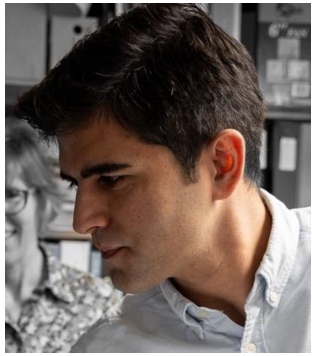
Entanglement growth in the disordered Fermi-Hubbard model

Rachel Wortis

Ahad Nokhostin Helm



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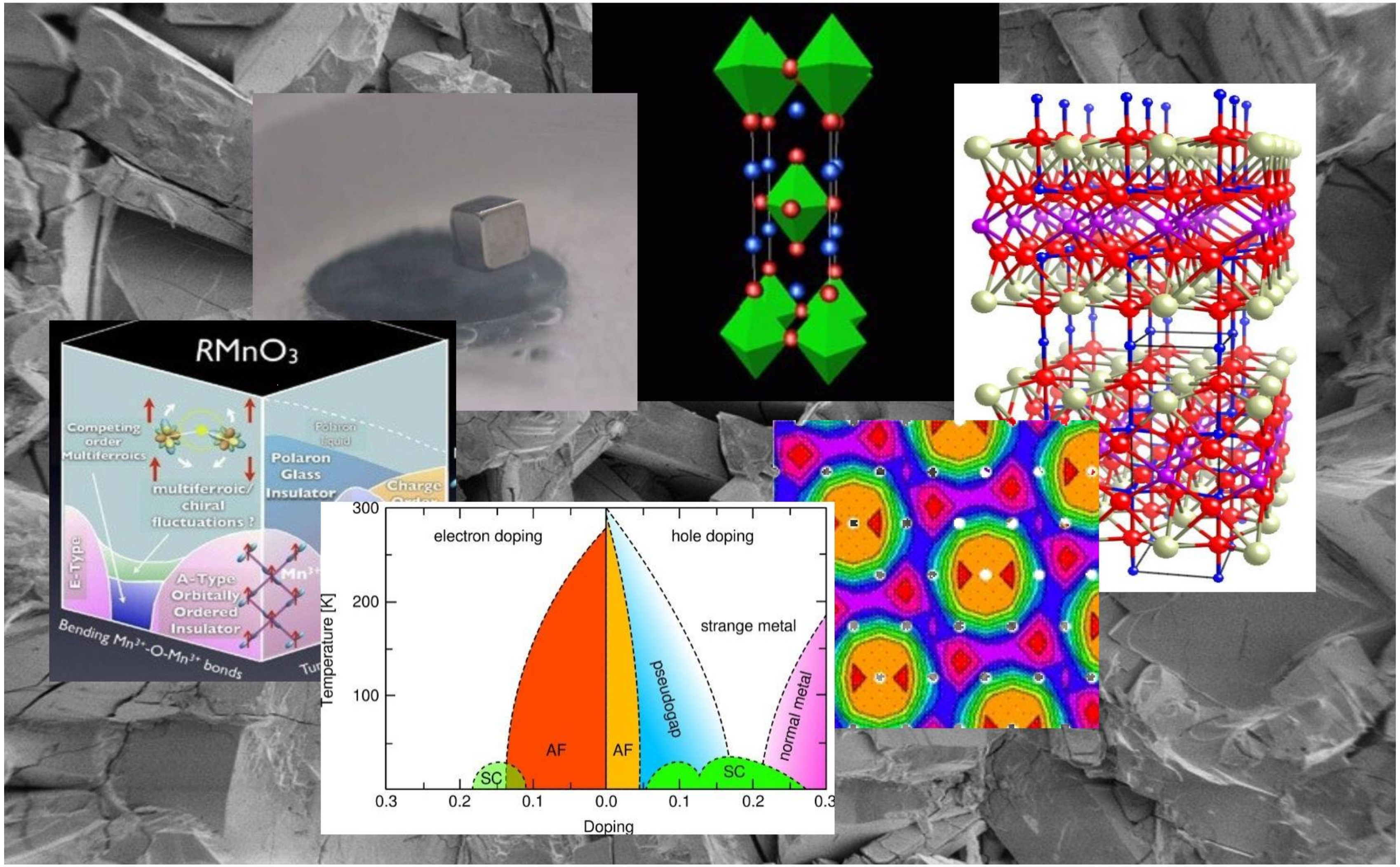


~~What does disorder do to strongly correlated systems?~~

What do interactions do to disordered systems?

Outline

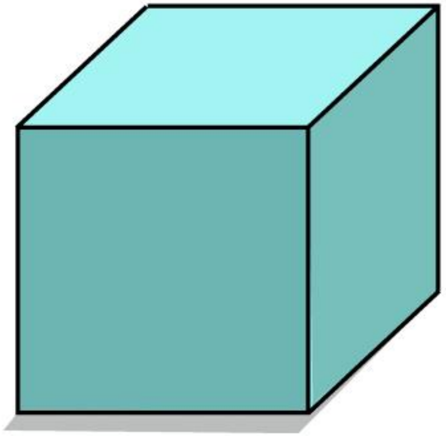
equilibrium in closed systems

What is it? How is it reached?

systems which don't equilibrate

the Fermi Hubbard model with disorder

equilibrium, classical
What is it?



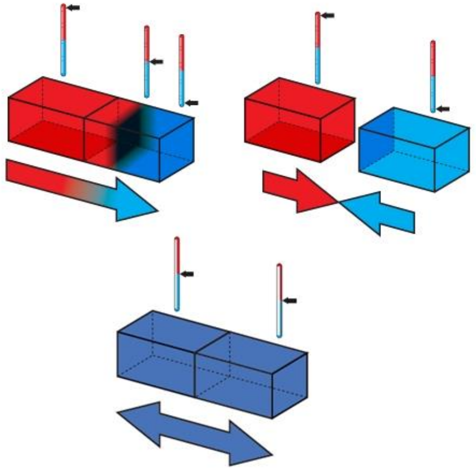
properties uniform in
space and time

time average = ensemble average

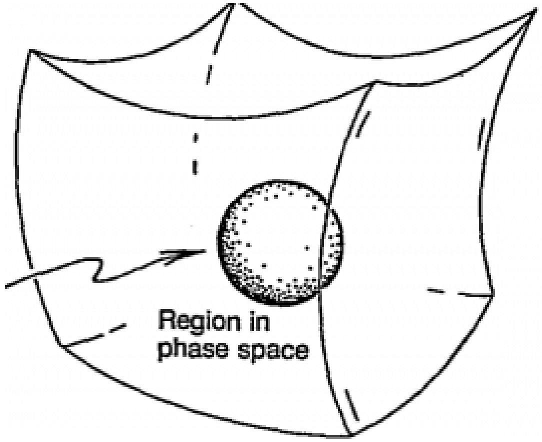
equilibrium, classical

How is it reached?

just wait

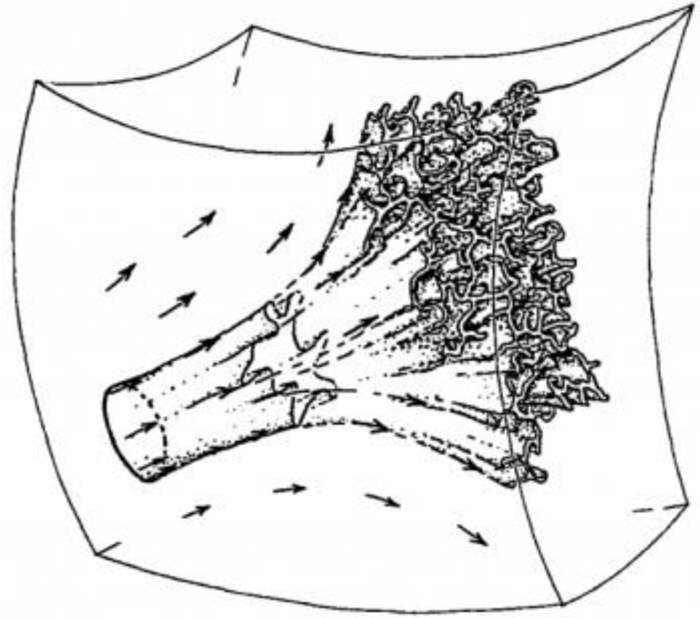


Hamiltonian dynamics



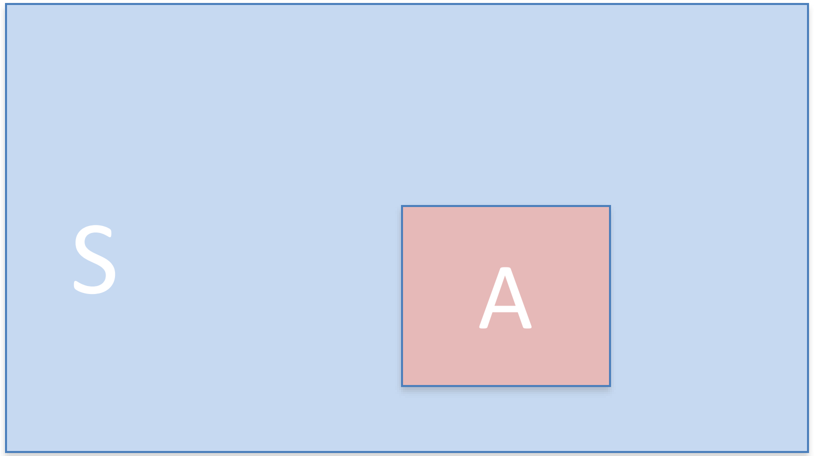
Liouville's theorem: phase space volume is conserved

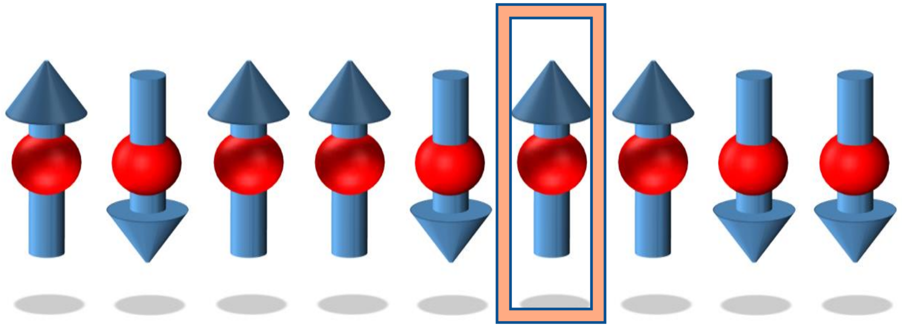
Liouville equation: in chaotic system, divergence of trajectories such that original volume enters any region of phase space you choose



equilibrium, quantum

What is it?





System S is in a single eigenstate with α
energy E_α

$$P_{Ai} \propto e^{-\epsilon_{Ai}/k_B T}$$

T comes from E_α

If the system were in contact with a thermal bath, what temperature of the bath would make the *average* energy of S equal to ? E_α

The rest of the system acts as
a thermal bath for the subsystem.

equilibrium, quantum

How is it reached?

If system starts in an eigenstate, there is no time dependence.

If system starts in a superposition of eigenstates, does it evolve toward equilibrium?

eigenstate thermalization hypothesis

$$\langle\hat{O}\rangle_{time} = \langle\hat{O}\rangle_{ensemble}$$

$$\langle\hat{O}\rangle = \langle\Psi(t)|\hat{O}|\Psi(t)\rangle = \sum_{\alpha,\beta} C_\alpha^* C_\beta O_{\alpha\beta} e^{-i(E_\beta - E_\alpha)t/\hbar}$$

$$\langle\hat{O}\rangle_{time} = \lim_{\tau\to\infty} \frac{1}{\tau}\int_0^\tau \langle\hat{O}\rangle dt = \sum_\alpha |C_\alpha|^2 O_{\alpha\alpha}$$

$$\langle\hat{O}\rangle_{ensemble} = \frac{1}{N} \sum_{\nu \text{ in energy window}} O_{\nu\nu}$$

Outline

equilibrium in closed systems

systems which don’t equilibrate

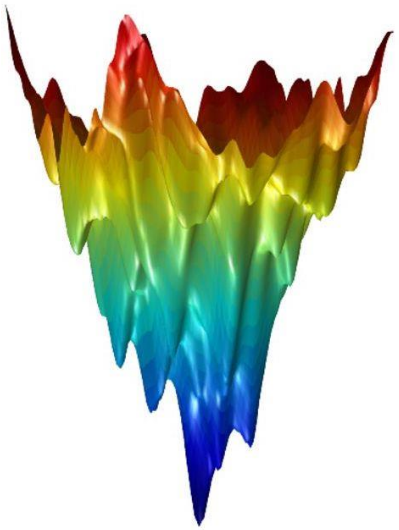
classical -> quantum single-ptcle -> quantum many-body

the Fermi Hubbard model with disorder

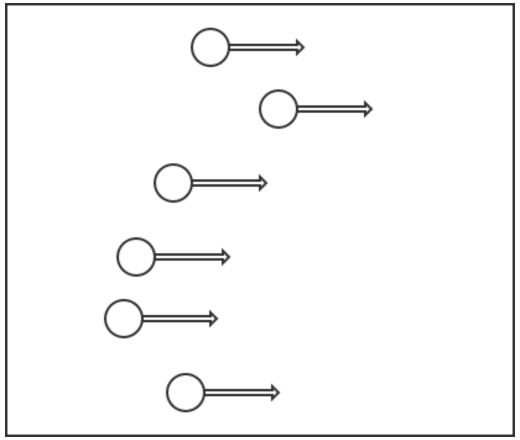
equilibrium, classical

What systems don't equilibrate?

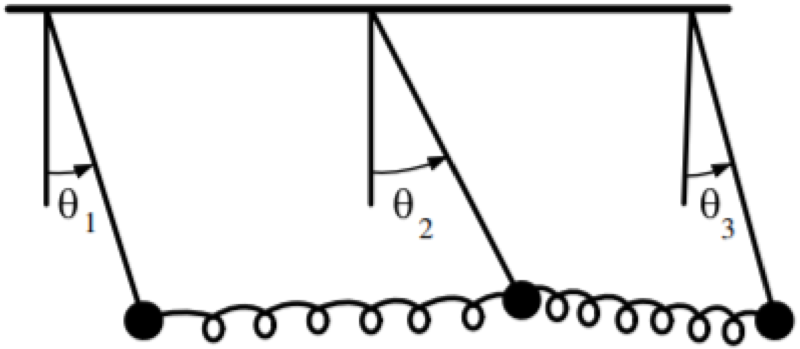
classical confinement



glassy



addition conserved quantities
(macroscopic number)
fine tuning



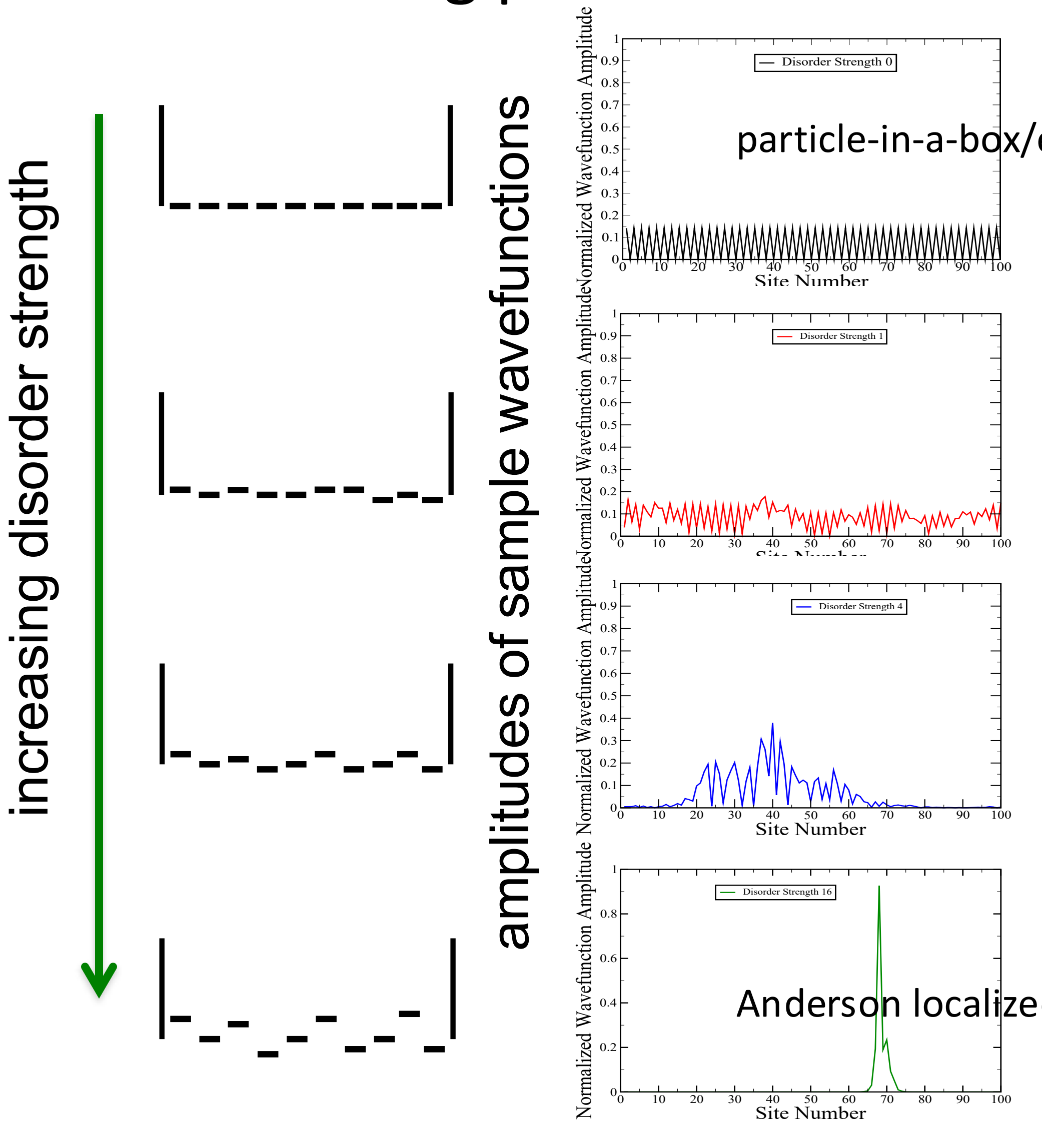
linearly coupled oscillators:
normal modes

non-ergodicity can persist for
sufficiently small
nonlinearities!
(KAM theorem)

What about quantum systems?

blocking equilibration with disorder

non-interacting particles



$$H = -t_h \sum_{\langle i,j \rangle} (c_i^\dagger c_j + c_j^\dagger c_i) + \sum_i \epsilon_i n_i$$

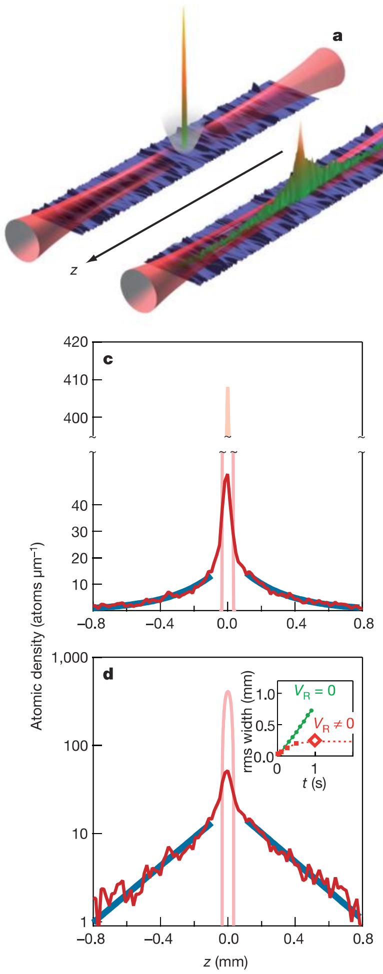
$$H = \sum_i h_i \tau_i^z$$

Absence of diffusion in certain random lattices
P.W. Anderson, Phys. Rev. **109**, 1492 (1958)

Scaling Theory of Localization: Absence of Quantum Diffusion in Two Dimensions
E. Abrahams, P. W. Anderson, D. C. Licciardello, and T. V. Ramakrishnan
Phys. Rev. Lett. **42**, 673 (1979)

Anderson localization

Anderson localization

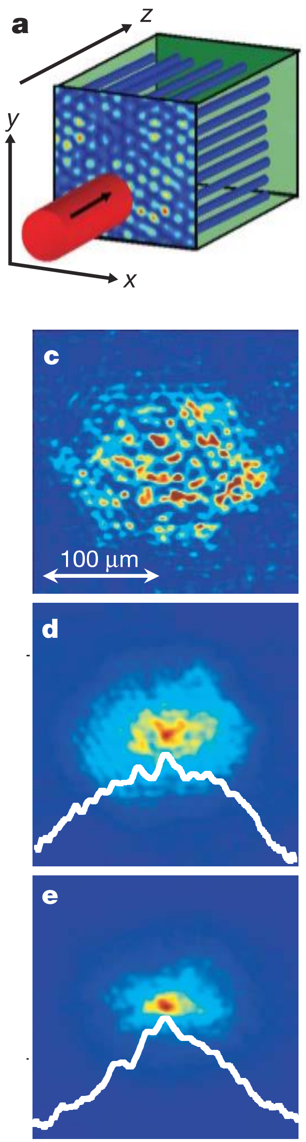


cold atoms

Billy, et al, Nature
453 891 (2008)

light

Schwartz, et al, Nature
446 52 (2007)

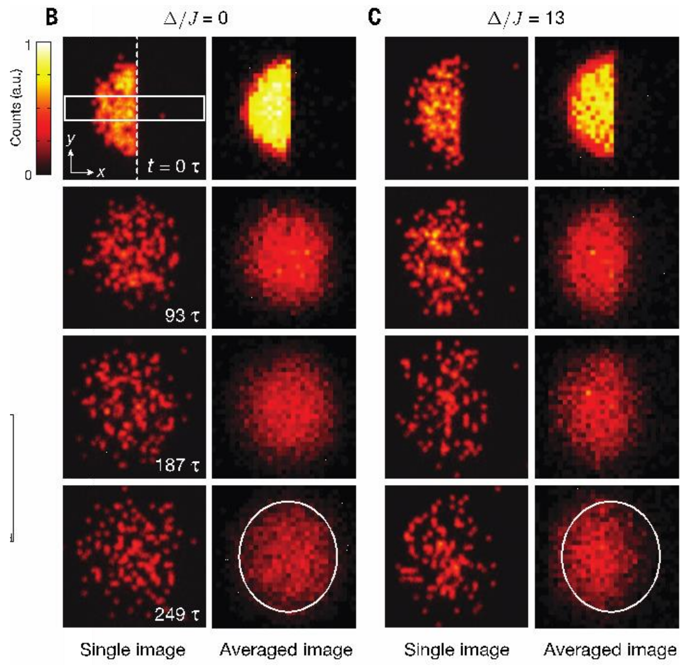


What does localization mean in an interacting system?

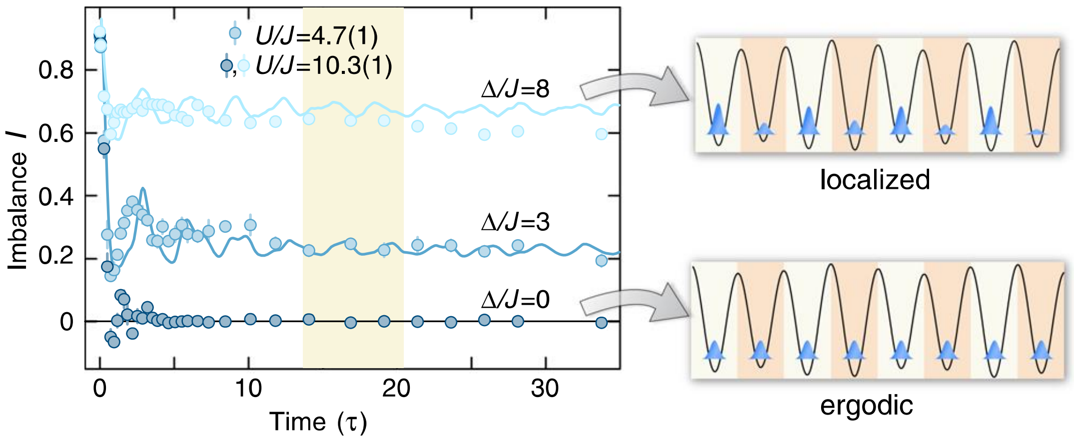
precursers: Fleishman & Anderson (1980); Finkelstein (1983); Giamarchi & Schulz (1988)

early work: Gornyi, Mirlin, & Polyakov (2005); Basko, Aleiner, & Altshuler (2006,2007)

reviews: Nandkishore & Huse, Annual Review of Cond. Matt. Phys. **6**, 15 (2015); Abanin, Altman, Bloch, & Serbyn, Rev. Mod. Phys. **91** 021001 (2019)



Choi, et al., Science **352** 1547 (2016)

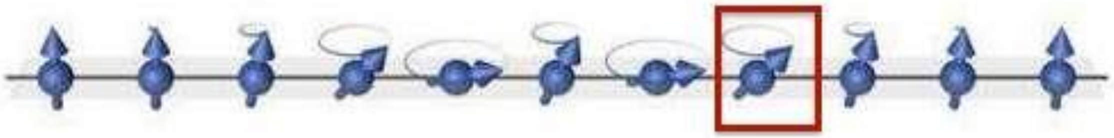


Schreiber, et al., Science **349** 6250 (2015)

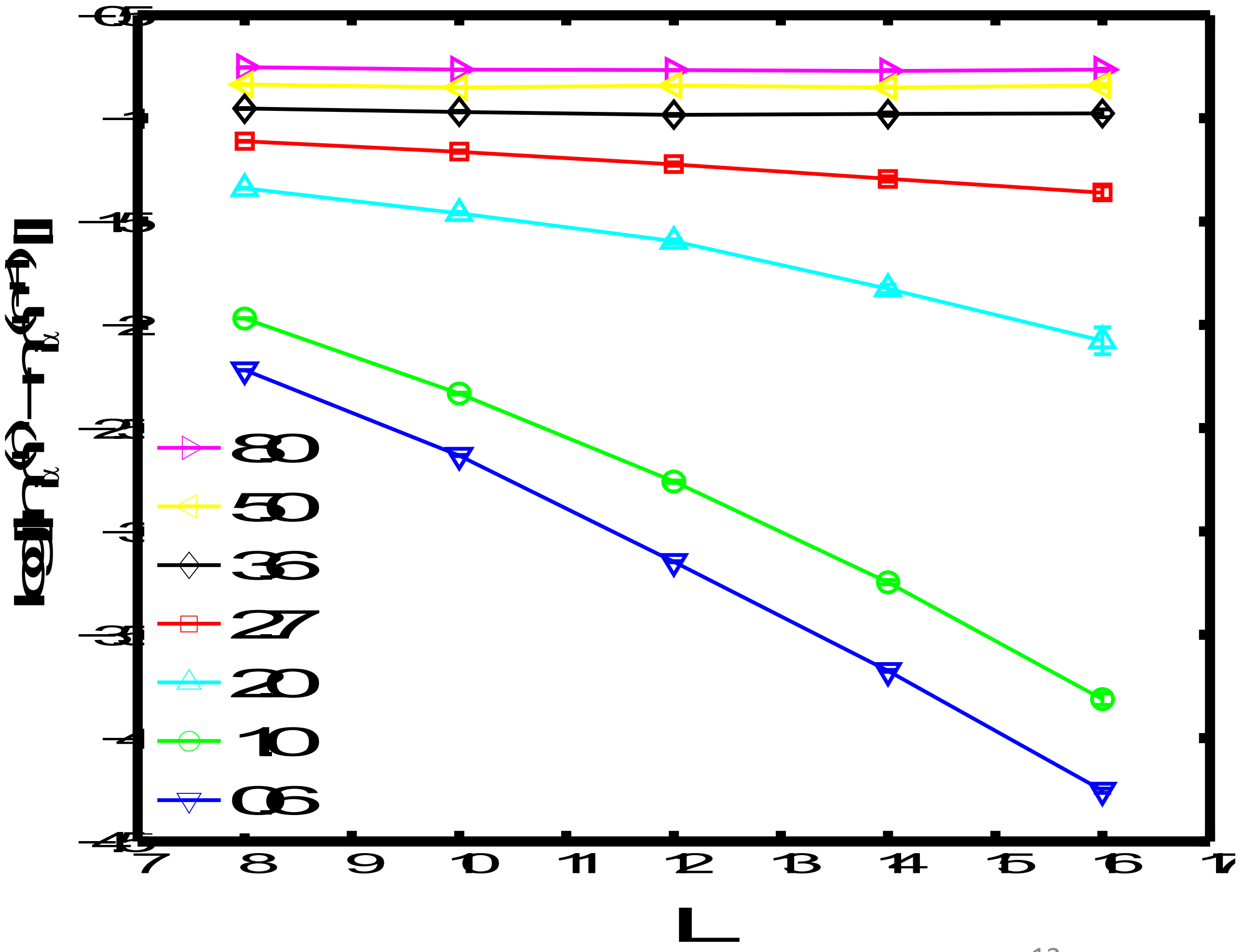
- lack of transport
- memory of initial conditions
- lack of thermalization
- macroscopic number of local conserved quantities
- Poisson statistics of level spacing
- discrete local spectrum
- entanglement entropy area law and logarithmic time dependence
- not just ground state property

What does localization mean in an interacting system?

Pal & Huse, PRB **82** 174411 (2010)



lack of thermalization: many-body localized system does not act as a thermal bath for a subsystem.



What does localization mean in an interacting system?

Hamiltonian in terms of local integrals of motion

$$H = \sum_i h_i \tau_i^z + \sum_{i,j} J_{ij} \tau_i^z \tau_j^z + \ldots$$

Huse, Nandkishore and Oganesyan, PRB **90**, 174202 (2014)

$$\tau_i^z$$

local integral of motion

questions

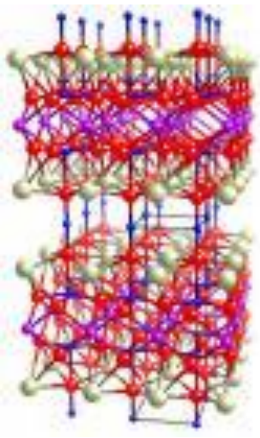
Is many-body localization stable in the thermodynamic limit?

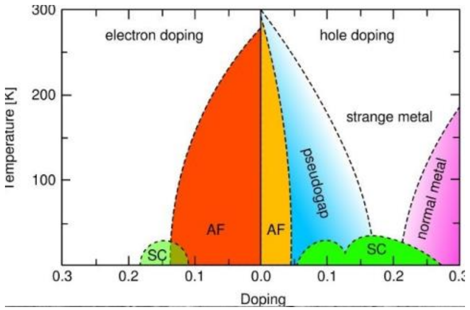
What is the nature of the transition? Is there an intermediate phase? A mobility edge?

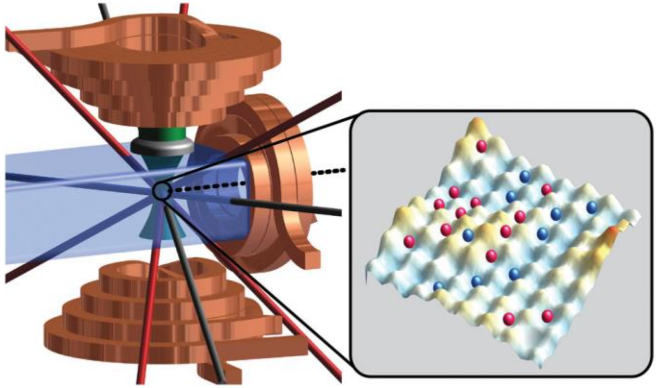
Is there localization for d>1?
Can localization persist in the presence of external coupling?

What novel nonequilibrium states might arise?
Can the local memory retention be used in quantum computing?

What new aspects of localization arise in multi-component systems?







Kondov PRL 2015

Fermi-Hubbard model with charge and spin disorder

$$H = -t_h \sum_{\langle i,j \rangle,\sigma} \left(c^\dagger_{i\sigma} c_{j\sigma} + c^\dagger_{j\sigma} c_{i\sigma} \right) + U \sum_i n_{i\uparrow} n_{i\downarrow}$$

$$+ \sum_i \epsilon_i d_i + \sum_i h_i m_i$$

charge density

$$d_i = n_{i\uparrow} + n_{i\downarrow}$$

charge disorder

$$\epsilon_i \in (-W_{ch}, W_{ch})$$

magnetization

$$m_i = n_{i\uparrow} - n_{i\downarrow}$$

spin disorder

$$h_i \in (-W_{sp}, W_{sp})$$

Outline

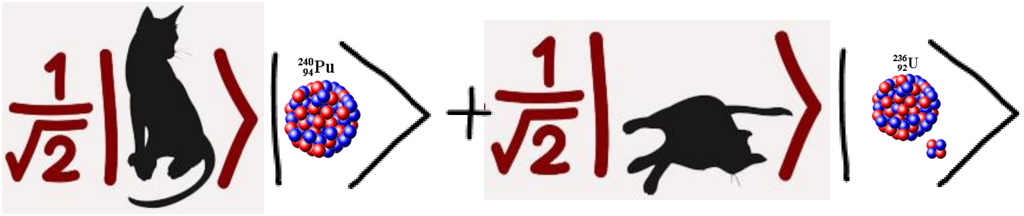
equilibrium in closed systems

systems which don't equilibrate

the Fermi Hubbard model with disorder

How does the entanglement grow with time?
What are the contributions of charge and spin to this growth?
How might disorder in charge and spin be used to control this growth?

Why study entanglement?



- Identifying distinct phases of matter
- Controlling entanglement for quantum computation
- Understanding how quantum systems approach thermal equilibrium

How to quantify entanglement?

entanglement entropy

$$\rho_A = \mathrm{Tr}_B\, \rho$$

$$S = -\mathrm{Tr}_A \left(\rho_A \ln \rho_A\right)$$

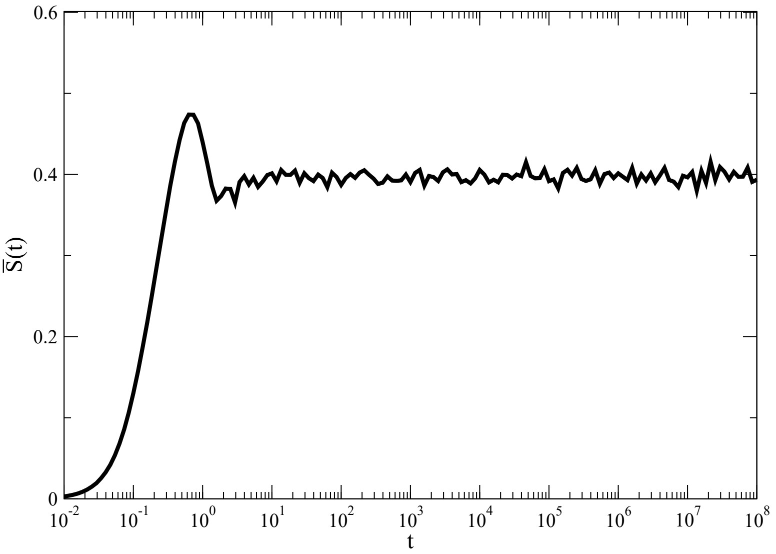
next: S(t) in noninteracting system -> S(t) in interacting system
-> understanding charge and spin contributions

Disorder impedes entanglement growth and equilibration

non-interacting tight-binding + disorder

$$H = \sum_i h_i \tau_i^z$$

entanglement entropy



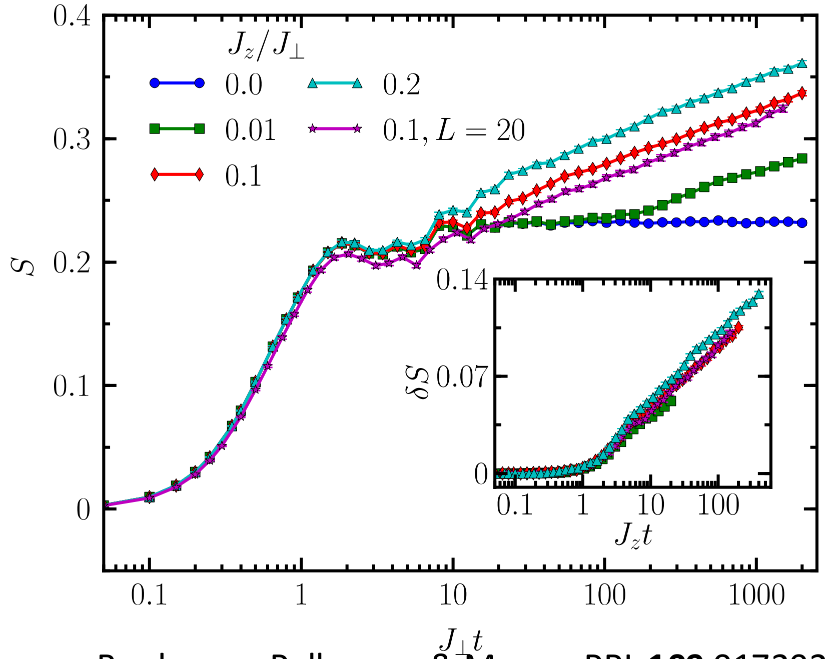
time

$$\tau_i^z$$

local integral of motion

interacting spinless Fermions + disorder

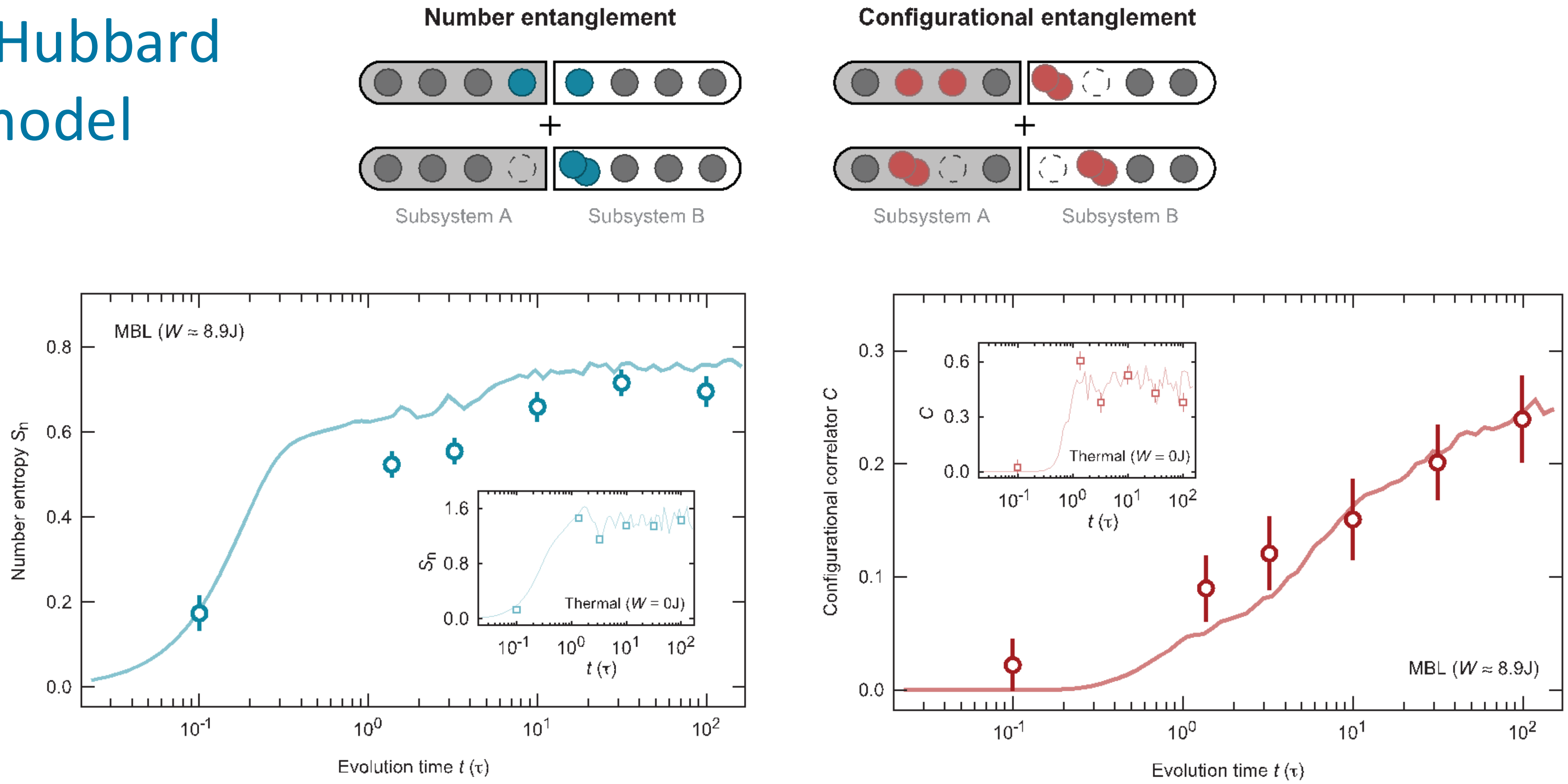
$$H = \sum_i h_i \tau_i^z + \sum_{i,j} J_{ij} \tau_i^z \tau_j^z + \ldots$$



Bardarson, Pollmann, & Moore, PRL **109** 017202 (2012)
Huse & Oganesyan, PRB **90** 174202 (2014)
Serbyn & Abanin, PRL **110** 260601 (2013)

Experiments in cold-atom systems

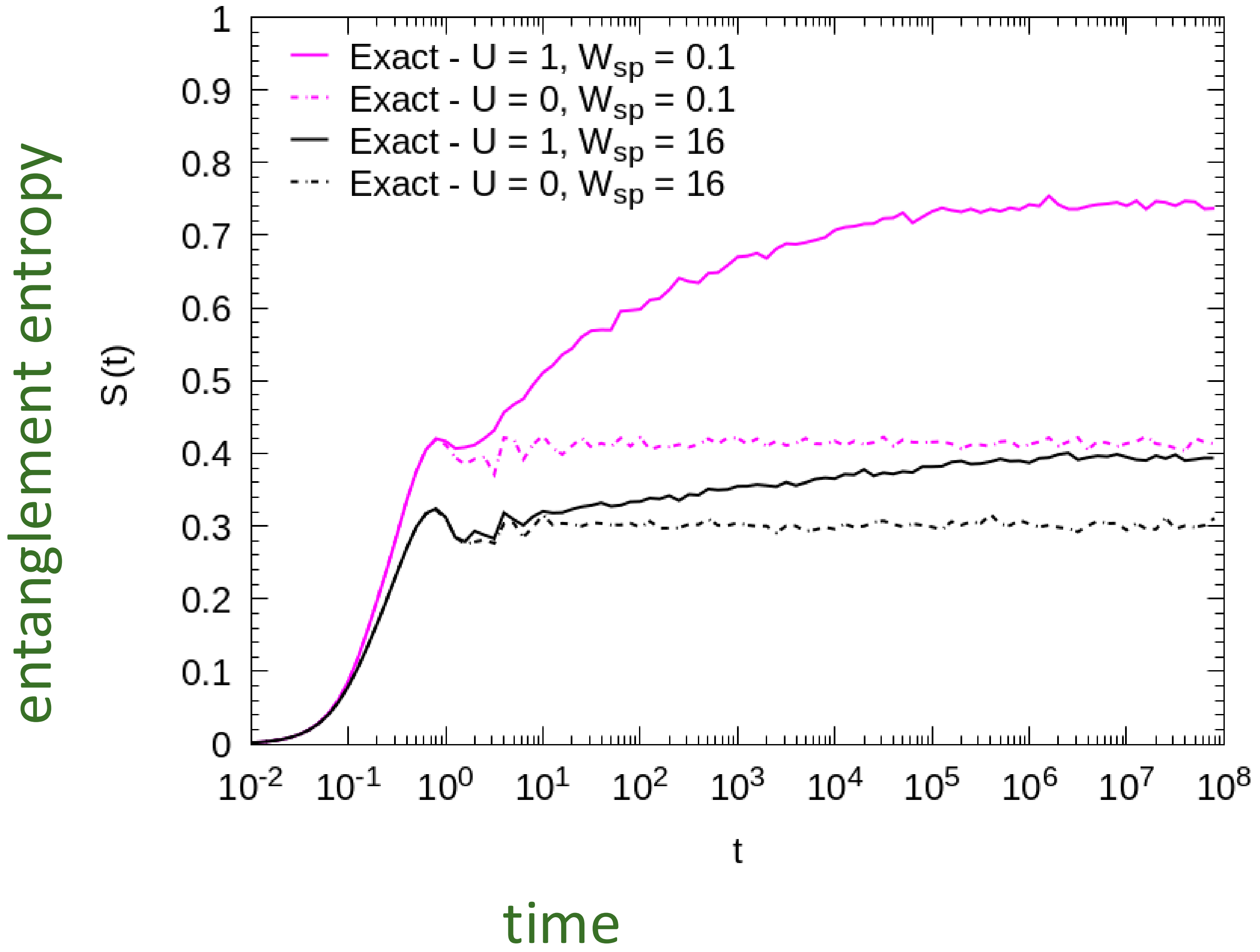
Bose-Hubbard model



Lukin, Rispoli, Schittko, Tai, Kaufman, Choi, Khemani, Leonard, Greiner
Science **364** 256 (2019)

Entanglement growth in disordered Fermi-Hubbard model

comparing W_{ch}=16,W_{sp}=16 with W_{ch}=16,W_{sp}=0.1



Lower W_{sp} causes small increase for U=0

but a large increase for U=1

Expressing the disordered Fermi-Hubbard model in terms of local integrals of motion

Hamiltonian in the usual form:

$$H = -t_h \sum_{\langle i,j \rangle,\sigma} \left(c^\dagger_{i\sigma} c_{j\sigma} + c^\dagger_{j\sigma} c_{i\sigma} \right) + U \sum_i n_{i\uparrow} n_{i\downarrow} + \sum_i \epsilon_i d_i + \sum_i h_i m_i$$

Originally proposed Hamiltonian in terms of local integrals of motion:

$$H = \sum_i h_i \tau_i^z + \sum_{i,j} J_{ij} \tau_i^z \tau_j^z + \ldots$$

Our version with charge and spin specific local integrals of motion:

$$H = J_0 I + \sum_i \left(J_i^c \mathfrak{d}_i + J_i^s \mathfrak{m}_i \right) + \sum_{ij} \left(J_{ij}^{cc} \mathfrak{d}_i \mathfrak{d}_j + J_{ij}^{cs} \mathfrak{d}_i \mathfrak{m}_j + J_{ij}^{ss} \mathfrak{m}_i \mathfrak{m}_j \right) + \ldots$$

Charge and spin-specific local integrals of motion

making integrals of motion

Huse (2014); Chandran (2015)

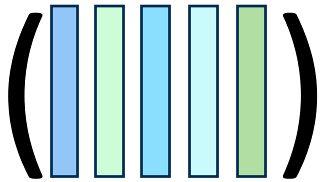
Eigenstates of H → Q such that $Q^{+}HQ$ is diagonal.

If A is diagonal in Fock basis, then QAQ^{+} diagonal in energy basis.

making them local

Leipner-Johns (2019)

Choose local A, e.g. $n_{i\sigma}$

Choose Q close to the identity   Q = 

charge and spin character

Leipner-Johns (2019)

Construct from local charge

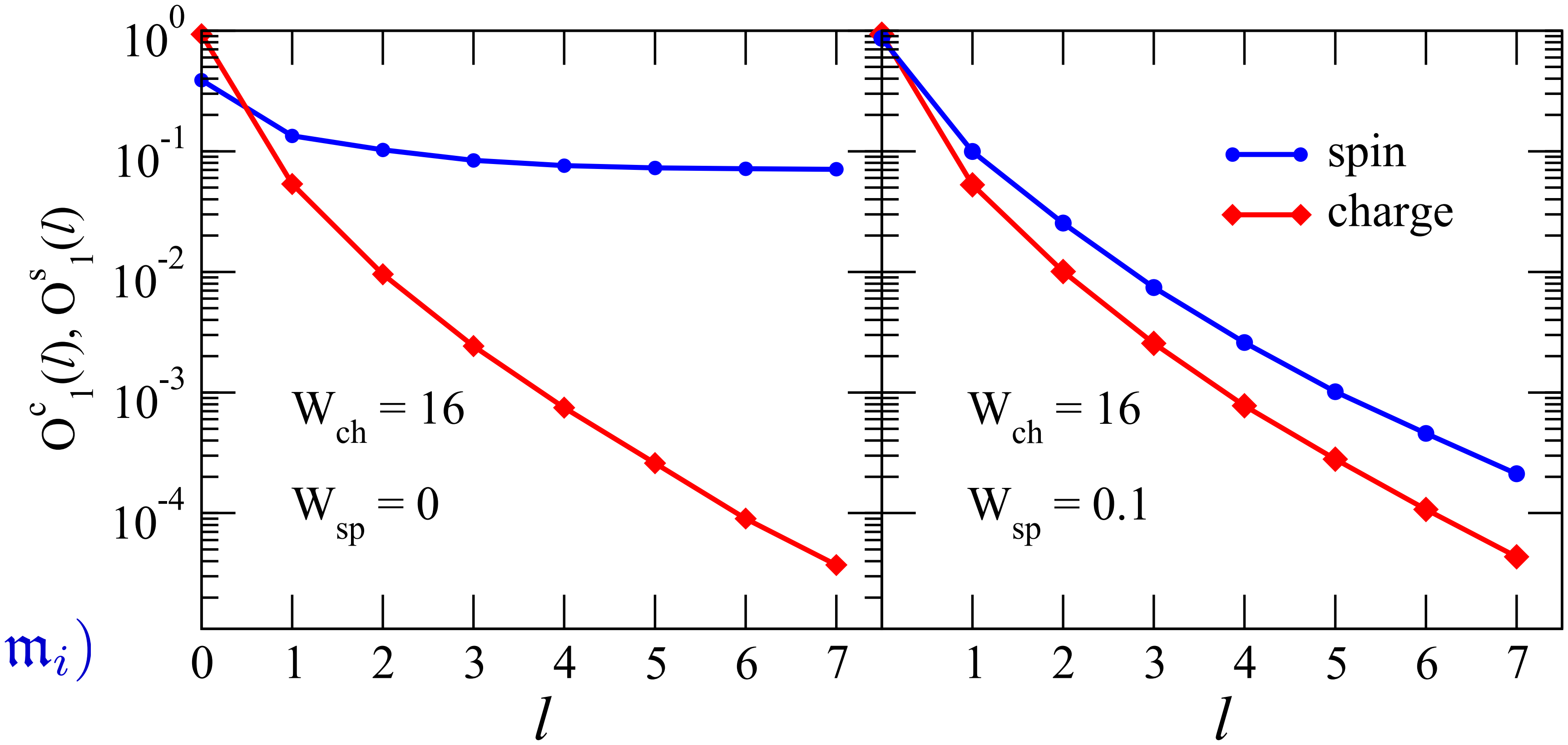
and magnetization operators

$$d_i = n_{i\uparrow} + n_{i\downarrow} \qquad \mathfrak{d}_i = Q\tilde{d}_i Q^\dagger$$

$$m_i = n_{i\uparrow} - n_{i\downarrow} \qquad \mathfrak{m}_i = Q\tilde{m}_i Q^\dagger$$

Locality of integrals of motion

Phys. Rev. B **100**, 125132 (2019)



overlap vs distance

$$O_i^s(\ell) = \frac{1}{N}\mathrm{Tr}(\tilde{m}_{i\pm\ell}\mathfrak{m}_i)$$

$$O_i^c(\ell) = \frac{1}{N}\mathrm{Tr}(\tilde{d}_{i\pm\ell}\mathfrak{d}_i)$$

Zero spin disorder:
No localization of spin

Very weak spin disorder:
Localization of spin is almost as strong as that of charge

The Hamiltonian written in terms of LIOMs

$$H = J_0 I + \sum_i (J_i^c \mathfrak{d}_i + J_i^s \mathfrak{m}_i) + \sum_{ij} (J_{ij}^{cc} \mathfrak{d}_i \mathfrak{d}_j + J_{ij}^{cs} \mathfrak{d}_i \mathfrak{m}_j + J_{ij}^{ss} \mathfrak{m}_i \mathfrak{m}_j) + \ldots$$

1st-order terms

2nd-order terms

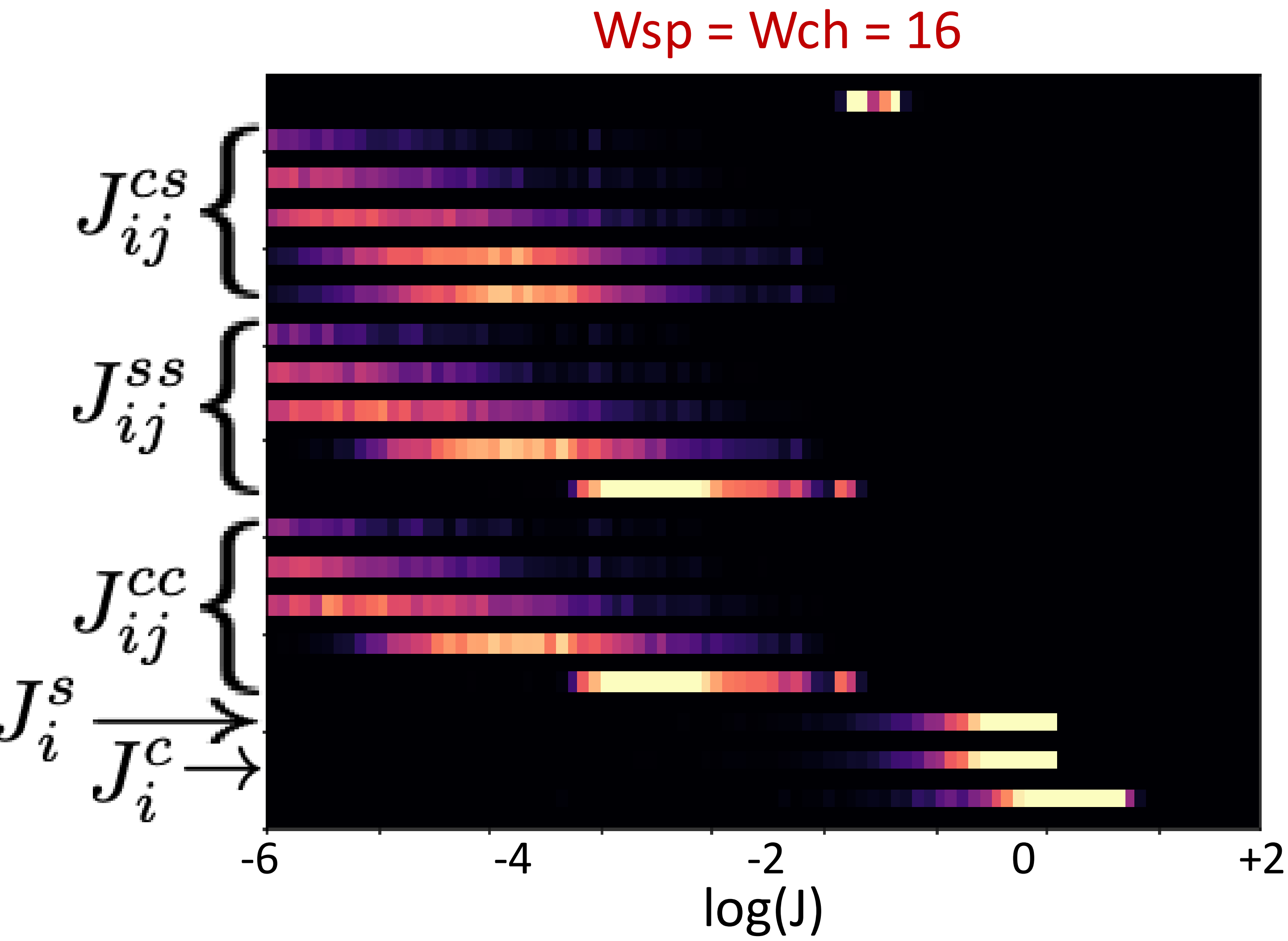
Next:

Results: distribution of J values → entanglement entropy

(1) Wsp=Wch (2) Wsp<<Wch

Distribution of coefficients in the LIOM Hamiltonian

$$H = J_0 I + \sum_i (J_i^c \mathfrak{d}_i + J_i^s \mathfrak{m}_i) + \sum_{ij} (J_{ij}^{cc} \mathfrak{d}_i \mathfrak{d}_j + J_{ij}^{cs} \mathfrak{d}_i \mathfrak{m}_j + J_{ij}^{ss} \mathfrak{m}_i \mathfrak{m}_j) + \ldots$$



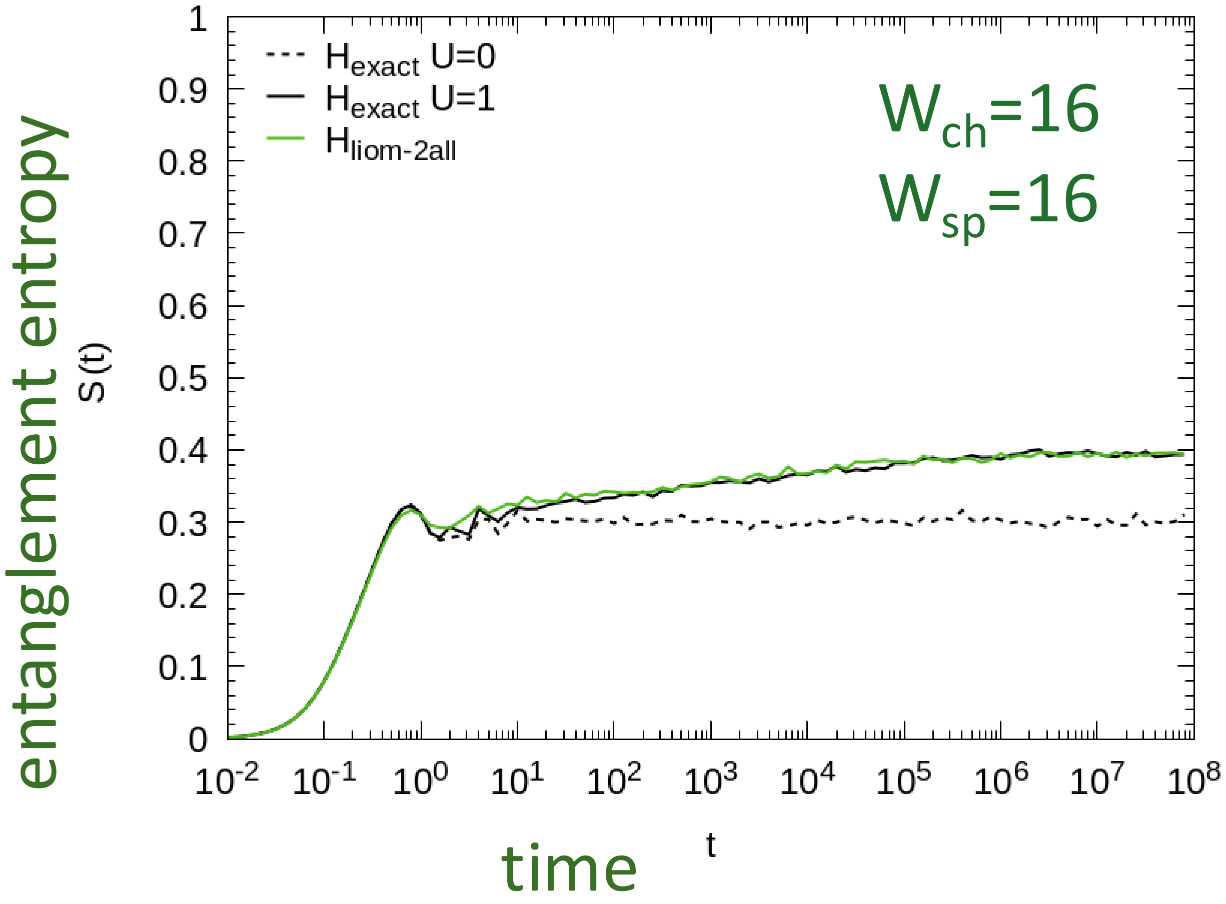
Charge and spin values the same

2nd-order coefficients decay with distance

Charge-spin coupling 10x smaller than charge-charge or spin-spin

Entanglement growth in disordered Fermi-Hubbard model

comparing exact with up to 2nd-order terms in LIOM Hamiltonian



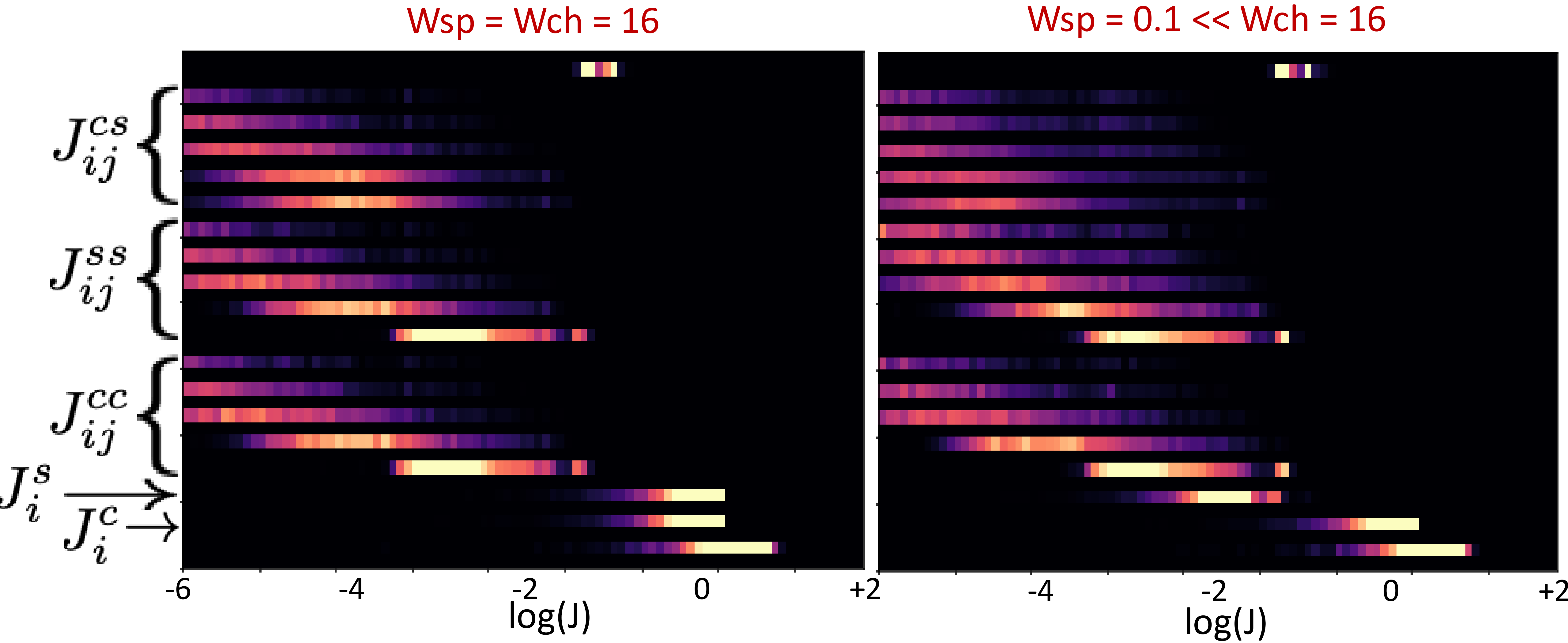
2nd-order terms in the LIOM Hamiltonian capture most of the physics

log(t) growth same as seen in single-component systems

weak coupling between charge and spin not apparent

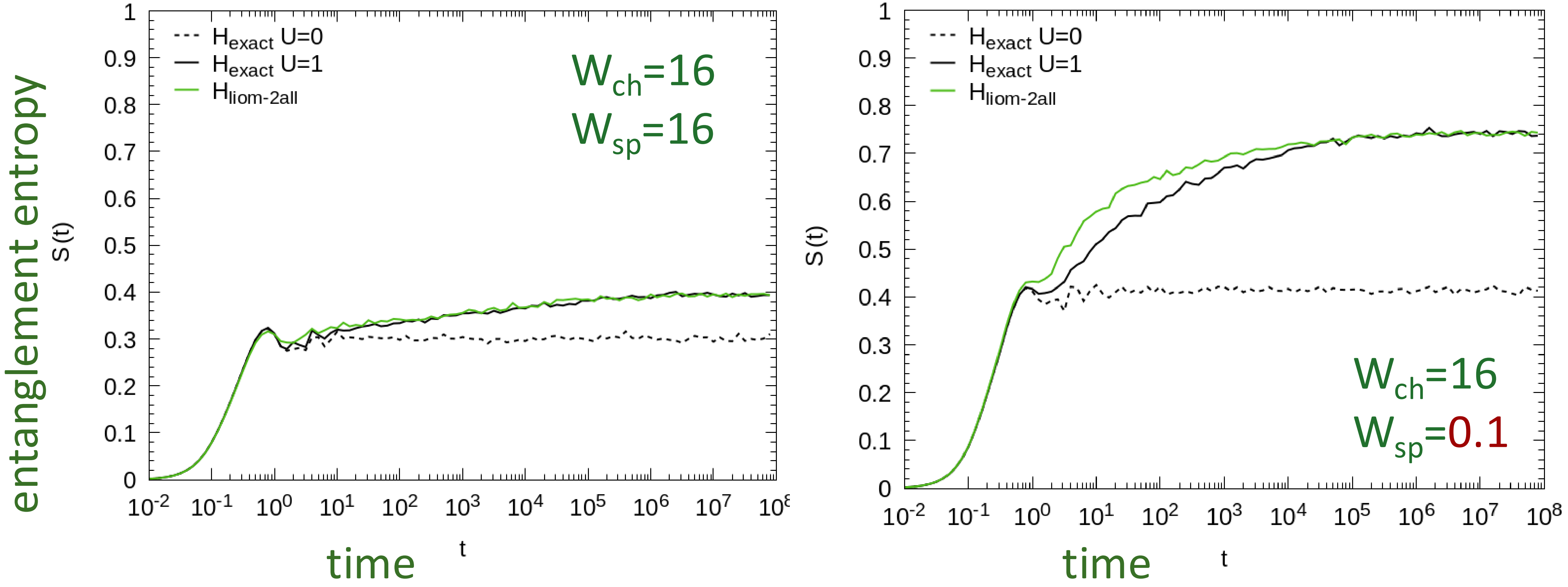
Distribution of coefficients in the LIOM Hamiltonian

$$H = J_0 I + \sum_i (J_i^c \mathfrak{d}_i + J_i^s \mathfrak{m}_i) + \sum_{ij} (J_{ij}^{cc} \mathfrak{d}_i \mathfrak{d}_j + J_{ij}^{cs} \mathfrak{d}_i \mathfrak{m}_j + J_{ij}^{ss} \mathfrak{m}_i \mathfrak{m}_j) + \ldots$$



Entanglement growth in disordered Fermi-Hubbard model

comparing exact with up to 2nd-order terms in LIOM Hamiltonian



Wsp<< Wch: long-term growth no longer log(t)

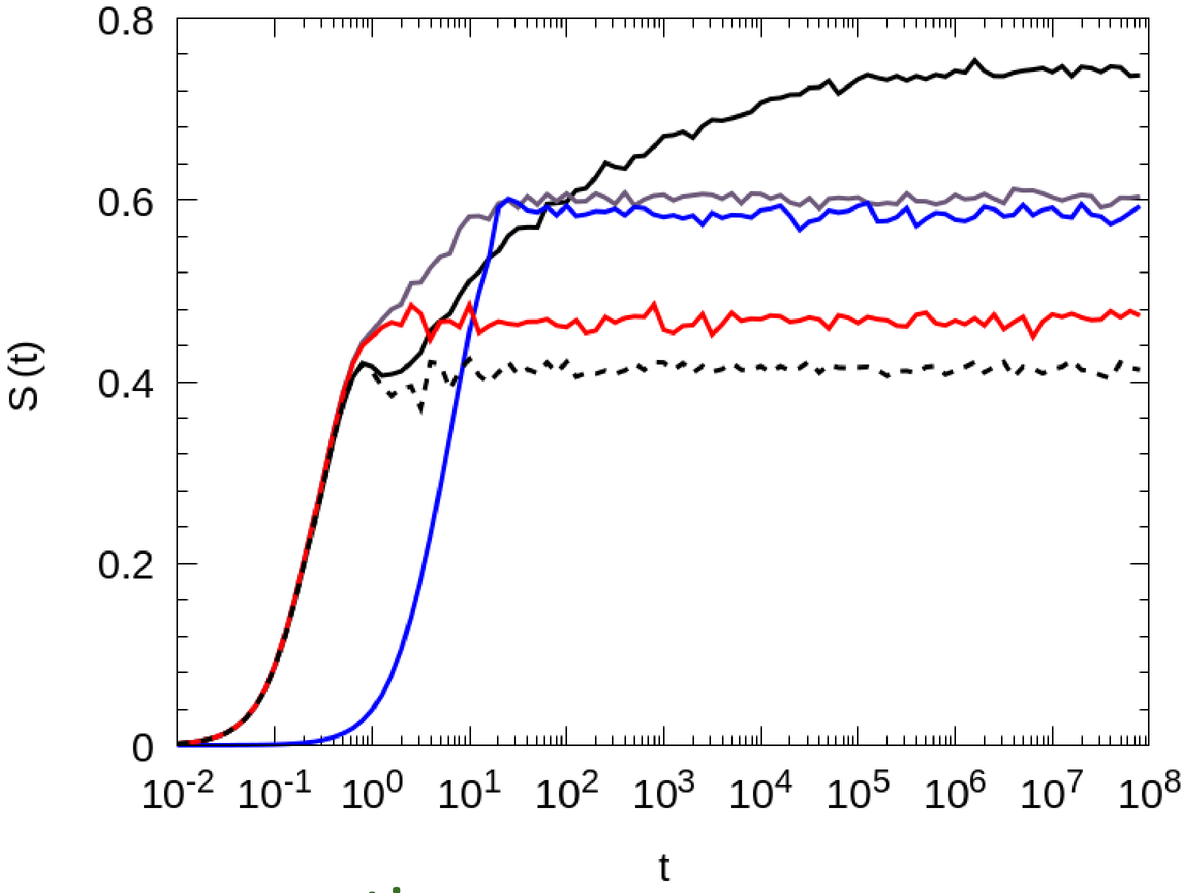
2nd-order terms still capture most physics, but not quite as well

Entanglement growth in disordered Fermi-Hubbard model

separating 1st-order terms into charge and spin

W_{ch}=16
W_{sp}=0.1

entanglement entropy



time

exact

all 1st order
1st-order spin only
1st-order charge only

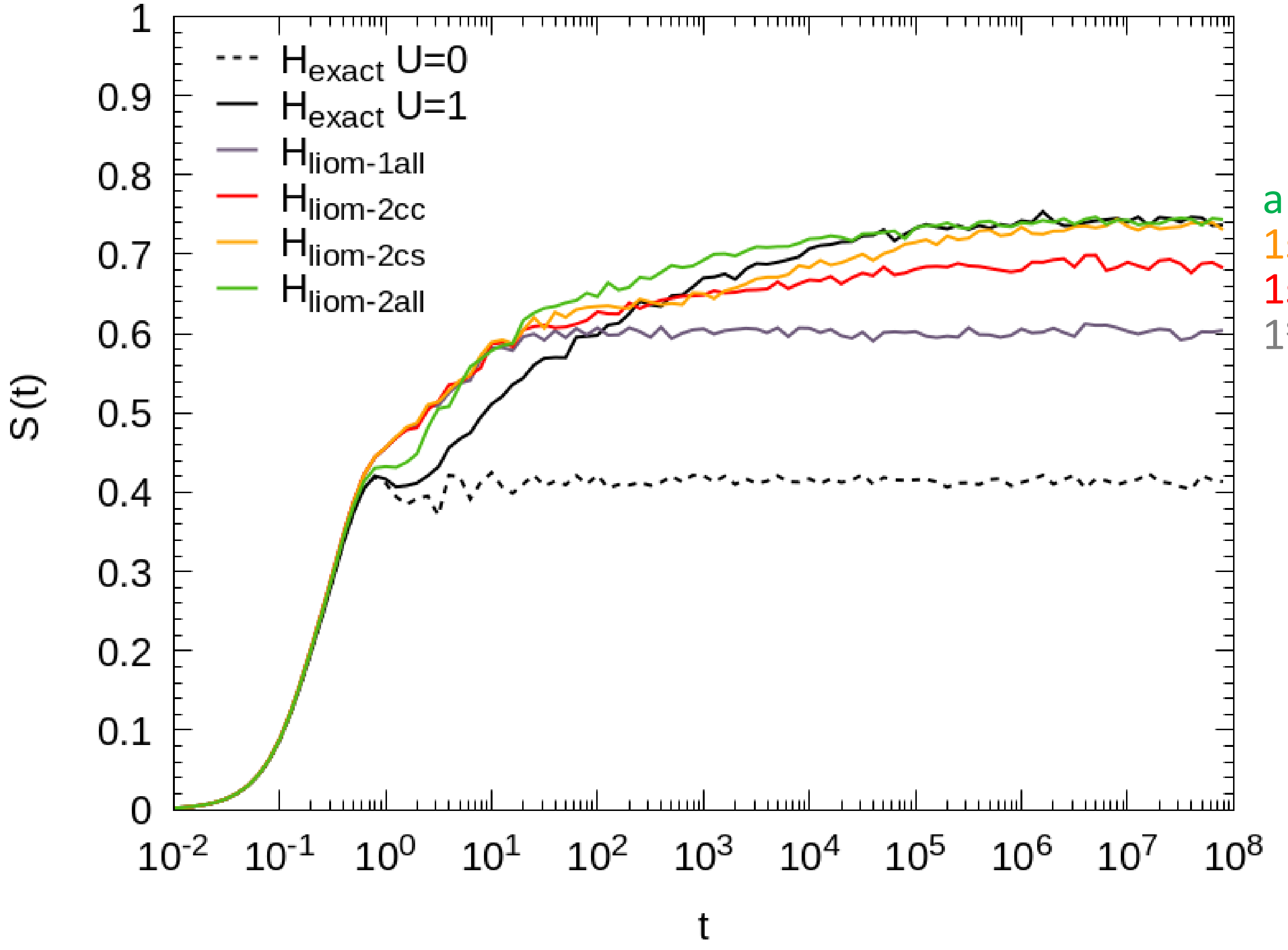
origin of two-stage rise:
W_{ch}>W_{sp} →
localization length charge < spin
→ charge entangles faster
to lower saturation value

Entanglement growth in disordered Fermi-Hubbard model

contributions of different 2nd-order terms

$W_{ch}=16$
$W_{sp}=0.1$

entanglement entropy



time

all 2nd-order
1st + charge-spin
1st + charge-charge
1st order only

charge-spin coupling contributes at later times than charge-charge and spin-spin

↑
↓

average charge-spin coupling strength is much less than average charge-charge and spin-spin coupling strength

Summary

Disorder can prevent equilibration

The Fermi-Hubbard model can have disorder in charge and spin

Localization in spin depends on disorder in both spin and charge

We study entanglement growth, distinguishing the contributions from charge and from spin by expressing the Hamiltonian in terms of optimally local charge and spin integrals of motion

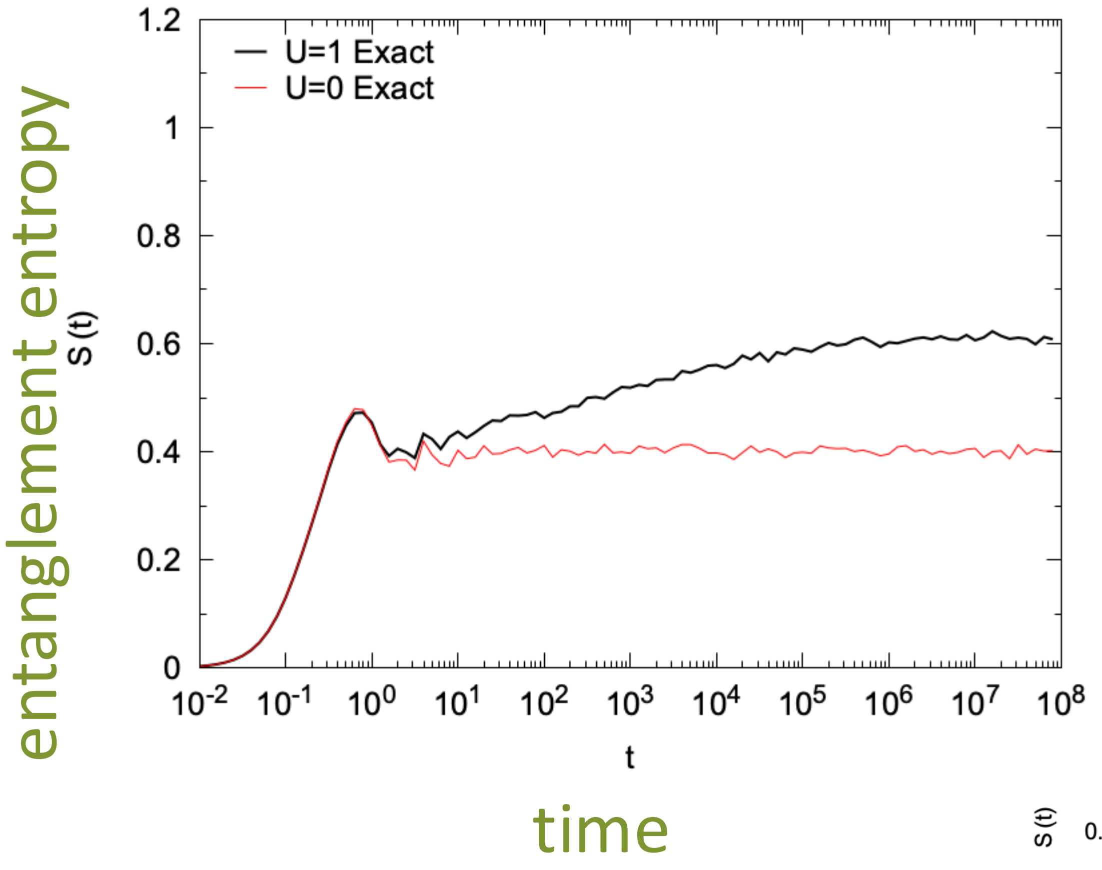
The Hamiltonian truncated at 2nd order captures most of the physics

Charge-charge and spin-spin coefficients decay exponentially with distance
Charge-spin coupling is much weaker than charge-charge or spin-spin

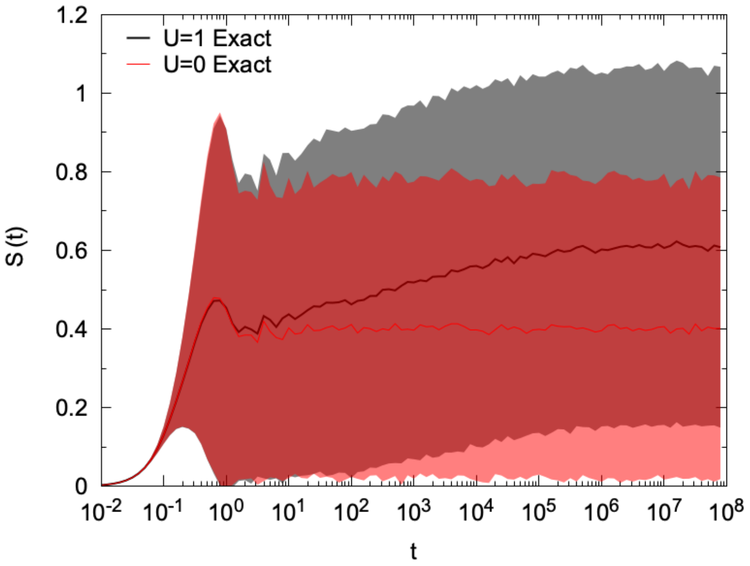
rwortis@trentu.ca

Entanglement growth in disordered Fermi-Hubbard model

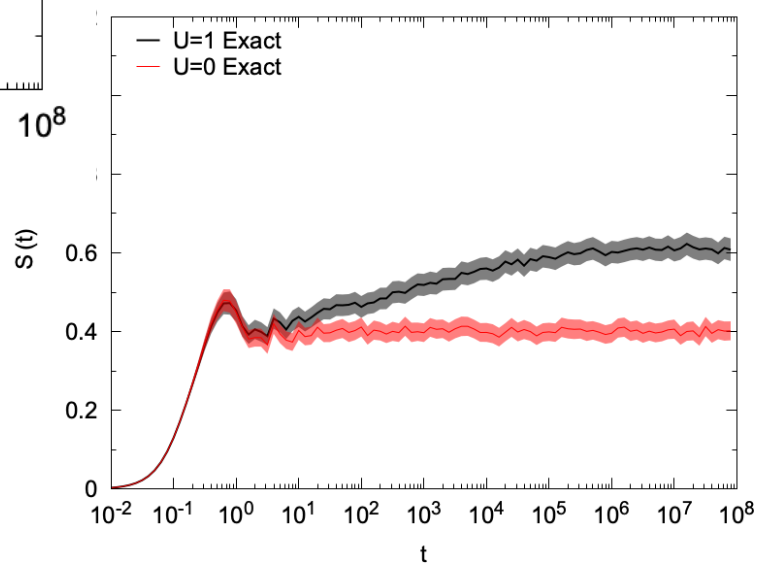
strong disorder in both charge and spin



standard devation

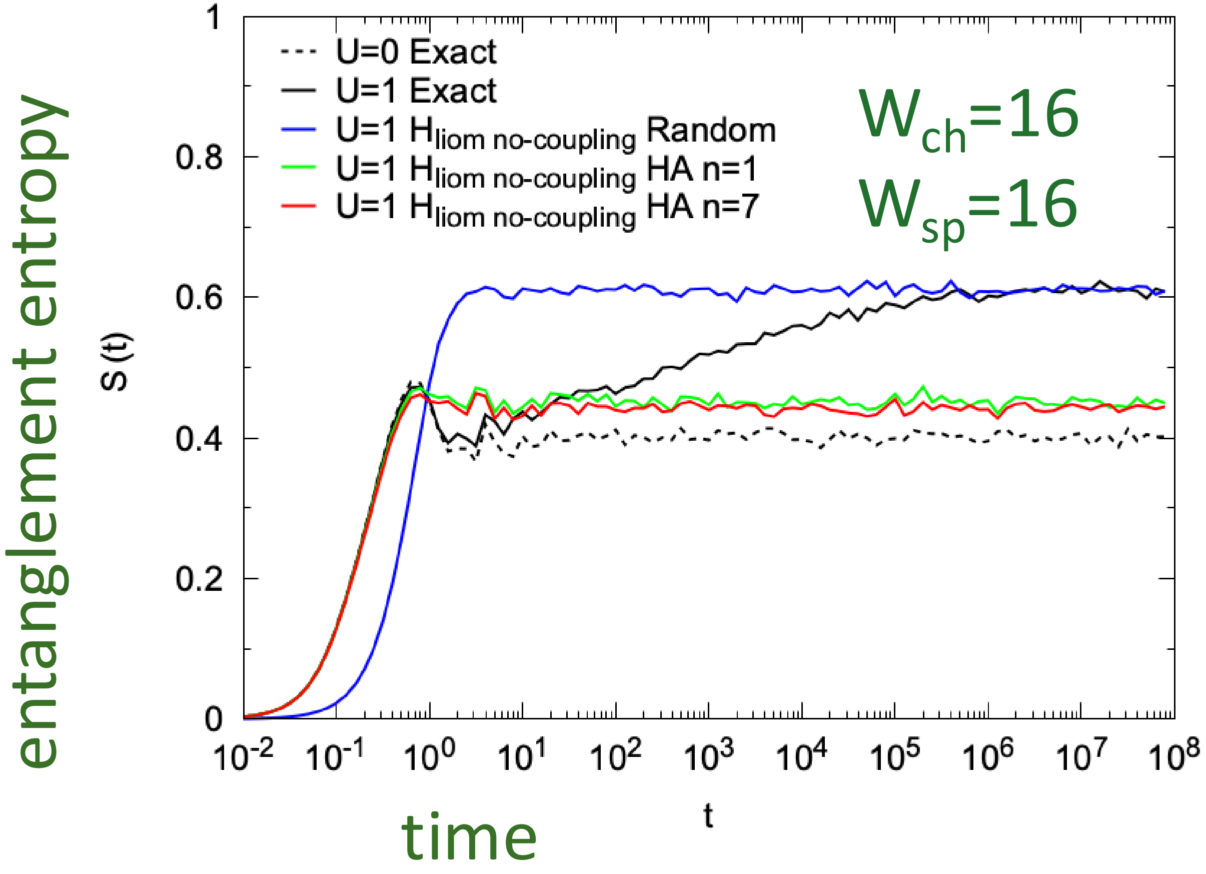


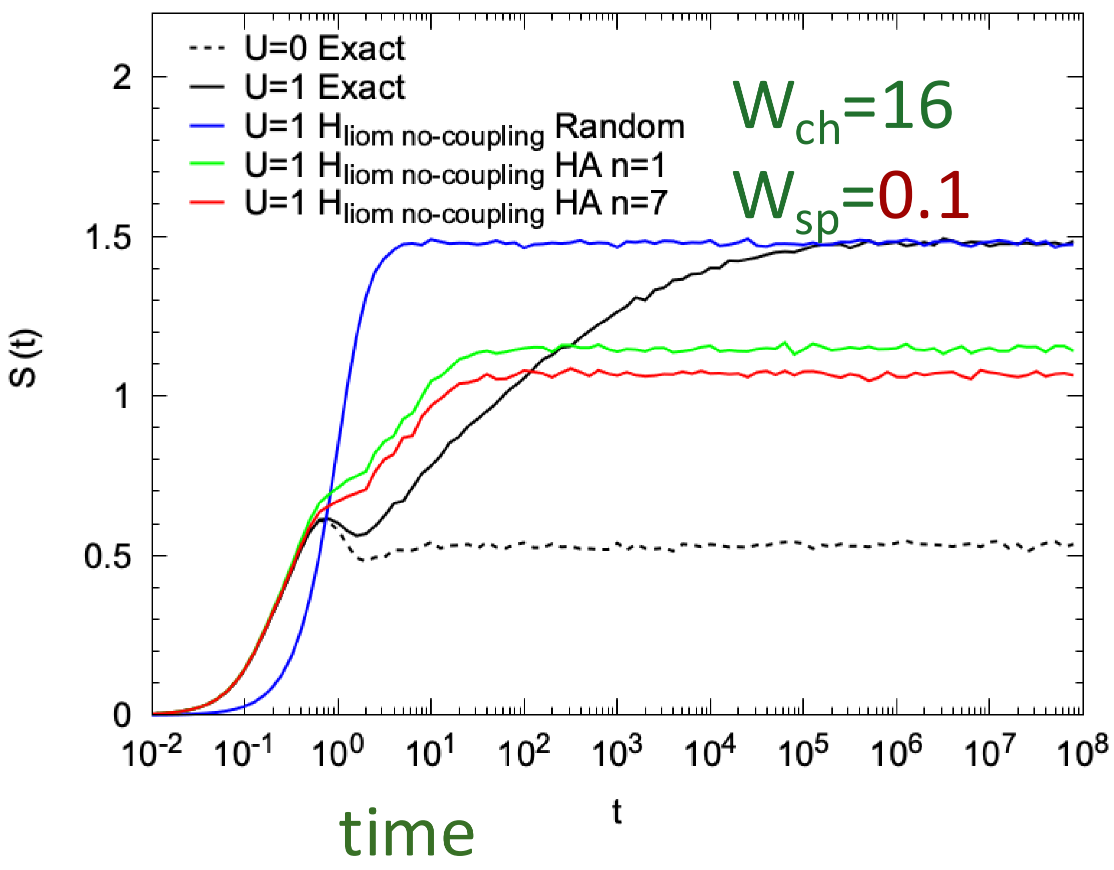
standard error of the mean



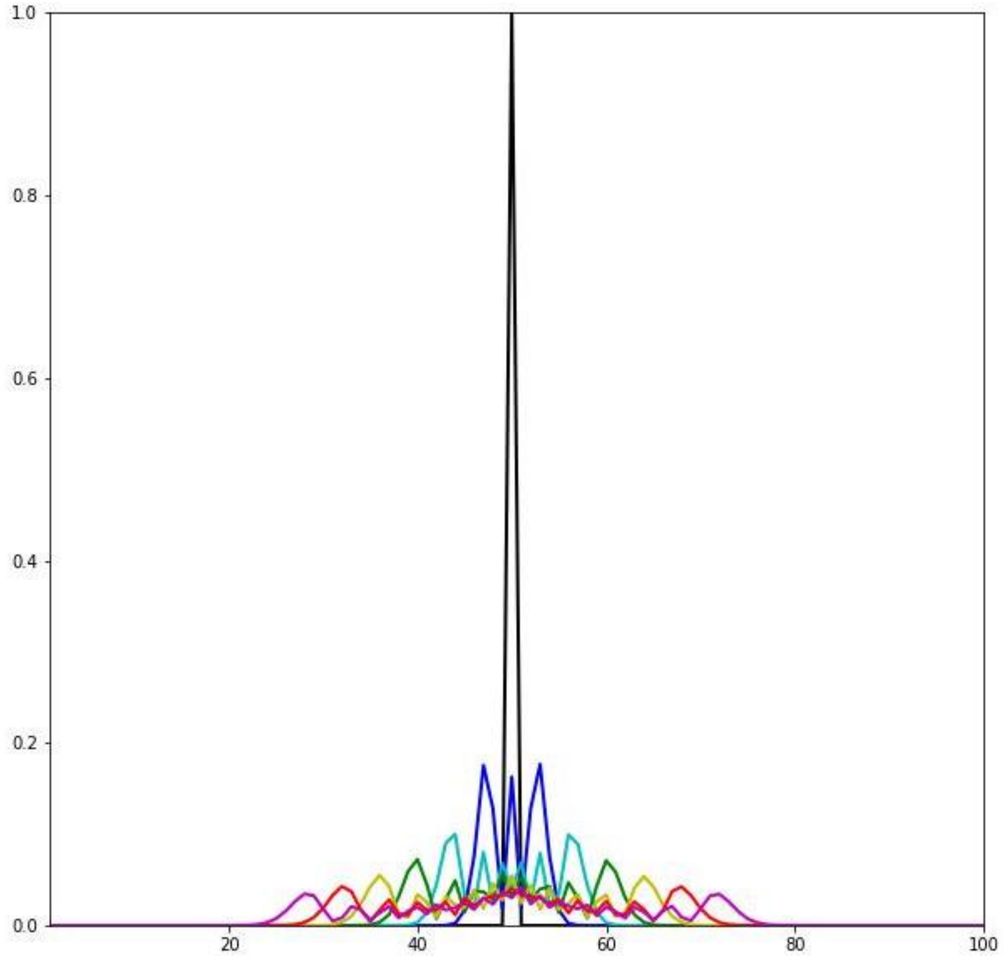
Entanglement growth in disordered Fermi-Hubbard model

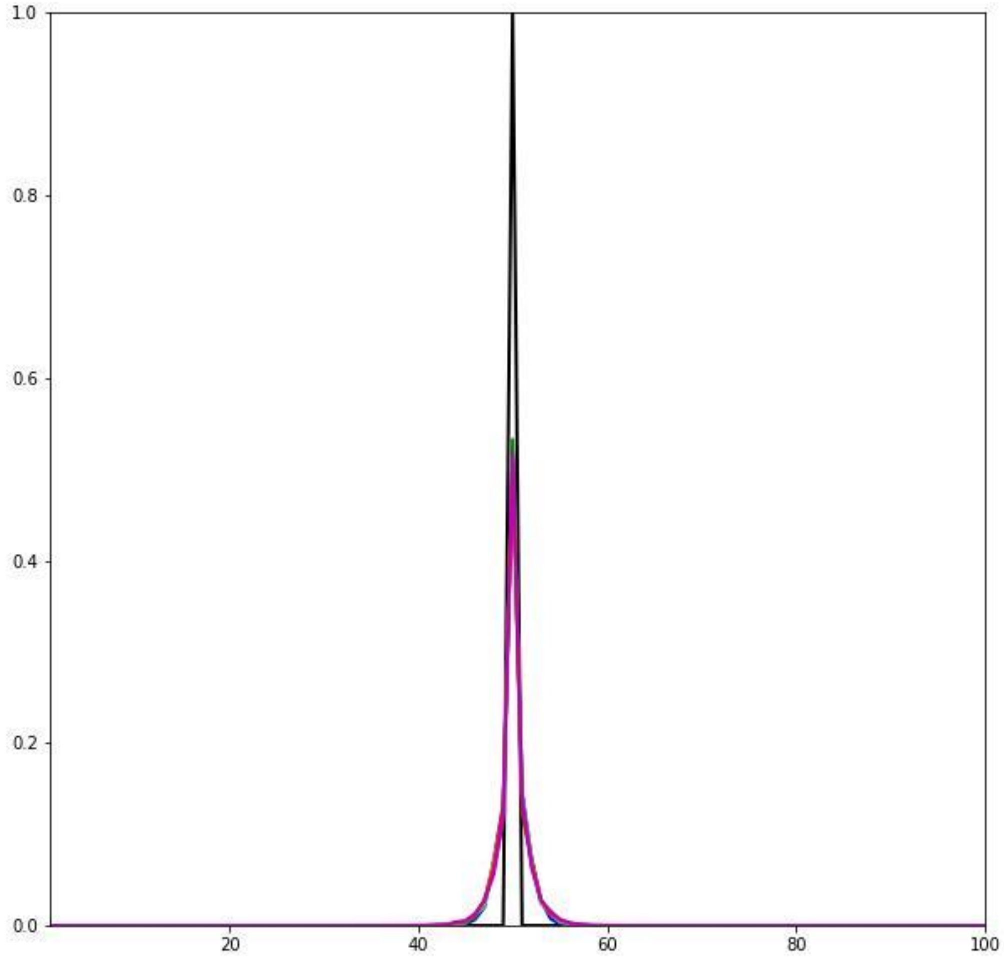
Does optimizing locality of integrals of motion make a difference?





Optimizing the locality of the LIOMs is making a difference;
but with W_{sp}=0.1 extra entanglement remains


1.0
0.8
0.6
0.4
0.2
0.0
20
40
60
80
100


1.0
0.8
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80
100

Strong disorder in both channels not required for localization