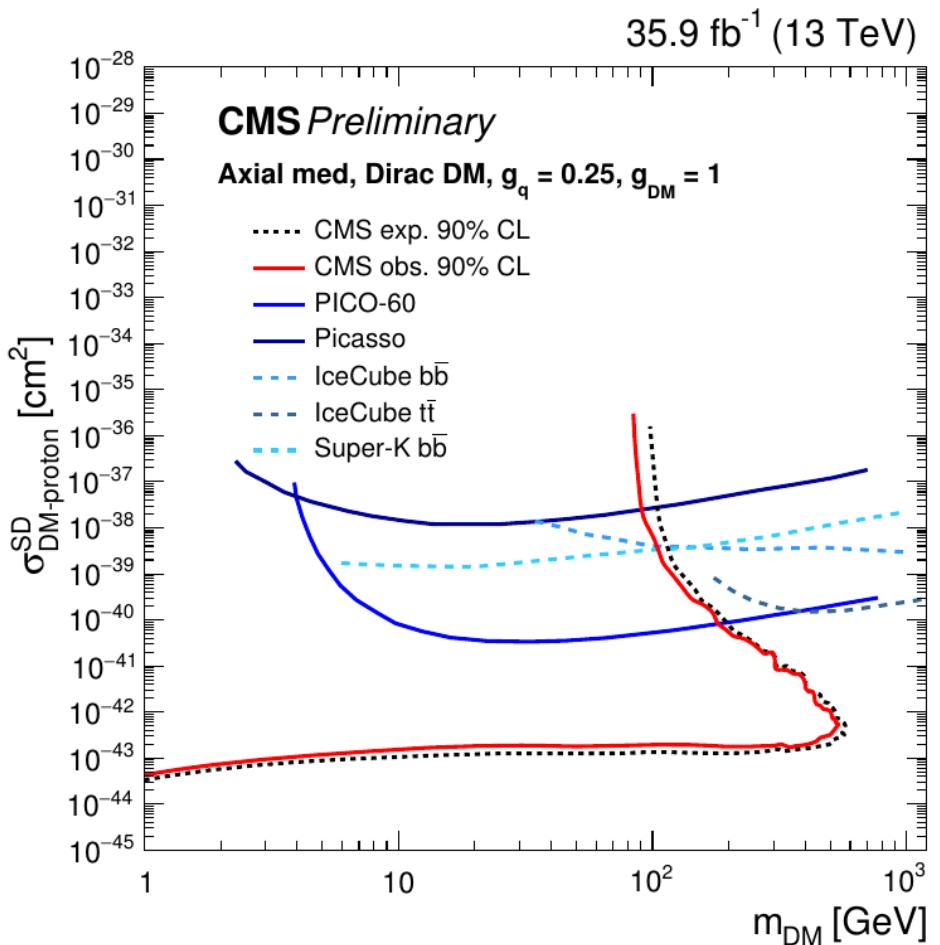
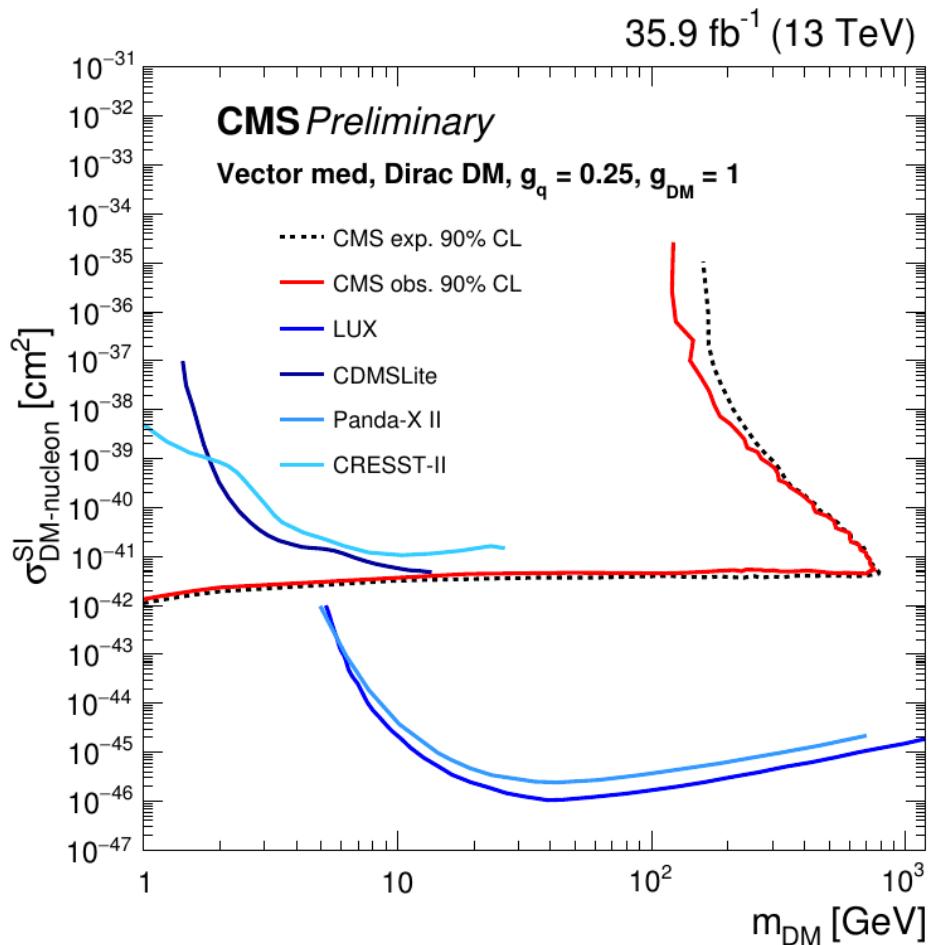
Limits on both spin-1 and spin-0 mediators

- Vector/Axial exclusion (this slide) up to 1.8 TeV
- Pseudoscalar (backup) up to 400 GeV



Reinterpret as invisible Higgs : $\text{BR}(h \rightarrow \text{inv.}) < 0.53$ (0.4 exp.)

And recast as limits on SI/SD DM-nucleon cross section (1603.04156)



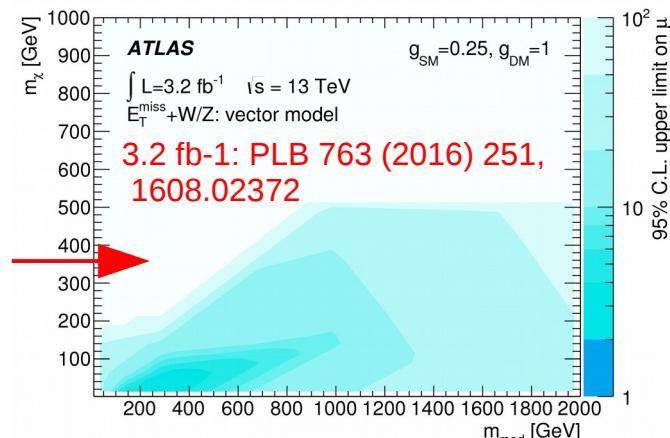
Low-mDM reach complementary to direct detection!

ATLAS Monojet

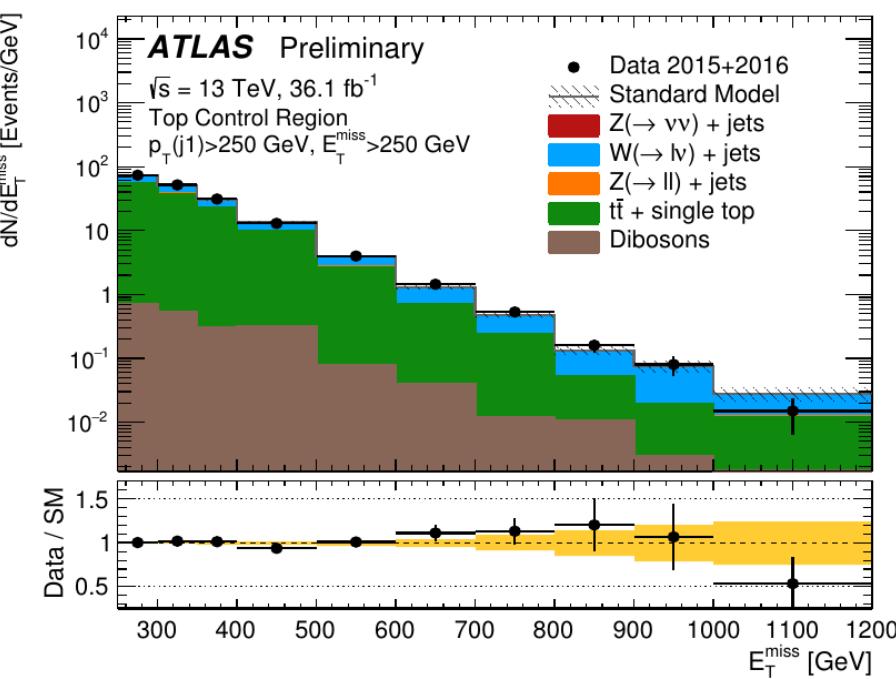
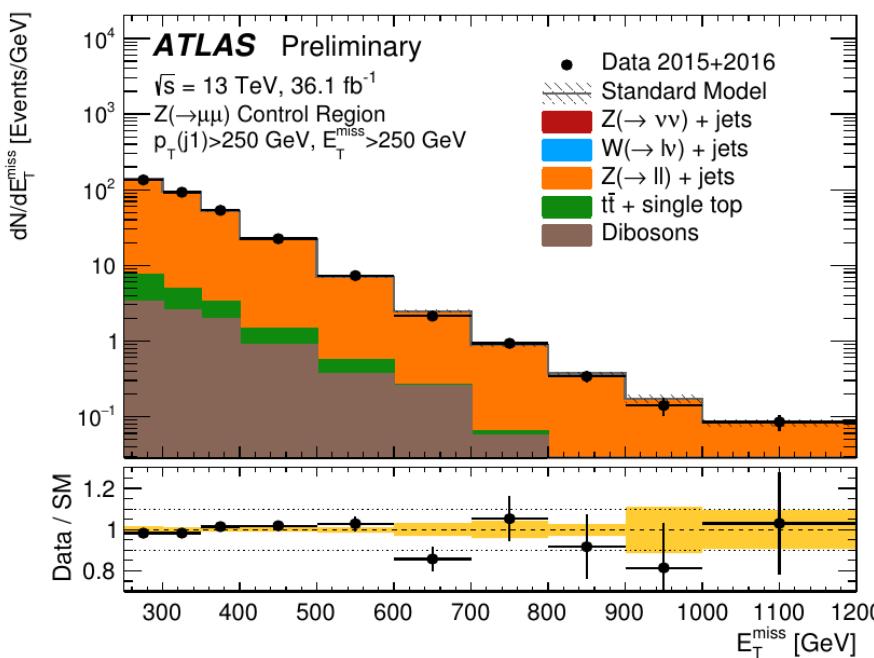
36.1 fb⁻¹ : ATLAS-CONF-2017-060, 3.2 fb⁻¹: PRD 94 (2016) 032005, 1604.07773

Similar monojet search strategy pursued in ATLAS:

- p_T^{AK4} , $E_T^{\text{miss}} > 250 \text{ GeV}$, $\Delta\phi > 0.4 \text{ radian}$, vetos
- Simultaneous binned likelihood fit to E_T^{miss}
- No mono-V category, dedicated mono-W search
- No $Z(\text{ee}) + \text{jets}$, $\gamma + \text{jets}$ CRs, adds $t\bar{t}$ CR

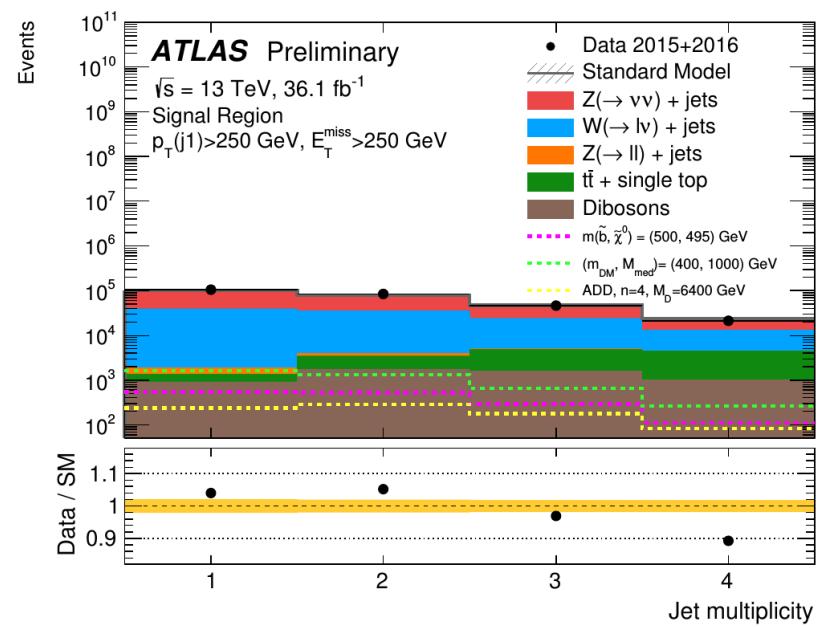
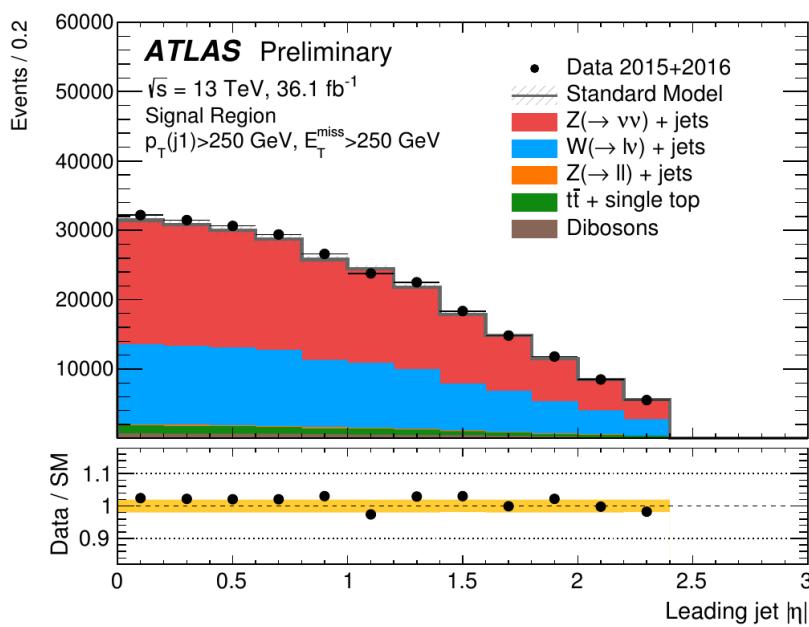
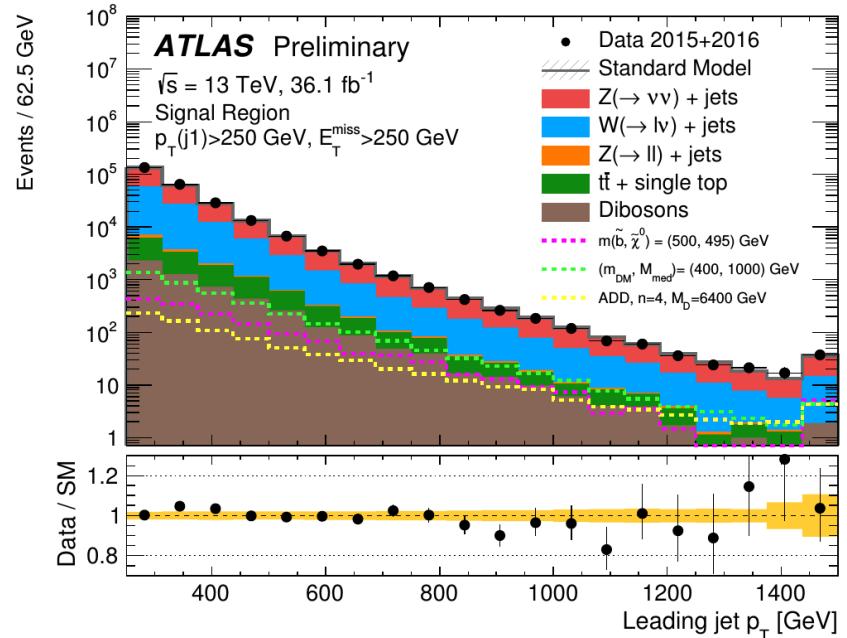
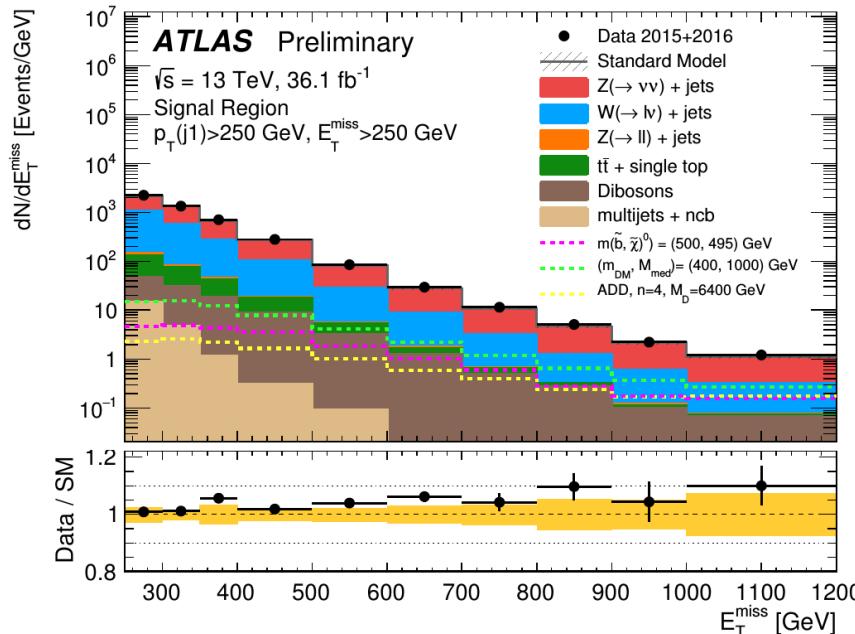


Good agreement in $Z(\text{ll}) + \text{jets}$ & $W(\text{lv}) + \text{jets}$ control regions



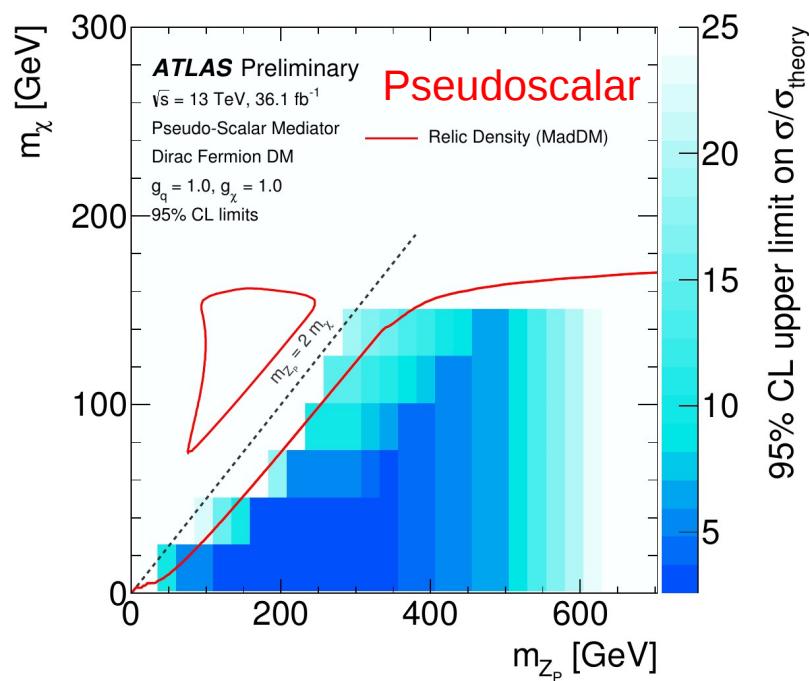
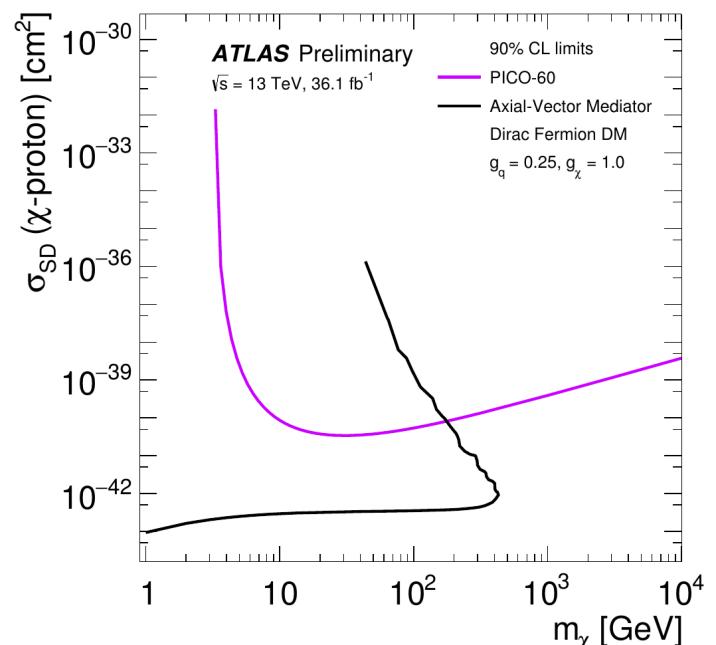
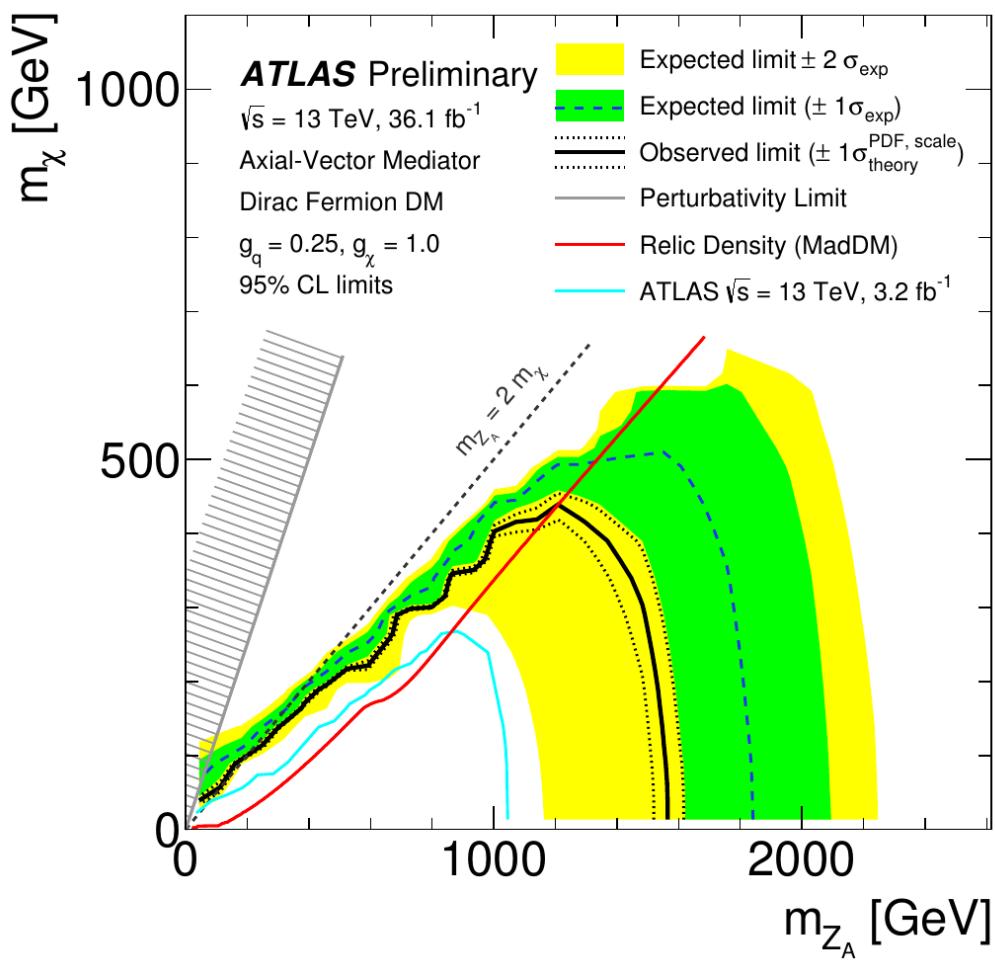
ATLAS Monojet

And good agreement in the signal region ...



Limits on both spin-1 and spin-0 mediators

- Axial-vector exclusion up to 1.55 TeV
- Not yet sensitive to pseudoscalars



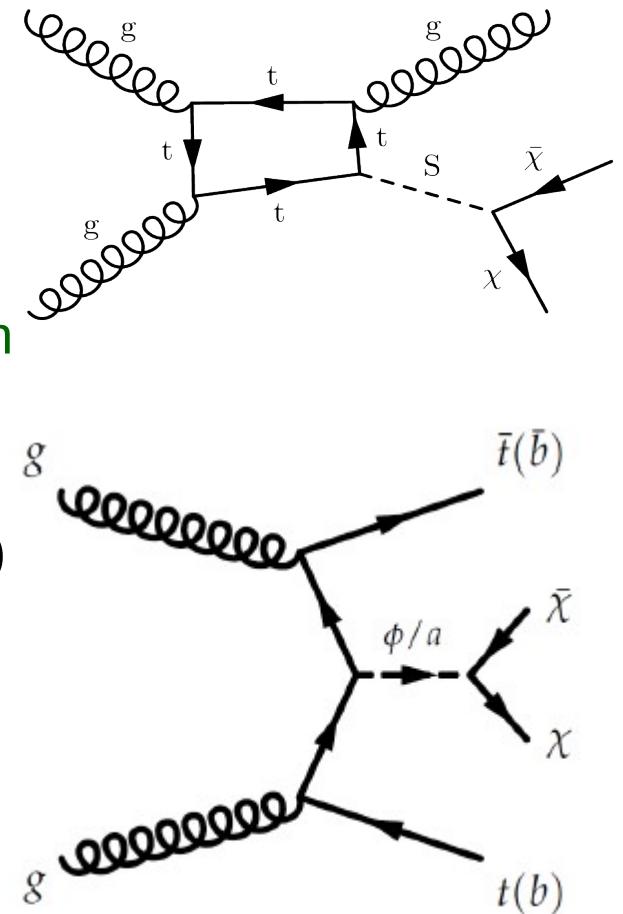
ttbar + DM

Monojet drives sensitivity to spin-1 mediator scenarios

- Picture more nuanced for spin-0 models ...
 - MFV \rightarrow mediator has Yukawa coupling
 - Monojet through heavy quark loops
- Implies tree-level couplings to top and bottom
 - Same mediator as in monojet
 - Yukawa enhancement \rightarrow tt+DM competitive with monojet at low mMed!
- Can also anticipate a “monotop” signature ...
 - Assumes specialized signal model (see backup)

DM+ heavy quarks = rich signatures!

- tt final states: all-hadronic, semileptonic, dileptonic
 - Produces leptons, high-pT jets, b jets, E_T^{miss}
- Many experimental handles \rightarrow many viable DM search strategies

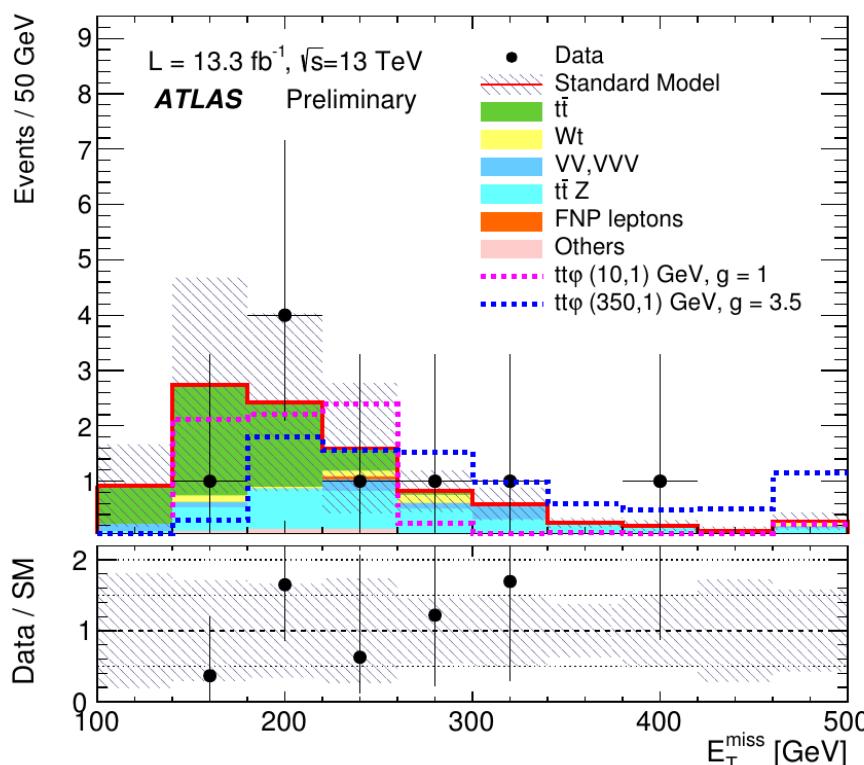


Backgrounds : mostly SM ttbar (with a lost lepton), single top, ttV

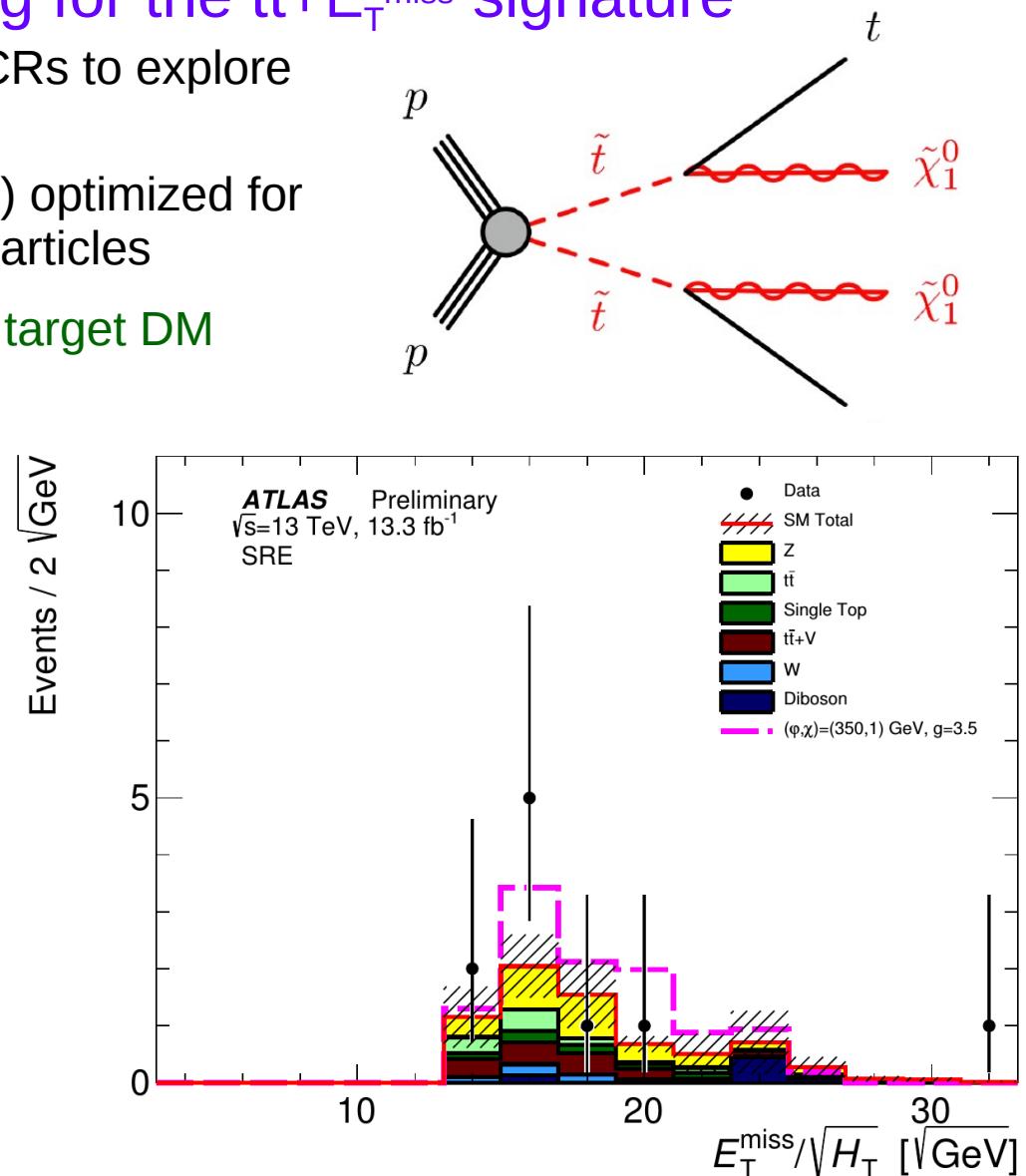
ATLAS tt/bb + DM

SUSY stop searches also looking for the tt+E_T^{miss} signature

- These generally involve many SRs & CRs to explore wide range of SUSY scenarios
- Leverage SUSY observables (eg: mT2) optimized for selecting E_T^{miss} from decays of heavy particles
- Extend SUSY search with regions that target DM production, add DM interpretation



ATLAS-CONF-2016-076 (13.3 fb-1)
ttbar (dilepton) + E_T^{miss}



ATLAS-CONF-2016-076 (13.3 fb-1)
ttbar (dilepton) + E_T^{miss}

ATLAS tt/bb + DM

ATLAS-CONF-2016-086 (13.3 fb-1)

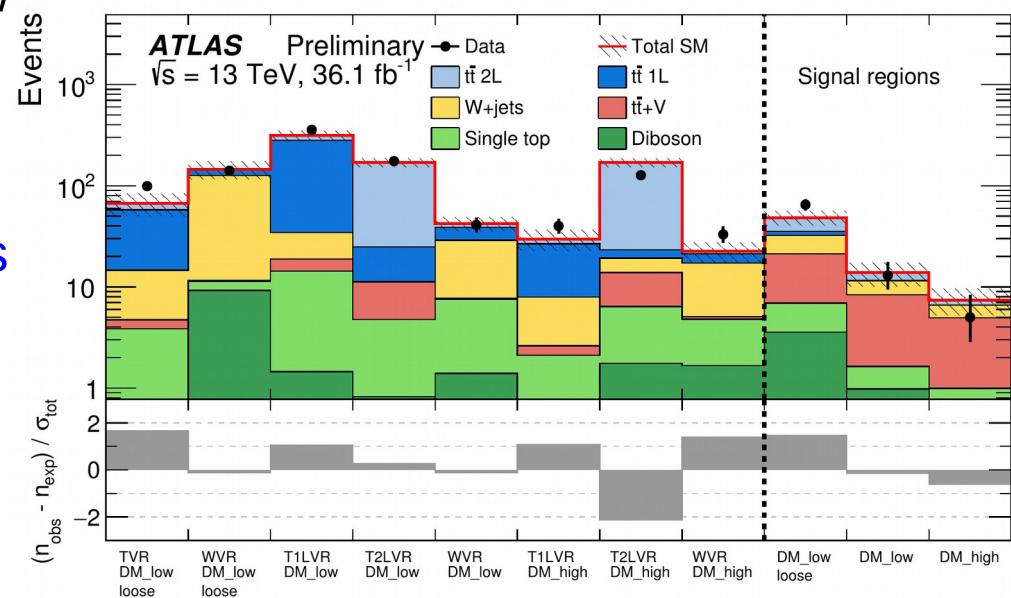
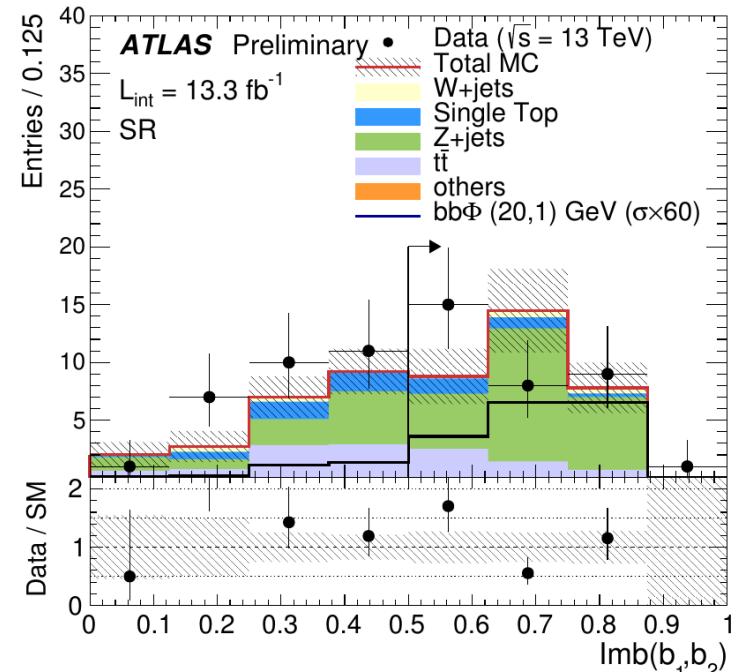
Dedicated bb+ E_T^{miss} search

- Sensitive to models (eg: 2HDM w/ large $\tan\beta$) in which coupling to down-type quarks enhanced
- Select events with large pT imbalance between 2 high-pT b-tagged jets
- 3 CRs to control Z+jets, W+jets and ttbar

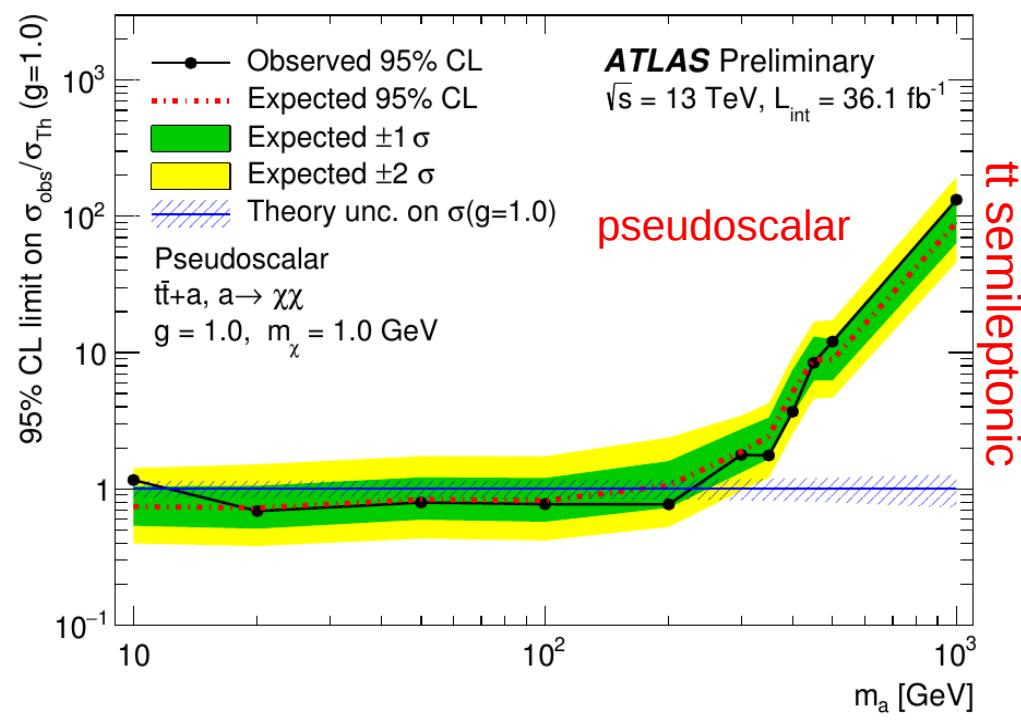
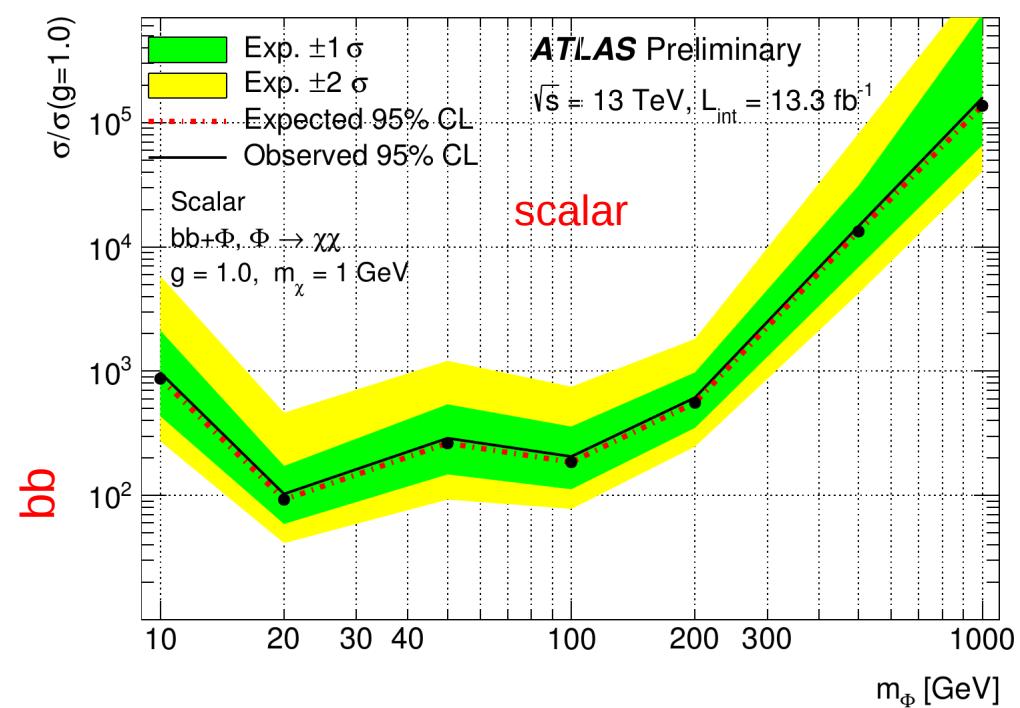
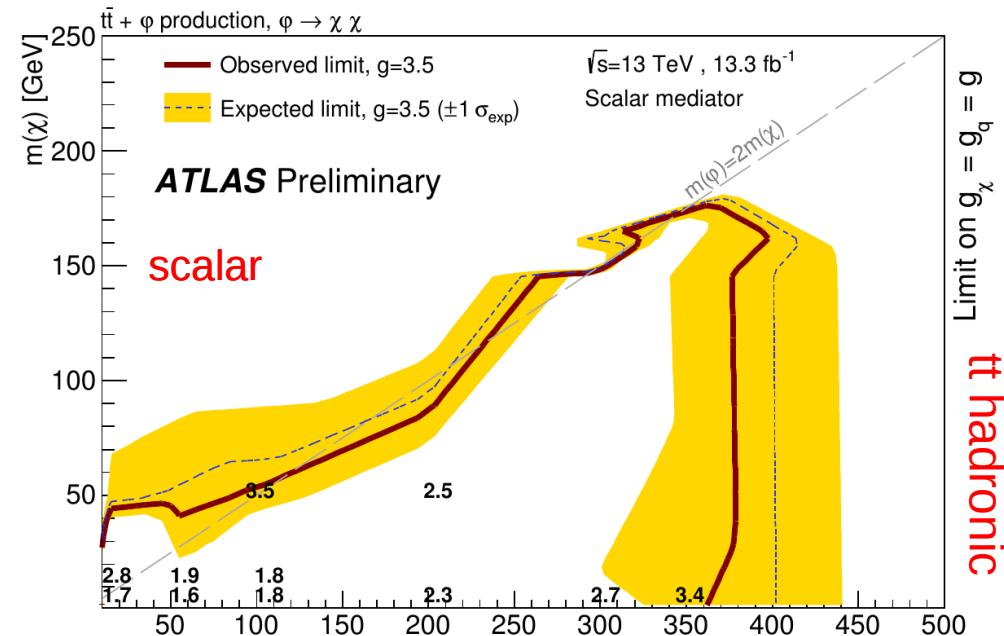
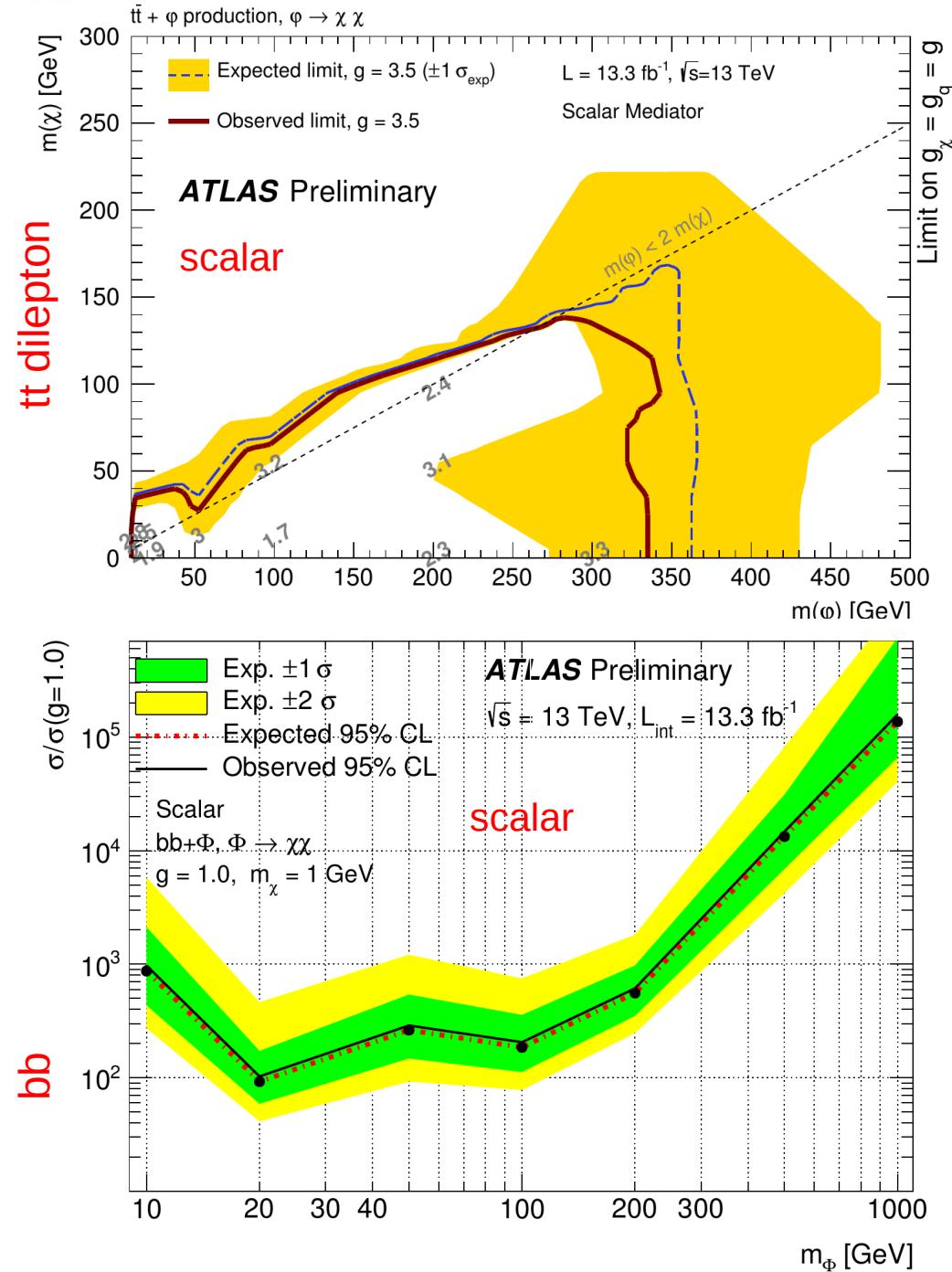
ATLAS-CONF-2017-037 (36.1 fb-1)

Update: tt(semileptonic)+ E_T^{miss} search

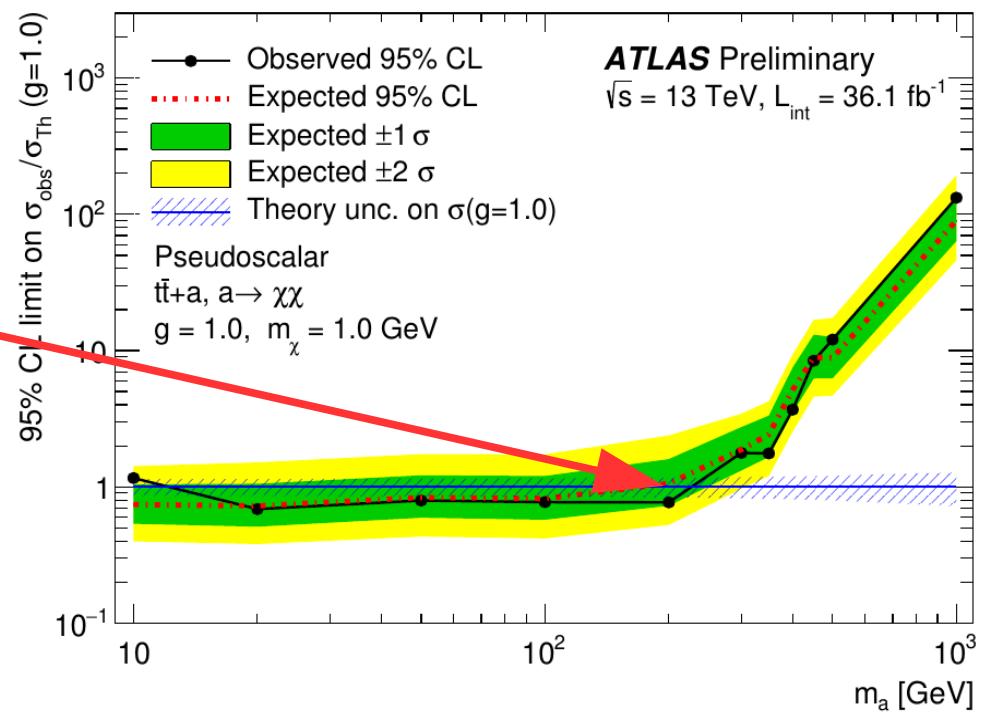
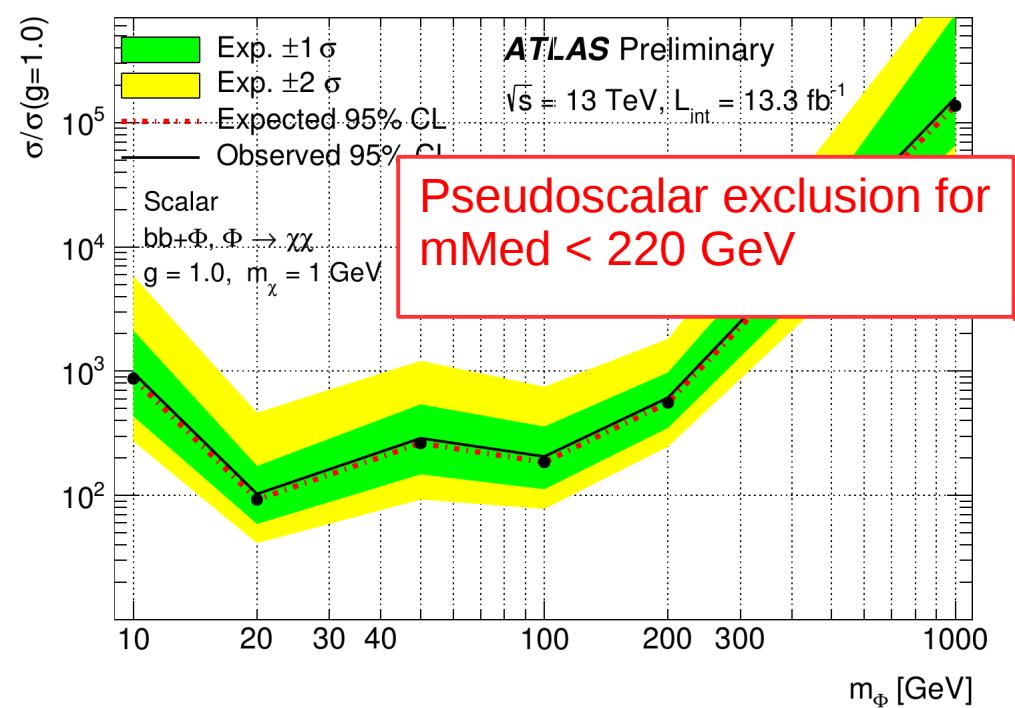
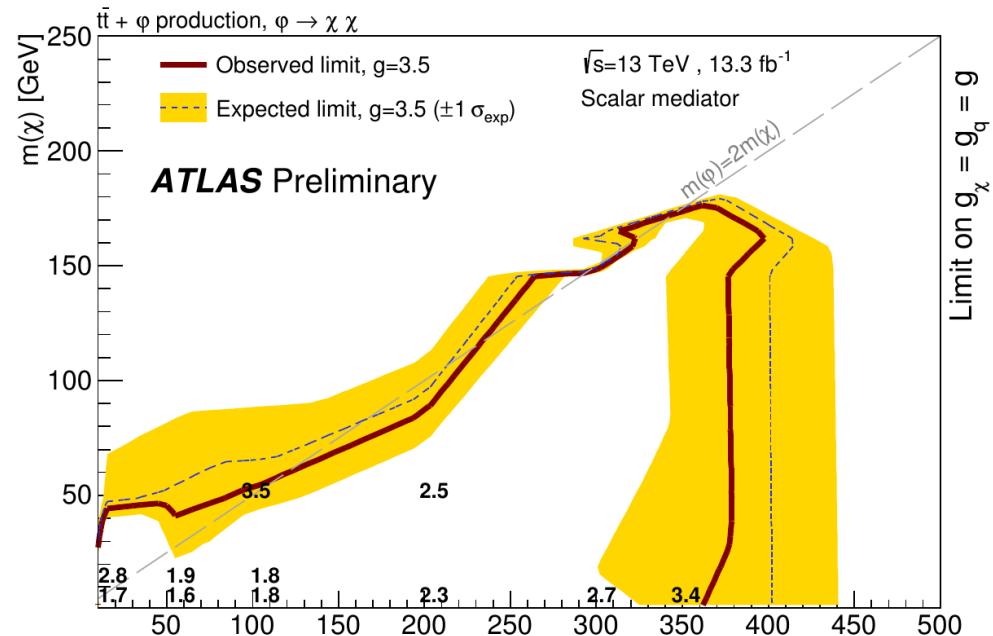
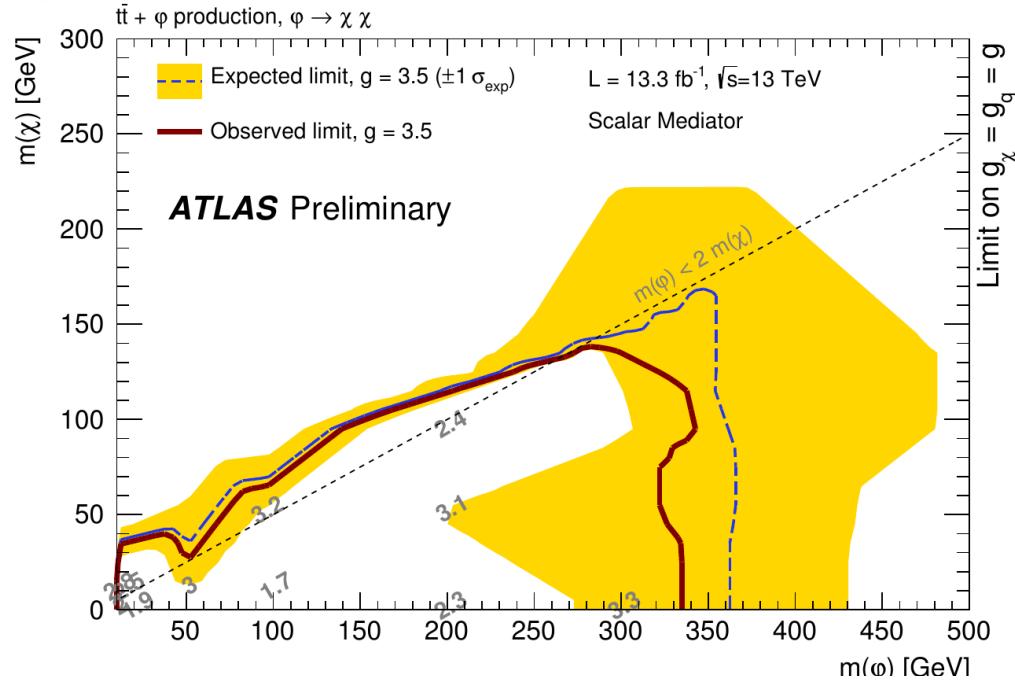
- DM categories provide sensitivity to low (~ 20 GeV) and high (~ 300 GeV) mass DM mediators
- New SRs use boosted top-tagging discriminant to identify hadronic decays of high-pT top quarks
- ttbar normalized via CR fit, signal extraction from 3 bin cut & count analysis



ATLAS tt/bb + DM Limits



ATLAS tt/bb + DM Limits



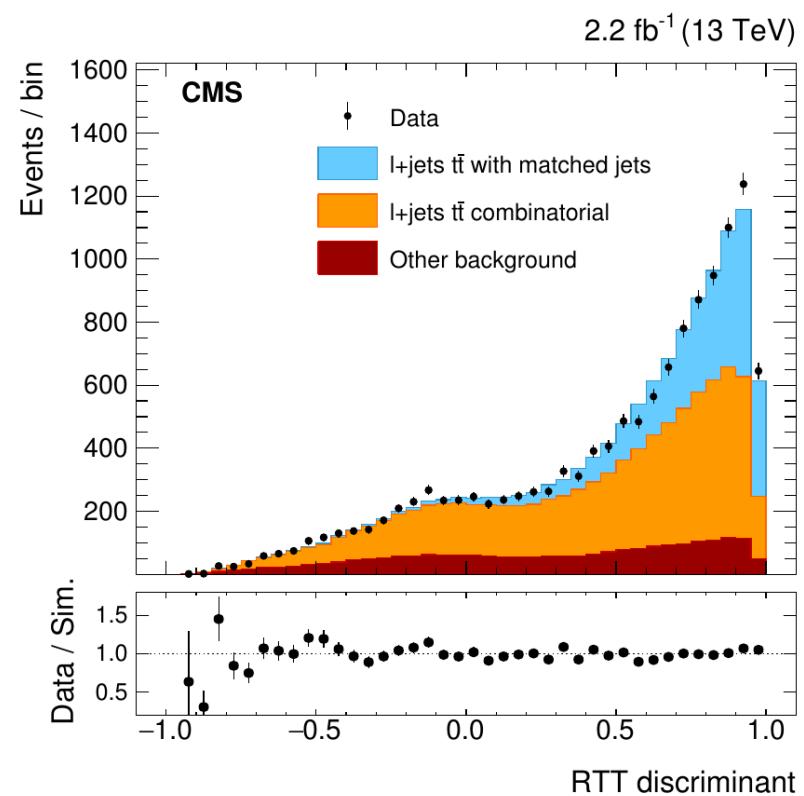
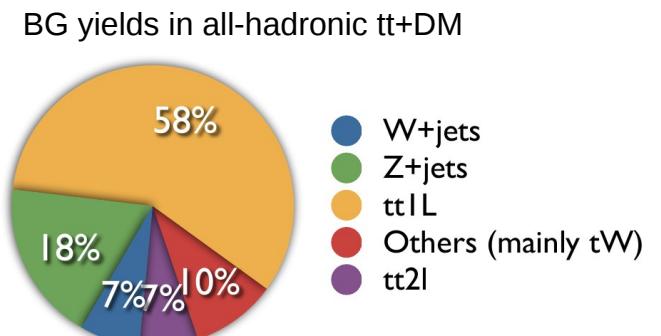
2.2 fb⁻¹ : 1706.02581, CMS-PAS-EXO-16-005, CMS-PAS-EXO-16-028

Combined search using all tt+E_T^{miss} and bb+E_T^{miss} channels

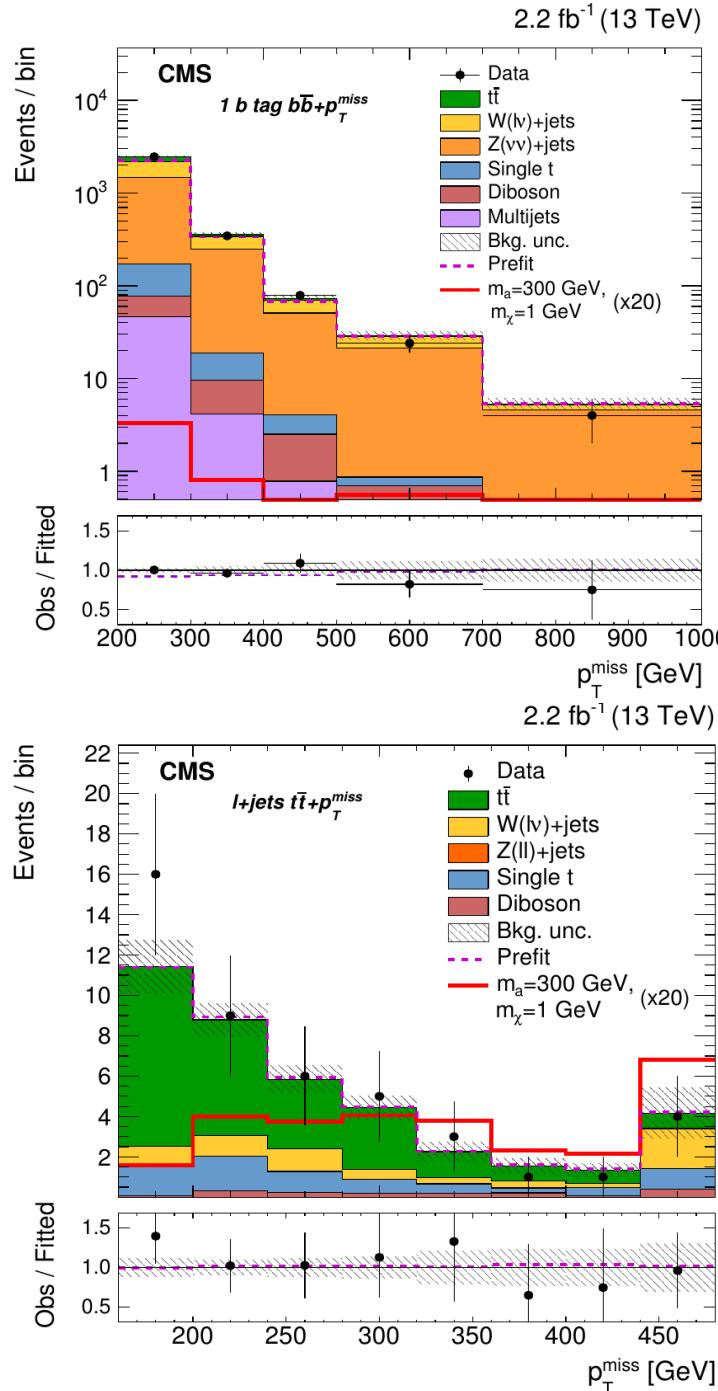
- $E_T^{\text{miss}} > 200$ for bb & all-hadronic tt, $E_T^{\text{miss}} > 160$ GeV for semileptonic tt, $E_T^{\text{miss}} > 50$ GeV for dileptonic tt
- Employs novel *resolved top quark tagger* to reconstruct low/moderate pT hadronic decays
 - Top pT is soft in for mediator masses for which there is LHC sensitivity
 - BG from SM tt with missing lepton
 - Categorize signal and bkg according to number of top tags
- Simultaneous E_T^{miss} fit using 8 SRs + 19 CRs

Search uses just 2.2 fb⁻¹ from Run2

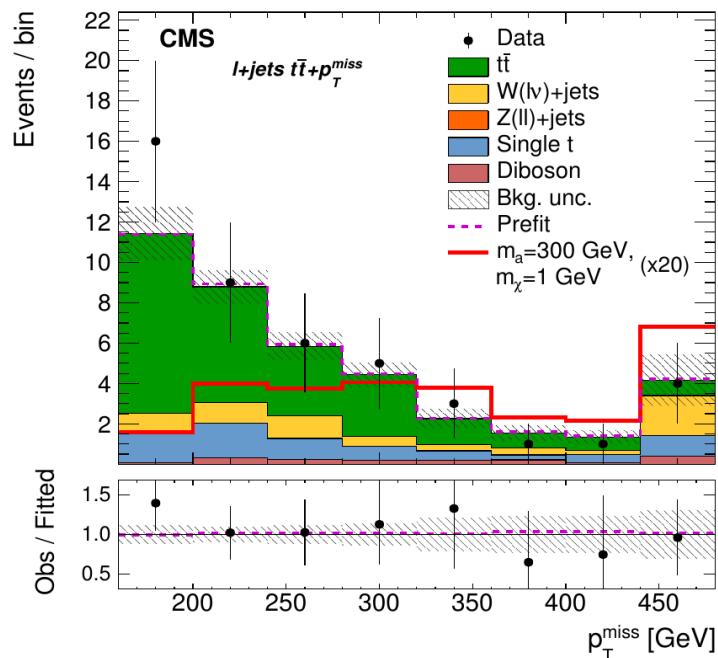
- Analysis of full 35.9 fb⁻¹ in progress



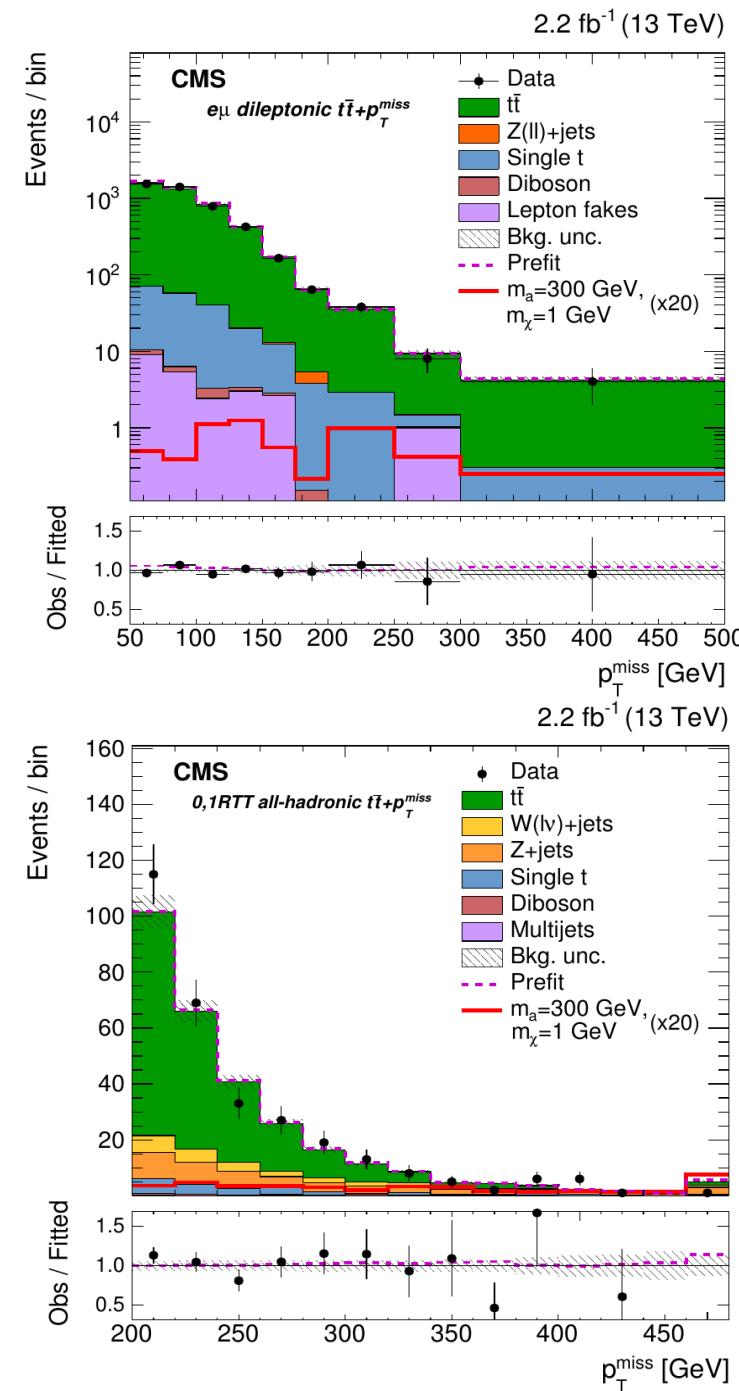
bb signal region



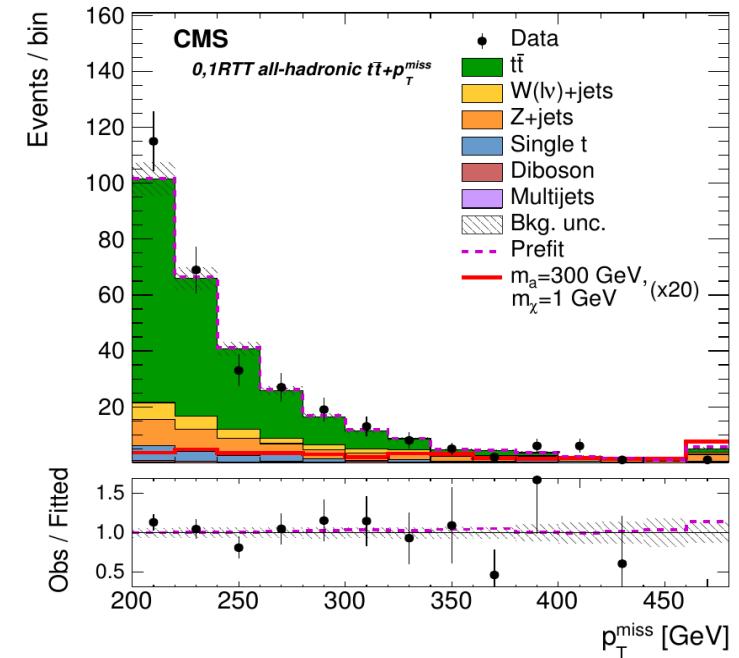
semileptonic signal region



dileptonic eμ signal region

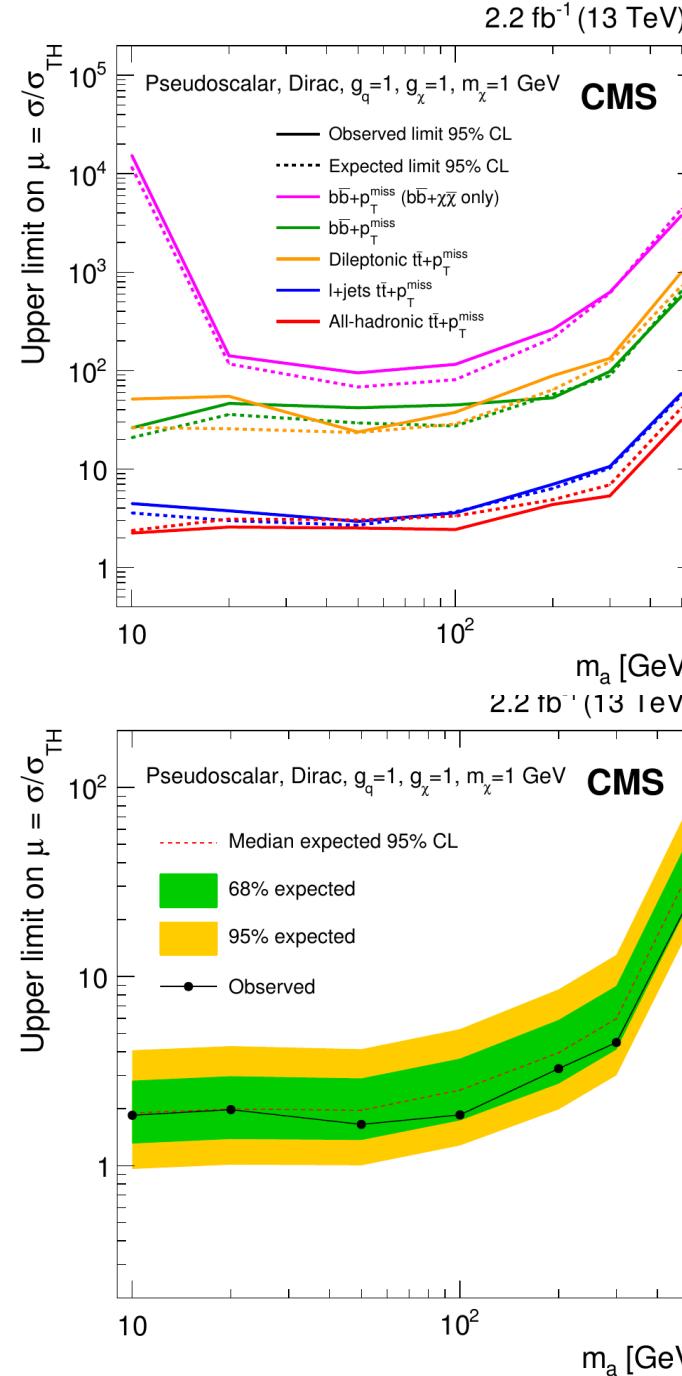
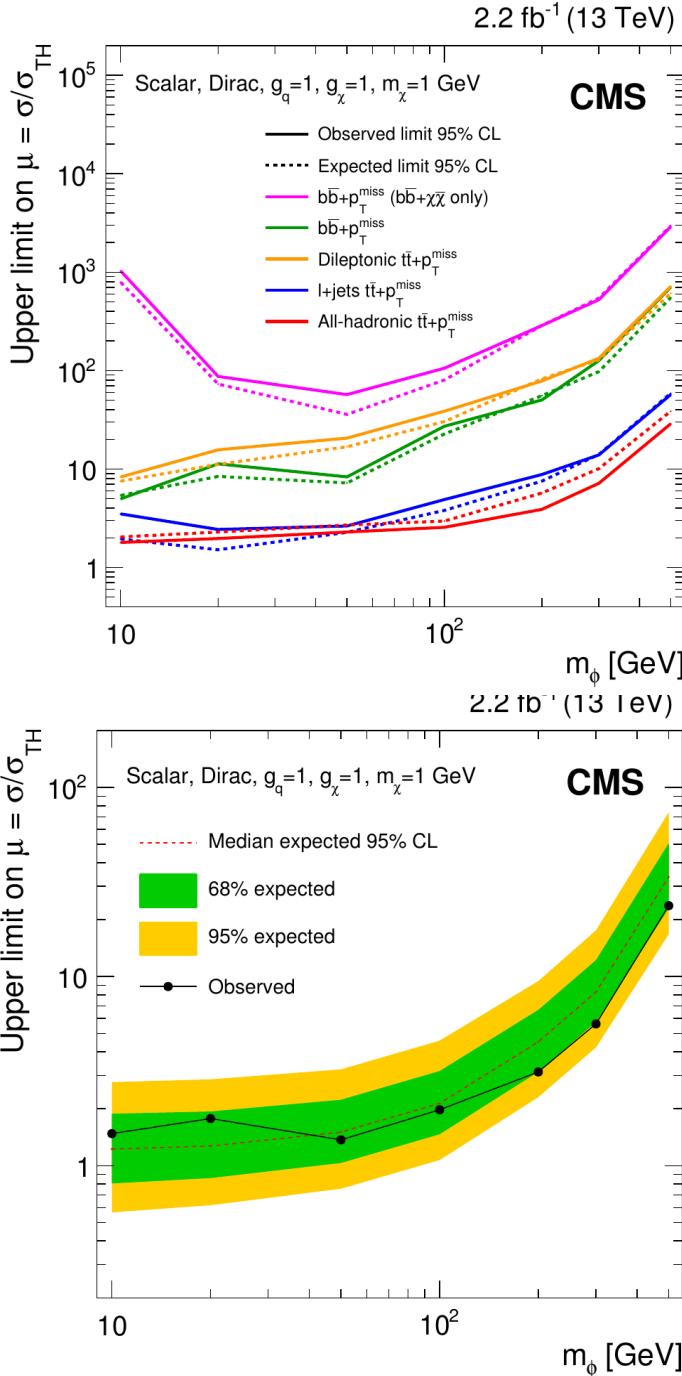


all-hadronic 01RTT signal region



CMS tt/bb + DM Limits

Full combination, scalar



Per-channel scalar

Per-channel pseudoscalar

Full combination, scalar

Full combination, pseudoscalar

Direct Mediator Searches

If mediator couples to quarks, then also decay to SM particles

- Search for the DM mediators directly via traditional LHC “bump hunts”
 - Dijet (+ISR), dilepton, di-bjet, etc ... eg:

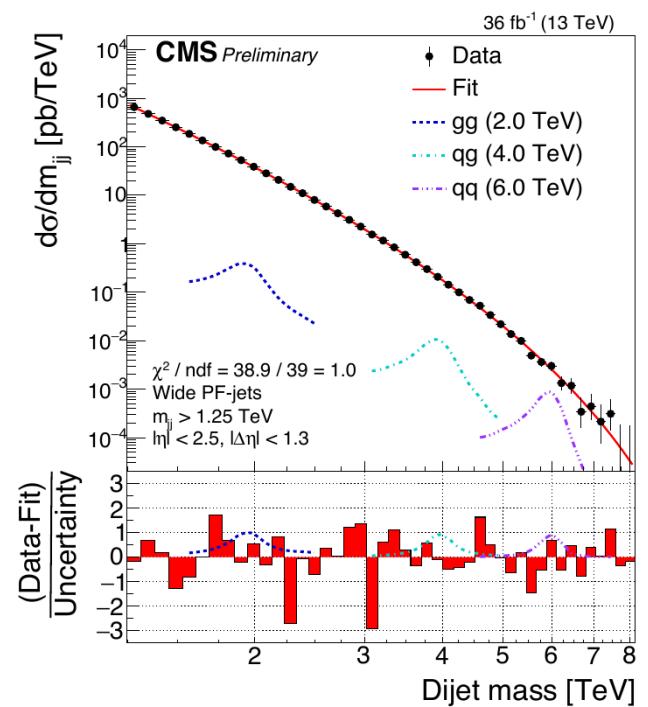
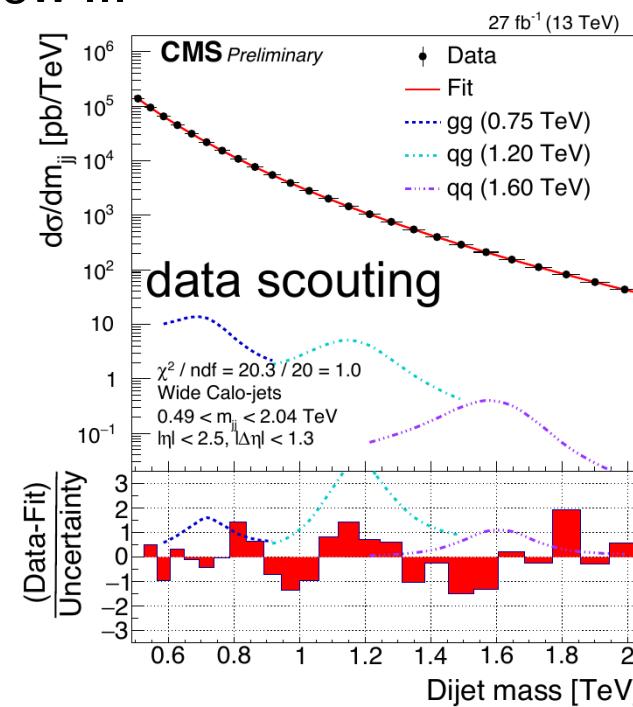
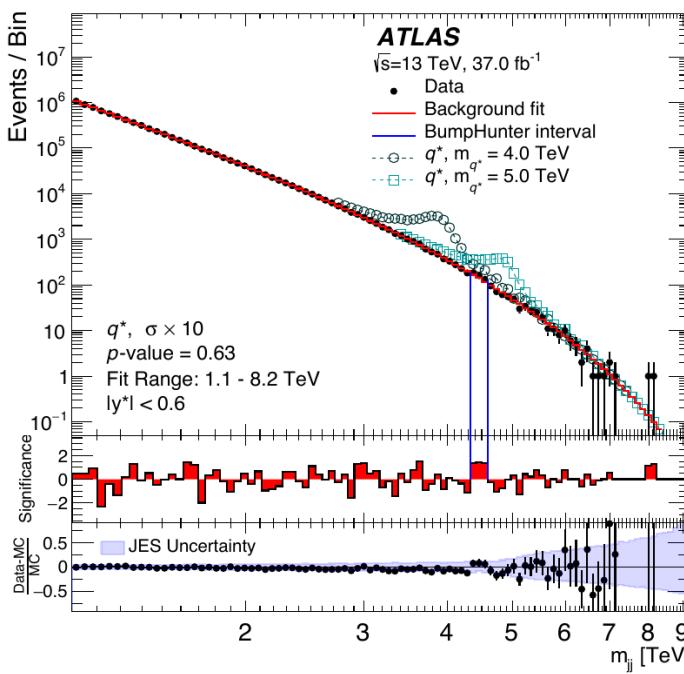
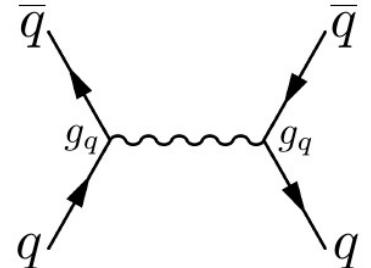
Dijet : 15.7 fb⁻¹ ATLAS-CONF-2016-069, 27 & 36 fb⁻¹ CMS-PAS-EXO-16-056

Dijet angular, 3.6 fb⁻¹ (ATLAS) PLB 754 (2016) 302-322, 36 fb⁻¹ CMS-PAS-EXO-16-046

Boosted dijet : 3.2 fb⁻¹ (bjets) ATLAS-CONF-2016-031, 36 fb⁻¹ CMS-PAS-EXO-17-001

Dilepton :: 36 fb⁻¹ (ATLAS) 1707.02424, 2.9+19.7 fb⁻¹ (CMS) PLB 768 (2017) 57

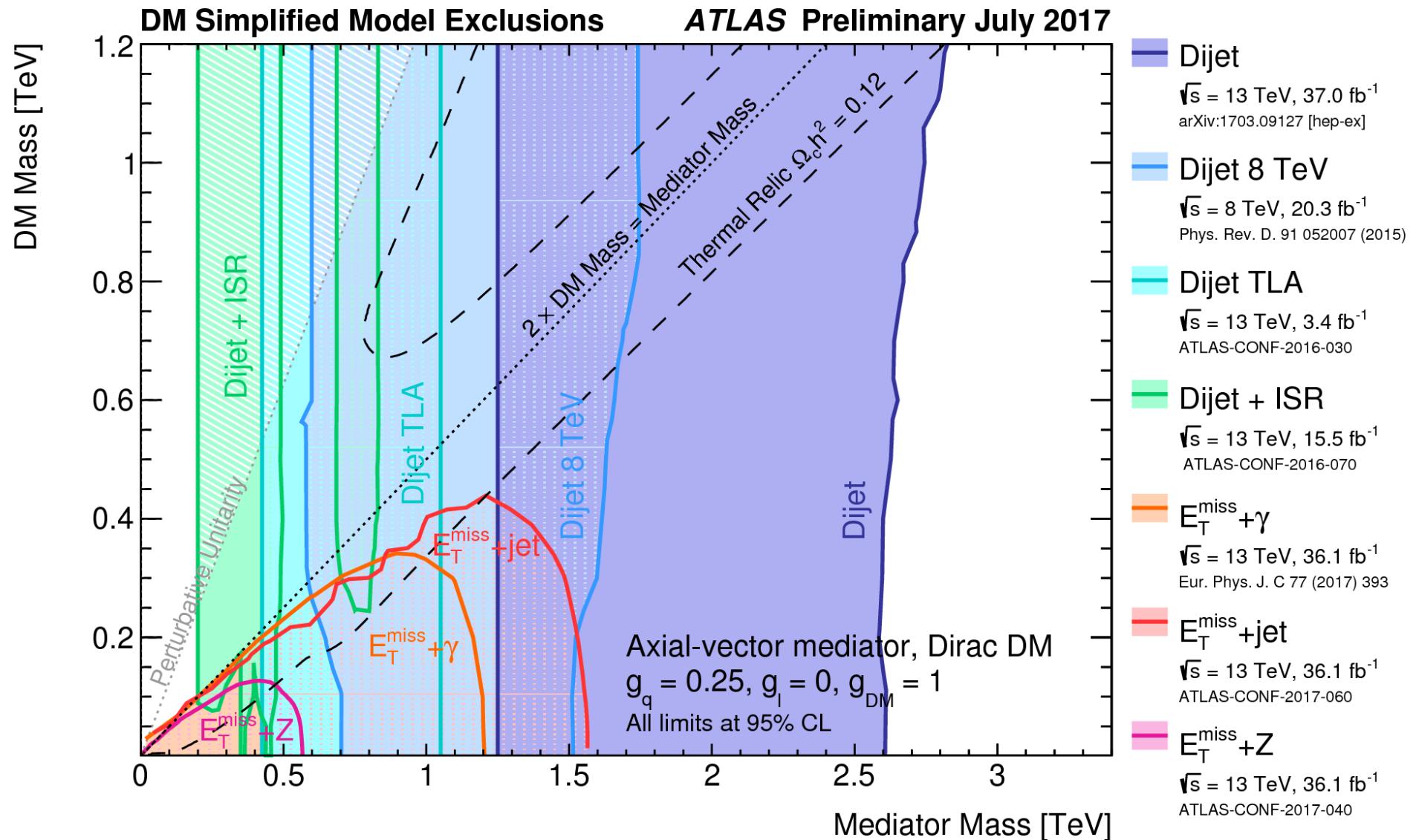
-
- New techniques (data scouting [CMS], Trigger Level Analysis [ATLAS]) allows searches to now push to lower mediator masses
- Dijet search results below ...



Collider DM Summaries

Comprehensive picture of LHC sensitivity to DM simplified models

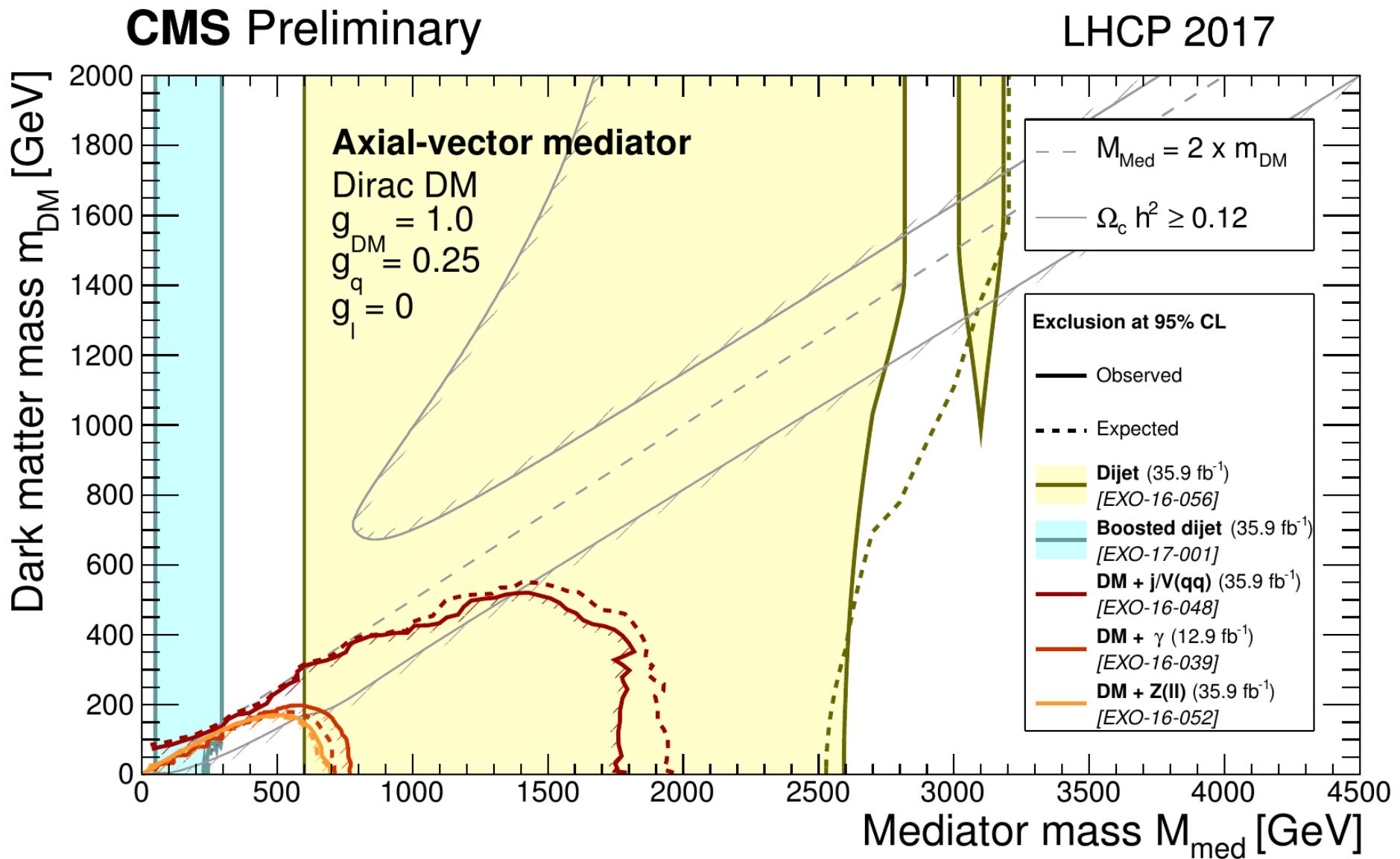
- Axial-vector mediator shown here (see [ATLAS Exotica Summaries](#))



Collider DM Summaries

Comprehensive picture of LHC sensitivity to DM simplified models

- Axial-vector mediator shown here (see [CMS DM Summaries](#))



Robust program of $E_T^{\text{miss}} + X$ DM searches at the LHC

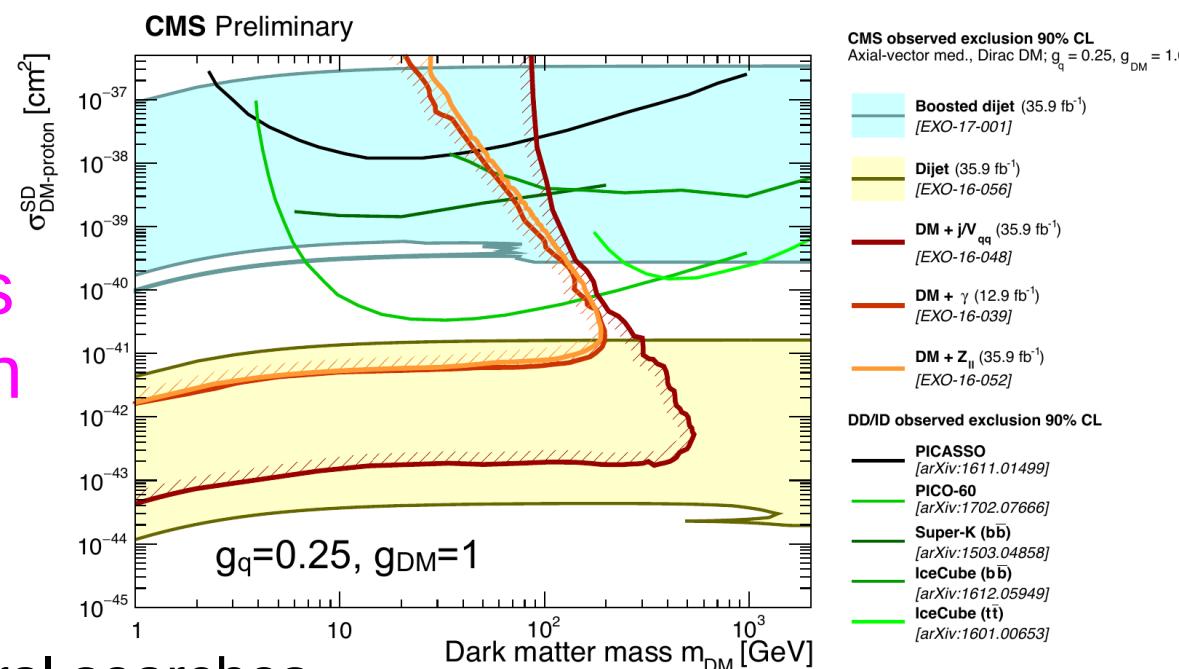
Run 2 results pushing into new territory, limits on

- Multi-Tev spin-1 mediators
- Low-mass spin-0 mediators

Complementary strengths
vs direct/indirect detection

On the horizon:

- Large bump in stats for several searches
- Stronger interplay between DM channels
- New methods for treating SM systematics (eg: arxiv: 1705.04664)
- Interpretations with somewhat-less-simplified models (eg: 1701.07427)



**“SOMEBODY
CALL FOR
BACKUP!”,**





CMS mono-X Searches



X	Dataset	Documentation
jet + hadronic V	36.1 fb-1	EXO-16-048
photon	12.9 fb-1	1706.03794
Z(l ^l)	35.9 fb-1	CMS-EXO-16-052
Higgs ($\gamma\gamma$)	36.1 fb-1	CMS-EXO-16-054
Higgs (bb), with $\gamma\gamma$	2.3 fb-1	1703.05236
tt (hadronic + semileptonic + dileptonic) & bb	2.2 fb-1	1706.02581
t hadronic	36.1 fb-1	EXO-16-051

Direct Mediator Production	Dataset	Documentation
dijets	36.1 fb-1	CMS-EXO-16-056
dijets (angular)	36.1 fb-1	CMS-EXO-16-046
boosted dijets	36.1 fb-1	CMS-EXO-17-001
dilepton	13.1 fb-1	CMS-EXO-16-031



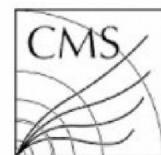
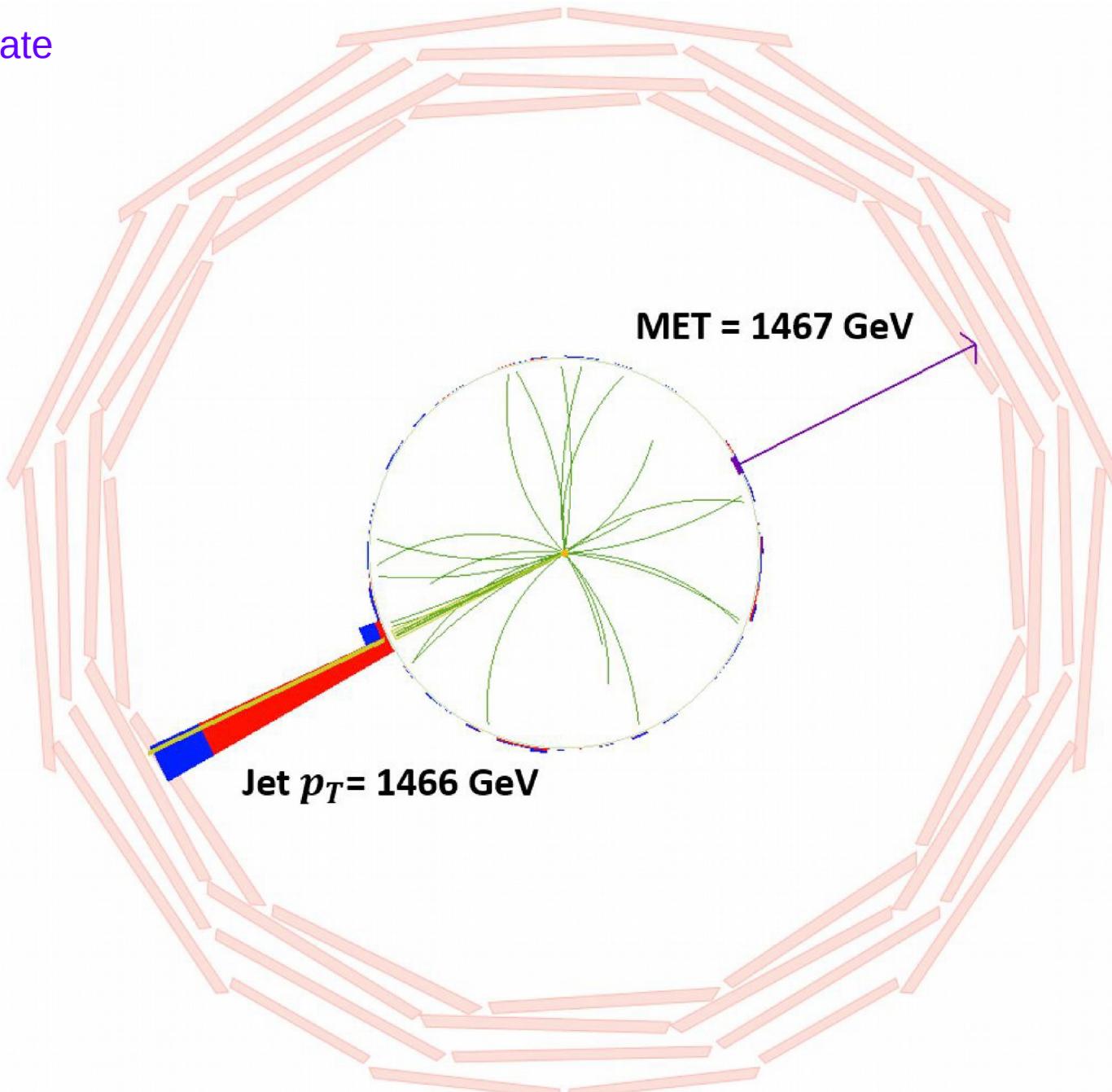
ATLAS Mono-X Searches



X	Dataset	Documentation
jet	36.1 fb-1	ATLAS-CONF-2017-060
photon	36.1 fb-1	EPJC 77 (2017) 393 1704.03848
W & Z (hadronic)	3.2 fb-1	PLB 763 (2016) 251
Z (leptonic)	36.1 fb-1	ATLAS-CONF-2017-040
Higgs ($\gamma\gamma$)	36.1 fb-1	1706.03948
Higgs (bb)	36.1 fb-1	1707.01302
tt (hadronic)	36.1 fb-1	ATLAS-CONF-2016-077
tt (semileptonic)	36.1 fb-1	ATLAS-CONF-2017-037
tt (dilepton)	13.3 fb-1	ATLAS-CONF-2016-076
bb	13.1 fb-1	ATLAS-CONF-2016-086

Direct Mediator Production	Dataset	Documentation
dijets	3.5+33.5 fb-1	1703.09127
dijet ISR	15.5 fb-1	ATLAS-CONF-2016-070
dijet TLA	3.4 fb-1	ATLAS-CONF-2016-030
dilepton	36.1 fb-1	ATLAS-CONF-2017-027

Monojet candidate
at $\sqrt{s} = 13$ TeV

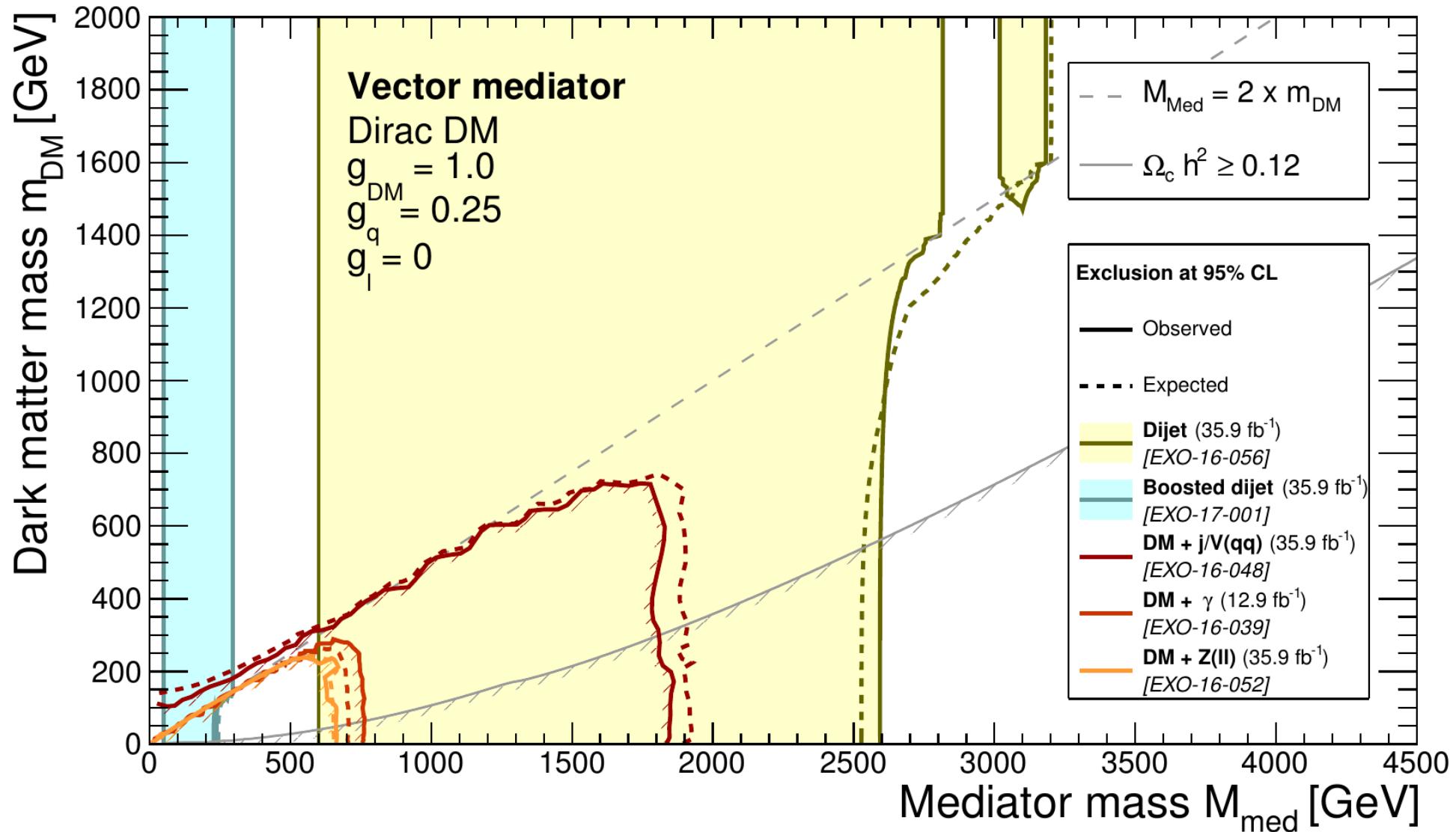


CMS Experiment at LHC, CERN
Data recorded: Sat Oct 3 06:58:12 2015 CEST
Run/Event: 258159 / 550030997
Lumi section: 434

More CMS summary plots

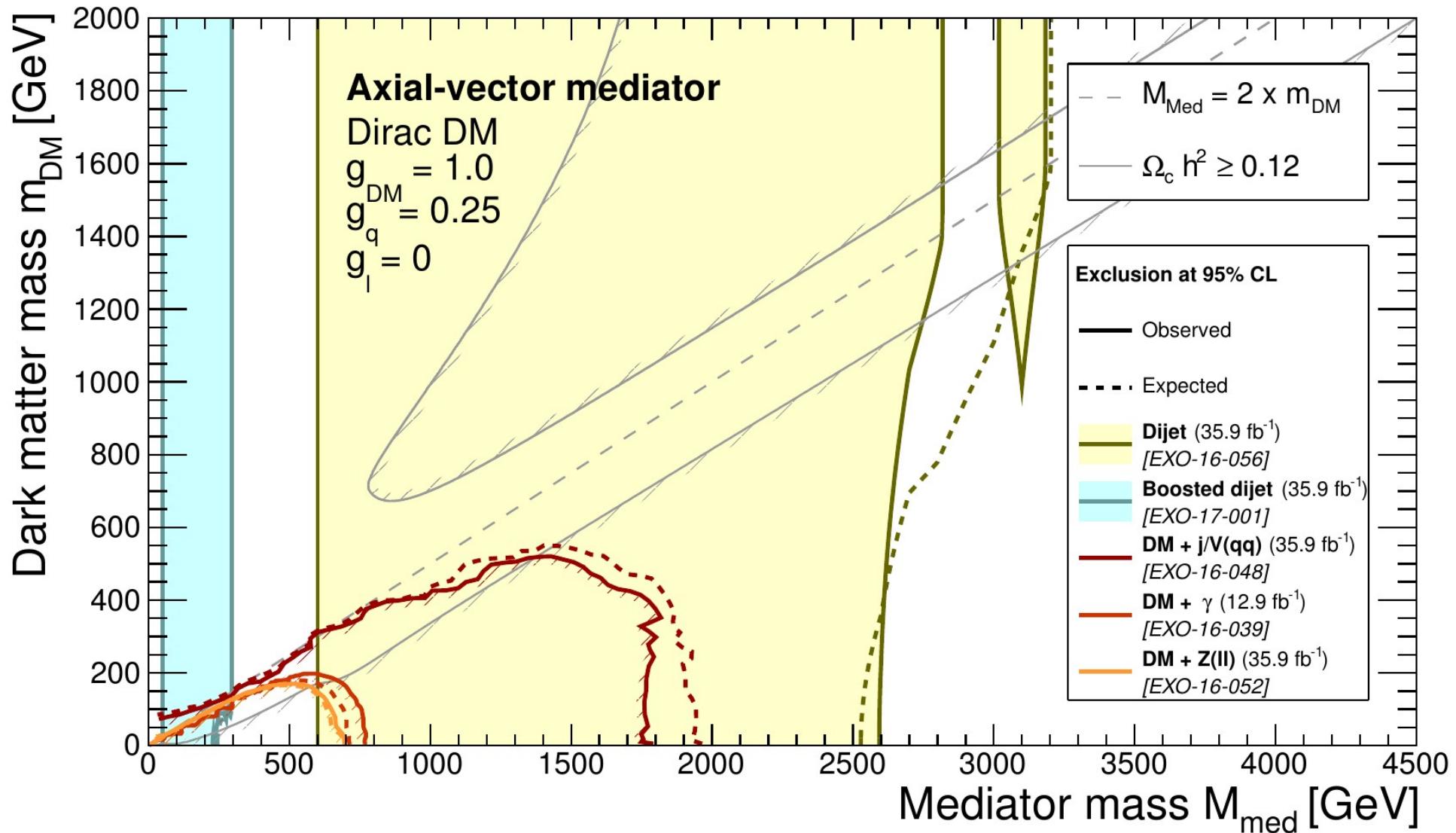
CMS Preliminary

LHCP 2017



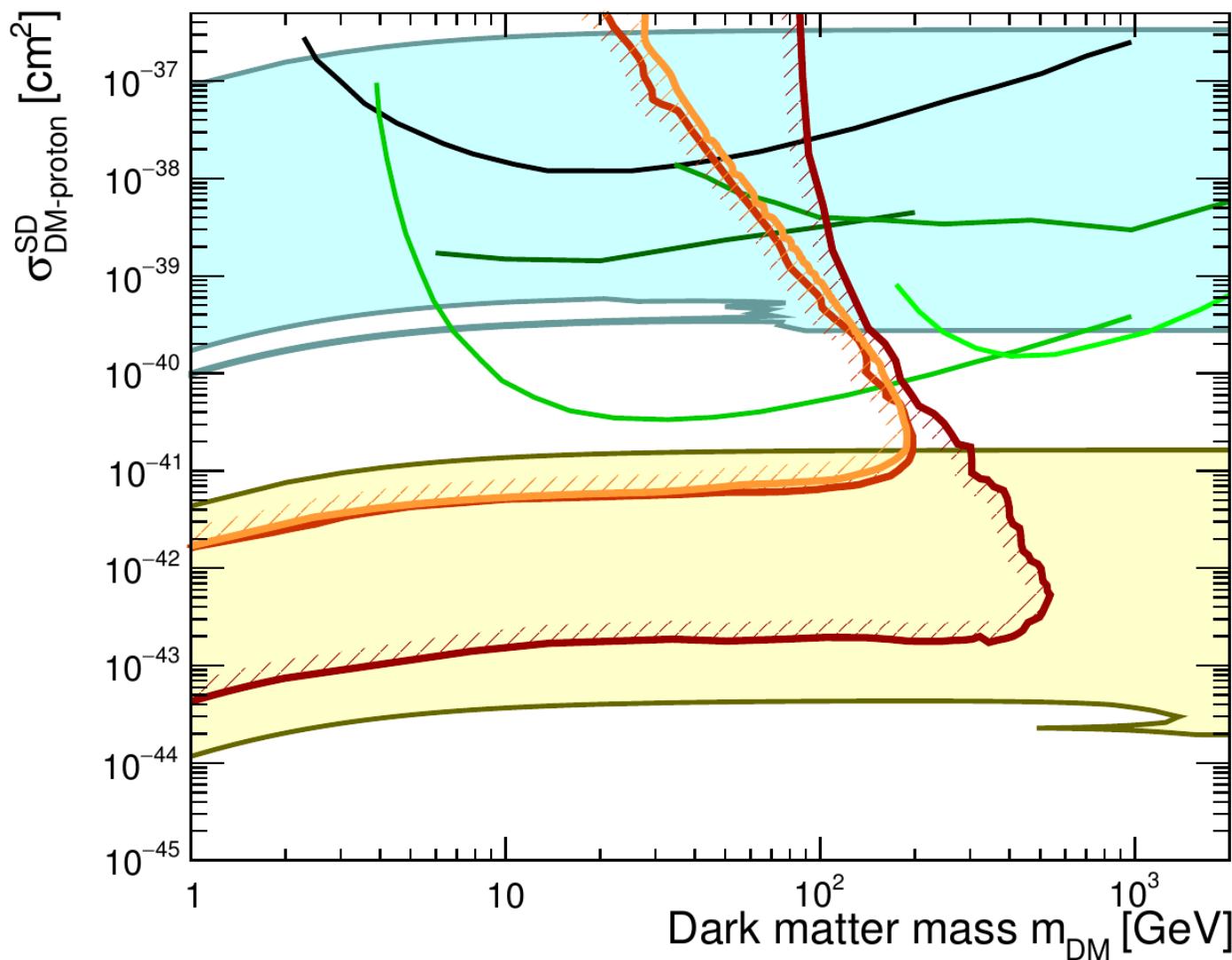
CMS Preliminary

LHCP 2017



CMS Preliminary

LHCP 2017



CMS observed exclusion 90% CL
Axial-vector med., Dirac DM; $g_q = 0.25$, $g_{\text{DM}} = 1.0$

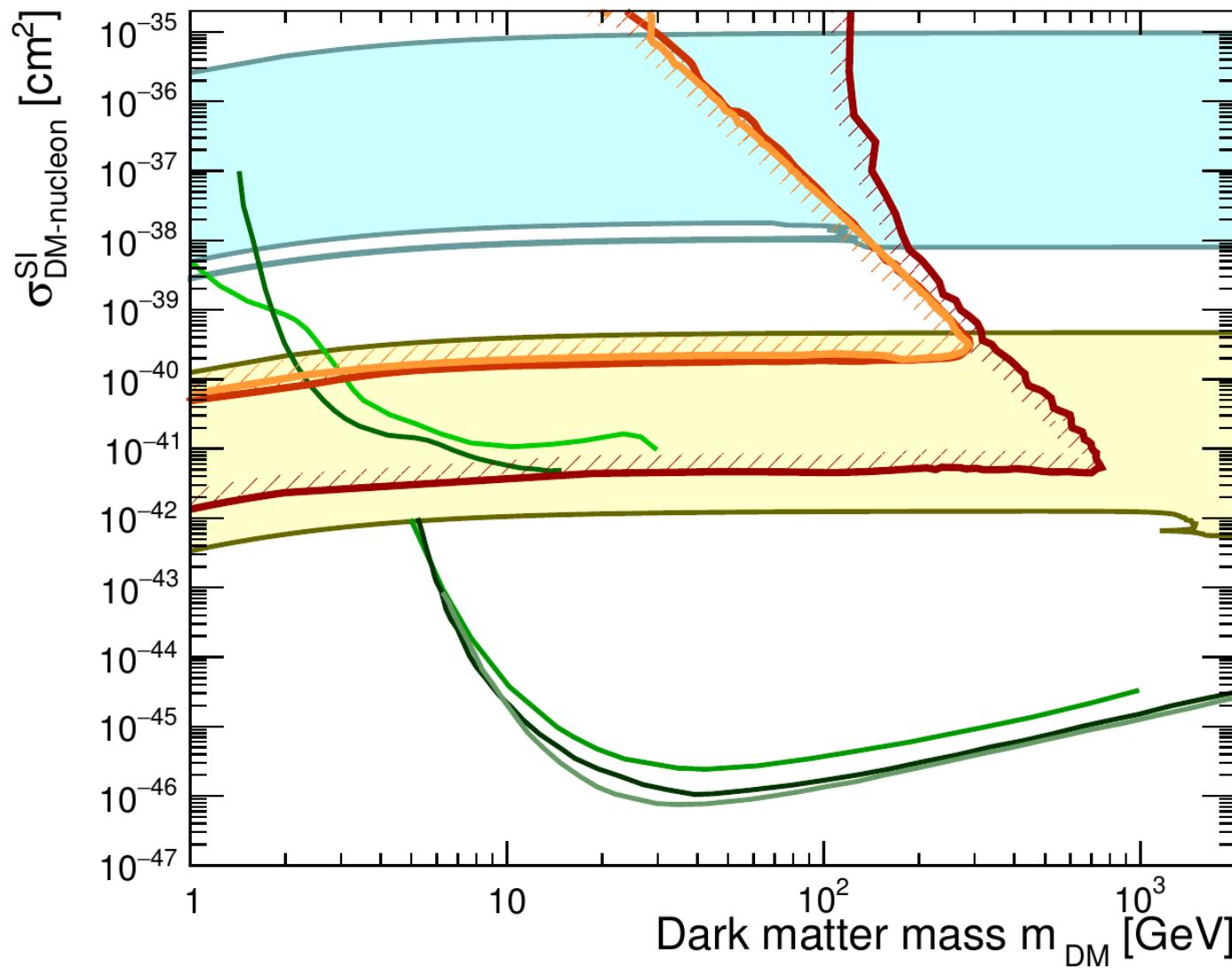
- Boosted dijet (35.9 fb^{-1})
[EXO-17-001]
- Dijet (35.9 fb^{-1})
[EXO-16-056]
- $\text{DM} + j/V_{qq}$ (35.9 fb^{-1})
[EXO-16-048]
- $\text{DM} + \gamma$ (12.9 fb^{-1})
[EXO-16-039]
- $\text{DM} + Z_{II}$ (35.9 fb^{-1})
[EXO-16-052]

DD/ID observed exclusion 90% CL

- PICASSO
[arXiv:1611.01499]
- PICO-60
[arXiv:1702.07666]
- Super-K ($b\bar{b}$)
[arXiv:1503.04858]
- IceCube ($b\bar{b}$)
[arXiv:1612.05949]
- IceCube ($t\bar{t}$)
[arXiv:1601.00653]

CMS Preliminary

LHCP 2017

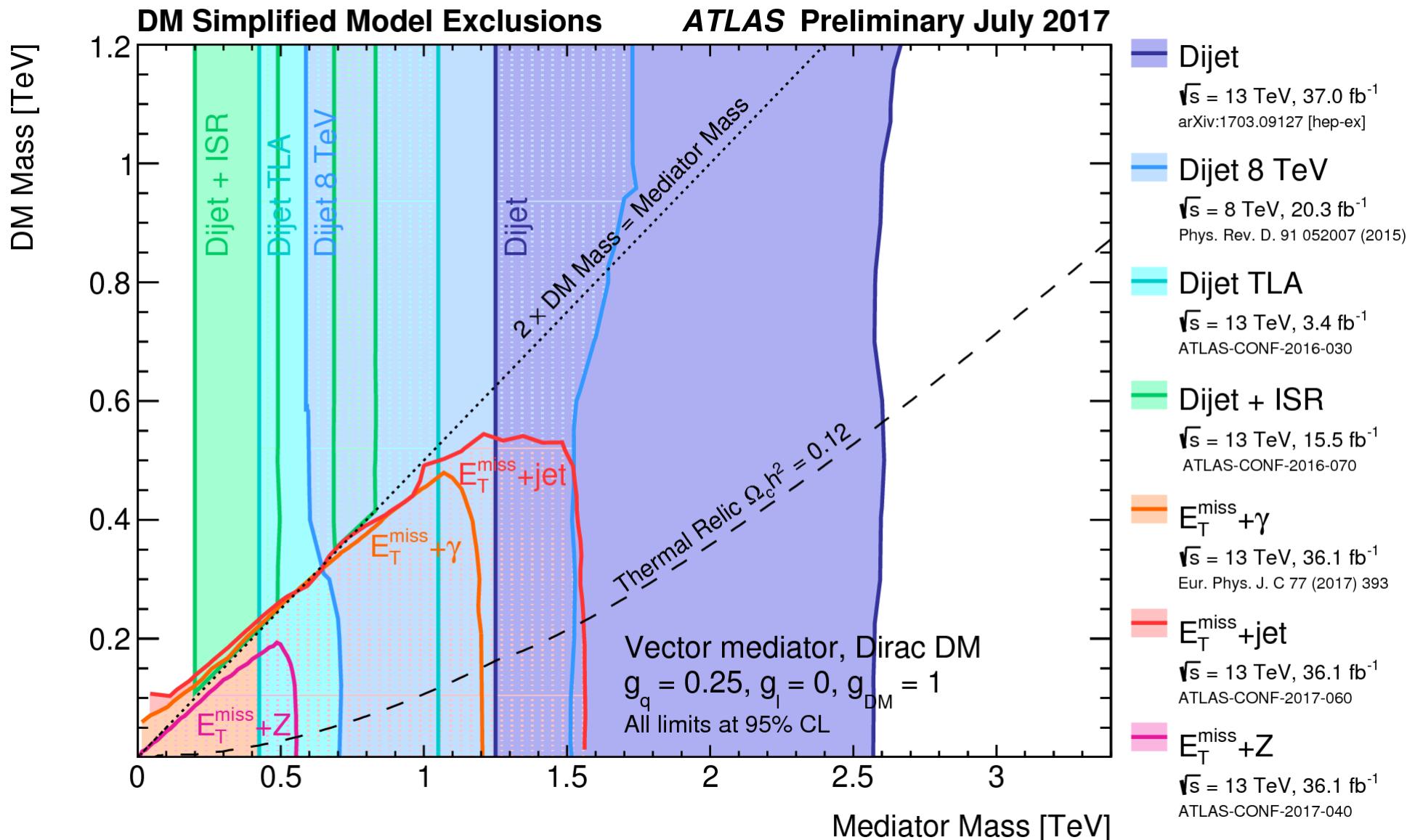


CMS observed exclusion 90% CL
Vector med., Dirac DM; $g_q = 0.25$, $g_{DM} = 1.0$

- Boosted dijet (35.9 fb^{-1})
[EXO-17-001]
- Dijet (35.9 fb^{-1})
[EXO-16-056]
- DM + j/V_{qq} (35.9 fb^{-1})
[EXO-16-048]
- DM + γ (12.9 fb^{-1})
[EXO-16-039]
- DM + Z_H (35.9 fb^{-1})
[EXO-16-052]

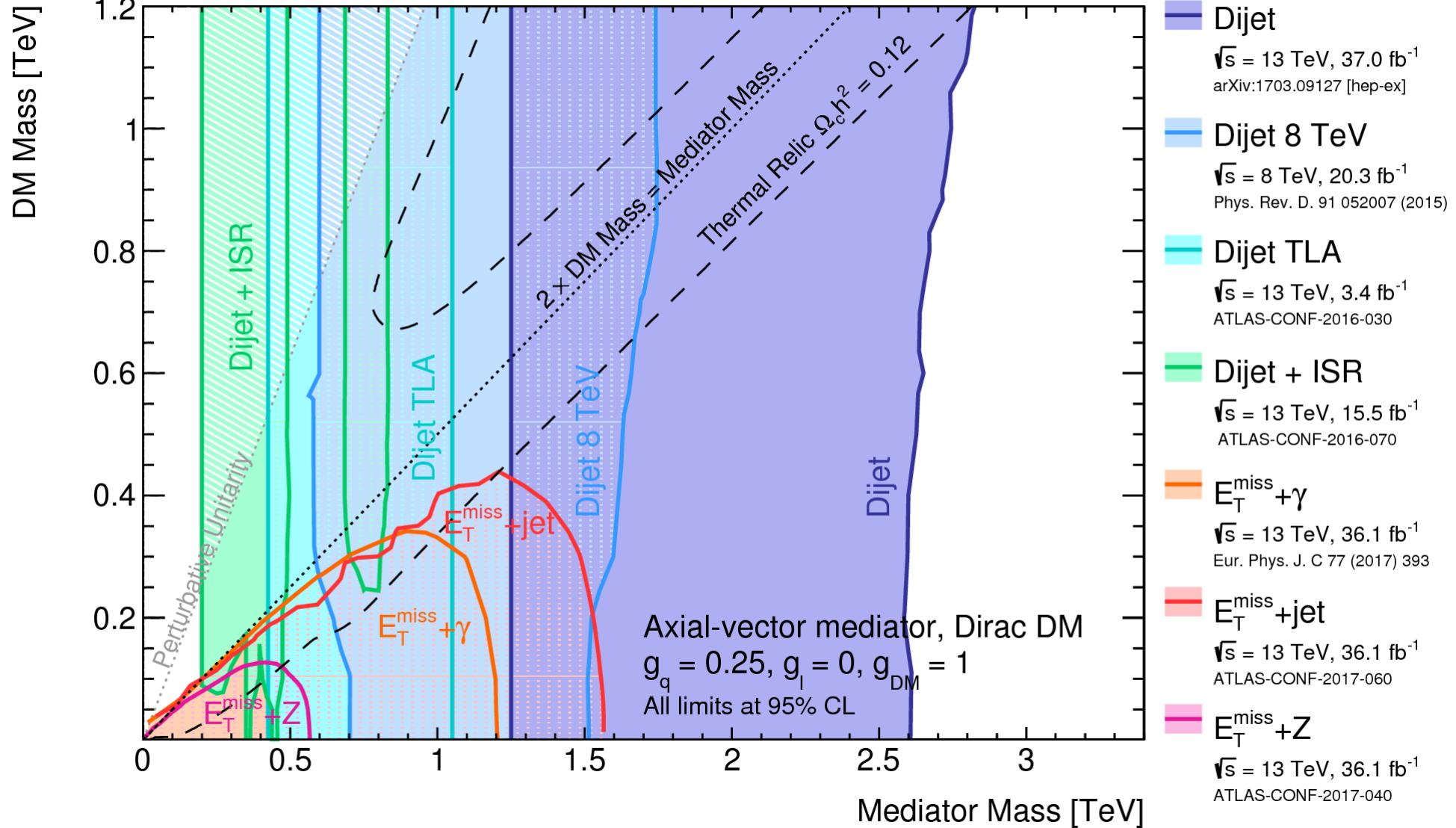
- DD observed exclusion 90% CL**
- CRESST-II
[arXiv:1509.01515]
 - CDMSlite
[arXiv:1509.02448]
 - PandaX-II
[arXiv:1607.07400]
 - LUX
[arXiv:1608.07648]
 - XENON1T
[arXiv:1705.06655]

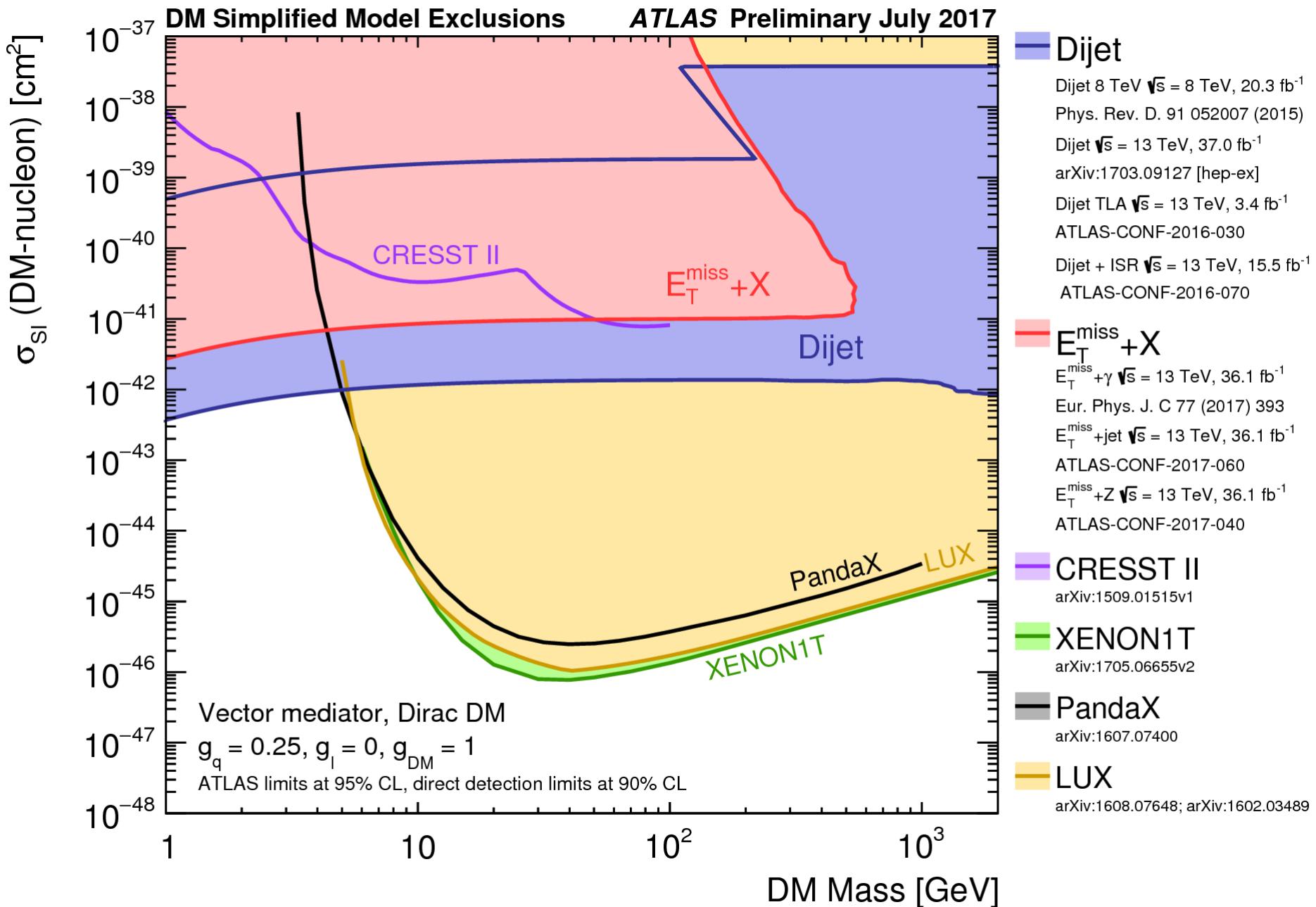
More ATLAS summary plots

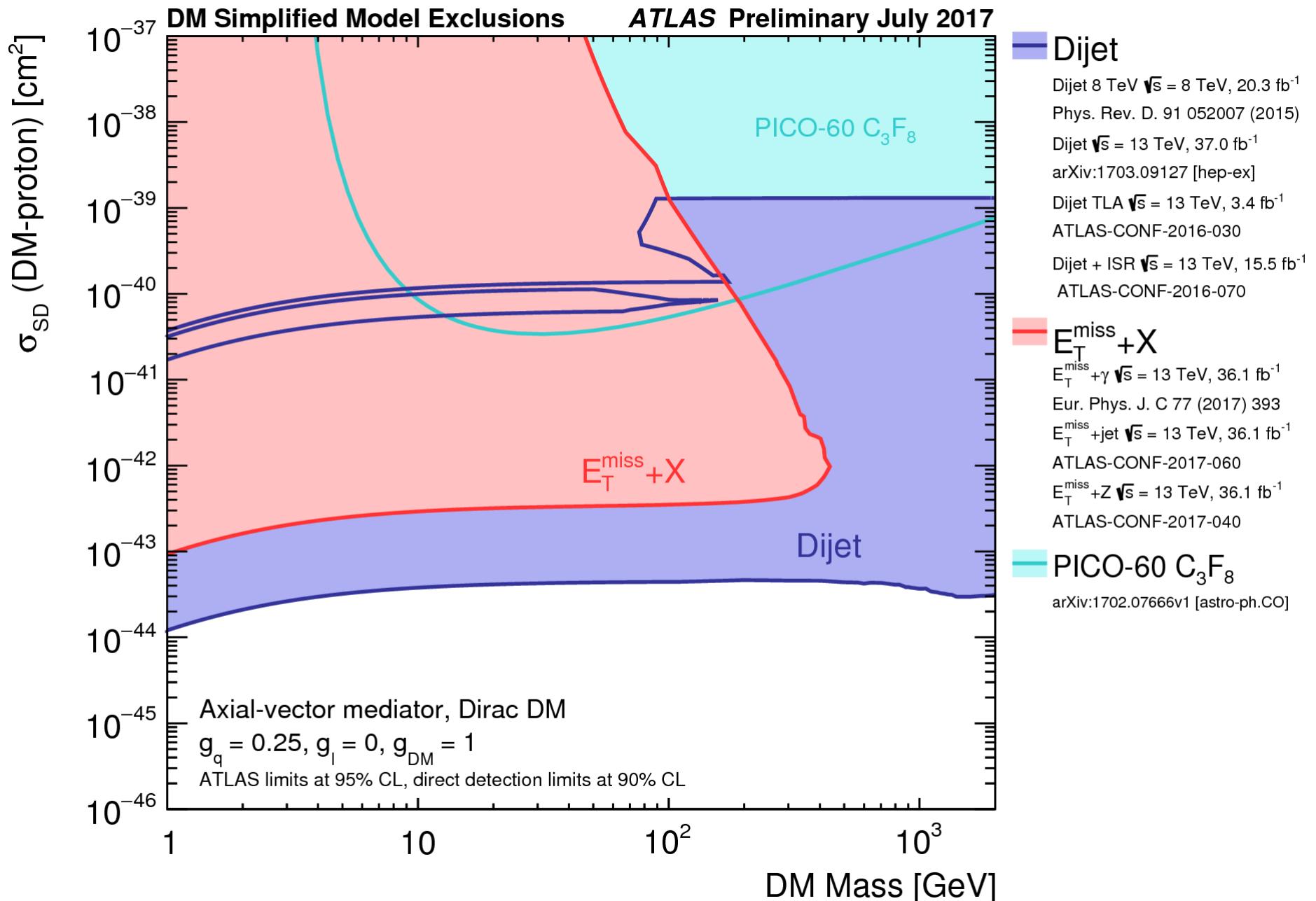


DM Simplified Model Exclusions

ATLAS Preliminary July 2017



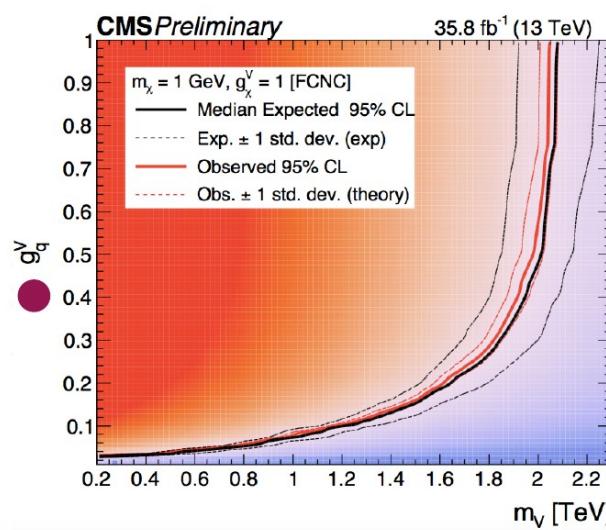
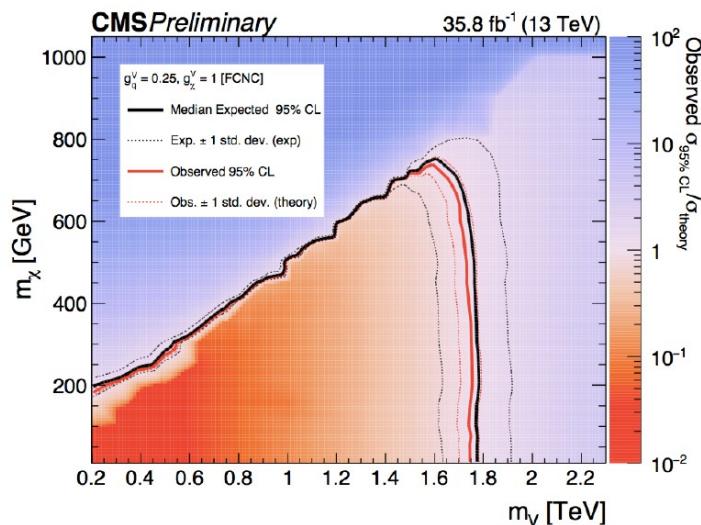
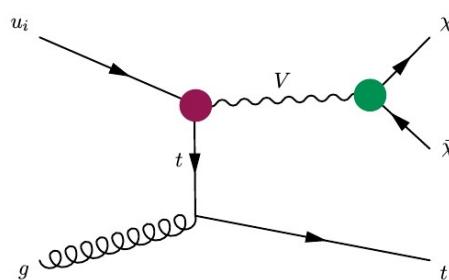




non-resonant model

$g_V = 0.25$

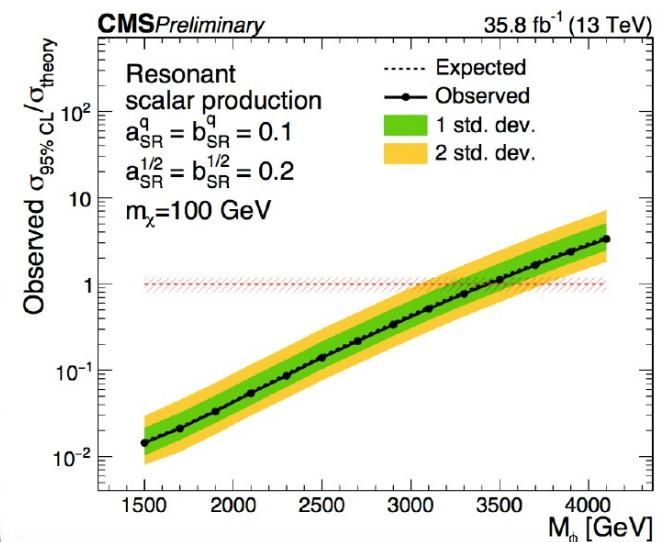
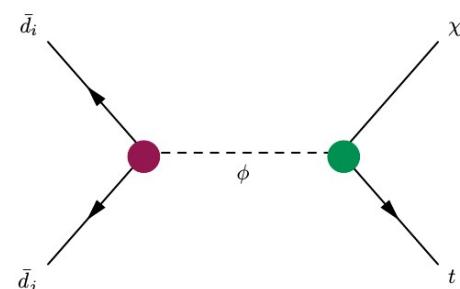
$g_\chi = 1$



resonant model

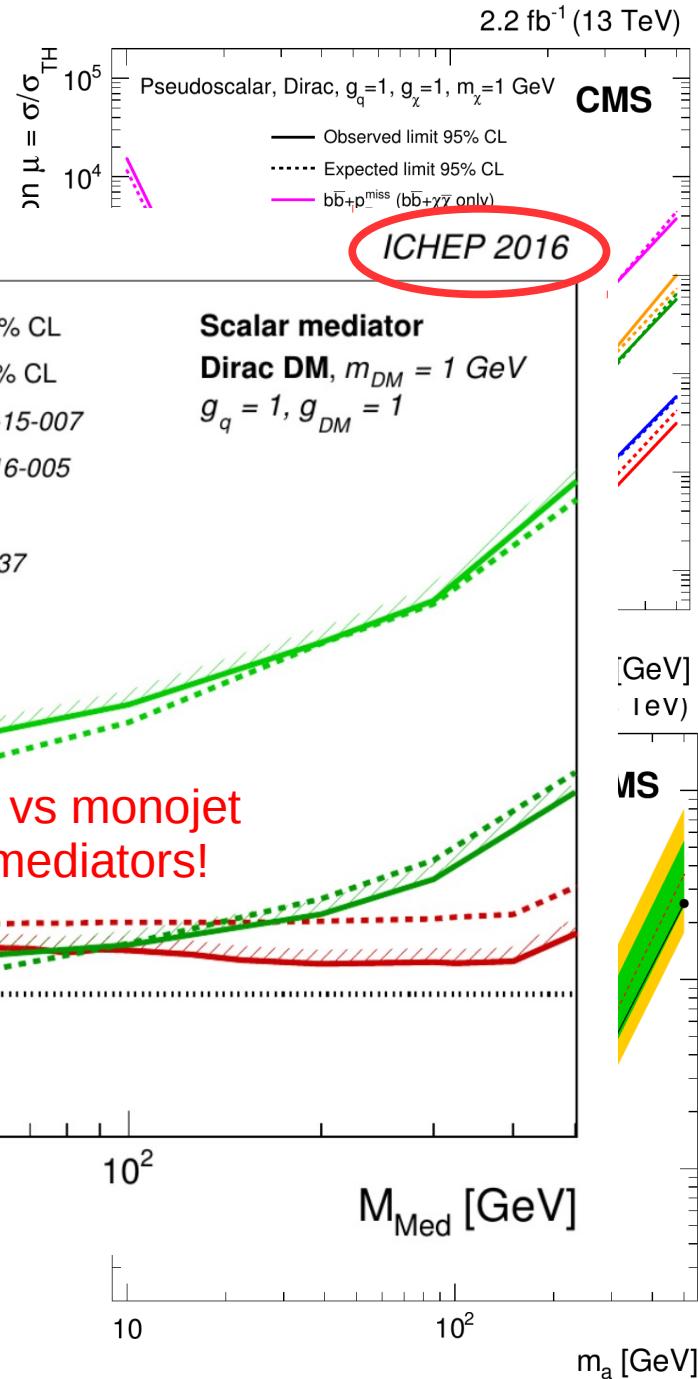
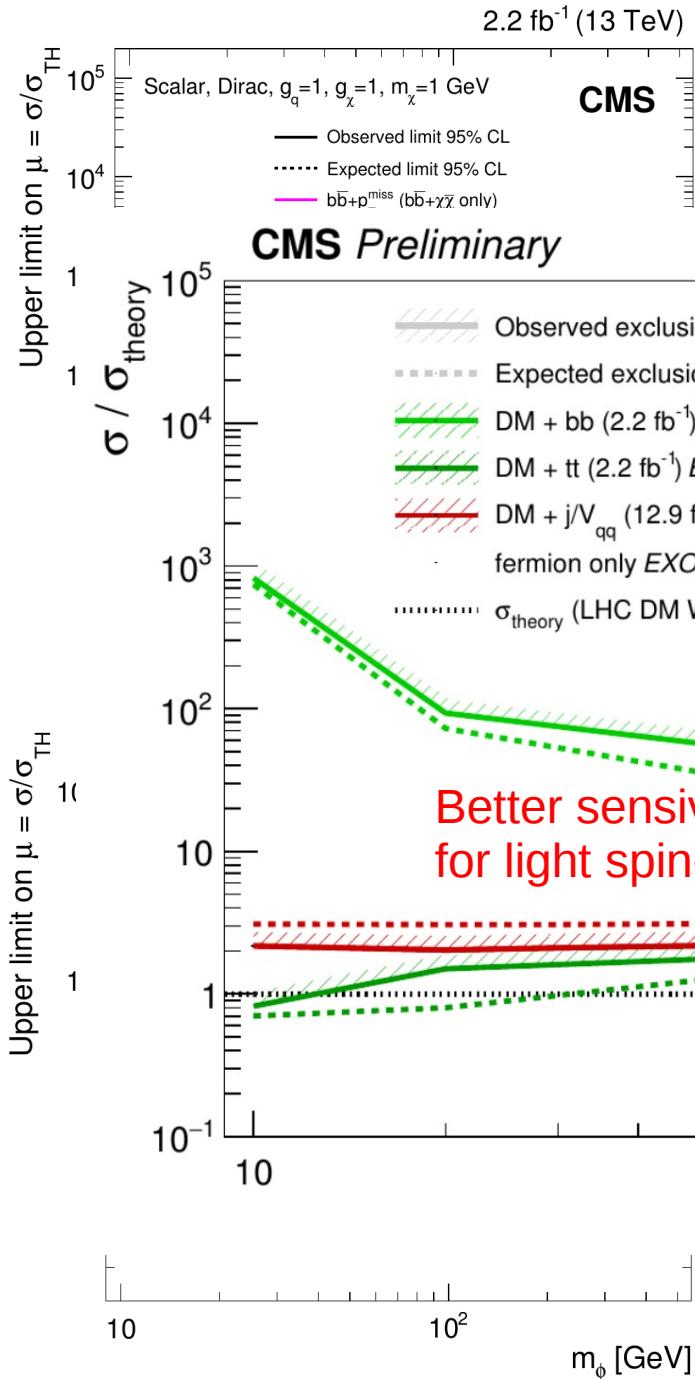
$a_q = b_q = 0.1$

$a_{1/2} = b_{1/2} = 0.2$



CMS tt/bb + DM Limits

Full combination, scalar



Per-channel scalar

Per-channel pseudoscalar

Full combination, pseudoscalar

SI/SD Translation

$$\begin{aligned}\sigma_{\text{SI}}^0 &= \frac{9 g_{\text{DM}}^2 g_q^2 \mu_{n\chi}^2}{\pi M_{\text{med}}^4} \\ &\approx 1.1 \times 10^{-39} \text{ cm}^2 \cdot \left(\frac{g_{\text{DM}} g_q}{1} \right)^2 \left(\frac{1 \text{ TeV}}{M_{\text{med}}} \right)^4 \left(\frac{\mu_{n\chi}}{1 \text{ GeV}} \right)^2 . \\ \sigma_{\text{SD}}^0 &= \frac{3 g_{\text{DM}}^2 g_q^2 (\Delta_u + \Delta_d + \Delta_s)^2 \mu_{n\chi}^2}{\pi M_{\text{med}}^4} \\ &\approx 4.6 \times 10^{-41} \text{ cm}^2 \cdot \left(\frac{g_{\text{DM}} g_q}{1} \right)^2 \left(\frac{1 \text{ TeV}}{M_{\text{med}}} \right)^4 \left(\frac{\mu_{n\chi}}{1 \text{ GeV}} \right)^2\end{aligned}$$

$$\begin{aligned}\Gamma_{\min}^V &= \frac{g_\chi^2 M_{\text{med}}}{12\pi} \left(1 + \frac{2m_\chi^2}{M_{\text{med}}^2}\right) \beta_{DM} \theta(M_{\text{med}} - 2m_\chi) \\ &\quad + \sum_q \frac{3g_q^2 M_{\text{med}}}{12\pi} \left(1 + \frac{2m_q^2}{M_{\text{med}}^2}\right) \beta_q \theta(M_{\text{med}} - 2m_q),\end{aligned}$$

$$\begin{aligned}\Gamma_{\min}^A &= \frac{g_\chi^2 M_{\text{med}}}{12\pi} \beta_{DM}^3 \theta(M_{\text{med}} - 2m_\chi) \\ &\quad + \sum_q \frac{3g_q^2 M_{\text{med}}}{12\pi} \beta_q^3 \theta(M_{\text{med}} - 2m_q).\end{aligned}$$

$$\begin{aligned}\Gamma_{\phi,a} &= \sum_f N_c \frac{y_f^2 g_q^2 m_{\phi,a}}{16\pi} \left(1 - \frac{4m_f^2}{m_{\phi,a}^2}\right)^{x/2} + \frac{g_\chi^2 m_{\phi,a}}{8\pi} \left(1 - \frac{4m_\chi^2}{m_{\phi,a}^2}\right)^{x/2} \\ &\quad + \frac{\alpha_s^2 y_t^2 g_q^2 m_{\phi,a}^3}{32\pi^3 v^2} \left| f_{\phi,a} \left(\frac{4m_t^2}{m_{\phi,a}^2} \right) \right|^2\end{aligned}$$

$$\begin{aligned}f_\phi(\tau) &= \tau \left[1 + (1 - \tau) \arctan^2 \left(\frac{1}{\sqrt{\tau - 1}} \right) \right], \\ f_a(\tau) &= \tau \arctan^2 \left(\frac{1}{\sqrt{\tau - 1}} \right)\end{aligned}$$