Ultrastrong coupling, nonselective measurement and quantum Zeno dynamics

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⟨Quantum

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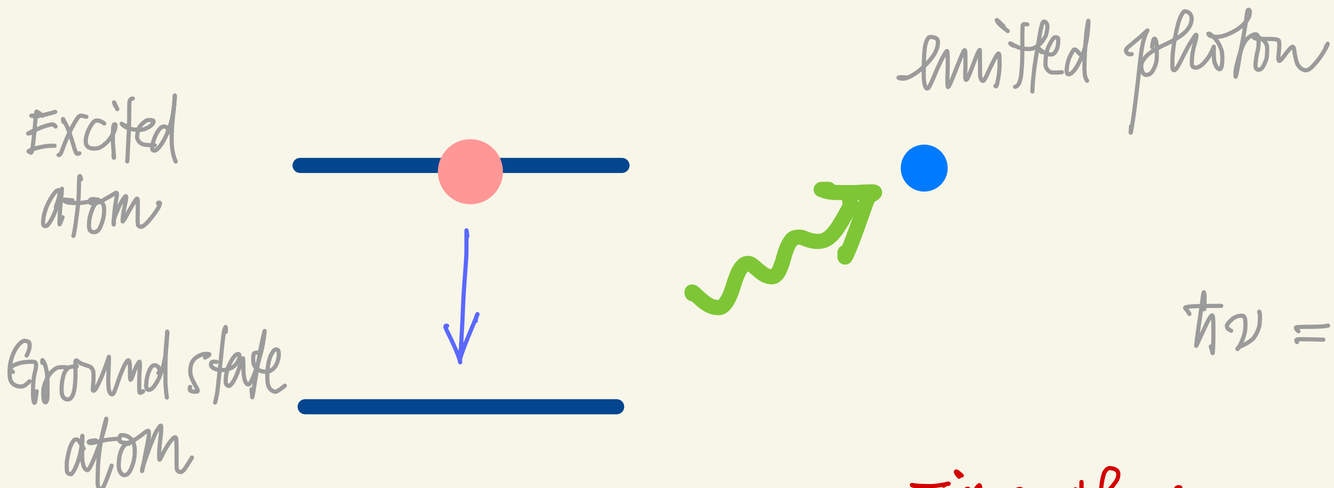
Quantum Zeno effect

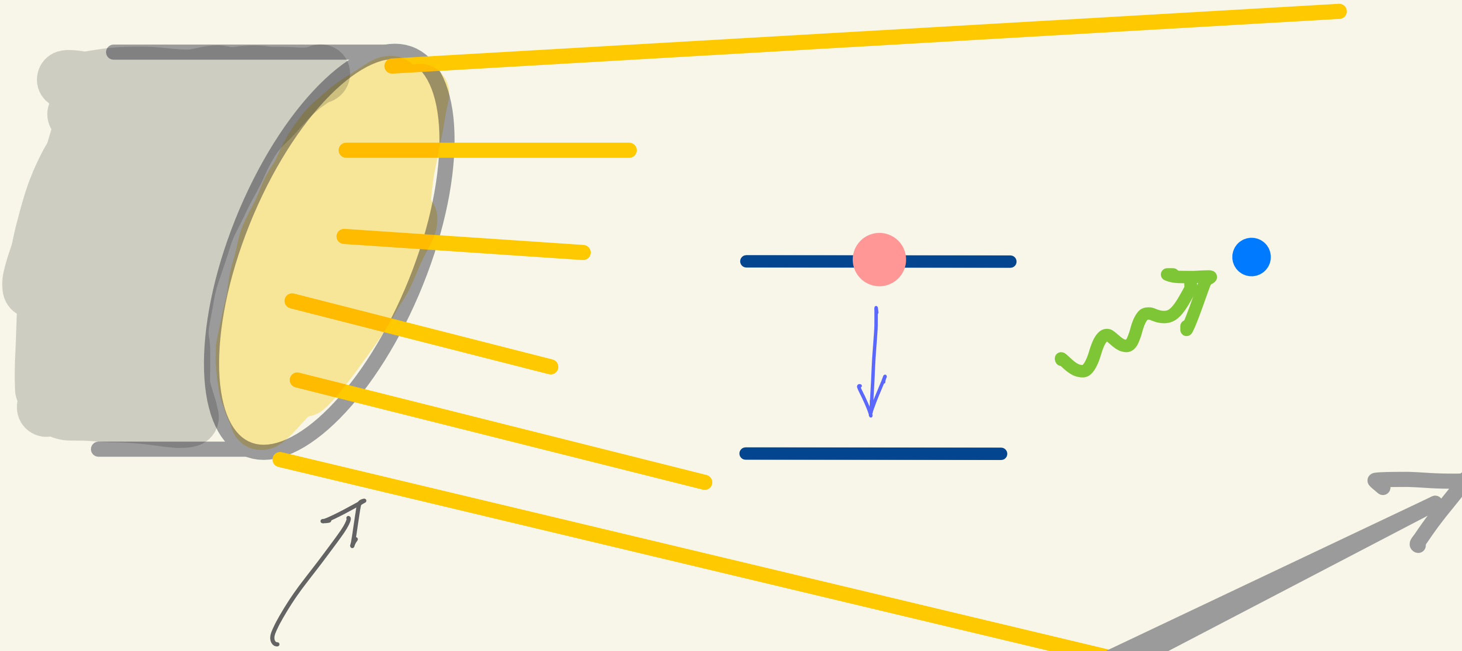
If a quantum system is measured frequently then its evolution is slowed down.

Physical reality : atomic de-excitation



$\hbar\nu = \Delta E$

Time of process : τ_0



Time of process:

$$\boxed{\tau_Z \gg \tau_0}$$

Shine light on affair

= observe = measure

Experimentally verified!

Mathematical description

Quantum theory:

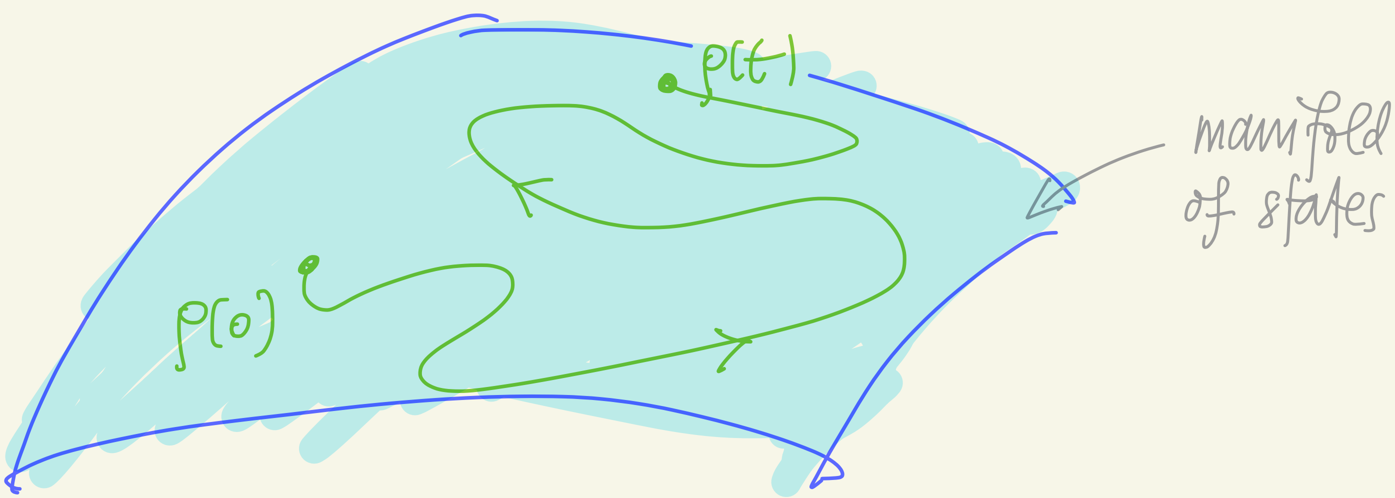
① states of a system are given by a density matrix ρ on a Hilbert space $\mathcal{H}$.

ρ operator on $\mathcal{H}$, positive ($\rho \geq 0$), normalized ($\mathrm{tr}\, \rho = 1$)

② **Dynamics** of isolated system: given by **Schrödinger equation**

$$\rho(t) = \mathcal{U}_t\, \rho(0) = e^{-itH}\, \rho(0)\, e^{itH}$$

H : Hamiltonian



③ Physical quantities $\leftrightarrow$ hermitian operators M on $\mathcal{H}$

"observables"

$$M = \sum_j m_j P_j$$ spectral decomposition

eigenvalues eigenprojections

Measurement outcome is m_j with probability $p_j = \operatorname{tr}(P_j \rho)$

If m_j is measured $\Rightarrow$ state collapses to new density matrix

$$\frac{P_j \rho P_j}{p_j}$$ $\Rightarrow$ Non-selective measurement process

$$\rho \mapsto \mathcal{J}\rho = \sum_j P_j \rho P_j$$

Process of evolution interspersed with measurements

$$\rho \longmapsto \rho(t,N) = \left(\mathcal{P}\, \mathcal{U}_{t/N}\right)^N \rho$$

[Misra-Sudarshan '76 , Facchi-Pascazio '00,'08, many more]

$$\lim_{N\to\infty} \rho(t,N) = e^{-itH_Z} \left(\sum_j P_j \rho P_j \right) e^{itH_Z}$$

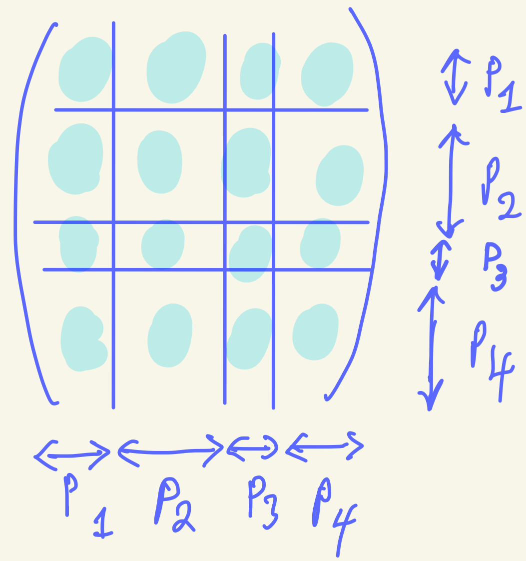
where $$H_Z = \sum_j P_j H P_j$$

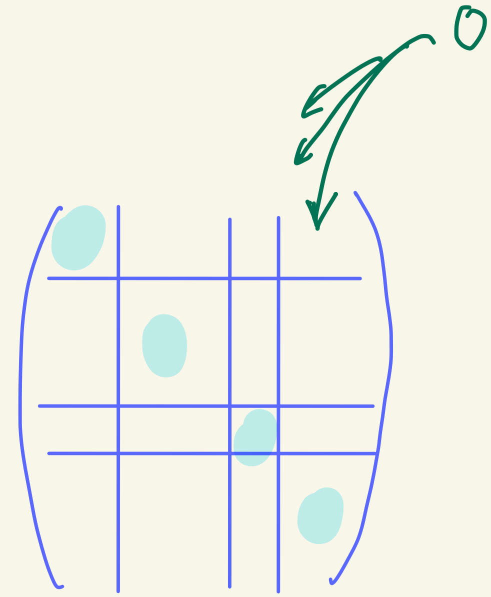
In the limit of infinitely frequent measurements the dynamics is stopped

within so-called Zeno subspaces Ran P_n

Block picture:

$$\mathcal{H} = \bigoplus_j \mathrm{Ran}\, P_j$$

$H =$ 

$\longmapsto$ $H_2 =$ 

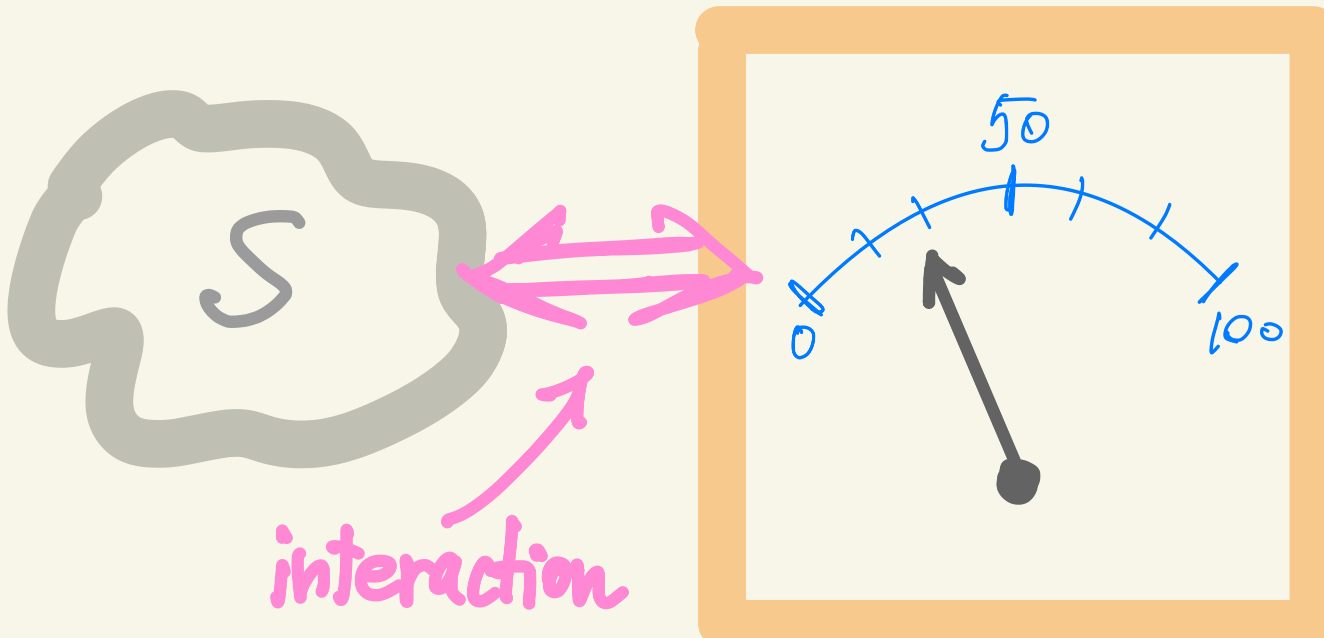
Off-diagonal blocks erased $(=0)$ by taking $\sum_j P_j \circ P_j$

Happens for state ρ & dynamics H.

DECOUPLED subspaces. Dim $= 1 \Rightarrow$ <u>no</u> dynamics!

But how do you actually measure a system?

→ You must bring it in contact with a <u>measurement apparatus</u>.



Open Quantum system

One would then think:

Strong interaction ↔ Zeno effect

We show that this is correct!

The model

"Spin-Boson" type.

$$\mathcal{H} = \mathcal{H}_S \otimes \mathcal{H}_R$$

$\mathbb{C}^N \otimes$ Bosonic Fock space over $L^2(\mathbb{R}^3, d^3k)$

$$H = H_S + H_R + \lambda G \otimes \varphi(g)$$

$$G = \sum_n \gamma_n P_n$$

eigenvalues

eigenprojections

$$H_R = \int_{\mathbb{R}^3} \omega(k)\, a^*(k) a(k)\, d^3k$$

$$\varphi(g) = \int_{\mathbb{R}^3} \left(g(k)\, a^*(k) + h.c. \right) d^3k$$

Dynamics of system part: marginal of full SR state:

$$\rho_S(t) = \mathrm{Tr}_R \left(e^{-itH} \rho_S(0) \otimes \underline{\rho_R(0)} \, e^{itH} \right)$$

↖ assumed: Gaussian state.

$\rho_S(0)$: arbitrary system state

$\rho_R(0)$: Gaussian reservoir state: $\langle W(f) \rangle_{\rho_R(0)} = e^{-\langle f, Cf \rangle}$

Conditions: $g(k)$ and $\omega(k)$ continuous in a neighborhood of some $k_0 \in \mathbb{R}^3$

$g(k_0) \neq 0$ & $\omega(k)$ not constant around k_0

Our theorem: $\forall t > 0$,

$$\boxed{\lim_{|\lambda| \to \infty} \rho_S(t) = e^{-itH_Z} \left(\sum_n P_n \, \rho_S(0) \, P_n \right) e^{itH_Z}}$$

$$\boxed{H_Z = \sum_n P_n H_S P_n}$$

- ultrastrong coupling with R induces Zeno dynamics
  — measurement of coupling operator G
- The system dynamics does not depend on details of reservoir !!!

Main tool in proof: Dyson series

$$H = H_S + \underbrace{H_R + \lambda\, G \otimes \varphi}_{=K} = H_S + K$$

① Compare dynamics generated by H and by K:

$$e^{-itH} = \left[\mathbb{1} + \sum_{n \geq 1} i^n \int_{0 \leq t_n \leq \dots \leq t_1 \leq t} H_S(t_n) \cdots H_S(t_1) \right] e^{-itK}$$

where $H_S(t) = e^{-itK} H_S \, e^{itK}$.

Then $\quad \rho_S(t) = \mathrm{tr}_R \; e^{-itH} \left(\rho_S(0) \otimes \rho_R(0) \right) e^{itH}$

$$= \mathrm{tr}_R \left[\mathbb{1} + \sum_{n \geq 1} i^n \int \cdots \right] e^{-itK} \left(\rho_S(0) \otimes \rho_R(0) \right) e^{itK} \left[\mathbb{1} + \sum_{n \geq 1} (-i)^n \int \cdots^* \right]$$

Only terms $e^{itK} \times e^{-itK}$ are present

2. Dynamics generated by e^{itK} is "easy" for $|\lambda| \to \infty$.

3. Take $|\lambda| \to \infty$ in each term of series

4. Resume the series to get final result.

Core point of ②:

For X: operator on system

$$e^{itK} X e^{-itK}$$

$$= \sum_{m,n} \underbrace{P_m X P_n}_{S} \otimes \underbrace{e^{it(H_R + \lambda \gamma_m \varphi)} e^{-it(H_R + \lambda \gamma_n \varphi)}}_{R}$$

$$\boxed{K = H_R + \lambda G \otimes \varphi, \quad G = \sum_n \gamma_n P_n}$$

core point of 3 : Take marginal over reservoir :

$$\mathrm{tr}_R\left(\rho_R^{(0)}\, e^{it(H_R + a\gamma_m \varphi)} e^{-it(H_R + a\gamma_n \varphi)} \right) \sim e^{-|a^2| |\gamma_m - \gamma_n|^2 f(t)}$$

some nonzero function of t

$$\xrightarrow{|a|\to\infty} \begin{cases} 0 & \gamma_m \neq \gamma_n \\ 1 & \gamma_m = \gamma_n \end{cases}$$

This selects terms $m=n$ only $(|a| \to \infty)$

$$\Rightarrow \lim_{|a|\to\infty} \mathrm{tr}_R\left(\rho_R^{(0)}\, e^{itK} X e^{-itK} \right) = \sum_n P_n X P_n$$

Using this property in every term of (double) series

$$\lim_{|\lambda|\to\infty} \rho_S(t) = \lim_{|\lambda|\to\infty} \mathrm{tr}_R \left[\mathbb{1} + \sum_{n\geq 1} i^n \int \dots\right] e^{-itK} \left(\rho_S(0)\otimes\rho_R(0)\right) e^{itK} \left[\mathbb{1} + \sum_{n\geq 1} (-i)^n \int \dots {}^*\right]$$

gives a new series which can be resummed to yield

$$\lim_{|\lambda|\to\infty} \rho_S(t) = e^{-itH_z} \left(\sum_n P_n \, \rho_S(0) \, P_n\right) e^{itH_z}$$

Upshot

Quantum Zeno dynamics ≡ Dynamics of system interspersed with frequent measurements $\mathcal{P}$. Limit of infinitely frequent measurements yields

$$\rho(t) = e^{-itH_Z}\left(\mathcal{P}\rho(0)\right)e^{itH_Z}$$

This formalism
- does not model actual measurement apparatus
- assumes measurement process is instantaneous (apply $\mathcal{P}$ by hand!)

⇒ We introduce open system model where system couples strongly to environment. In ultrastrong coupling limit ($|\lambda| \to \infty$) system dynamics is shown indeed to equal

EFFECT IS LARGELY INDEPENDENT OF DETAILS OF ENVIRONMENT.