Droplet creation in the walking droplet pilot wave system and in Bose-Einstein condensates

Walking droplets, vortex molecules, and Faraday instability

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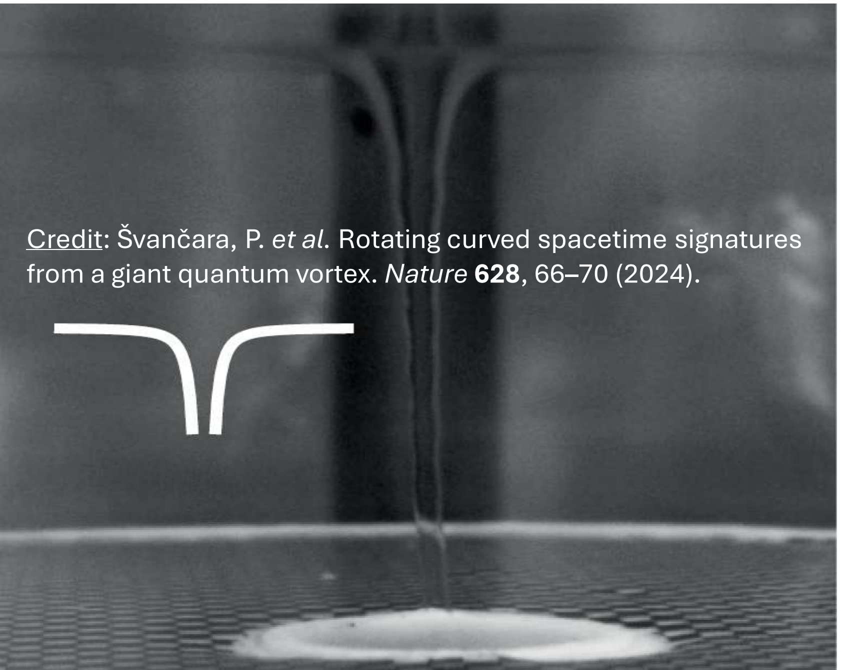
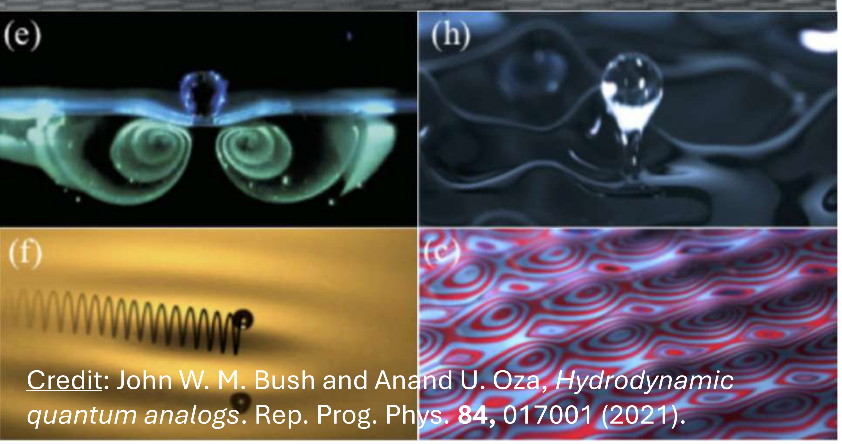
University of Saskatchewan



Credit: Švančara, P. *et al.* Rotating curved spacetime signatures from a giant quantum vortex. *Nature* **628**, 66–70 (2024).

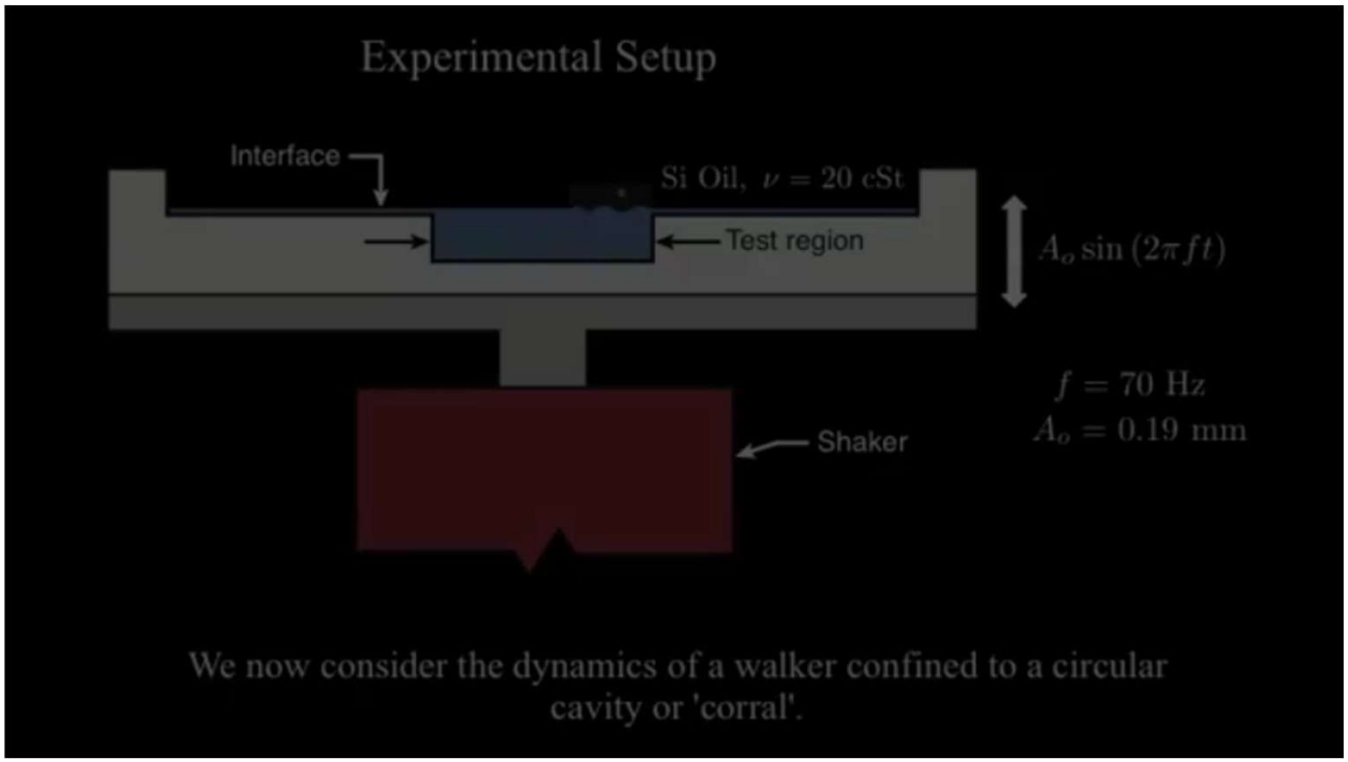


Credit: John W. M. Bush and Anand U. Oza, *Hydrodynamic quantum analogs*. Rep. Prog. Phys. **84**, 017001 (2021).

Analog models

- **Gravitational**: sonic black hole, Hawking radiation, superradiance, Penrose effect, ringdown, curved spacetime
- **Cosmological**: redshift, Hubble friction, false vacuum decay, cosmic strings
- **High energy**: electroweak & QCD theory, neutron stars
- **Quantum foundations (Hydrodynamic Quantum Analogs)**: diffraction through slits, tunneling, entanglement, multi-modal statistics, Landau levels and Zeeman splitting, spin states, spin-spin correlations, ...

Walking droplet system

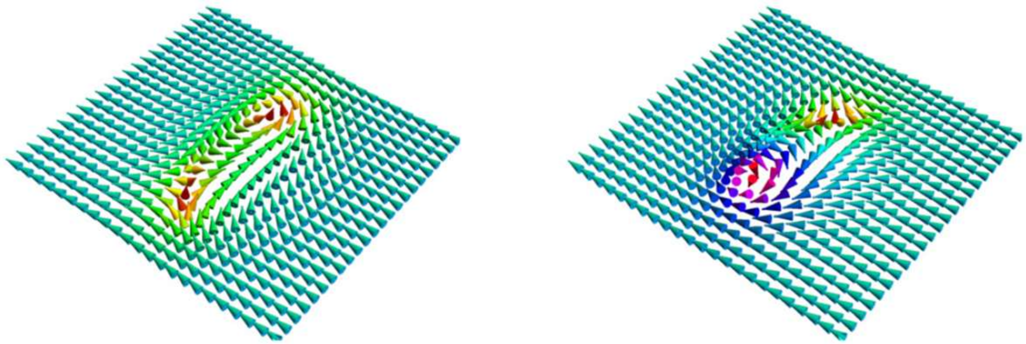


Quantum behaviours: single- and double-slit diffraction, tunneling, entanglement, quantized orbits, orbital-level splitting, multi-modal statistics, spin states, spin-spin correlations, ...

The quantum-like behaviours appear most strongly in the ***"high-memory"*** *regime, near the Faraday threshold. Walking droplet dynamics are* ***non-local (non-Markovian)!***

Rabi-coupled condensates

- BEC mixture of two different *hyperfine spin states* of same atom
- can introduce Rabi coupling, driven by external oscillatory EM field, at *transition frequency* ω_{rf} between the two states
- E.g. ^{87}Rb $|F,m\rangle = |1,1\rangle,\ |1,-1\rangle$
- **Vortices become bound into pairs**, with behaviour analogous to quark color confinement



(a) mesonic molecule $\bar{u}u$ (b) baryonic molecule ud

Pseudospin plots. Credit: Eto, Ikeno, and Nitta. *Physical Review Research* **2**, 033373 (2020).

Connection

Walking droplet system

- **exhibits quantum-like behaviours (of electrons / photons)**
- vertically *driven*
- droplet 'walks' in resonance with its guiding wave field
- *Faraday instability*

Rabi-coupled condensates

- **simulates quark confinement**
- *driven* by oscillatory EM field, in resonance with energy difference btwn the two states
- *Faraday instability*

Numerical Methods

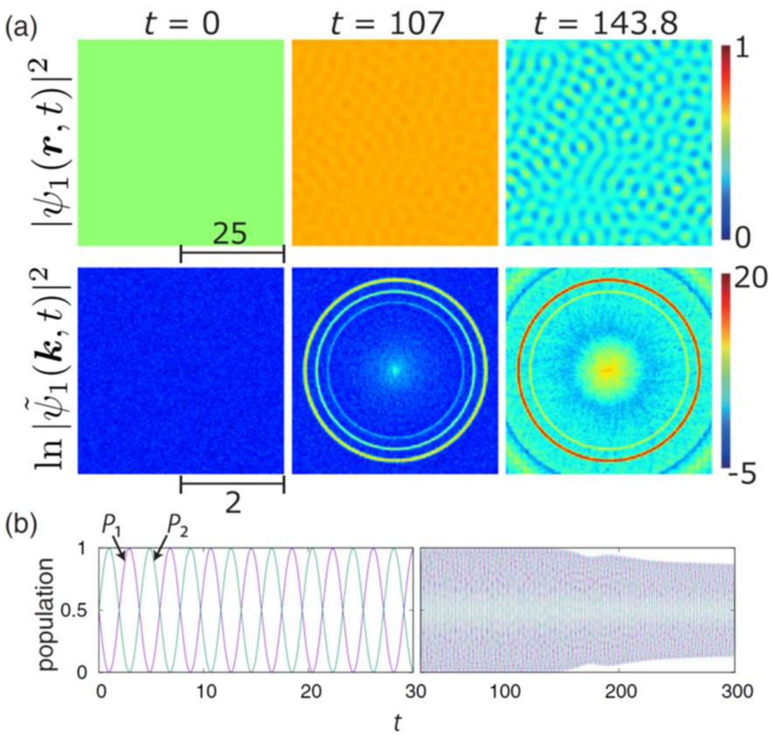
GP eq'ns:

$$i\hbar\frac{\partial}{\partial t}\Psi_1 = \left[-\frac{\hbar^2}{2m}\nabla^2 + V_1(\boldsymbol{x}) + \left(g_1|\Psi_1|^2 + g_{12}|\Psi_2|^2\right)\right]\Psi_1 - \hbar\omega\Psi_2$$

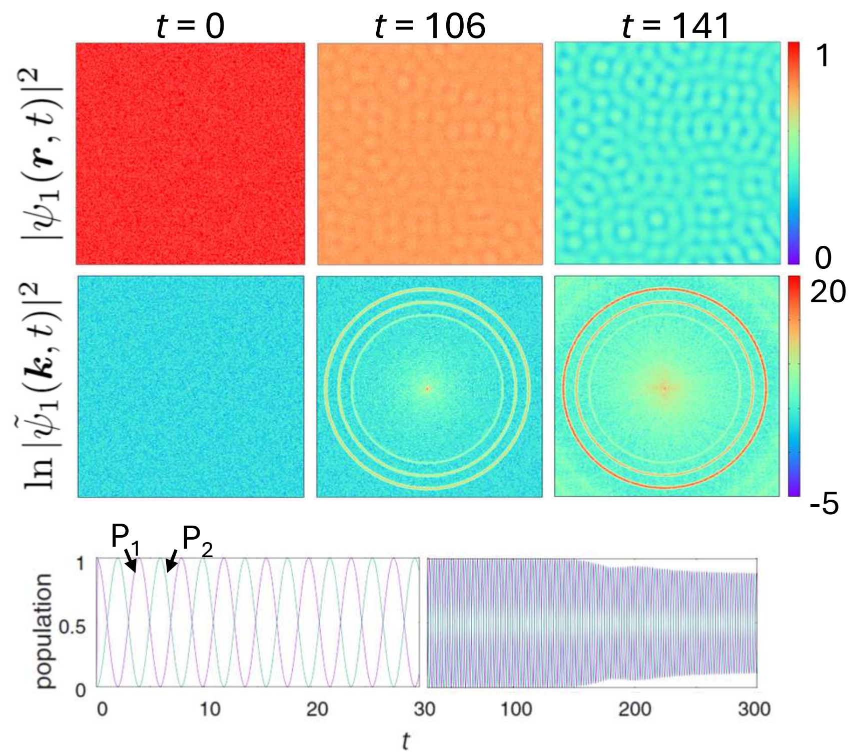
$$i\hbar\frac{\partial}{\partial t}\Psi_2 = \left[-\frac{\hbar^2}{2m}\nabla^2 + V_2(\boldsymbol{x}) + \left(g_2|\Psi_2|^2 + g_{12}|\Psi_1|^2\right)\right]\Psi_2 - \hbar\omega\Psi_1$$

- Two components, Ψ_1 and Ψ_2 ("Condensate wave functions")
- Inter- and intra-component couplings g_{12}, g_1, g_2 (from *mean field* scattering amplitudes)
- Rabi frequency ω (population transfer)
- Study real-time dynamics using a *time-splitting, pseudospectral method*

Faraday waves from Rabi coupling

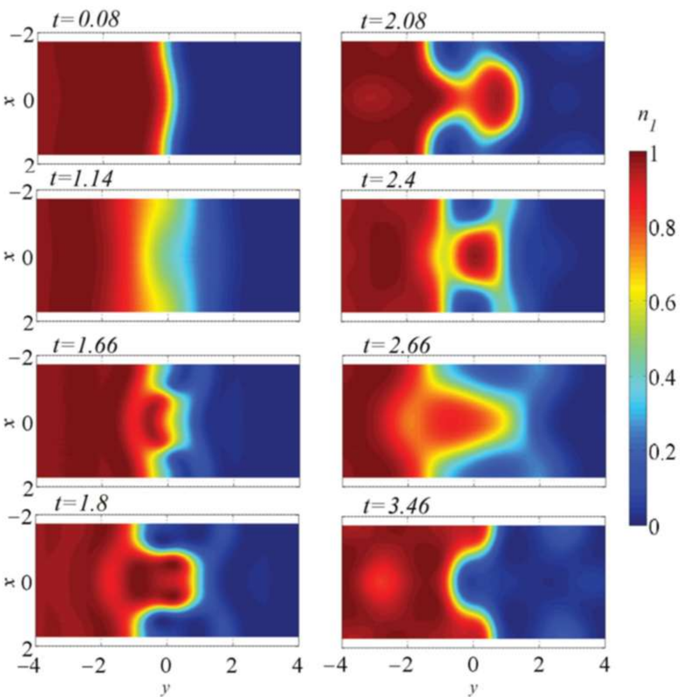


Credit: T. Chen et al, *Phys. Rev. A* **100**, 063610 (2019)



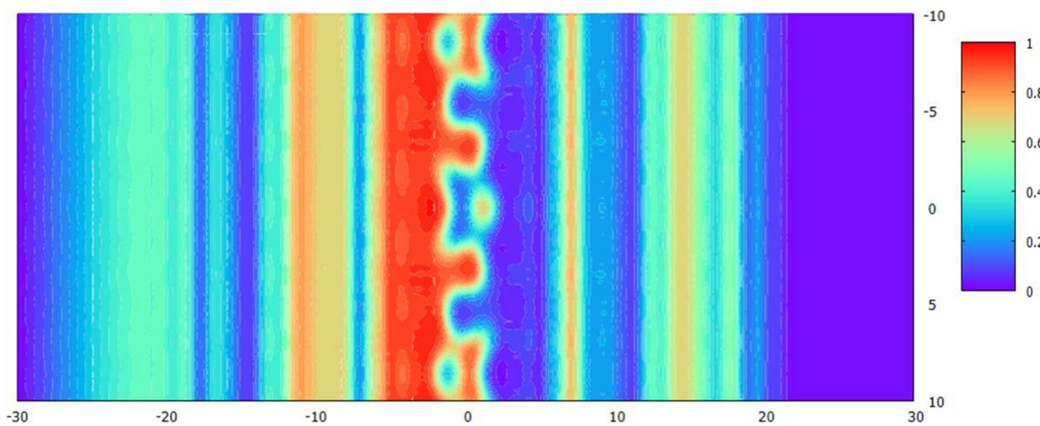
My results

Parametric resonance and droplet creation



Interface dynamics under an oscillatory driving force

Credit: D. Kobyakov et al, *Phys. Rev. A* **86**, 023614 (2012)



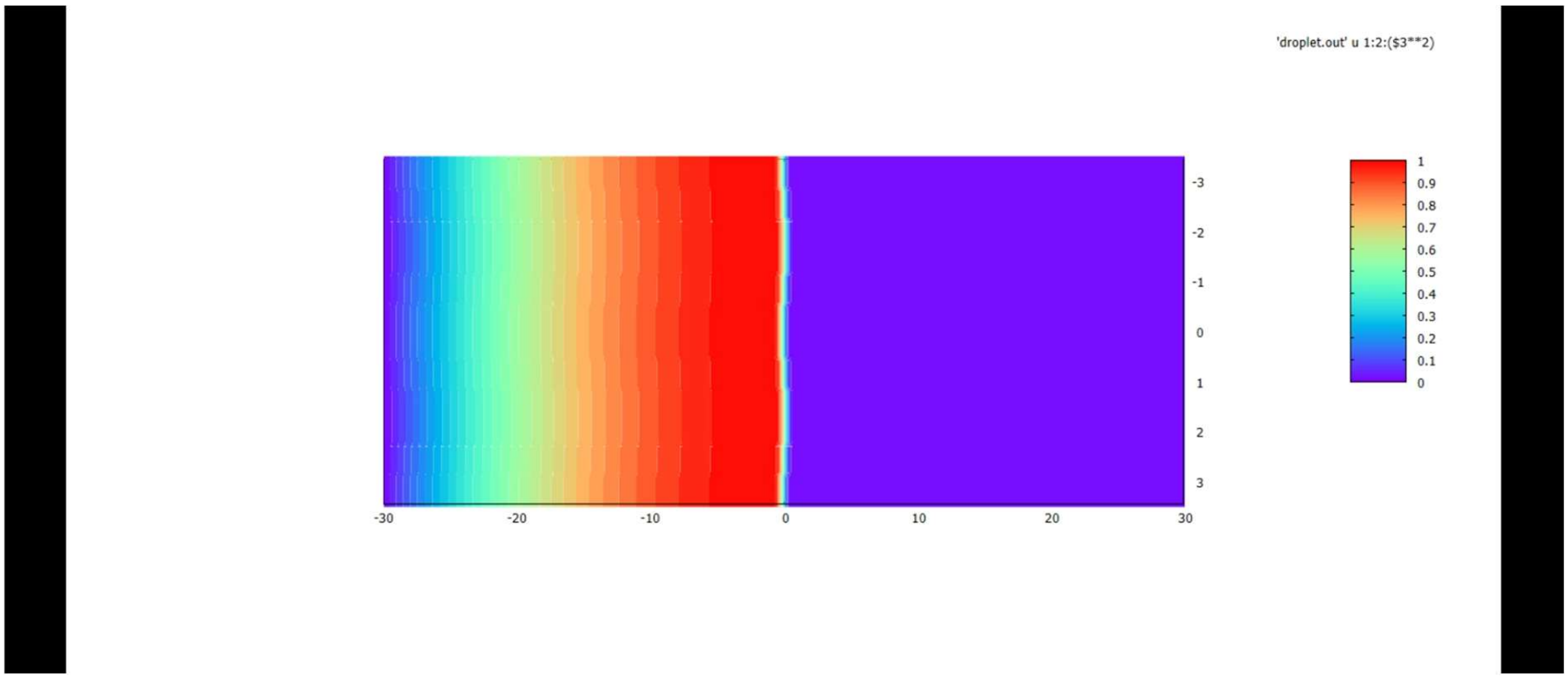
Interface dynamics with Rabi coupling
My results

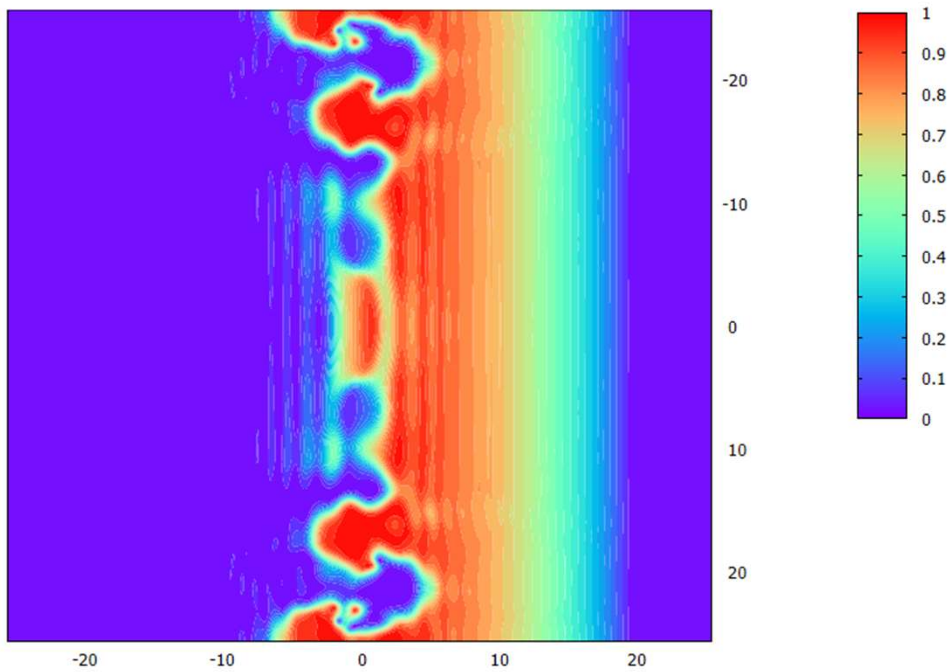
Summary

- **Walking droplets** exhibit many *quantum-like behaviours* (electrons / photons)
  - *Faraday instability and "memory" is crucial for these behaviours*
- Condensates with Rabi coupling host **"vortex molecules"**, an analog for *quark confinement*
- *Both systems intrinsic Faraday instability and droplet creation*

Thank you!

gamma = 0.01, omega = 0.8

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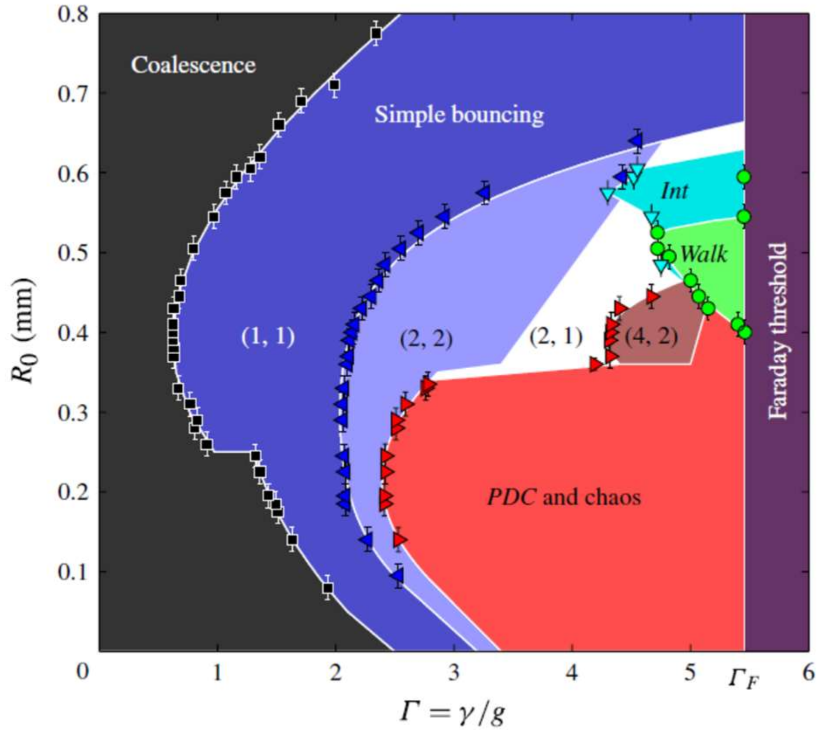
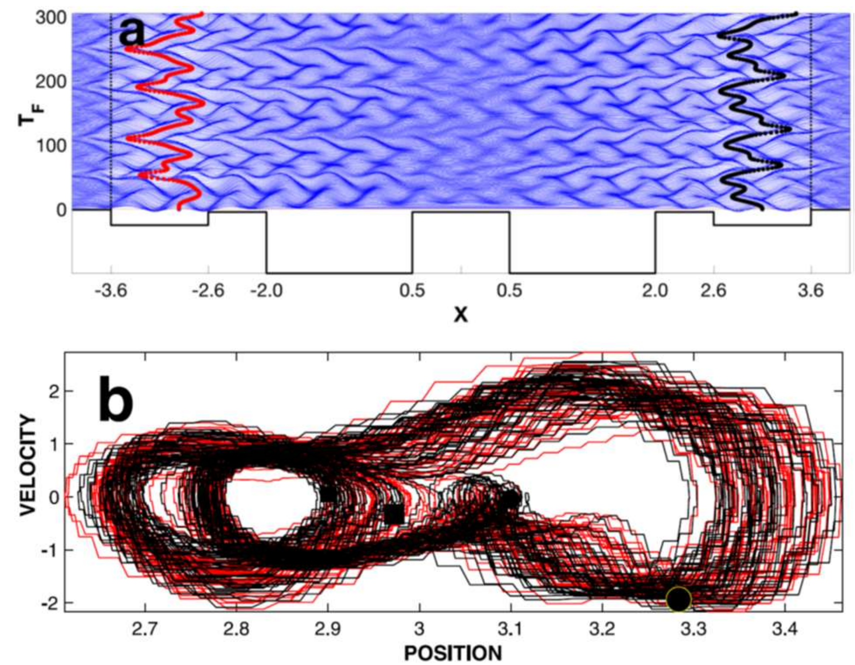
Walking droplet system



Different bouncing / walking modes (50 cSt).
Period-doubling cascade and chaos.

Credit: Jan Moláček and John W. M. Bush, *J. Fluid Mech.*, 2013

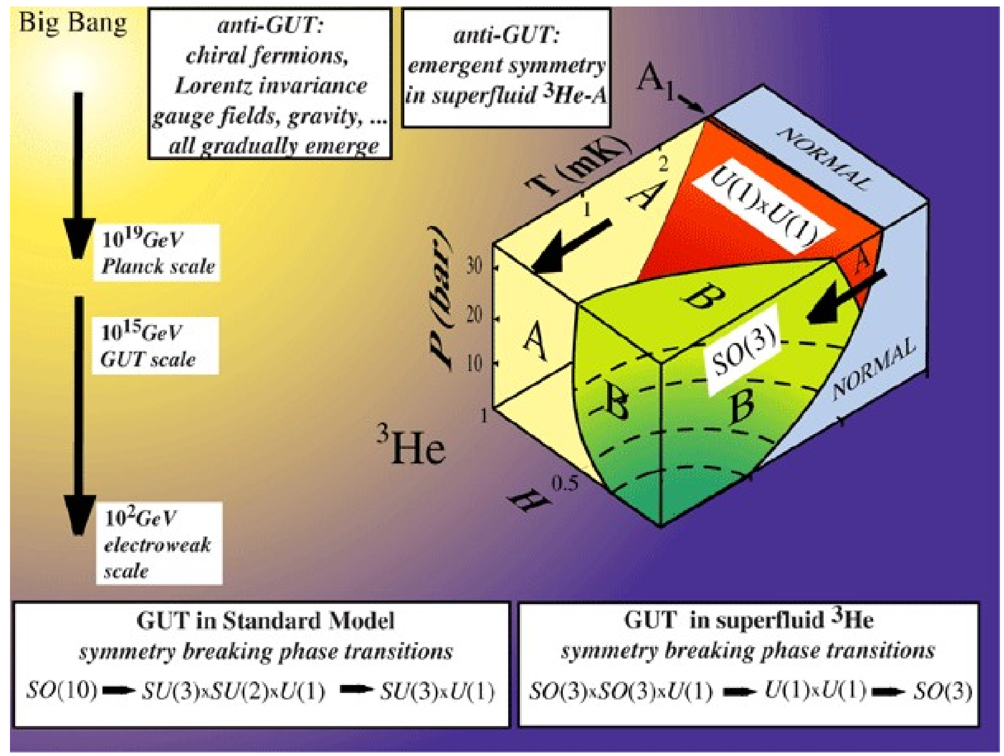


Phase-space locking (entanglement).

Credit: André Nachbin, *Chaos*, 2018

Superfluid ^{3}He-A and the Standard Model

- Grand Unification Theory (GUT): symmetries emerge as energy increases (particle physics)
- Anti-GUT: symmetries emerge as energy decreases
- In superfluid ^{3}He-A, as temperature reduced further, it acquires most of symmetries of the **Standard Model**
  - *Lorentz invariance, local gauge invariance, elements of general covariance, chiral fermions*



Credit: Volovik, *The Universe in a Helium Droplet*, 2003.

u- and d-vortices

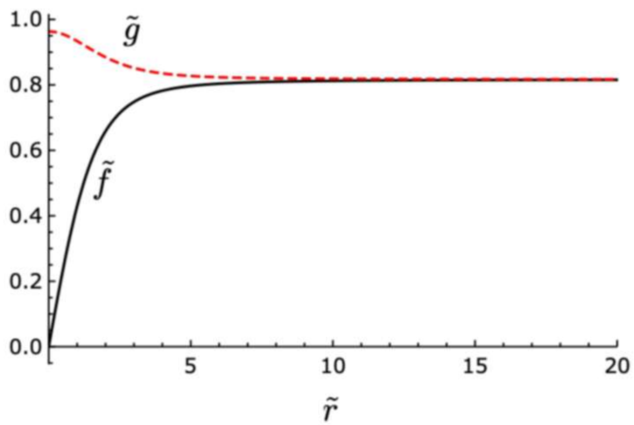
- When Rabi coupling $\omega = 0$, have separate u- and d-vortices (by analogy with up and down quarks)

u-vortex:

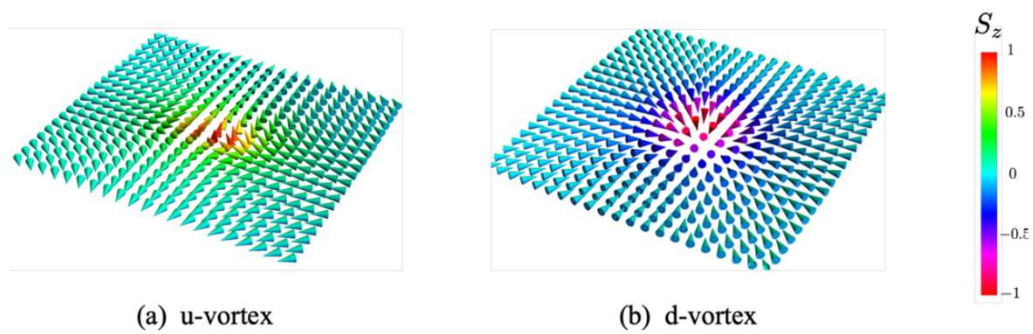
$$\tilde{\Psi}_1 = \tilde{f}(\tilde{r})e^{i\theta}, \qquad \tilde{\Psi}_2 = \tilde{g}(\tilde{r})$$

d-vortex:

$$\tilde{\Psi}_1 = \tilde{g}(\tilde{r}), \qquad \tilde{\Psi}_2 = \tilde{f}(\tilde{r})e^{i\theta}$$



Vortex profile functions.

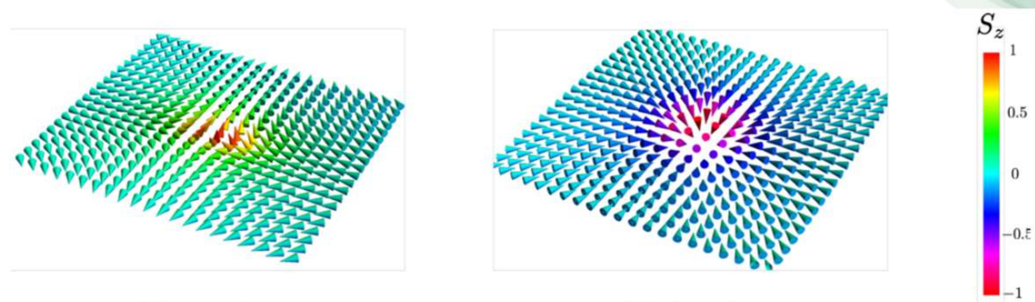


Pseudospin plots.

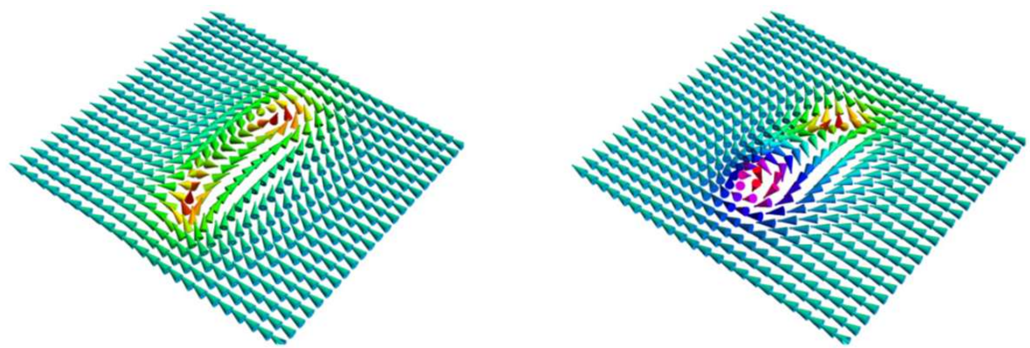
Credit: Eto, Ikeno, and Nitta. *Physical Review Research* **2**, 033373 (2020).

Pseudospin

$$\boldsymbol{S} = -\frac{\vec{\Psi}^\dagger \boldsymbol{\sigma} \vec{\Psi}}{\vec{\Psi}^\dagger \vec{\Psi}}, \qquad \vec{\Psi} = (\Psi_1, \Psi_2)^T$$



(a) u-vortex (b) d-vortex



(a) mesonic molecule $\bar{u}u$ (b) baryonic molecule ud

Symmetry breaking

GP eq'ns:

$$\left[i\hbar\frac{\partial}{\partial t}+\frac{\hbar^2}{2m}\nabla^2-\left(g_i|\Psi_i|^2+g_{12}|\Psi_{\hat{i}}|^2-\mu_i\right)\right]\Psi_i=-\hbar\omega\Psi_{\hat{i}}, \qquad i=1,2$$

- GP equations have full ***SU(2)*** symmetry only when $g_1 = g_2 = g_{12}$ (and $\omega = 0$)
- When g_1 or $g_2 \neq g_{12}$, symmetry broken to ***U(1)×U(1)***
- When Rabi coupling $\omega \neq 0$, symmetry broken to ***U(1)***

Symmetry breaking

$$U(1)_1 \times U(1)_2 : (\Psi_1, \Psi_2) \to (e^{i\alpha_1}\Psi_1,\ e^{i\alpha_2}\Psi_2) \quad \cong \quad [U(1)_S \times U(1)_R]/\mathbb{Z}_2 : (\Psi_1, \Psi_2) \to (e^{i\alpha}\Psi_1,\ e^{\pm i\alpha}\Psi_2)$$

$$n_i = \frac{1}{2\pi}\oint_C d\theta_i, \qquad i = 1, 2 \quad \longleftrightarrow$$

$$n_S = \frac{1}{2\pi}\oint_C d\theta_S = \frac{1}{2\pi}\oint_C \frac{1}{2}(d\theta_1 + d\theta_2) = \frac{n_1 + n_2}{2}$$

$$n_R = \frac{1}{2\pi}\oint_C d\theta_R = \frac{1}{2\pi}\oint_C \frac{1}{2}(d\theta_1 - d\theta_2) = \frac{n_1 - n_2}{2}$$

- Both n_1 and n_2 (or n_S and n_R) are good *topological numbers* when $\omega = 0$

$U(1)_S \equiv U(1)_{mass}$
$U(1)_R \equiv U(1)_{spin}$

n_S is the total mass circulation or winding ⟶ baryon number!

n_R is the total spin winding ⟶ color charge!

Symmetry breaking

$$U(1)_1 \times U(1)_2 : (\Psi_1, \Psi_2) \to (e^{i\alpha_1}\Psi_1,\ e^{i\alpha_2}\Psi_2) \quad \cong \quad [U(1)_S \times U(1)_R]/\mathbb{Z}_2 : (\Psi_1, \Psi_2) \to (e^{i\alpha}\Psi_1,\ e^{\pm i\alpha}\Psi_2)$$

$$n_i = \frac{1}{2\pi}\oint_C d\theta_i, \qquad i = 1, 2$$

$$n_S = \frac{1}{2\pi}\oint_C d\theta_S = \frac{1}{2\pi}\oint_C \frac{1}{2}(d\theta_1 + d\theta_2) = \frac{n_1 + n_2}{2}$$

$$n_R = \frac{1}{2\pi}\oint_C d\theta_R = \frac{1}{2\pi}\oint_C \frac{1}{2}(d\theta_1 - d\theta_2) = \frac{n_1 - n_2}{2}$$

- When $\omega \neq 0$, only n_S is a topological number

$U(1)_S \equiv U(1)_{mass}$
$U(1)_R \equiv U(1)_{spin}$

n_S is the total mass circulation or winding → baryon number!

n_R is the total spin winding

Topological numbers

- When $\omega \neq 0$, n_R is not conserved, but instead is half the total **SG soliton number** (for curve *C)*
- Vortex molecules show color confinement!

	n_1	n_2	n_S (baryon #)	n_R (color charge)
u	1	0	1/2	1/2
d	0	1	1/2	−1/2
ū	−1	0	−1/2	−1/2
d̄	0	−1	−1/2	1/2
ūu	0	0	0	0
d̄d	0	0	0	0
ud	1	1	1	0
ūd̄	−1	−1	−1	0
ūd	−1	1	0	−1
d̄u	1	−1	0	1

don't exist in BECs

Low-energy effective theory

$$\mathcal{L}_{\rm dp} = \frac{g_c^2}{32\pi^2}\partial_\mu\vartheta\partial^\mu\vartheta + c g_c^2 m_{\rm dp}^2 \cos(\vartheta)$$

dual photon

$$\tilde{\mathcal{L}}_{\rm GP} = \frac{\hbar^2}{4(g+g_{12})}\dot{\varphi}_+^2 + \frac{\hbar^2}{4(g-g_{12})}\dot{\varphi}_-^2 - \frac{\hbar^2 n}{4m}(\nabla\varphi_+)^2 - \frac{\hbar^2 n}{4m}(\nabla\varphi_-)^2 + 2\hbar\omega n\cos\varphi_-$$

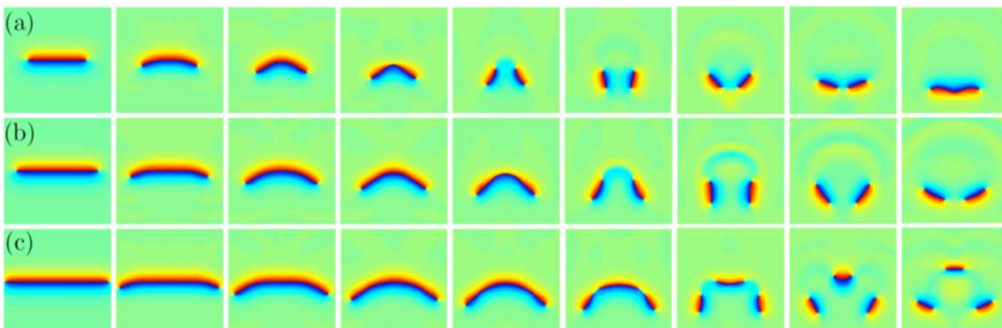
BEC ($\omega \neq 0$)

$$F_{\mu\nu} = \frac{e^2}{4\pi}\varepsilon_{\mu\nu\rho}\partial^\rho\vartheta, \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$\vartheta(z) = \sum_a q_a \mathrm{Im}\log(z - z_a)$$

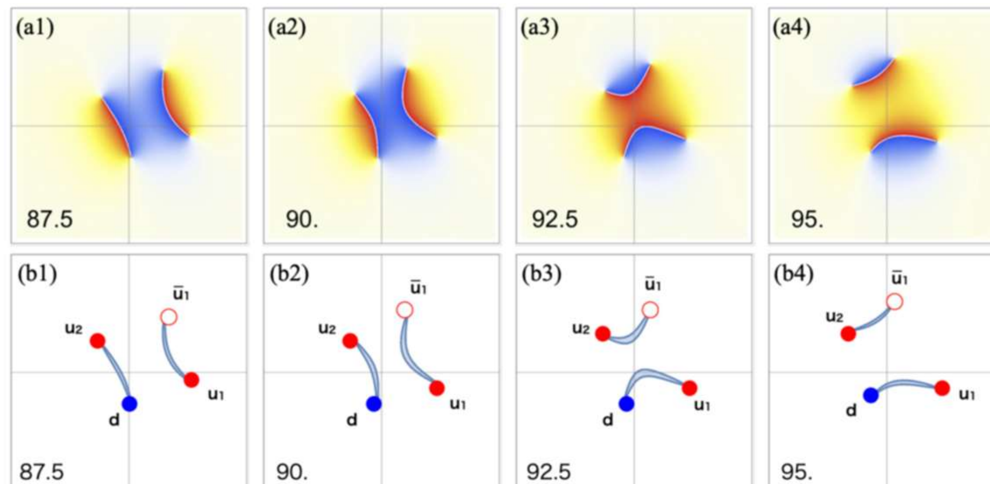
dual photon – electric charges q_a interchanged with vortices at points z_a

Vortex molecule dynamics



(a),(b) Meson splits into two baryons. (c) Longer meson splits into shorter meson and two baryons. Mesons travel in a straight line, and baryons rotate.

Credit: Minoru Eto and Muneto Nitta. *Physical Review A* **97**, 023613 (2018).



Collision between a baryon and a meson.

Credit: Eto, Ikeno, and Nitta. *Physical Review Research* **2**, 033373 (2020).

From scalar BEC...

$$\mathcal{E} = \int \left(\frac{\hbar^2}{2m} |\nabla \Phi|^2 + \frac{c_0}{2} |\Phi|^4 + 2c_0 \tilde{n} |\Phi|^2 - \mu_{\rm DE} |\Phi|^2 \right) d\mathbf{r}^2$$

...to superfluid universe

$$\mathcal{E} = \mathrm{GEM} + \int \left[\left(\frac{c_0}{2} \Psi_{\rm DM}^2 + 2c_0 \Psi_{\rm NM}^2 - \mu_{\rm DE} \right) \Psi_{\rm DM}^2 \right] d\mathbf{r}^2$$

$$\mathrm{GEM} = \int \frac{\hbar^2}{2m} |\nabla S(\mathbf{r})|^2 |\Phi(\mathbf{r})|^2 + \frac{\hbar^2}{2m} \left(\nabla |\Phi(\mathbf{r})| \right)^2 d\mathbf{r}^2$$