No static black hole horizons in the expanding universe

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Outline

1. Stationary black holes $\leftrightarrow$ event horizons
2. Dynamical black holes $\leftrightarrow$ apparent horizons
3. Can a static BH exist in an expanding universe?
4. No: it becomes a *naked spacetime singularity*. Evidence from radial light rays, freely-falling test masses, diverging Ricci scalar, test scalar field, BH thermodynamics.
5. BH are cosmologically coupled somehow, but how?

Black hole horizons: event and apparent

The BH concept is defined by the horizon

The technical definition of BH using event horizons:

(a connected component of) the boundary of the causal past of future null infinity

English translation:

shoot light rays, they will escape to infinity. Trace back *all* these null rays, at some point they will stop. There is a boundary (the EH) such that, if rays are emitted there, they never escape and don't make it to ∞.

So

the EH is the surface from which nothing will **ever** escape

Problem: the "ever" word. One needs to know the spacetime geometry all the way to the infinite future! Unless the BH is static, it's impossible (teleological definition). In simulations of collapse to a BH, codes crash long before infinity.

BH as defined by the event horizon

This definition is wonderful for proving mathematical theorems, but of little practical use unless the BH is stationary.

Realistic BHs are not stationary:

- they interact with their environment;
- they come in binaries;
- they are embedded in the expanding universe (more on this later);
- they emit Hawking radiation and lose mass, evolving (in a way that nobody has really calculated yet).

Enters the APPARENT HORIZON

So, abandon the EH for practical purposes. We need at least a proxy for the EH. Then, what is a BH?

Think spherical: take a 2D sphere of symmetry and the radial light rays (radial null geodesics) emanating from it, outgoing or ingoing. Look at the expansions of outgoing ($\Theta_{(+)}$) and ingoing ($\Theta_{(-)}$) bundles of light rays with tangents $u^a_{(\pm)}$: if $\Theta_{(\pm)} \equiv \nabla_c u^c_{(\pm)} > 0$ the bundle expands, if < 0 it is focused.

An **APPARENT HORIZON** (AH) is a surface where

$$\Theta_+ = 0$$
$$\Theta_- < 0$$

a marginally trapped surface, i.e., rays that want to come out cannot but ingoing rays fall in, and this is the outermost surface where this happens.
English translation:

the AH is the surface from which nothing can escape **now**

BH defined by the AH

Quasilocal definition of BH, does not require one to know the entire future (global structure) of spacetime.

Problem: the AH depends on the spacetime foliation, i.e., on the observer.

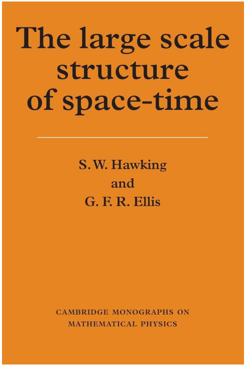
According to this definition, different observers may not even agree that there is a BH in spacetime.
Even for the prototypical BH, the Schwarzschild BH, one can find a foliation in which there is no AH! (Wald & Iyer '91; Schnetter & Krishnan '06)

Yet, EHs are useless for practical purposes. Simulations producing banks of templates for the detection of gravitational waves emitted by binaries (BH-BH or BH-NS) use AHs and trapped surfaces.

An important theorem:

Stationary AHs coincide with EHs

(S.W. Hawking & G.F.R. Ellis 1973, "The Yellow Peril" p. 323)



To recap:

our understanding of black holes has changed over time. We now use AHs, not EHs, to characterize BHs (and the foliation-dependence problem disappears for stationary horizons)

CAN A STATIC BH HORIZON EXIST IN AN EVOLVING UNIVERSE?

Realistic BHs are embedded in the dynamical universe with accelerated expansion. Can they be truly static?

BHs evolve on scales $\ll$ than the cosmological (Hubble) scale $\sim 14 \cdot 10^9$ years. On astrophysical scales (spatial and temporal), the effect of the cosmological expansion is completely negligible.

But ... what if we let tiny effects accumulate for, say, 10 billion years? Are they still negligible?

It is very hard to explain with known astrophysical channels, how supermassive BHs in galaxies grow so big so quickly at $z > 6$ (Inyoshi + ARAA 2020; Volonteri + Nat. Rev. Phys. 2021).

Moreover, LIGO detections of gravitational waves from binary mergers involving BHs report stellar mass BHs in the mass gap forbidden by well-established stellar evolution models (Abbott + PRL 2020; PRX 2023; Mehta + ApJ 2022)

A recent proposal: **cosmological coupling** — BHs interact with the Friedmann-Lemaître-Robertson-Walker (FLRW) universe in which they are embedded. After averaging out local inhomogeneities, the cosmological geometry is described by

$$ds^2 = -dt^2 + a^2(t)\left(dx^2 + dy^2 + dz^2\right)$$

1) Cosmological coupling proposes that BH masses scales as

$$\boxed{m(a) = m_i \left(\frac{a}{a_i}\right)^k, \quad k \approx 3}$$

this would solve the astrophysical problems (K. Croker + 2019-24)

2) If BHs are non-singular, as proposed by quantum gravity (completely speculative, no unique model), they should have a macroscopic de Sitter core instead of the central singularity and dark energy causing the present acceleration of the universe would be relegated to their interiors (K. Croker +)

Tentative evidence for cosmological coupling has been reported for supermassive BH populations in red elliptical galaxies in redshift range $0 < z \leqslant 2.5$ (Farrah et al. ApJ 2023). Lively debate, both theory and observations, since April 2023 ... (and lots of prejudice, even for *Newtonian* systems embedded in FLRW)

2) is outrageous (but well worth exploring); 1) is rather natural.

What is the theoretical support for cosmological coupling? Unfortunately, not much. There are very few exact solutions of the Einstein equation describing evolving BHs embedded in FLRW universes, and they have problems:

- Unclear how to define their physical mass in GR (Hawking quasilocal energy?)
- What is their boundary: AH?
- Negative energy densities for periods of time near the AH

- McVittie (1931) solution of the Einstein equations has AH decreasing with time (local t or t of the distant observer?)
- Unlike Schwarzschild, the exact solutions are not generic and are probably misleading.

Not useful! Need a more reliable approach.

Fundamental question:

Can an exactly static EH = AH be embedded in an expanding FLRW universe? In a dynamical background?

Perturbative analysys of Davidson, Rubin, Verbin PRD 2012 argues against static EH in $k = 0$ FLRW. Let's do it *exactly* and simply.

Begin with the problem of principle: assume that an *exactly static*, spherical BH horizon exists at r_{AH}, embedded in a dynamical spacetime (could be an expanding FLRW universe, but not necessary). Since it is static, EH=AH and what follows covers *any* BH horizon.

Without loss of generality, the spherical metric is

$$ds^2 = -T^2(t,r)dt^2 + a^2(t,r)\left(dr^2 + r^2 d\Omega_{(2)}^2\right)$$

in isotropic coordinates. Physical (areal) radius $R(t,r) = a(t,r)r$

If AHs exist, they are located by the roots of

$$\nabla^c R \nabla_c R = 0$$

or

$$-\frac{\dot{R}^2}{T^2} + \frac{R'^2}{a^2} = 0$$

(the same condition is obtained by imposing that $R =$ const. is a null surface $\rightarrow$ again, AH=EH)

Examine ingoing radial light rays with tangents

$$u^a \equiv dx^a/d\lambda = \left(u^0, u^1, 0, 0\right),$$

$\lambda =$ affine parameter, coordinate velocity $= u^1/u^0$. They obey

$$\frac{du^a}{d\lambda^2} + \Gamma^a_{bc} u^b u^c = 0$$

where

$$\Gamma^0_{00} = \frac{\dot{T}}{T}, \quad \Gamma^0_{01} = \frac{T'}{T}, \quad \Gamma^0_{11} = \frac{a\dot{a}}{T^2}$$

giving

$$\frac{d}{d\lambda}\left(\frac{1}{u^0}\right) = \frac{\dot{T}}{T} \pm \frac{2T'}{a} + \frac{\dot{a}}{a}\,,$$

$$\frac{d}{d\lambda}\left(\frac{1}{u^1}\right) = \frac{T'}{T} \pm \frac{2\dot{a}}{T} + \frac{a'}{a}\,.$$

Near the static EH the geometry must resemble Schwarzschild, which is approximated by the Rindler metric

$$ds^2 \simeq ds^2_{\text{Rindler}} + \ldots = -\frac{x^2}{4}dt^2 + 4m^2\left(dx^2 + d\Omega^2_{2)}\right) + \ldots$$

Since the AH=EH is *exactly* static, $\dot{T} = \dot{a} = 0$ there and

$$\frac{d}{d\lambda}\left(\frac{1}{u^0}\right) \simeq \frac{2T'}{a} \simeq \frac{2}{a_{\text{AH}}}$$

giving

$$\longrightarrow u^0 = \frac{C}{\lambda - \lambda_0} > 0\,,$$

Either:

- $u^0 \to 0$ as $\lambda \to +\infty$, "time stops" (if $\lambda > \lambda_0$ and $C > 0$), or
- the radial ray stops at the finite value λ_0 (if $\lambda < \lambda_0$ and $C < 0$), with $u^0 \to \infty$

Meanwhile, $u^1 \simeq \pm T\, u^0/a \to 0$ and

the coordinate velocity $\to 0$ as $x \to 0$,

i.e., ingoing light stops at the EH and outgoing rays cannot begin there.

GEODESIC INCOMPLETENESS = SPACETIME SINGULARITY

The would-be static EH=AH is a naked singularity.

Further evidence

Similar results for **timelike** radial geodesics.

Moreover, the Ricci scalar

$$\mathcal{R} \simeq -\frac{2}{a_{AH}^2}\left[\left(\frac{a'_{AH''}}{a_{AH}} + \frac{2}{r_{AH}}\right)\right]\frac{T'}{T} + \frac{T''}{a_{AH}\,T} \simeq \frac{1}{x} \rightarrow \infty$$

at the would-be horizon $\rightarrow$ SPACETIME SINGULARITY

Look at a **test scalar field** ϕ satisfying $\Box\phi = 0$. The s-modes $\phi(t, r)$ obey

$$\ddot{\phi} + \left(\frac{3\dot{a}}{a} + \frac{\dot{T}}{T}\right)\dot{\phi} - \frac{T^2}{a^2}\,\phi'' - \frac{T}{a^2}\left(T' + \frac{T\,a'}{a} + \frac{2T}{r}\right)\phi' = 0$$

at the static EH. $\dot{a}$, $\dot{T}$, $T \rightarrow 0$ and $\phi \sim At + \phi_0 \rightarrow \infty$ at late times. The energy-momentum tensor of s-waves has trace

$$T^c{}_c = \left(\frac{\dot{\phi}}{T}\right)^2 \rightarrow +\infty \quad \text{as } x \rightarrow 0$$

BH thermodynamics has problems with a static EH in a FLRW universe

In the presence of a test quantum field, a Schwarzschild BH emits Hawking radiation (blackbody) at temperature

$$T_H = \frac{\hbar c^3}{8\pi G k_B m}$$

The EH must be in equilibrium with this radiation. The Sultana-Dyer solution of GR is obtained by conformally transforming the Schwarzschild BH. Dimensional arguments (VF PRD 2007) and the renormalization of the scalar field stress-energy tensor (Saida + CQG 2007) suggest that this evolving BH scales has evolving temperature

$$T = \frac{\hbar c^3}{8\pi G k_B m\, a},$$

incompatible with constant T_H.

CONCLUSIONS

- **Black hole horizons are cosmologically coupled, somehow. We still don't know *how* they evolve.**
- **Since they evolve, do we choose EHs or AHs?**
- **We did not use the Einstein equations** → conclusion holds in all theories of gravity in which vacuum BHs are Schwarzschild.
- **How does the mass evolve with a? Is it $m \sim a^3$?**
- Will result still hold for non-singular (quantum) BHs?
- Will tentative observational evidence for cosmological coupling disappear or be strengthened?

Stay tuned ...

THANK YOU