Studying nuclear matter under extreme conditions using supercomputing: on Quark Gluon Plasma properties from large-scale Bayesian analysis

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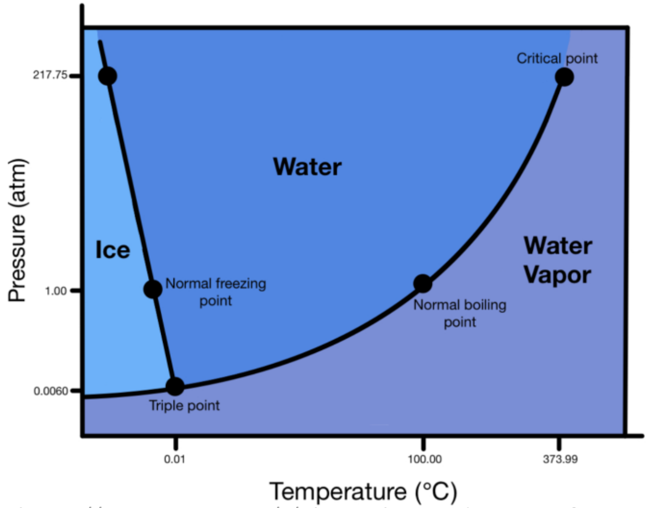
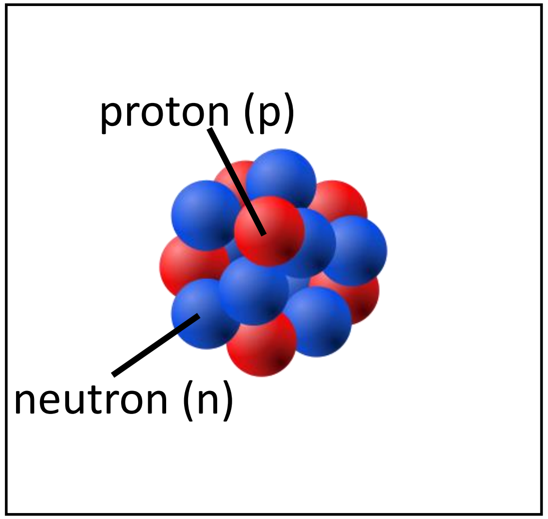
High energy nuclear collisions & nuclear equation of state

$T \uparrow$



Ref.: https://www.expii.com/t/phase-change-diagram-of-water-overview-importance-8031

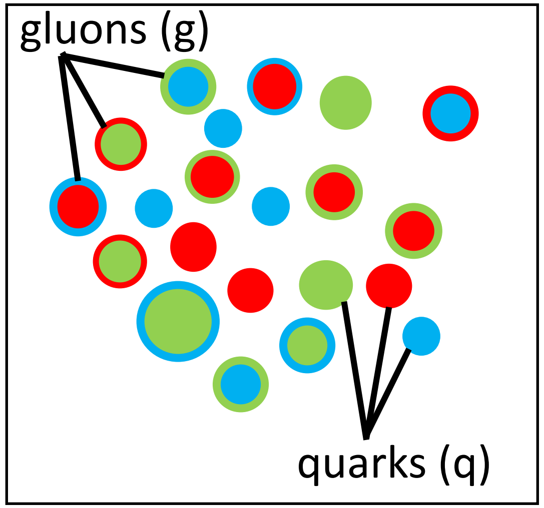
Ref.: https://en.wikipedia.org/wiki/Atomic_nucleus



$T \sim 10^2$ $Kelvin$ $(10^{-2}\ eV)$

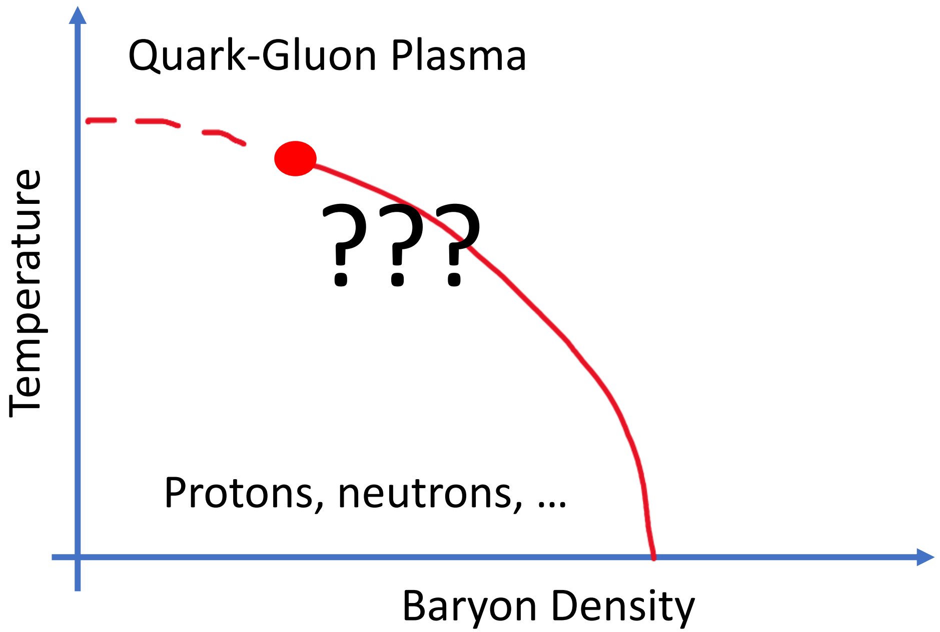
nucleus

$T \uparrow$

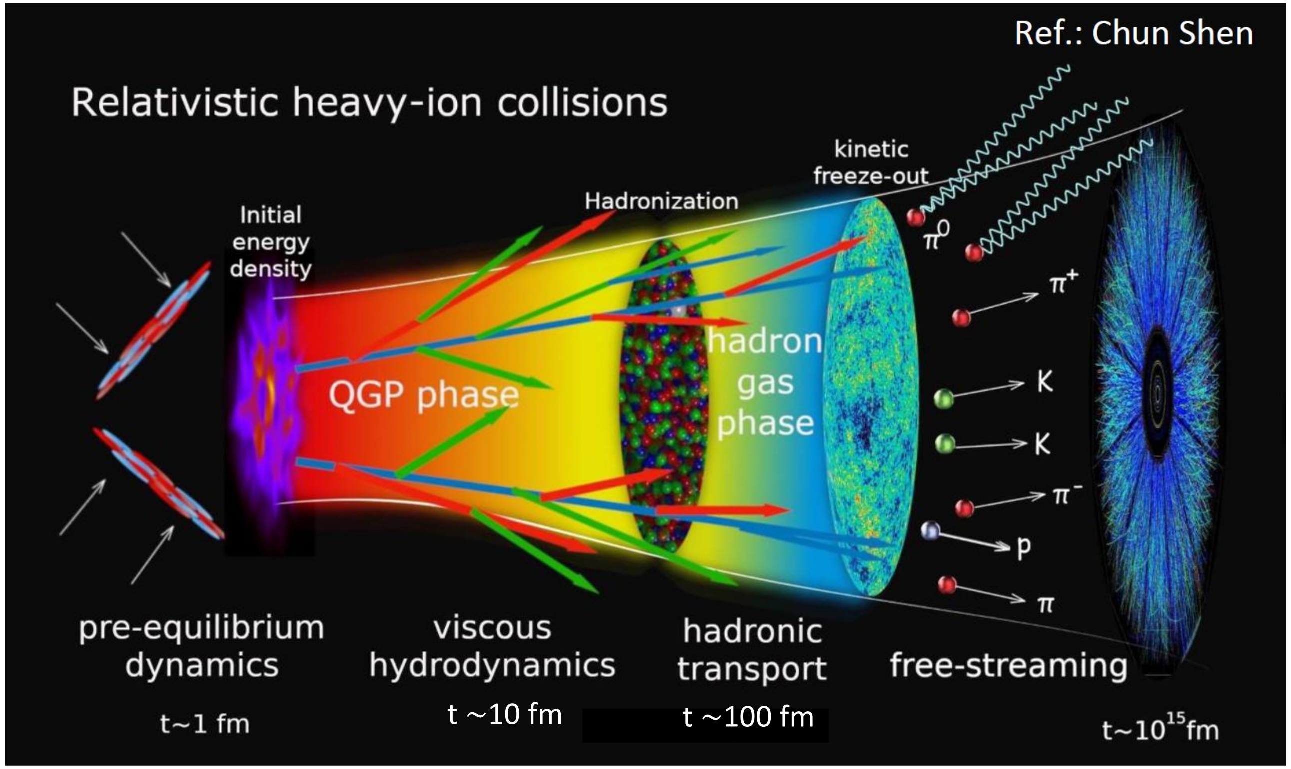


$T \sim 10^{12}$ $Kelvin$ $(\sim 10^8\ eV)$

Quark Gluons Plasma



Evolution of the nuclear medium as seen through jets



- 3 stages of heavy-ion collisions:
  - Pre-equilibrium stage: production of hard jets and onset of hydrodynamics (i.e. hydrodynamization)
  - Evolution of the nuclear fluid: viscous hydro
  - Last stage: Boltzmann transport/free-streaming
- Most of $T^{\mu\nu}$ is described by hydrodynamics and Boltzmann hadronic transport.

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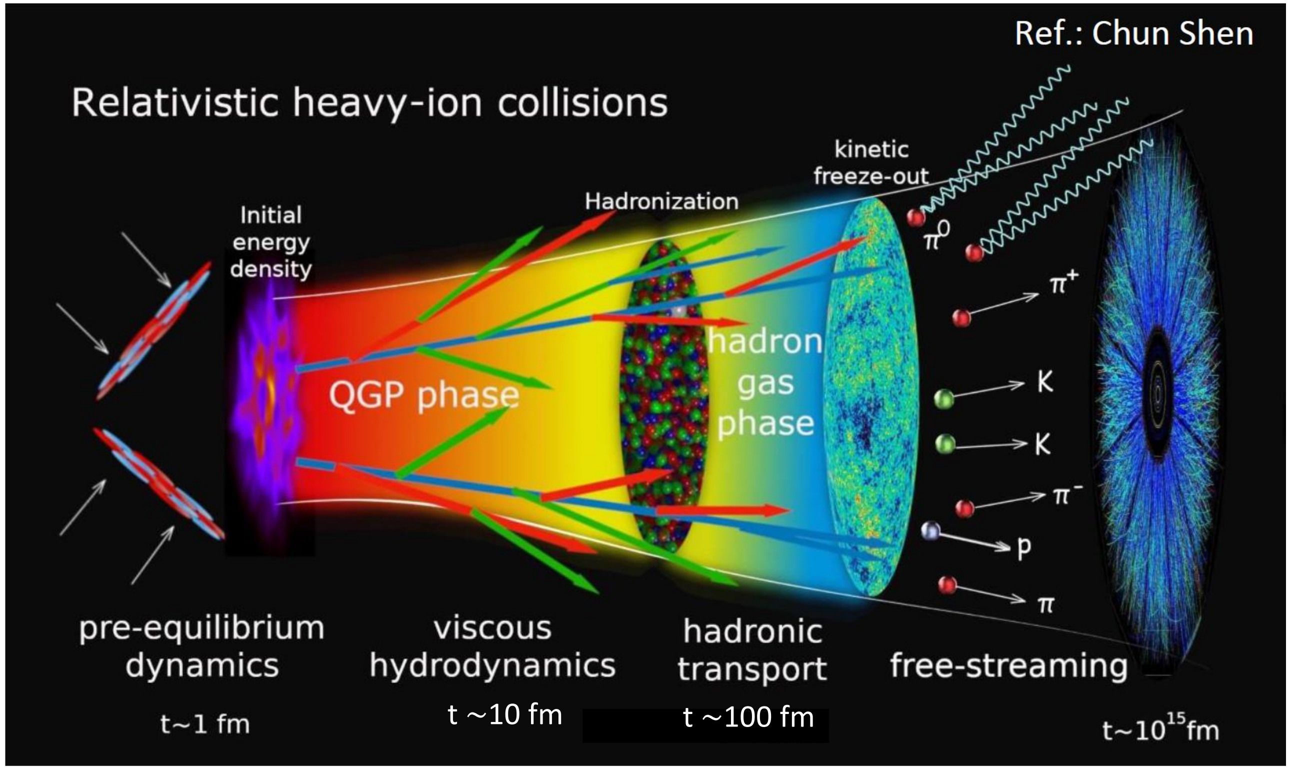

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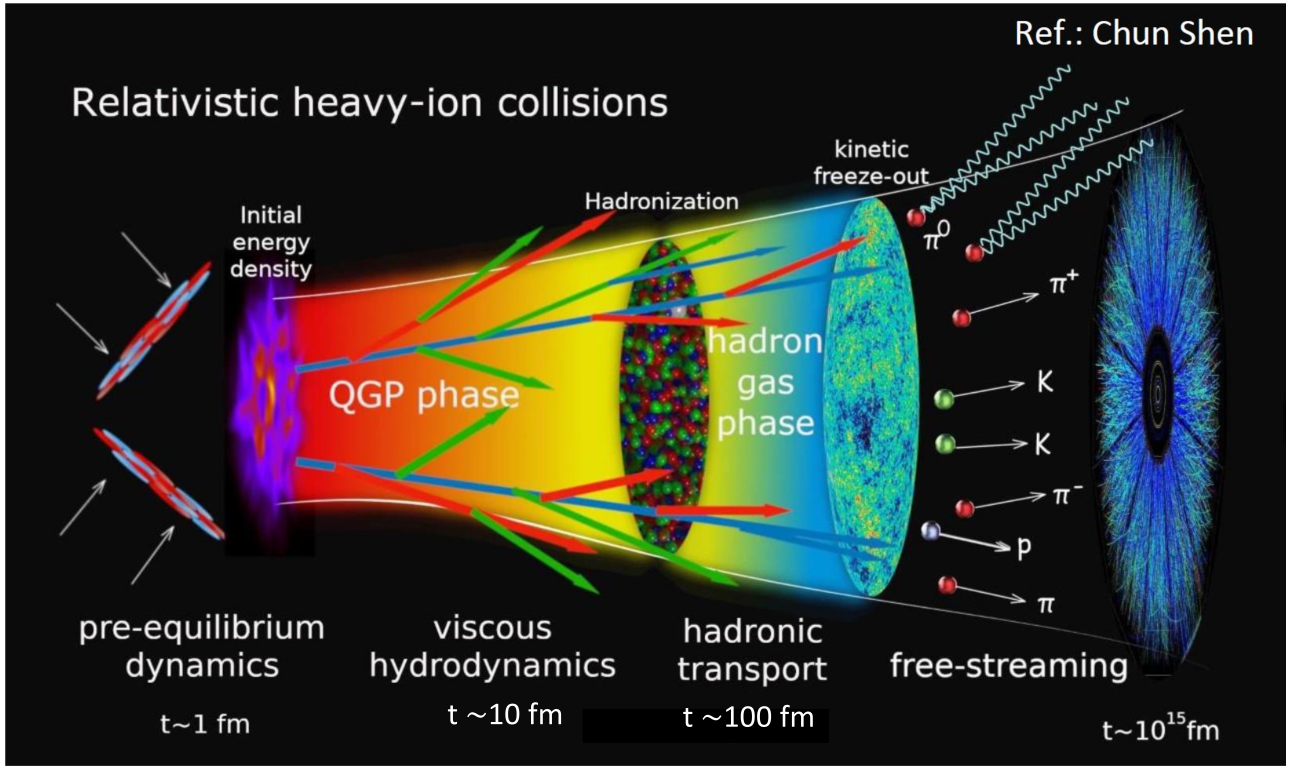

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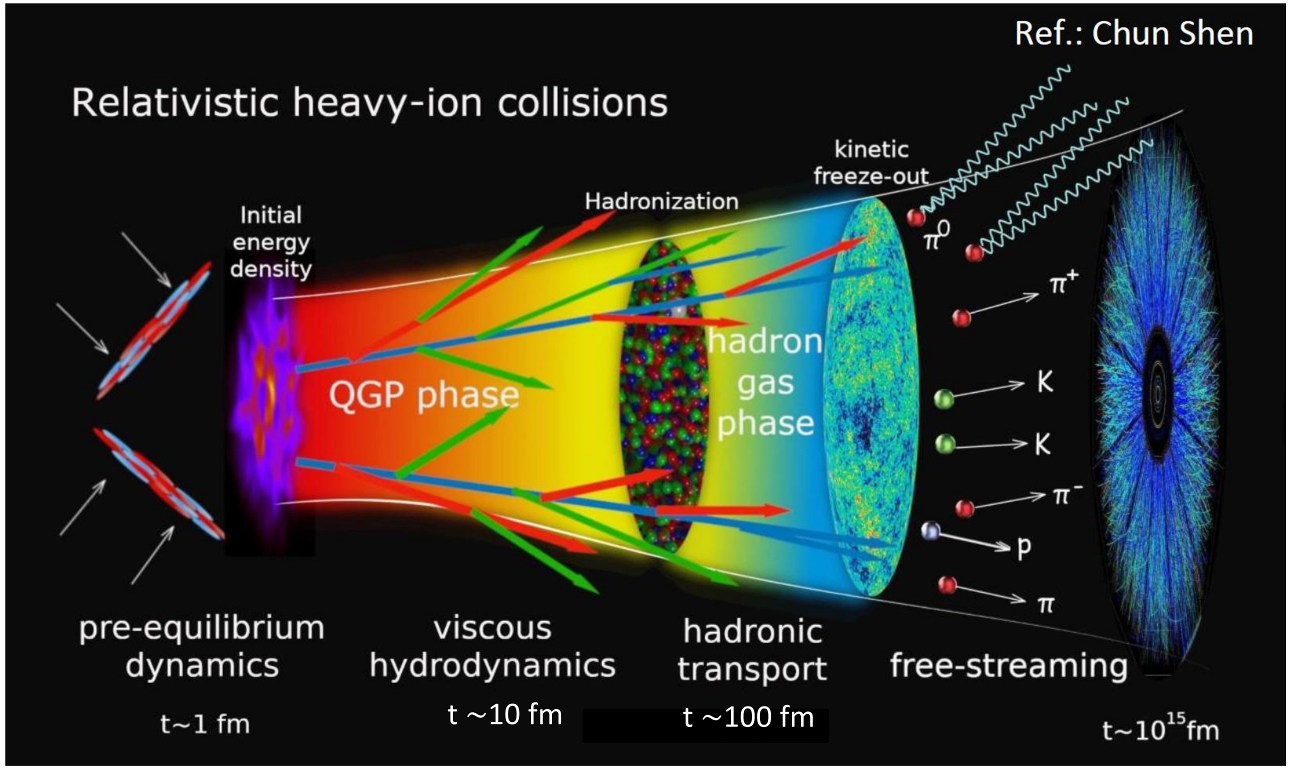

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- To help simulate these different aspects of heavy-ion collisions, the JETSCAPE (Jet Energy-loss Tomography with a Statistically and Computationally Advanced Program Envelope) framework was established.

Overview of fluid dynamics

- Fluid dynamics is a set of conservation equations, i.e. $\partial_\mu T^{\mu\nu} = 0 \; and \; \partial_\mu J^{\mu}_{B,S,Q} = 0$
- What does it mean to have a viscous fluid?

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- Dissipation of sound waves

Wave propagation of perturbations at speed of sound c_s

$$Pert. \propto \exp\left[i(c_s kt - \vec{k}\cdot\vec{x}) - \frac{4\eta}{3s}\frac{k}{2T}kt \right]$$

Decay/Dissipation of perturbations

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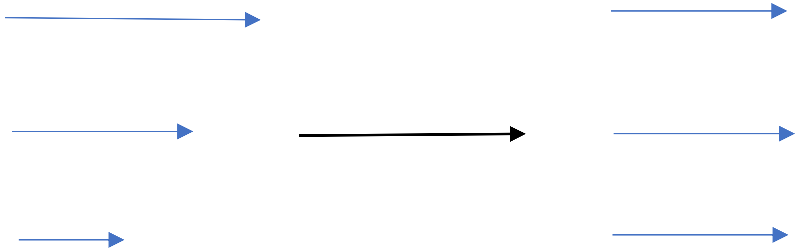
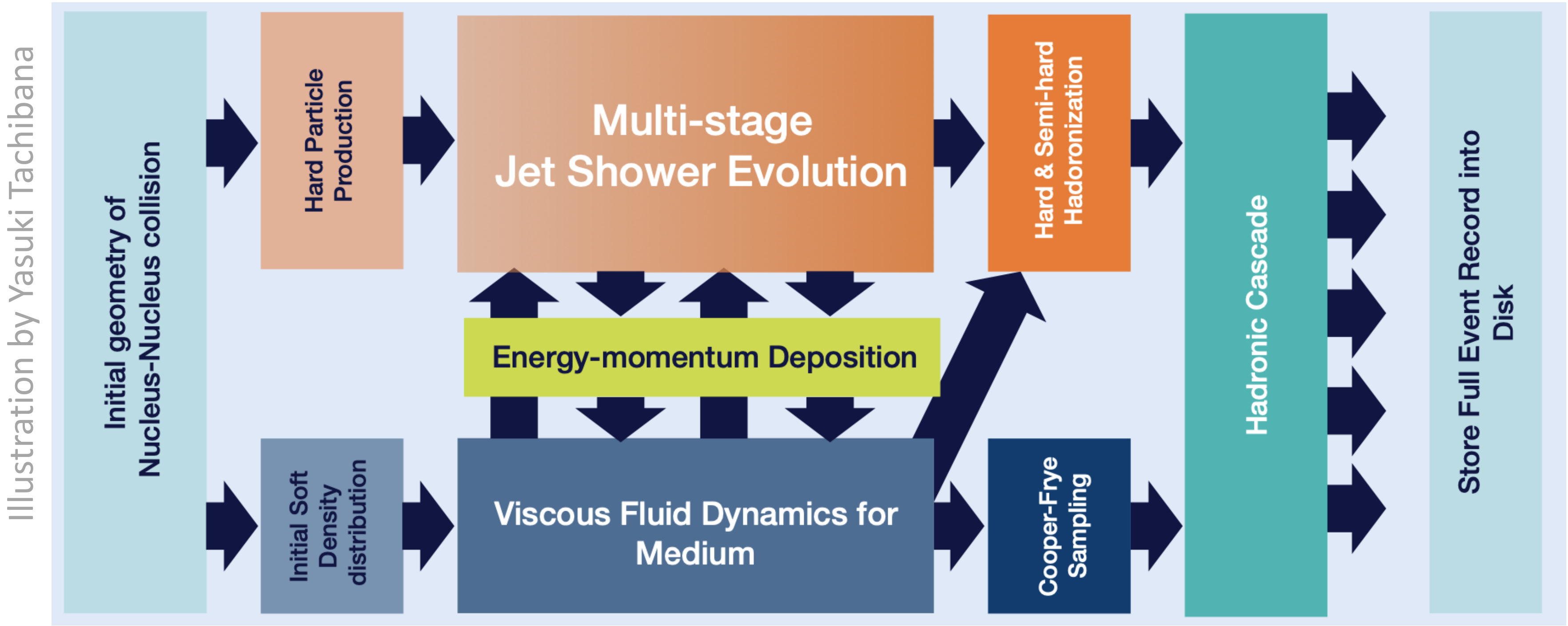
Decay/Dissipation of perturbations

- η introduces friction between fluid layers
  - η shear viscosity
  - s entropy density



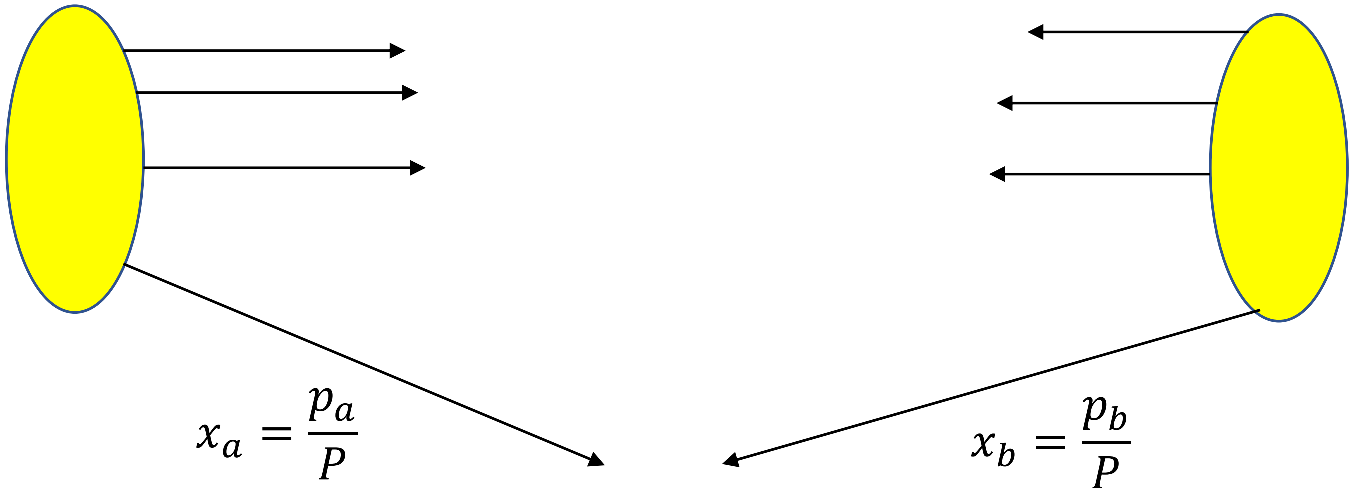
- How big is η in the QGP?

Simulating soft the nuclear medium



- JETSCAPE Collaboration provides a software framework describes:
  - Hydrodynamical simulations and Boltzamann transport simulations
  - Jets Monte Carlo event generators inside the nuclear medium
  - Provides a set of Bayesian tools to characterize the QGP, i.e. $\eta(\mathrm{T})$, $\zeta(T)$, $\hat{q}(T)$

Factorization at work: Leading order diagram

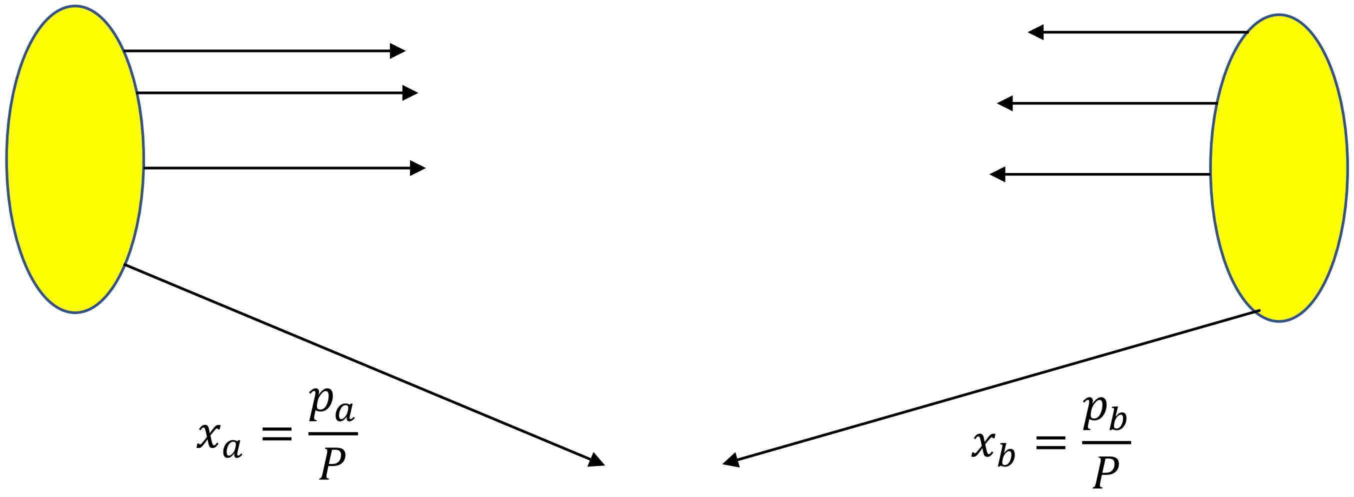


- Start by considering jets in the vacuum.
- The probability (or cross section) for producing a jet is given by

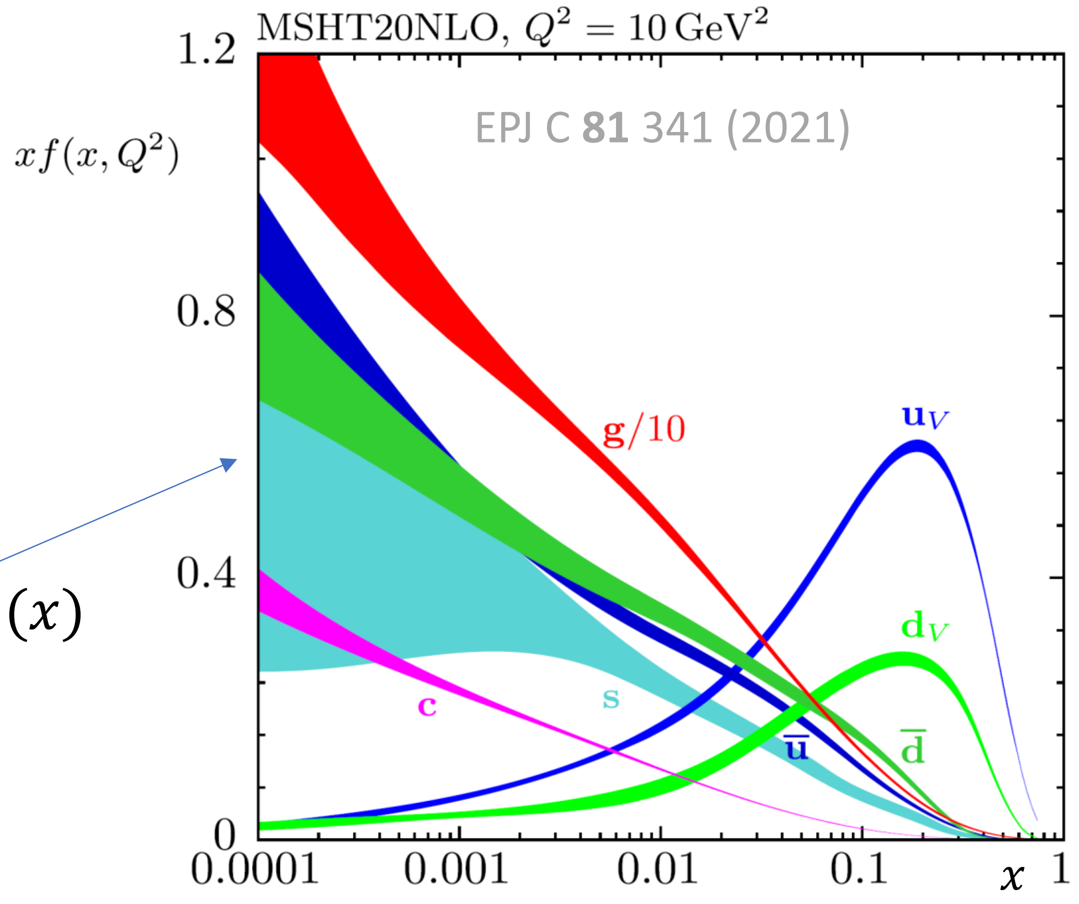
$$\frac{d\sigma_1^h}{dy dp_{T_1}} \sim \int dx_a dx_b G(x_a) G(x_b) \left[\frac{d\hat{\sigma}}{d\hat{t}}\right] D(z_1)$$

x

Factorization at work: Leading order diagram



- Parton Distribution Function (PDF) G: Prob. of finding a parton from the hardon
  - a non-perturbative process, most easily measured in $e + p$ experiment



$$G(x) \propto \sum_i x f_i(x)$$

$$\frac{d\sigma_1^h}{dy\,dp_{T_1}} \sim \int dx_a dx_b\, G(x_a)G(x_b) \left[\frac{d\hat{\sigma}}{d\hat{t}}\right] D(z_1)$$

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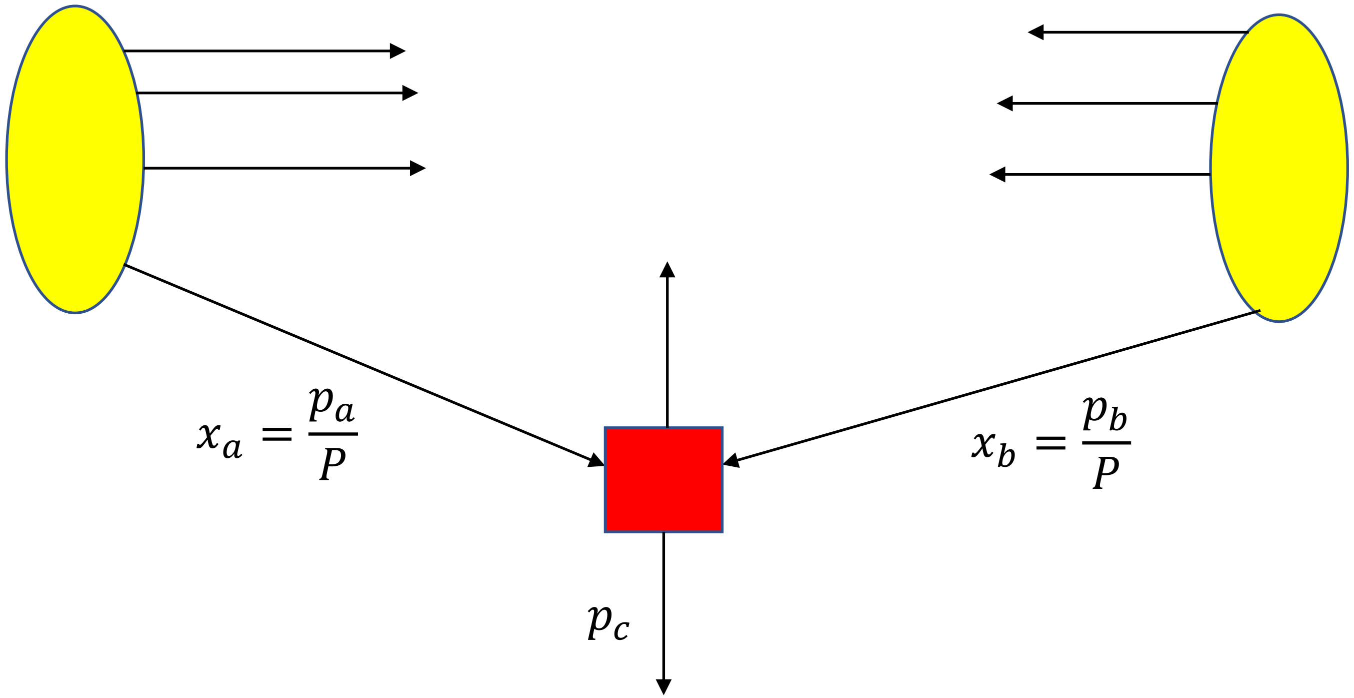

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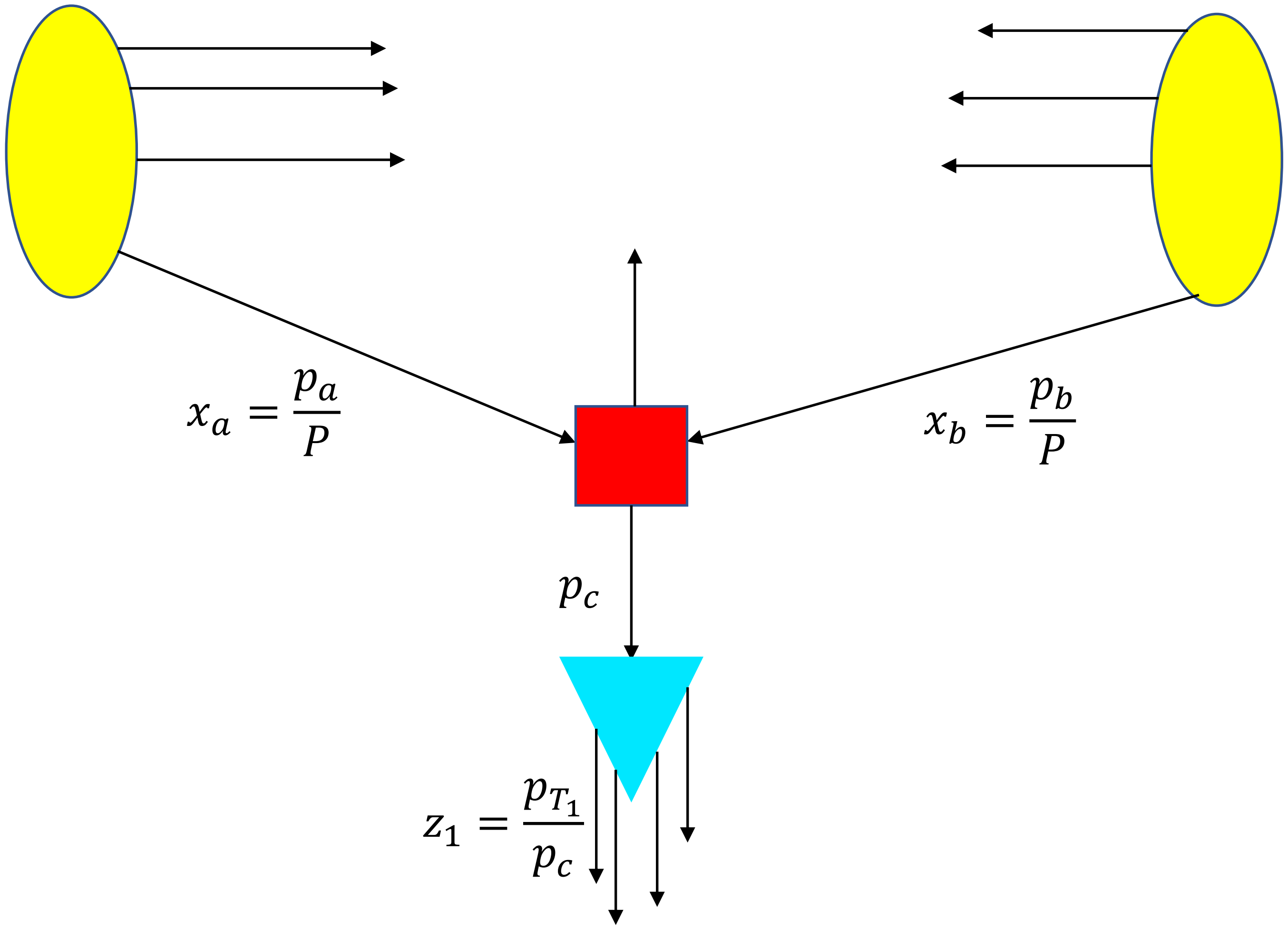

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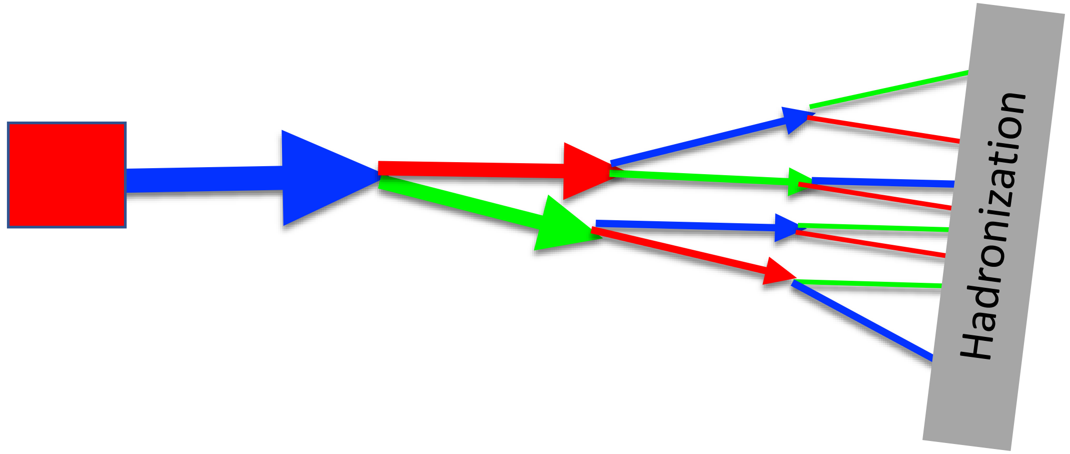
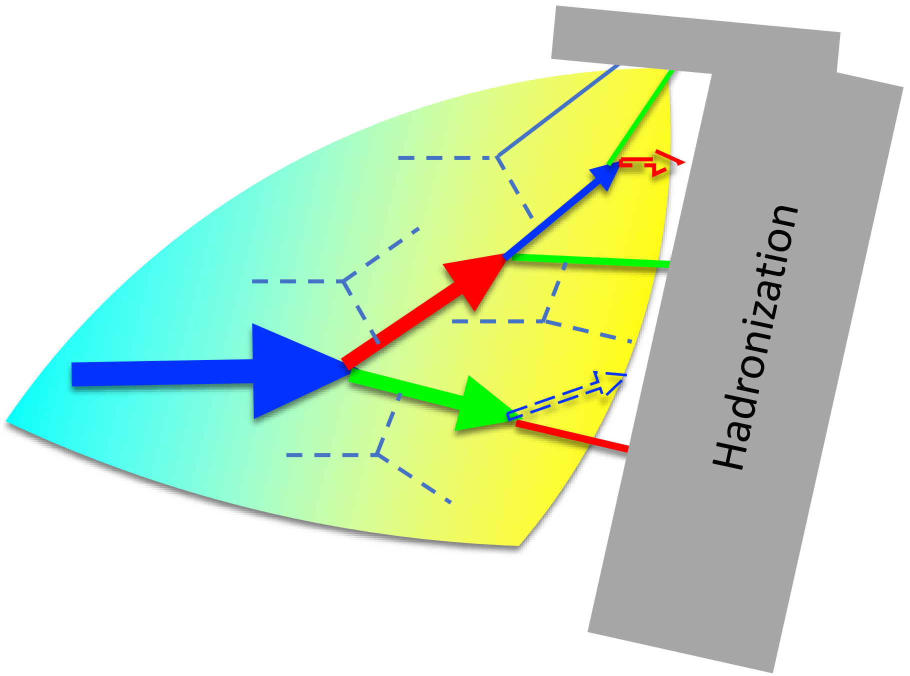
- Perturbative scattering process $\frac{d\hat{\sigma}}{d\hat{t}}$ generates highly virtual (short-lived) particles that produce a shower known as a jet.
- The showering and hadronization of quarks and gluons is encapsulated in the Fragmentation Function (FF) D: converts partons into hadrons
  - non-perturbative process, most easily measured in $e^+ + e^-$

Monte Carlo jet shower simulation in vacuum



- Monte Carlo simulations (e.g., Pythia) develop a shower at the quark/gluon level in vacuum by adding multiple splits.
- In vacuum, the particles in a jet after hadronization occupy a narrow cone.

Modified splitting inside the QGP



- In the nuclear medium, the particles in a jet after hadronization occupy a wider cone.
- The transport coefficient $\hat{q}$ is used to measure (transverse) momentum broadening of partons in the QGP.
  - This transport coefficients is akin to the spatial Brownian diffusion coefficient.

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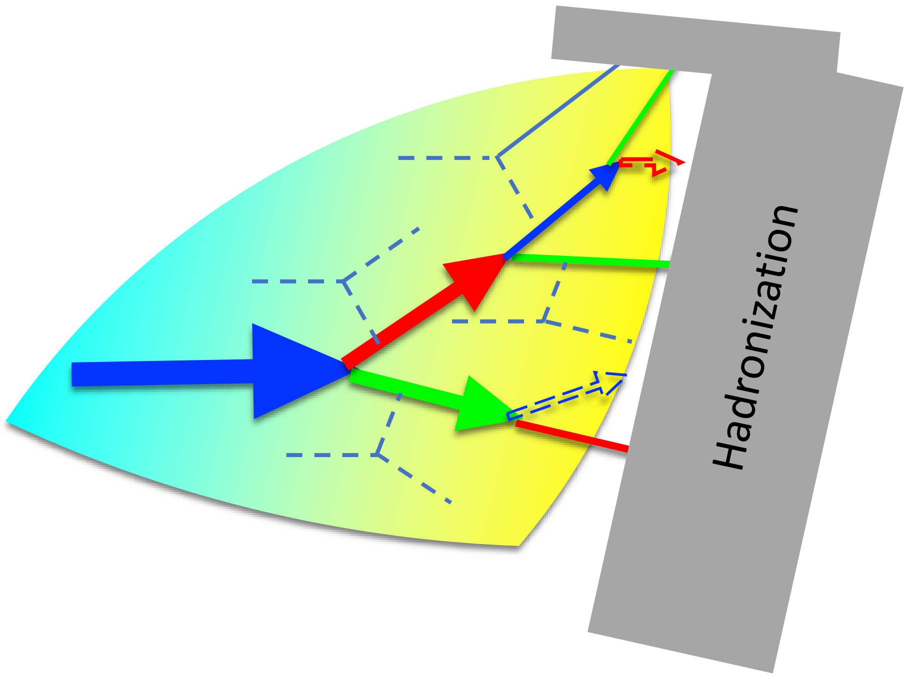

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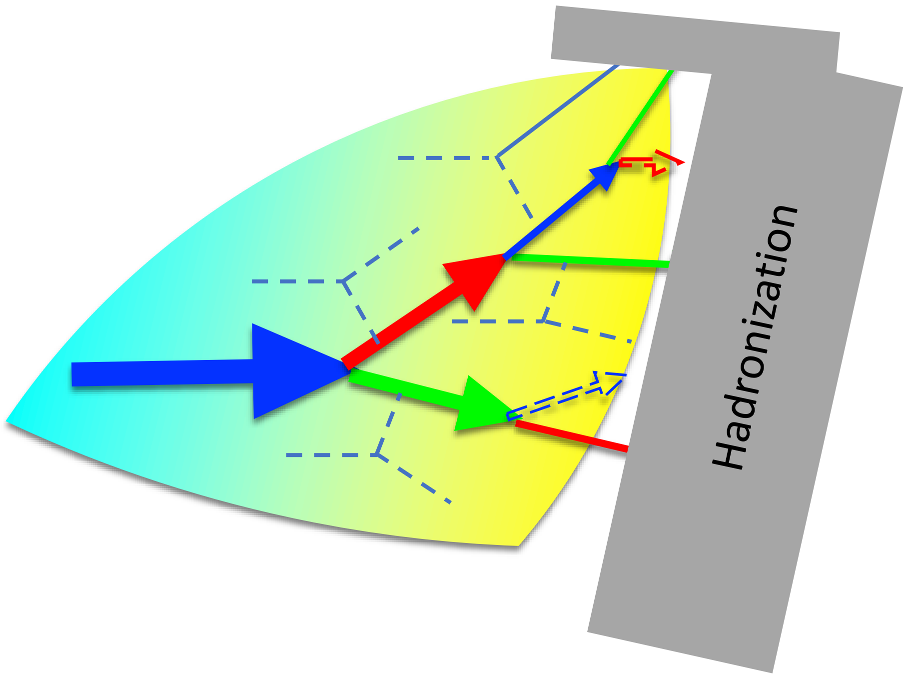

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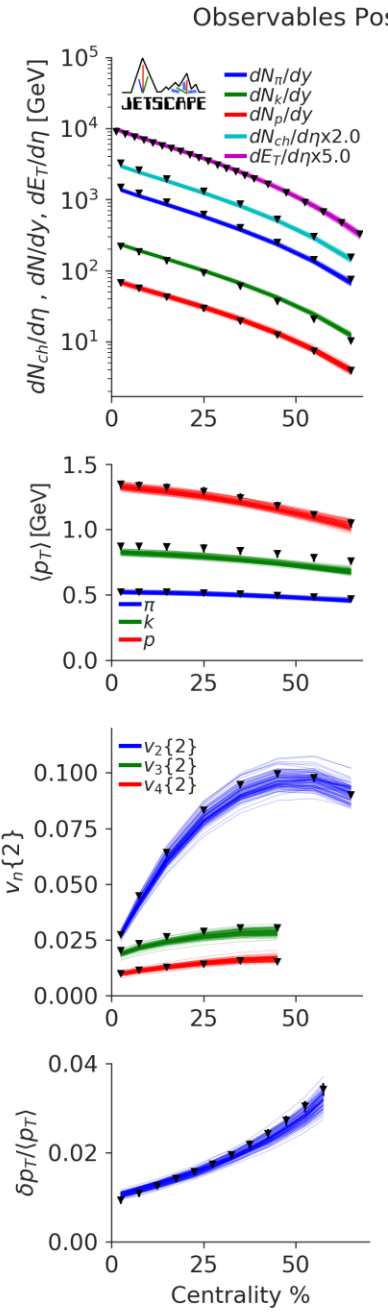
- Lukas Opitz poster #25 (DNP): hadronization in a viscous medium.

Comparisons w/ experimental data using Bayesian calibration



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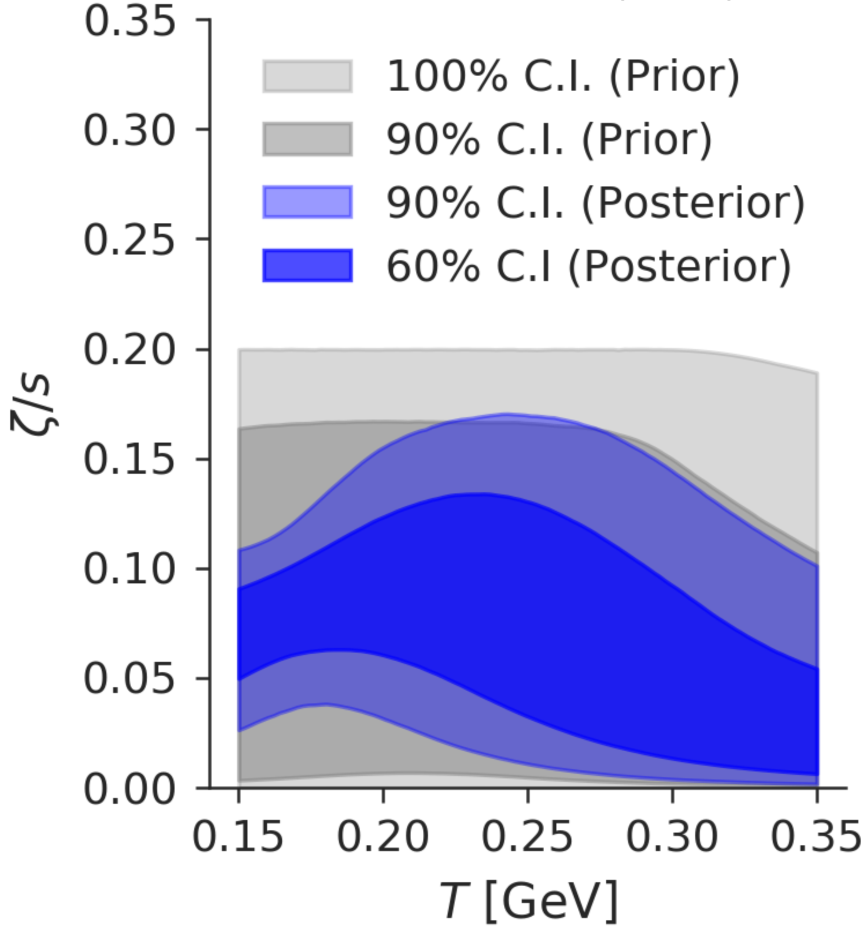

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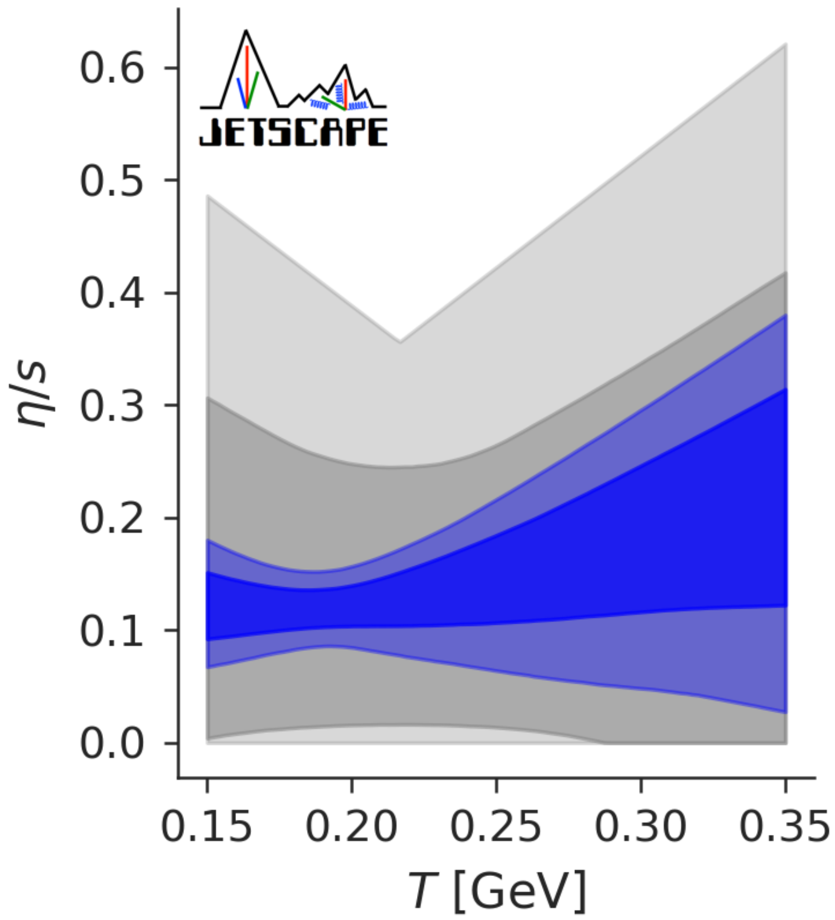
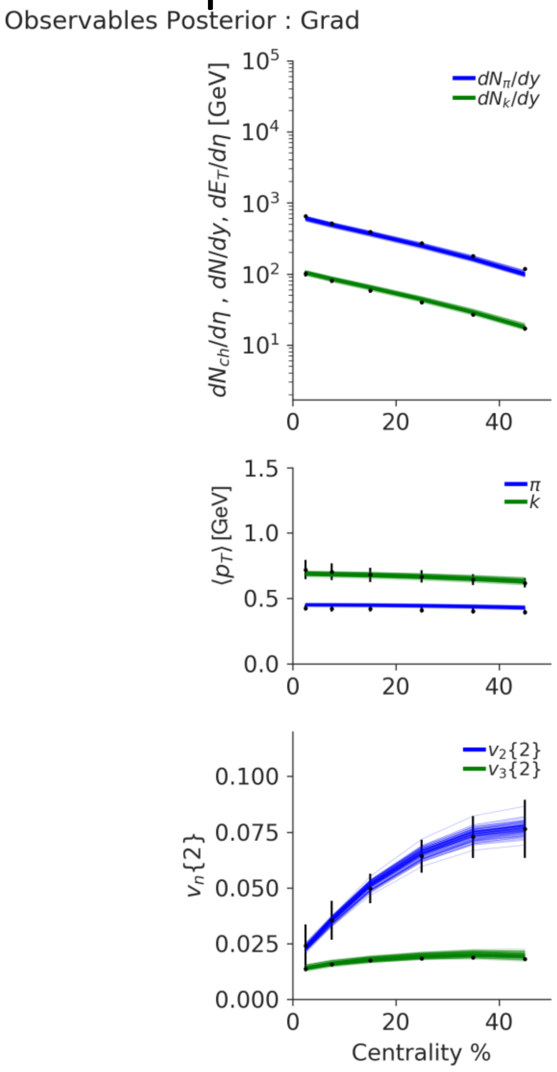


- Constraints on viscosities using only ALICE LHC data

Comparisons w/ experimental data using Bayesian calibration



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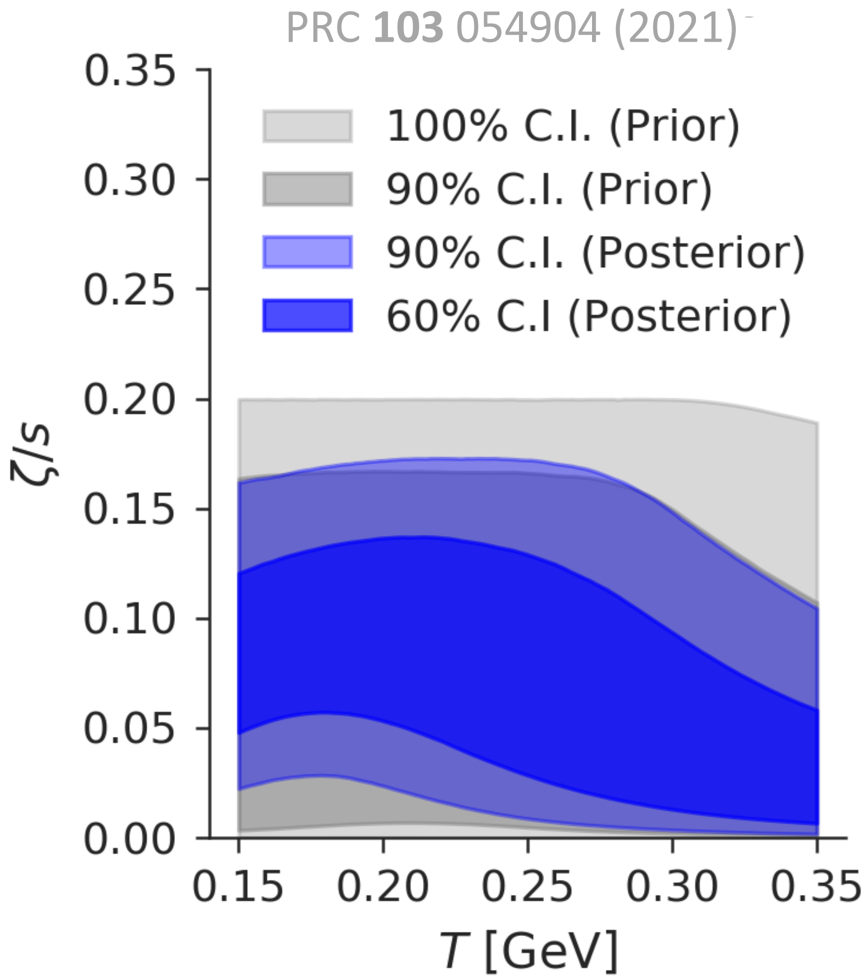
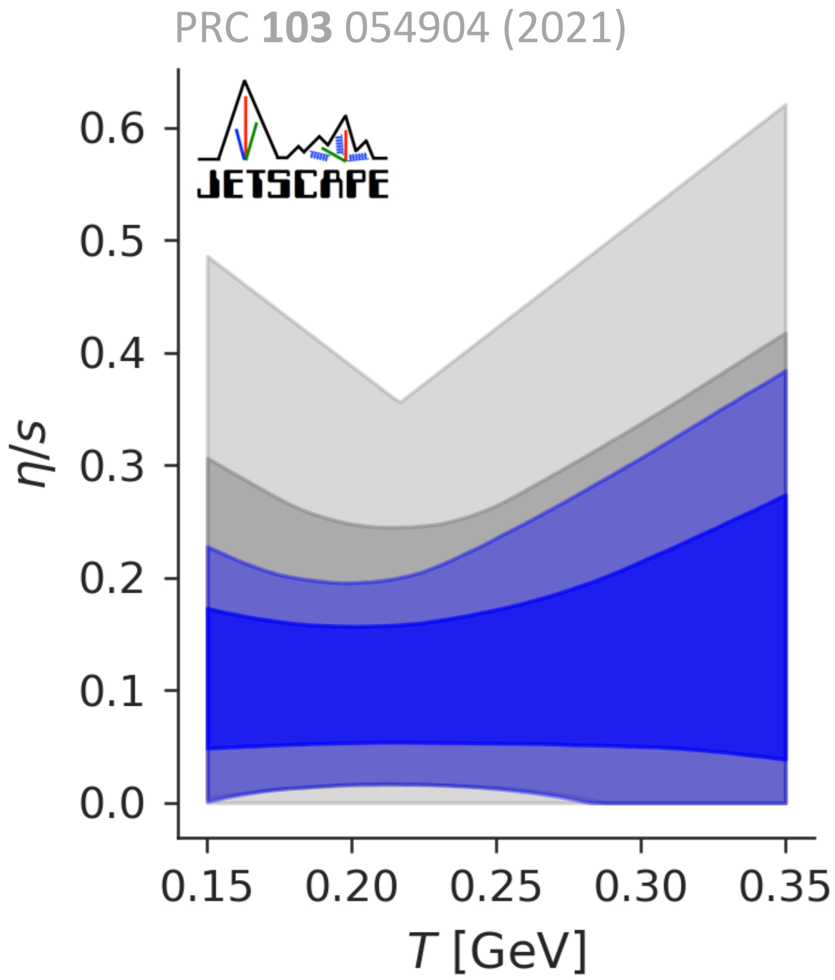
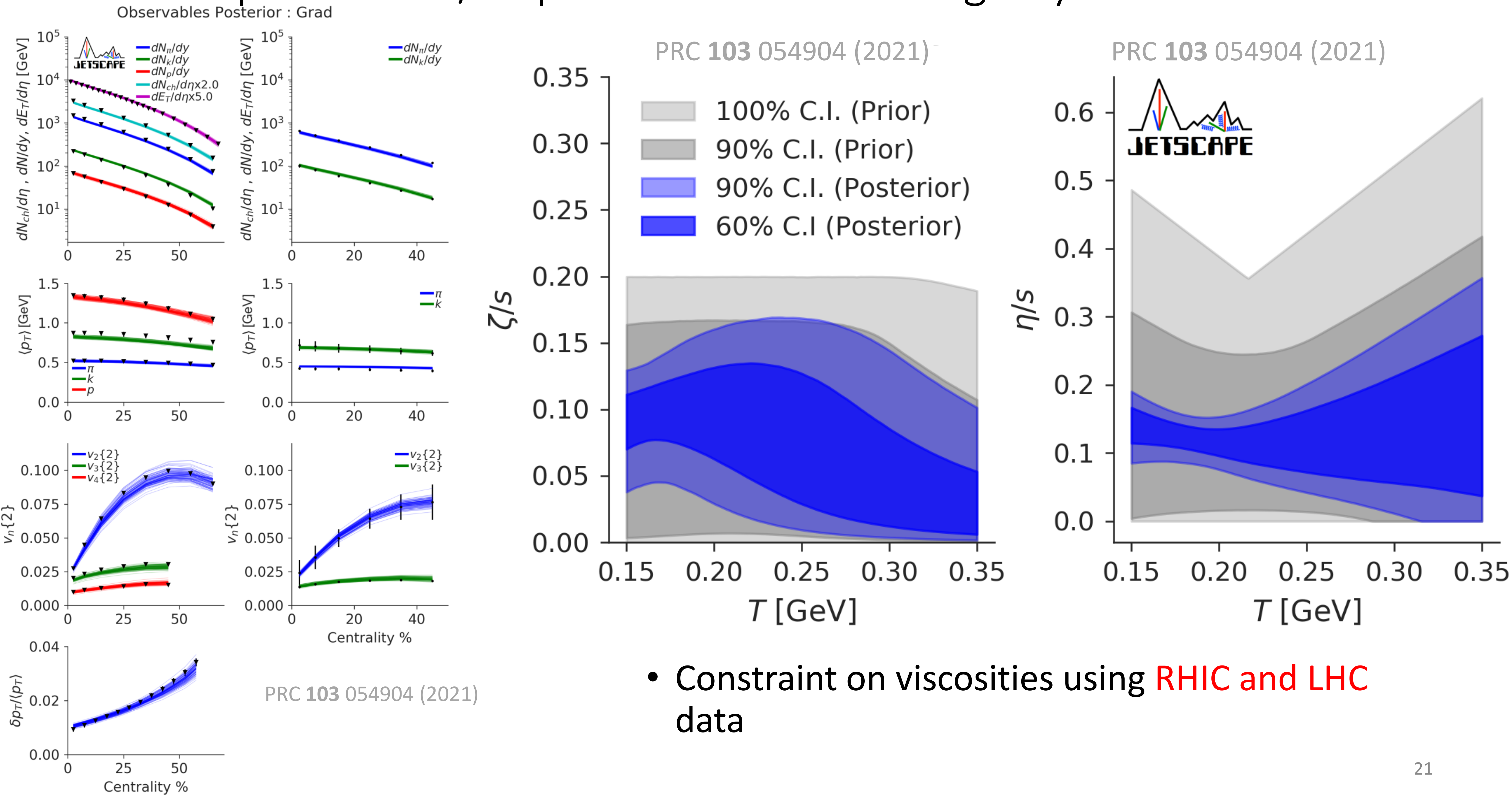




- Constraints on viscosities using only STAR RHIC

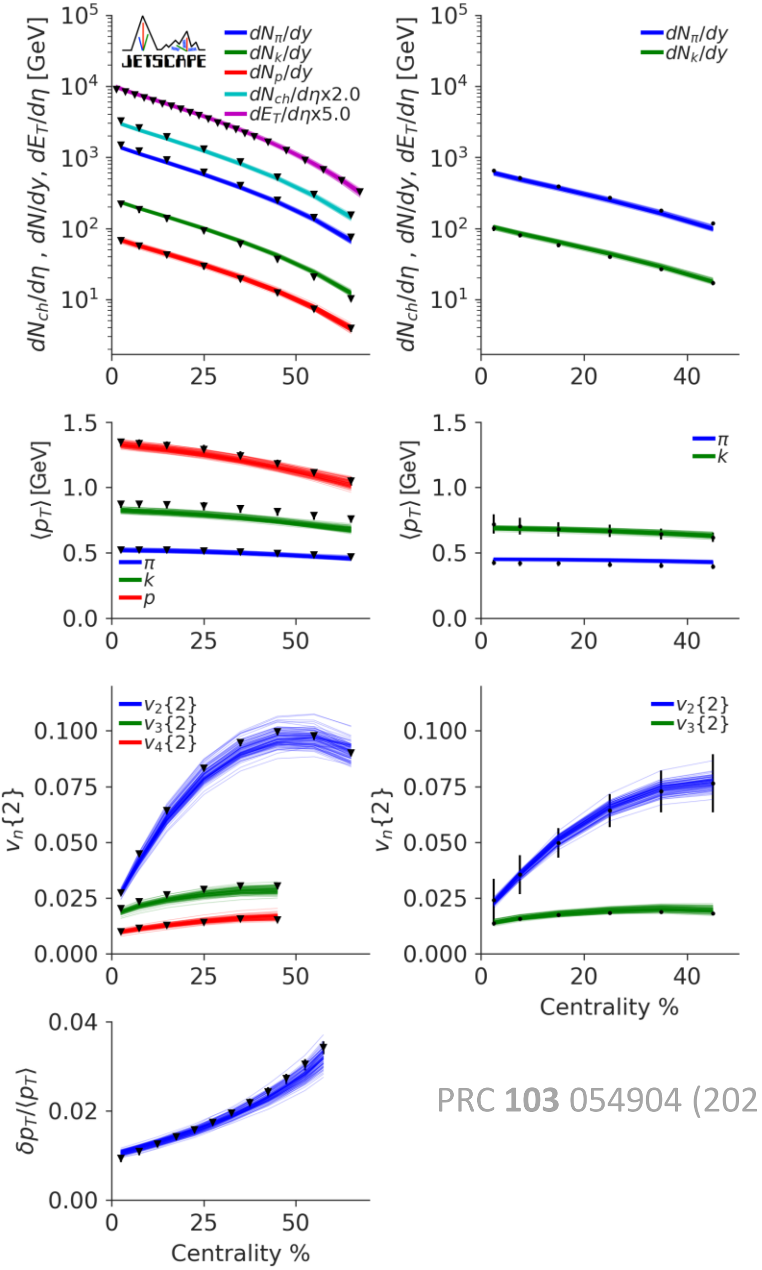
Comparisons w/ experimental data using Bayesian calibration



- Constraint on viscosities using RHIC and LHC data

Comparisons w/ experimental data using Bayesian calibration

Observables Posterior : Grad



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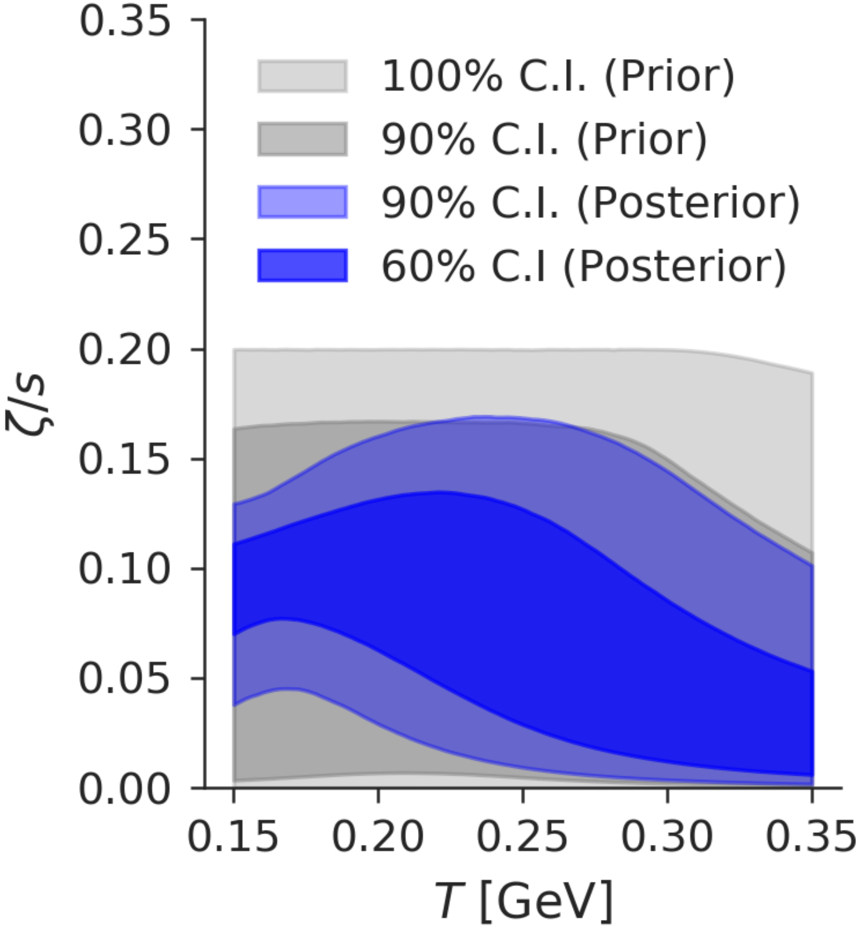

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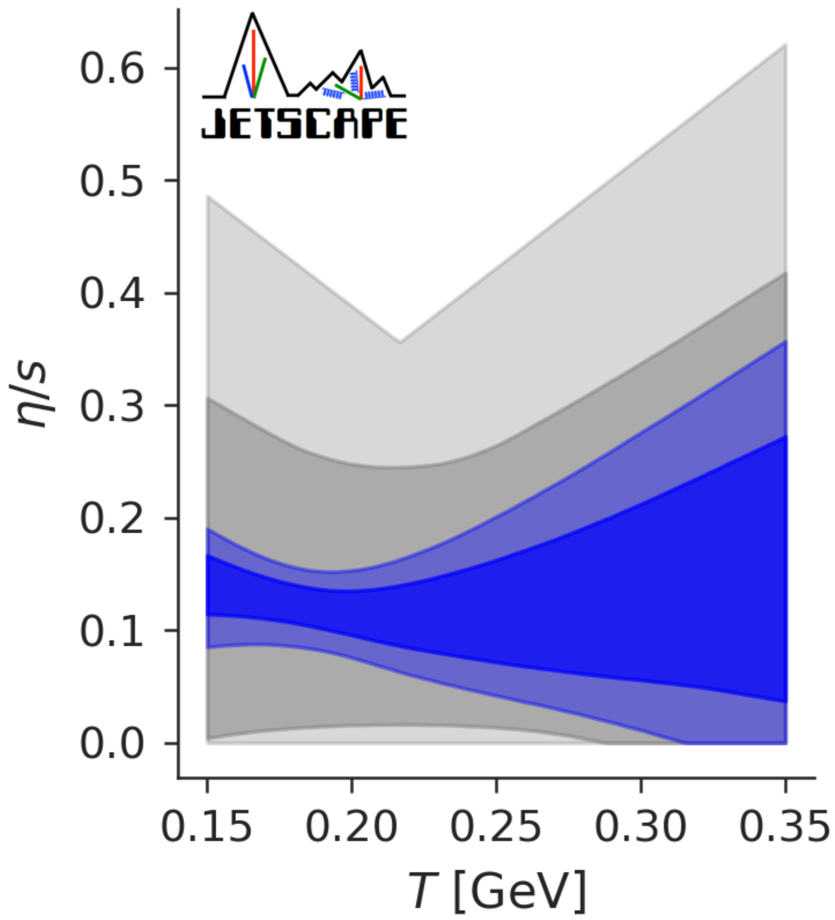
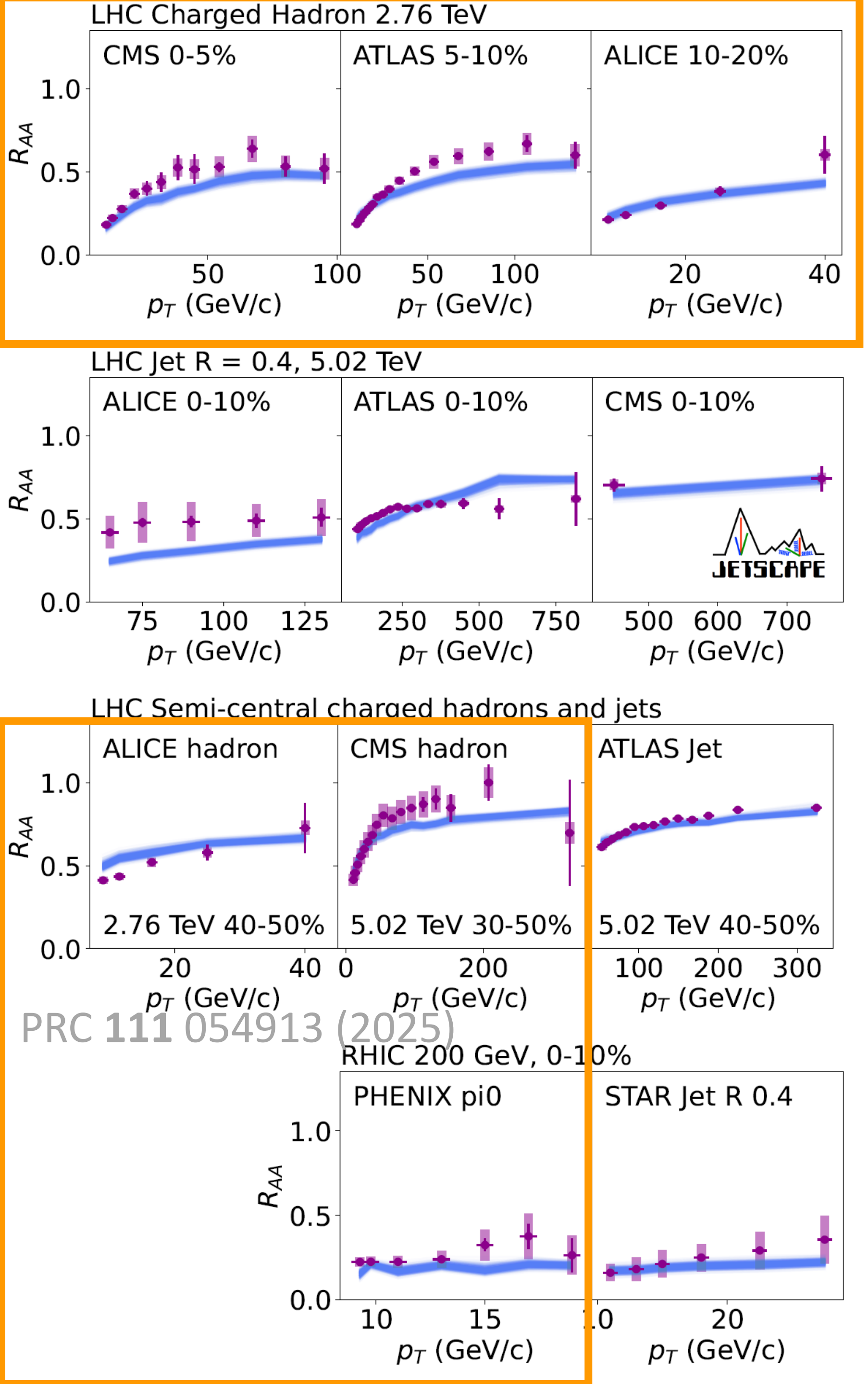
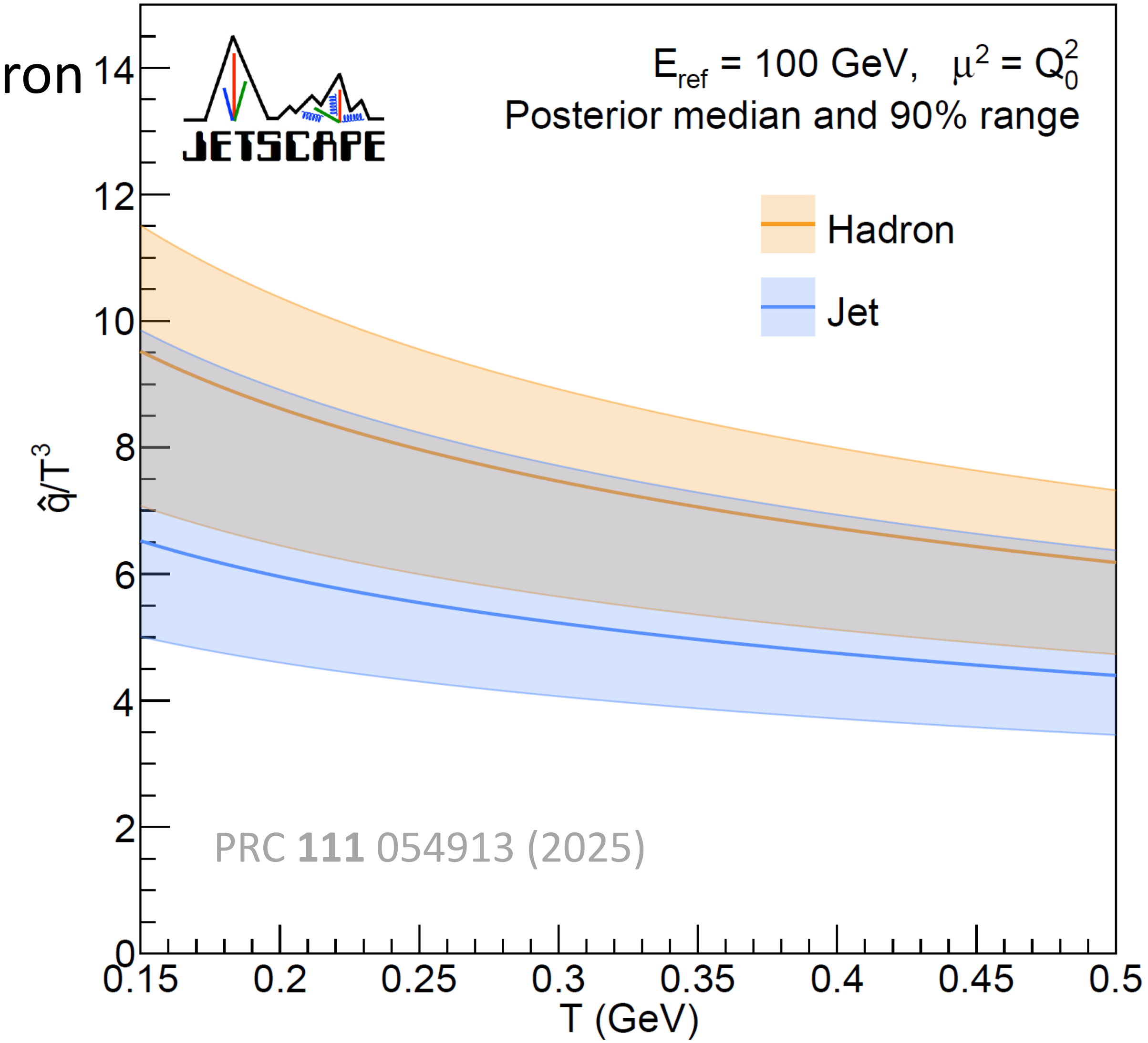


- $\mathcal{O}(10^6)$ core-hours used herein.

JETS in the QGP



- Orange:
  Charged Hadron



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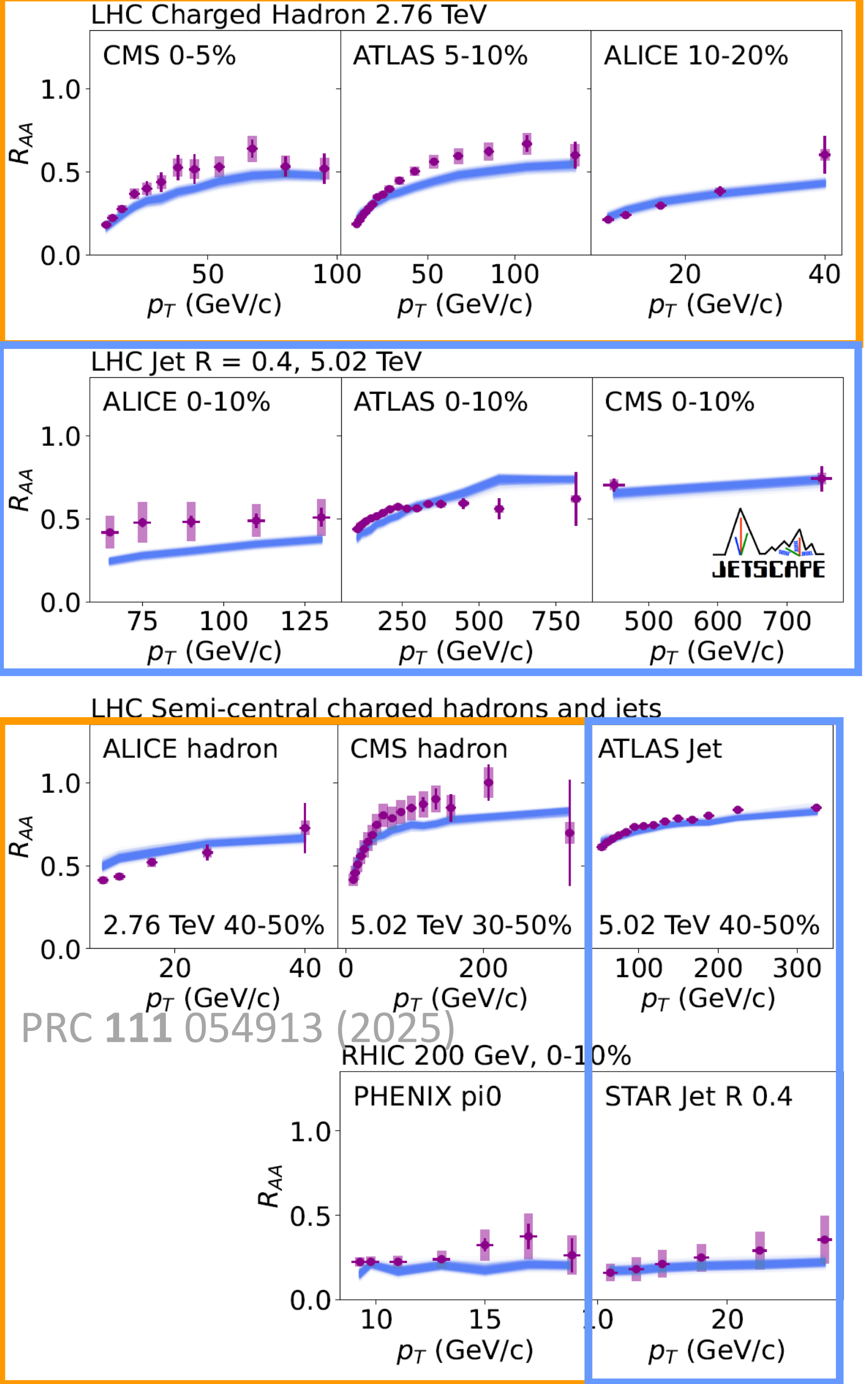
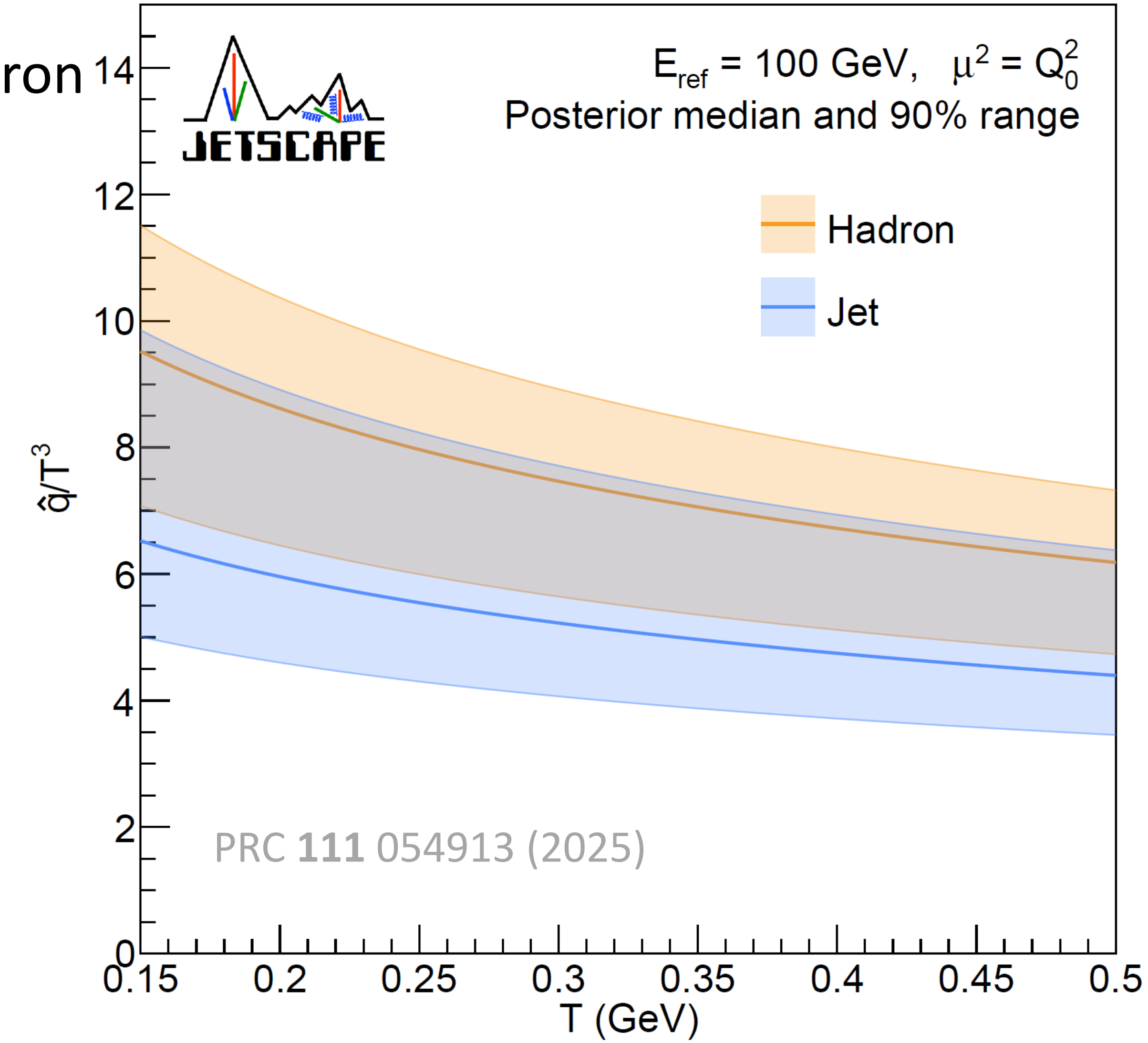


- Orange: Charged Hadron
- Blue: Jet



- Tension in $\hat{q}$ constraints between leading hadron vs reconstructed jet.

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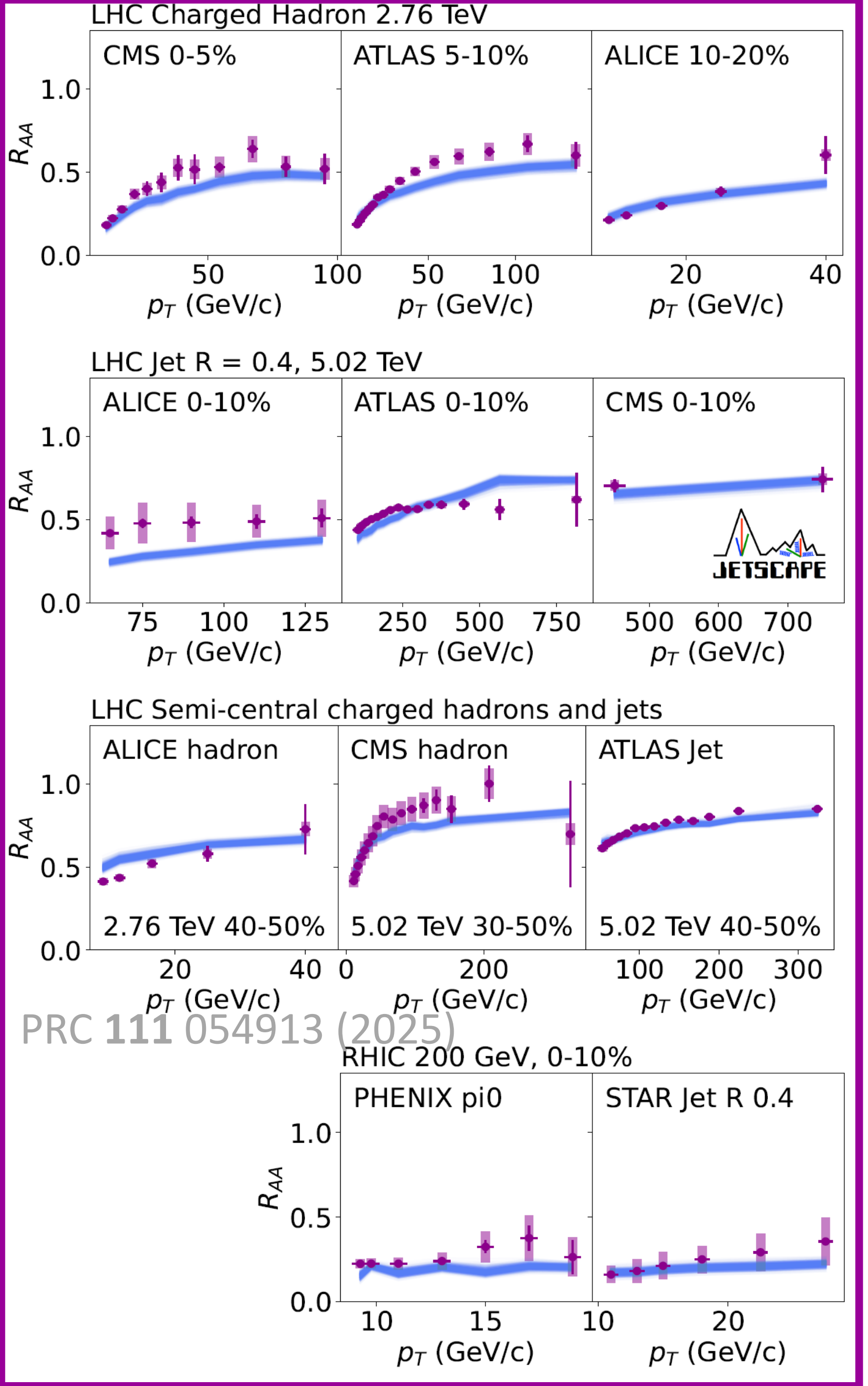
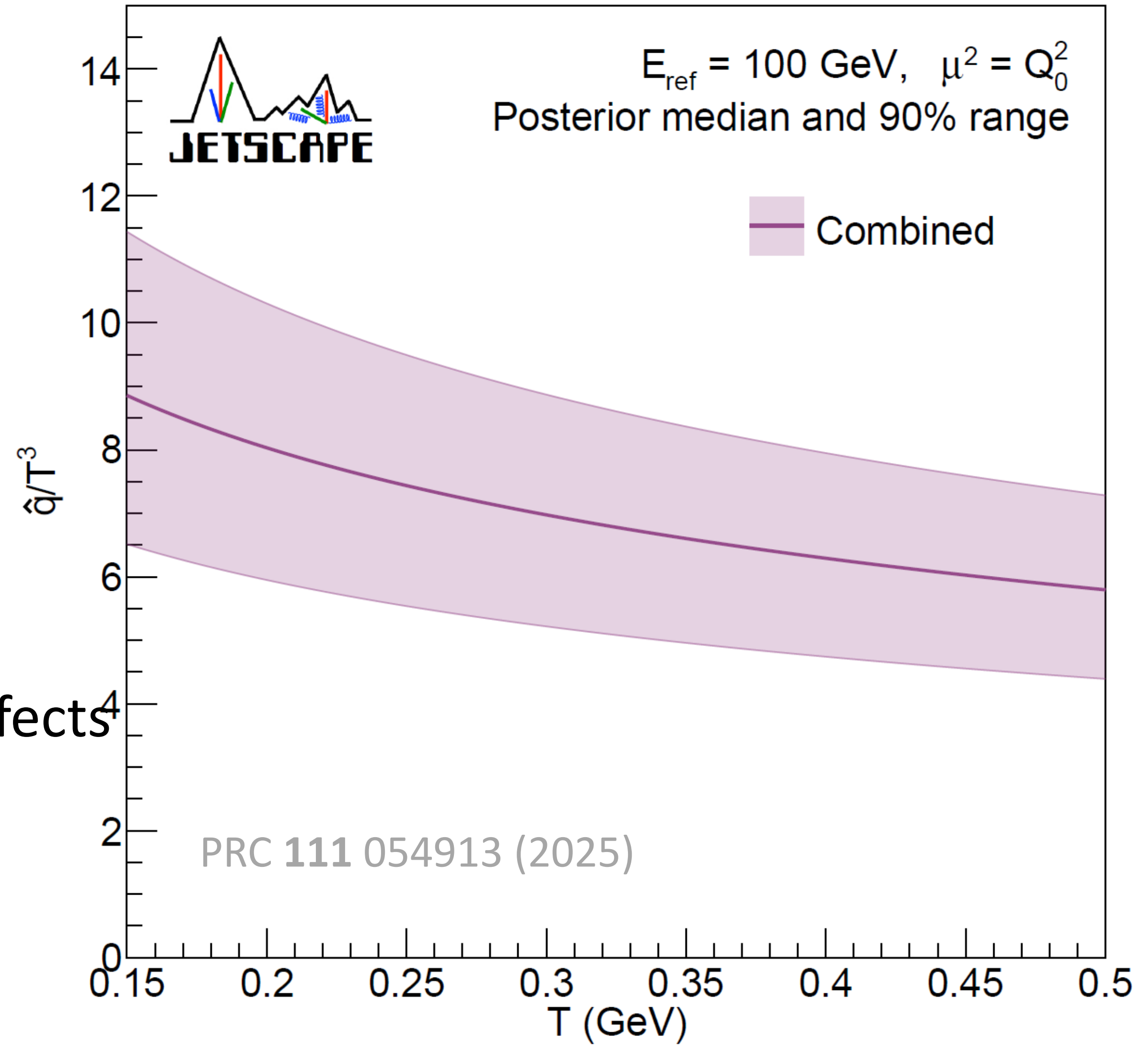


- Combined: Jet+Hadron

- No viscous effects on jets yet



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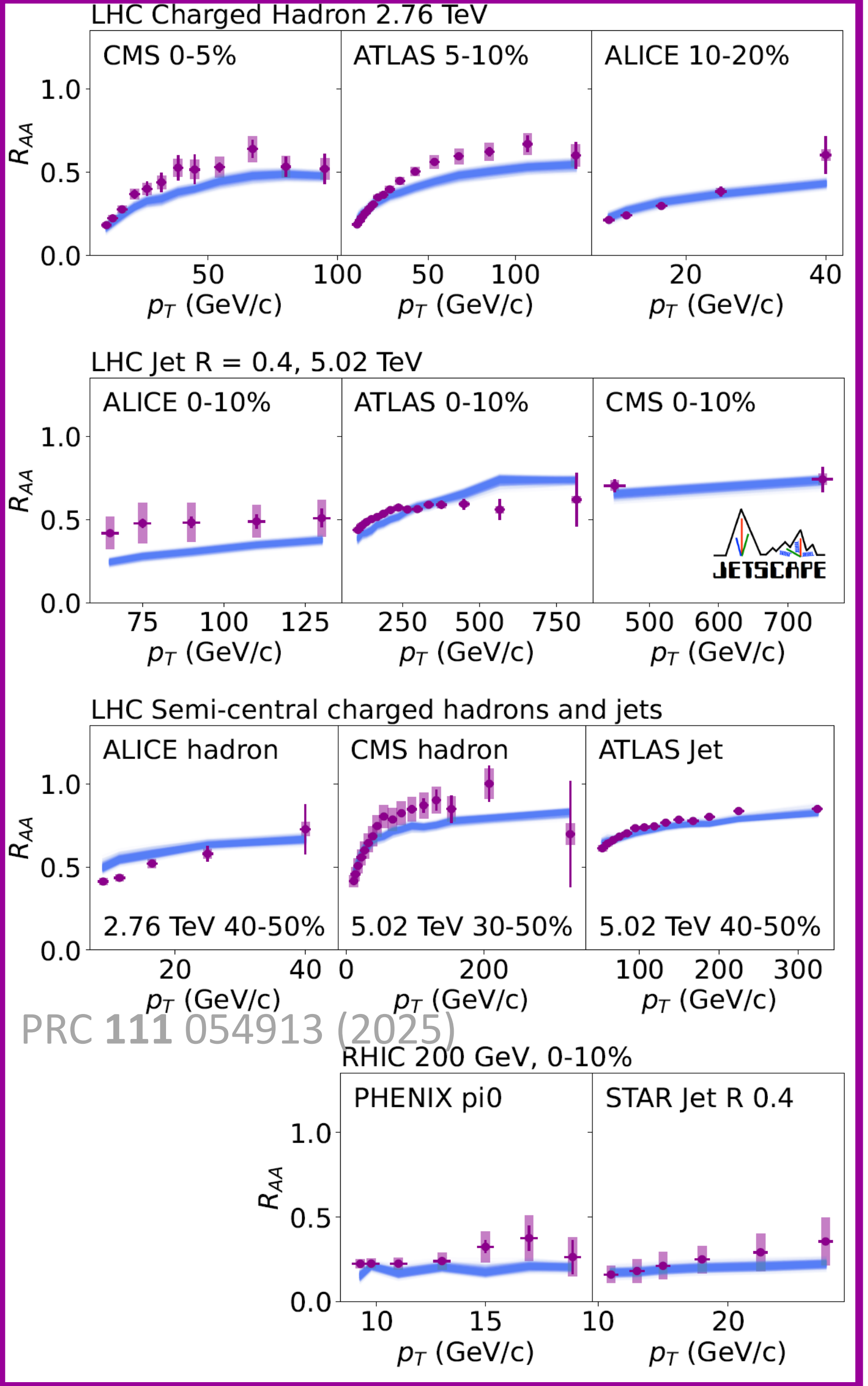

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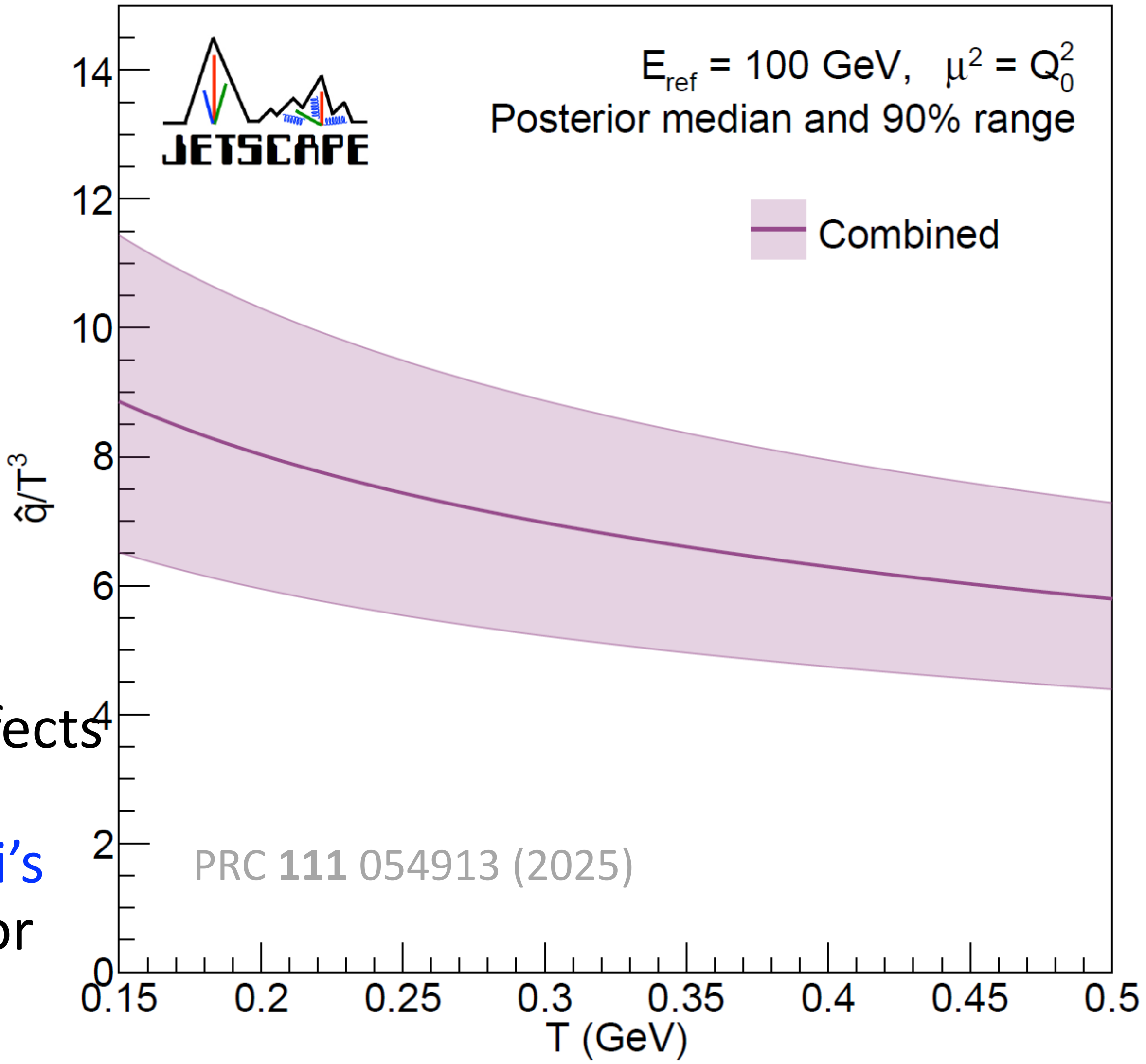

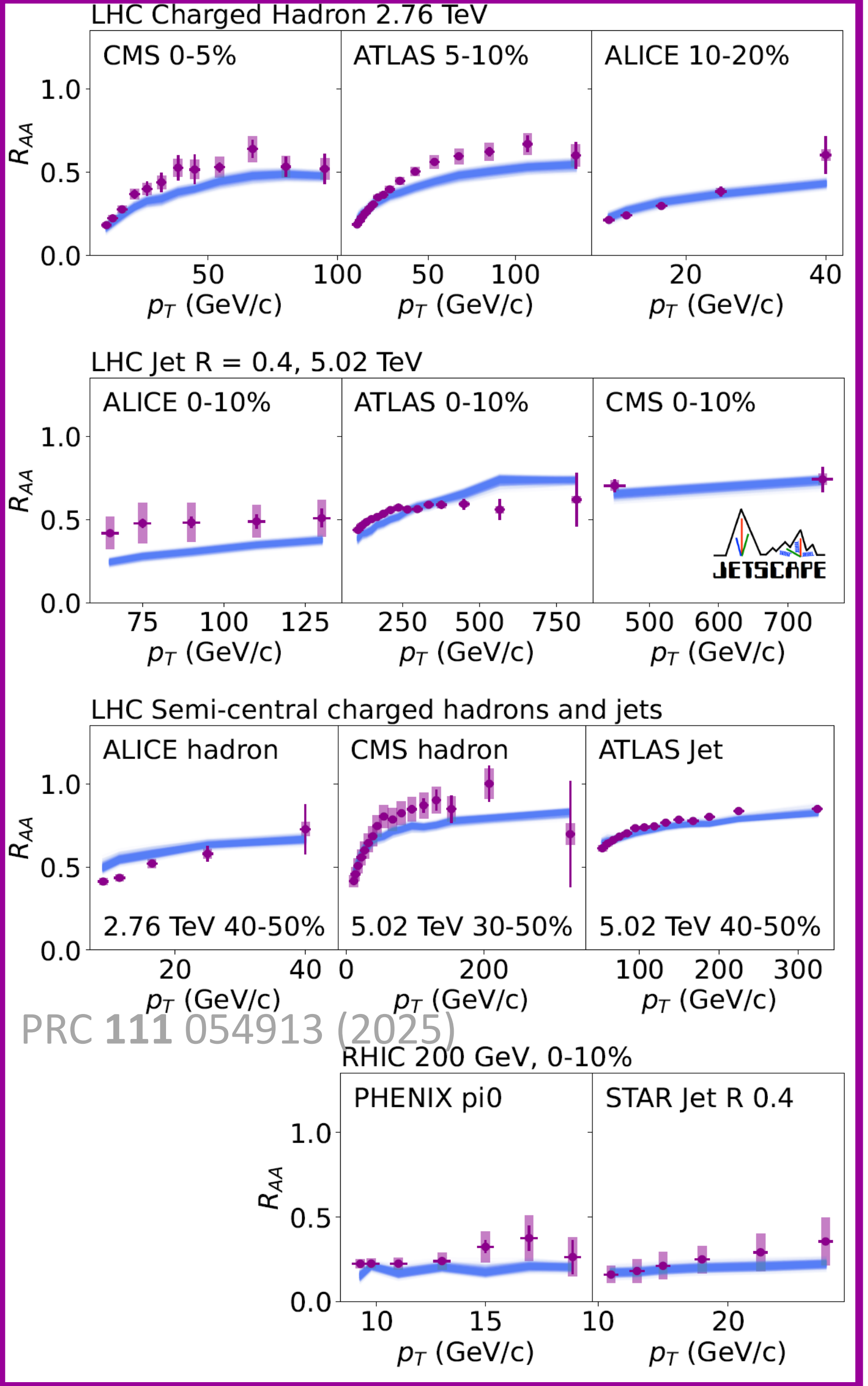

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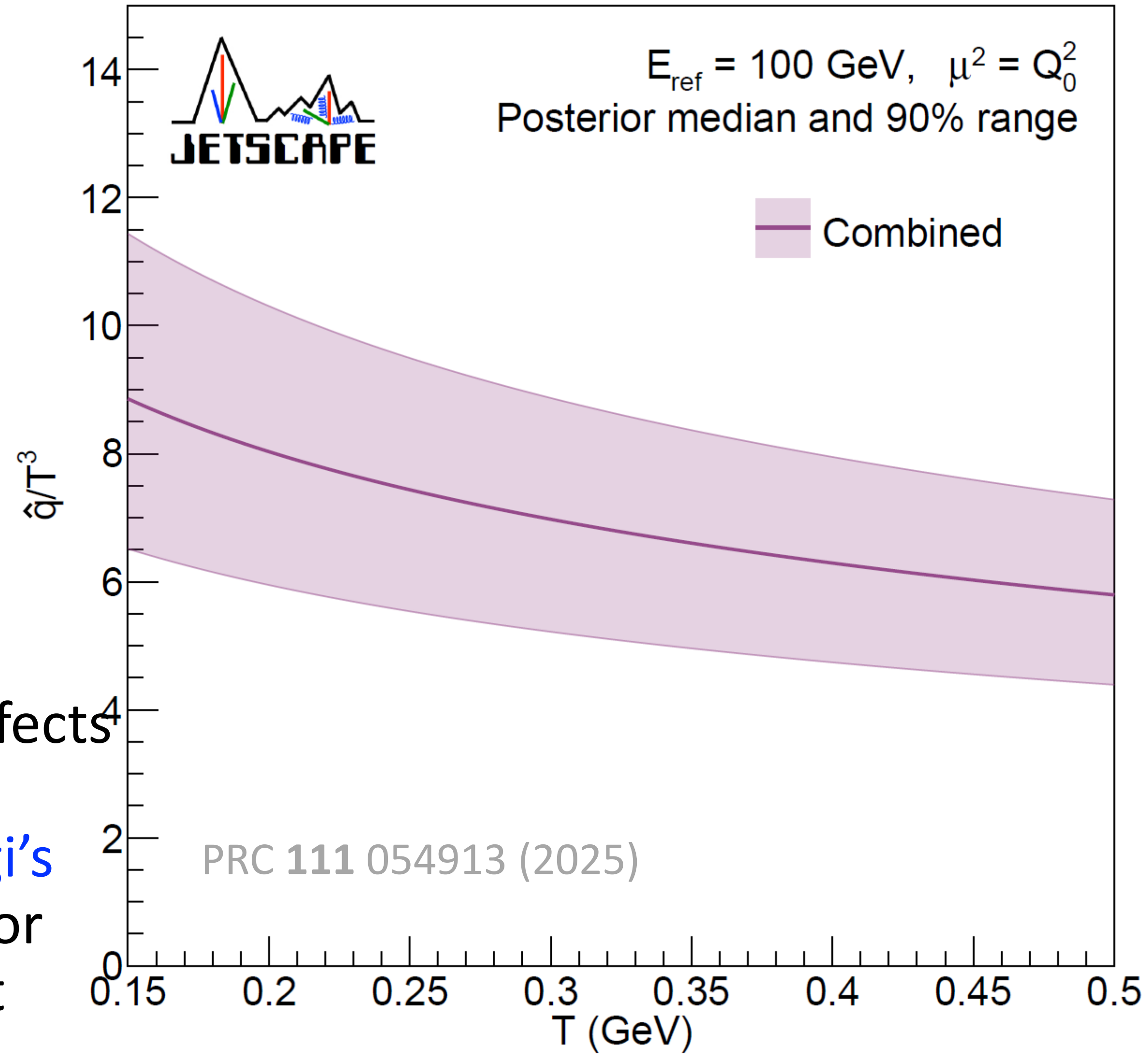

- $\mathcal{O}\left(10^7\right)$ core-hours used in this Bayesian analysis

Conclusion and Outlook

- Using hydrodynamics and Boltzmann transport, Bayesian constraints on the QGP viscosities was obtained.
- Using the MAP parameters to simulate the QGP medium, constraints on momentum diffusion coefficient $\hat{q}$ of jets in the QGP has been obtained through Bayesian model-to-data comparisons.
- Future Bayesian analysis will simultaneously constrain jets and the QGP fluid, requiring:
  - Optimized simulations to be developed that leverages existing accelerators (AVX in CPU, GPU, AI accelerators and so on)
- Great possibilities for physicists, mathematicians, computer scientist to work together and tackle numerically challenging problems and uncover deep insights abouts the QGP.

Backup

Modelling specific bulk (ζ/s) and shear (η/s) viscosities

- Bulk and shear viscosities were parametrized using 4-parameter functions

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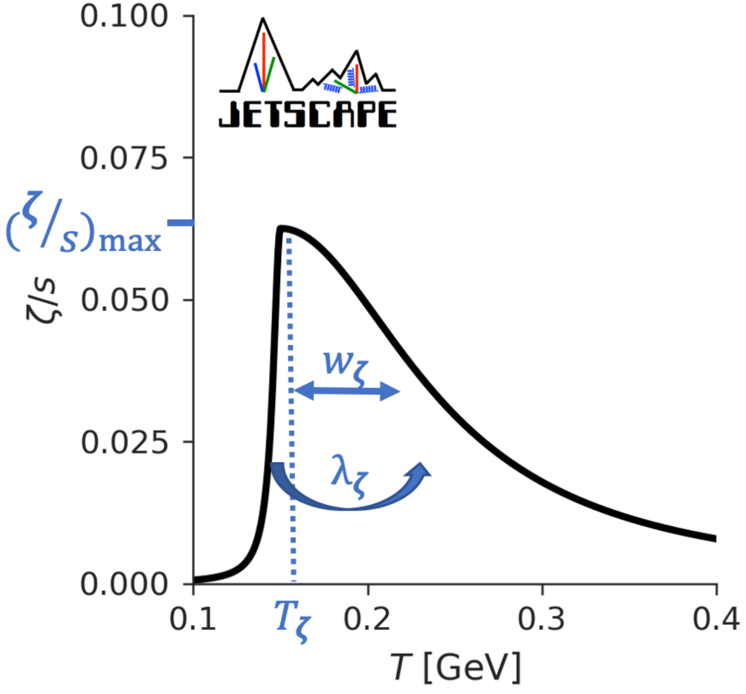

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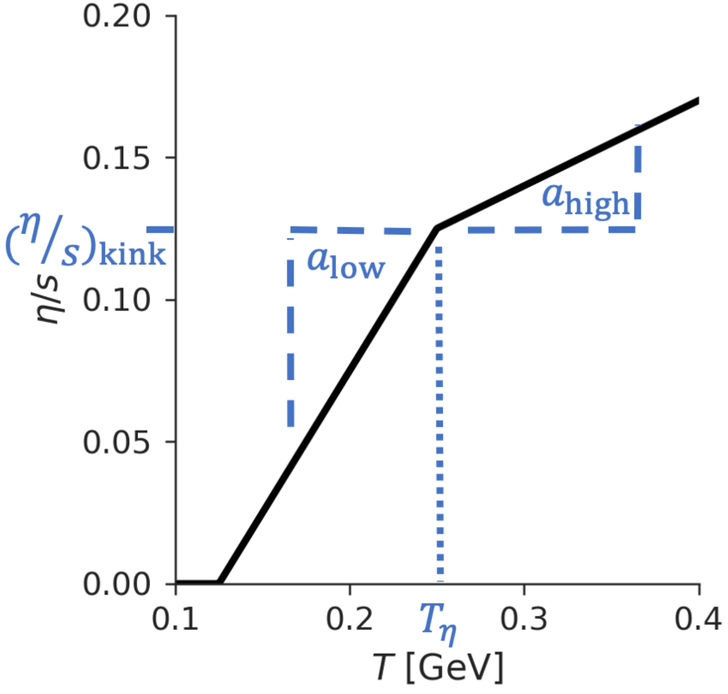

$$\frac{\zeta}{s}(T) = \frac{\left(\frac{\zeta}{s}\right)_{max} \Lambda^2}{\Lambda^2 + \left(T - T_\zeta\right)^2}$$

$$\Lambda = w_\zeta\left[1 + \lambda_\zeta\left(T - T_\zeta\right)\right]$$

$$\frac{\eta}{s}(T) = a_{low}\left(T - T_\eta\right)\Theta\left(T_\eta - T\right) + \left(\frac{\eta}{s}\right)_{kink} + a_{high}\left(T - T_\eta\right)\Theta\left(\mathrm{T} - \mathrm{T}_\eta\right)$$

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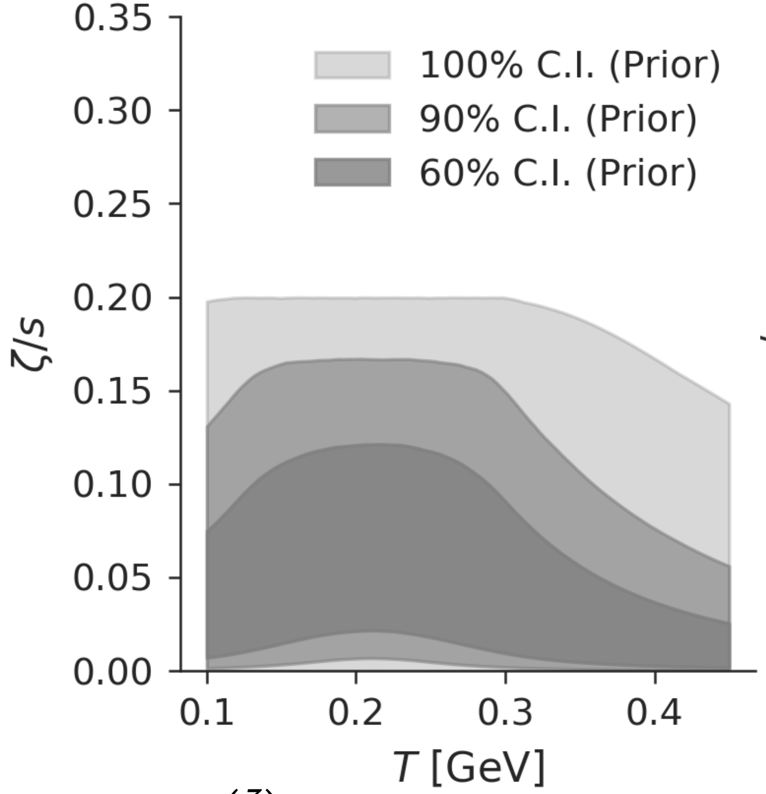

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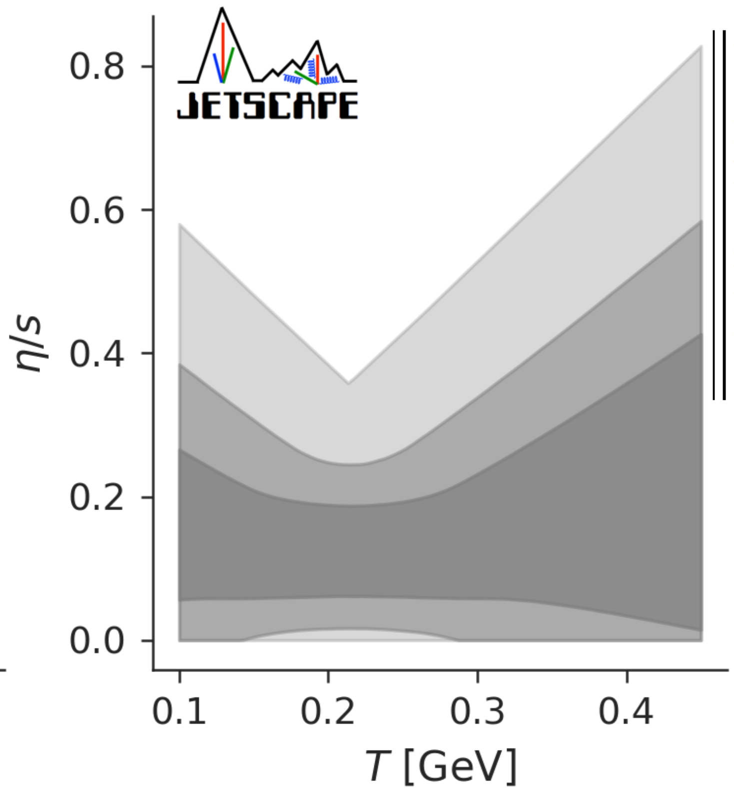

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temperature of (η/s) kink	T_η	[0.13, 0.3] GeV
(η/s) at kink	$(\eta/s)_{\text{kink}}$	[0.01, 0.2]
low temp. slope of (η/s)	a_{low}	[–2, 1] GeV^{-1}
high temp. slope of (η/s)	a_{high}	[–1, 2] GeV^{-1}
shear relaxation time factor	b_π	[2, 8]
maximum of (ζ/s)	$(\zeta/s)_{\max}$	[0.01, 0.25]
temperature of (ζ/s) peak	T_ζ	[0.12, 0.3] GeV
width of (ζ/s) peak	w_ζ	[0.025, 0.15] GeV
asymmetry of (ζ/s) peak	λ_ζ	[–0.8, 0.8]

$$\frac{\zeta}{s}(T) = \frac{\left(\frac{\zeta}{s}\right)_{\max}\Lambda^2}{\Lambda^2 + \left(T - T_\zeta\right)^2}$$

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$$\frac{\eta}{s}(T) = a_{\text{low}}\left(T - T_\eta\right)\Theta\left(T_\eta - T\right) + \left(\frac{\eta}{s}\right)_{\text{kink}} + a_{\text{high}}\left(T - T_\eta\right)\Theta\left(\mathrm{T} - \mathrm{T}_\eta\right)$$

An irreducible tensor decomposition of hydrodynamics

- In high-energy collisions (w/ negligible μ_B), what is flowing?... That can only be energy density ϵ

$$T^{\mu\nu}u_\nu = \epsilon u^\mu$$

Landau's flow definition

$u^\mu = \left(\gamma, \gamma\vec{\beta}\right)$ where

$\gamma = \left(1-\beta^2\right)^{-1/2}$ and $\vec{\beta} = \vec{v}/c$. Using natural units from how on $\Rightarrow c = 1$.

- Non-dissipative $T_0^{\mu\nu}$ can only take the form:

$$T_0^{\mu\nu} = \epsilon u^\mu u^\nu - P(\epsilon)\Delta^{\mu\nu} = \epsilon u^\mu u^\nu - P(\epsilon)(g^{\mu\nu} - u^\mu u^\nu)$$

- Including dissipation gives rise to dissipative corrections $\delta T^{\mu\nu}$ to $T_0^{\mu\nu}$, namely Π and $\pi^{\mu\nu}$

$$T^{\mu\nu} = T_0^{\mu\nu} + \delta T^{\mu\nu} = T_0^{\mu\nu} - \Pi\Delta^{\mu\nu} + \pi^{\mu\nu} = \epsilon u^\mu u^\nu - (P + \Pi)\Delta^{\mu\nu} + \pi^{\mu\nu}$$

where the viscous pressure are decomposed in terms of irreducible tensors, namely

radial deformations

$$\Pi = -\frac{1}{3}\Delta^{\mu\nu}T_{\mu\nu} - P(\epsilon)$$

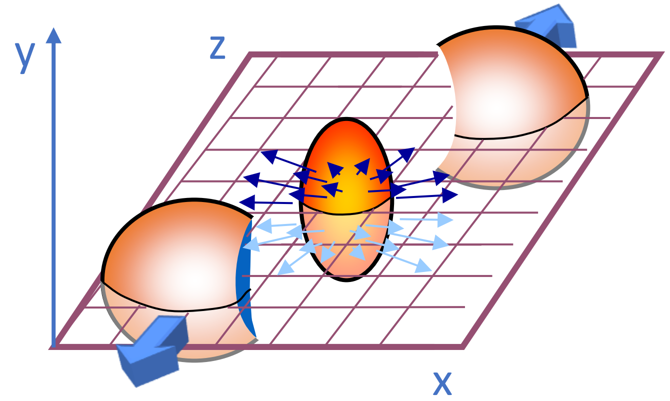
angular deformations

$$\pi^{\mu\nu} = T^{\langle\mu\nu\rangle} = \Delta^{\mu\nu}_{\alpha\beta}\, T^{\alpha\beta} = \left[\frac{1}{2}\left(\Delta^\mu_\alpha\Delta^\nu_\beta + \Delta^\mu_\beta\Delta^\nu_\alpha\right) - \frac{1}{3}\Delta^{\mu\nu}\Delta_{\alpha\beta}\right]T^{\alpha\beta}$$

w/ $\pi^\mu_\mu = 0$ and $u_\mu\pi^{\mu\nu} = 0$

A measure of anisotropic flow (v_n)

- Elliptic Flow



- A nucleus-nucleus collision is typically not head on; an almond-shape region of matter is created.
- To quantify this almond shape region, the centrality is introduced, where 0-10% being the 10% most head-on collisions, while 40-50% being semi-peripheral collisions shown.

- To describe the angular (ϕ) momentum distribution (in x-y plane, i.e. $\vec{p}_\perp$), use a Fourier decomposition (i.e. flow coefficients) v_n

$$\frac{dN}{dM d\eta_p p_\perp dp_\perp d\phi} = \frac{1}{2\pi} \frac{dN}{dM d\eta_p p_\perp dp_\perp} \left[1 + \sum_n v_n \cos(n\phi) \right]$$

$$\eta_p = \frac{1}{2} \mathrm{Log} \left[\frac{E_p + p^z}{E_p - p^z} \right]$$

- Second Fourier coefficient: elliptic flow (v_2) is the largest.