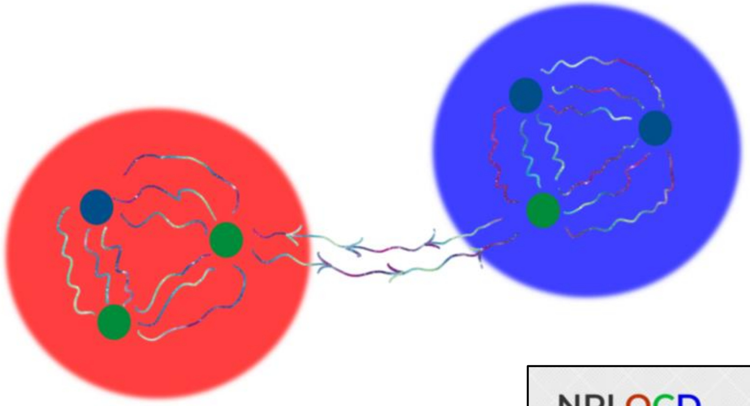
An *ab initio* framework for the modelling of electroweak processes in light nuclei

Michael Gennari

Collaborators

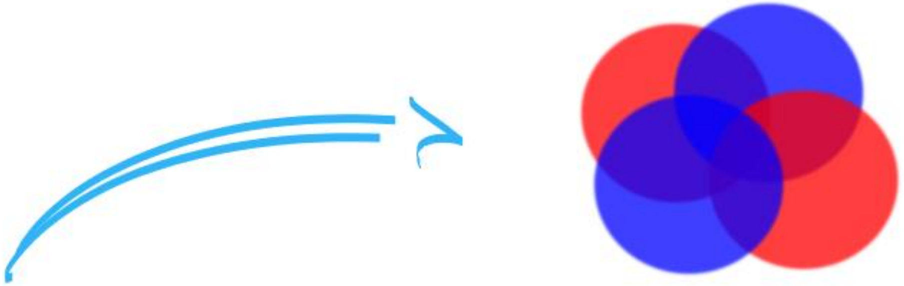
D. A. Najera, L. Jokiniemi, M. Drissi, C.-Y. Seng,
M. Gorchtein, P. Navratil

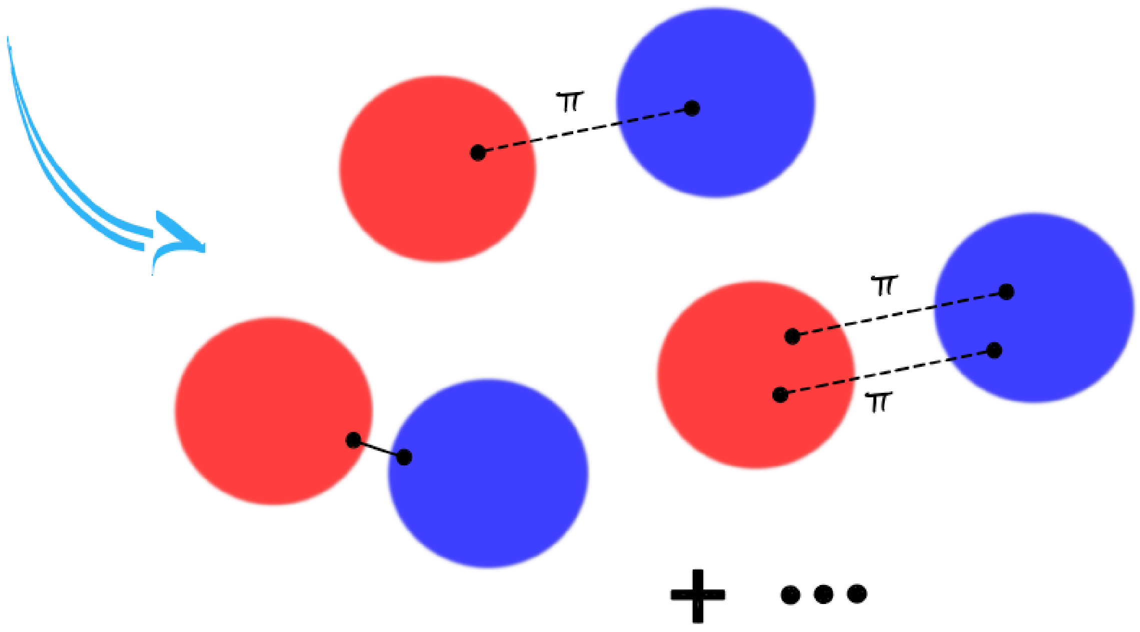
$$V = \sum_{i<j}^{A} V_{NN}^{(ij)} + \sum_{i<j<k}^{A} V_{NNN}^{(ijk)} + \cdots$$



NPLQCD
Nuclear Physics with Lattice QCD

$$SU(3)_{\mathrm{C}}$$





$$SU(2)_{\mathrm{L}} \times SU(2)_{\mathrm{R}} \sim SO(4)$$

Physica **96A** (1979) 327–340 © *North-Holland Publishing Co.*

PHENOMENOLOGICAL LAGRANGIANS*

STEVEN WEINBERG
Lyman Laboratory of Physics, Harvard University
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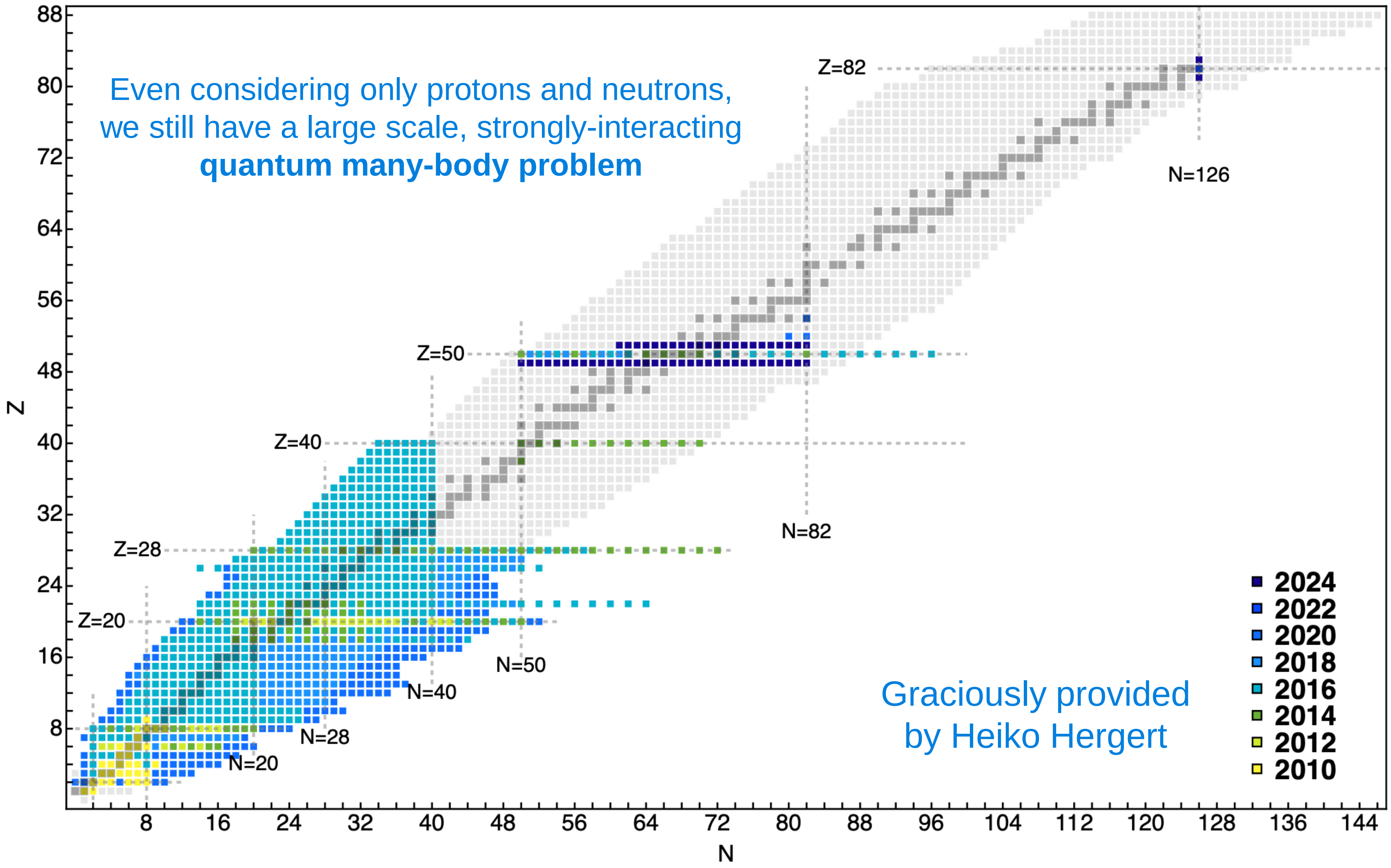
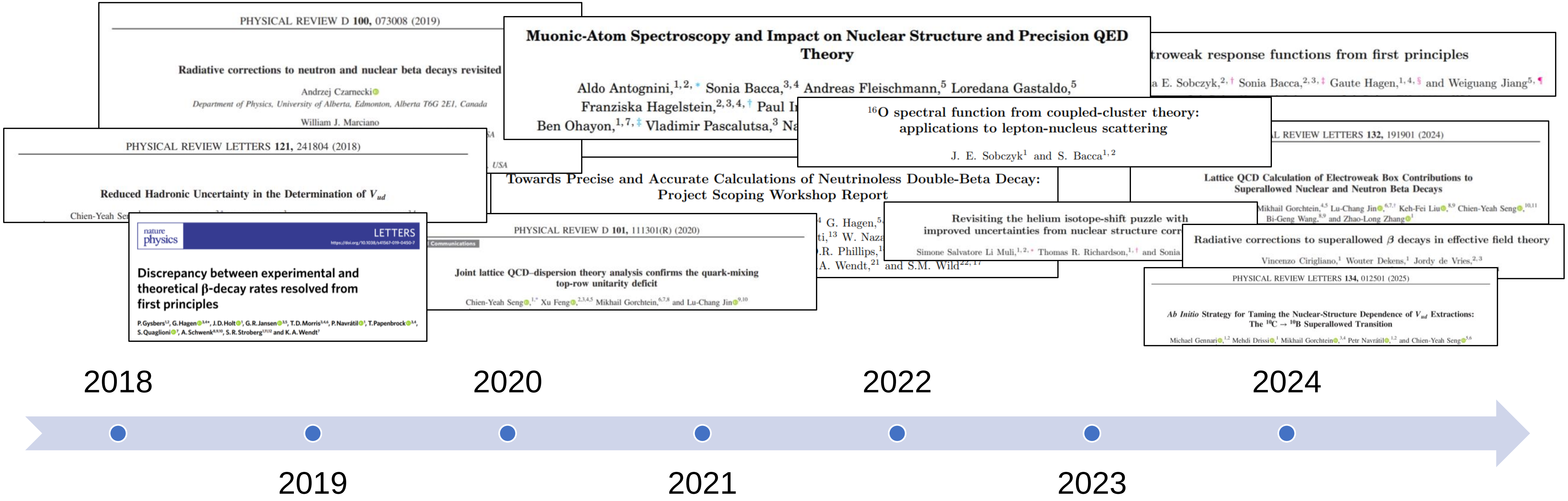
Harvard-Smithsonian Center for Astrophysics, Cambridge, Massachusetts 02138, USA

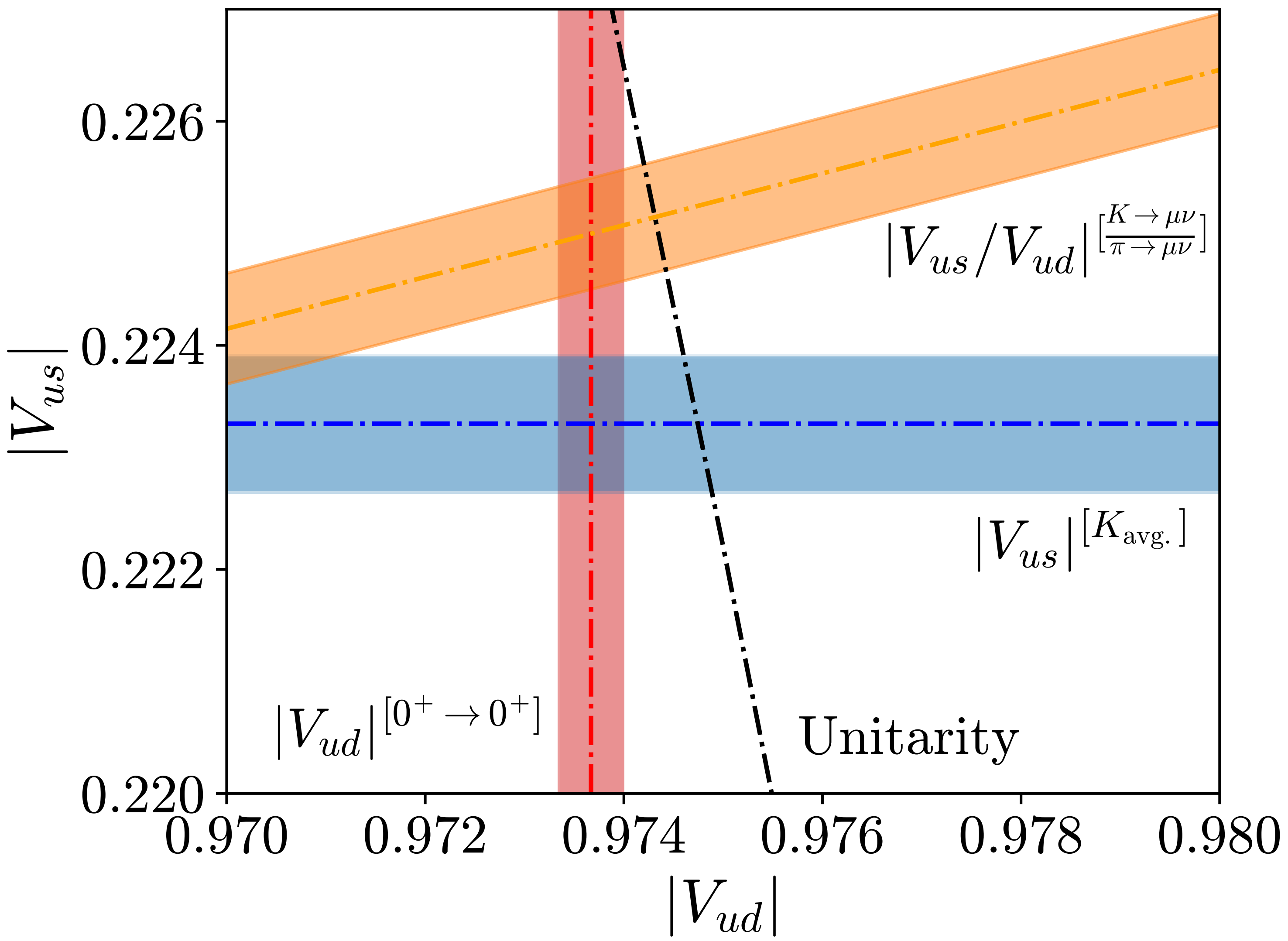
Even considering only protons and neutrons,
we still have a large scale, strongly-interacting
quantum many-body problem



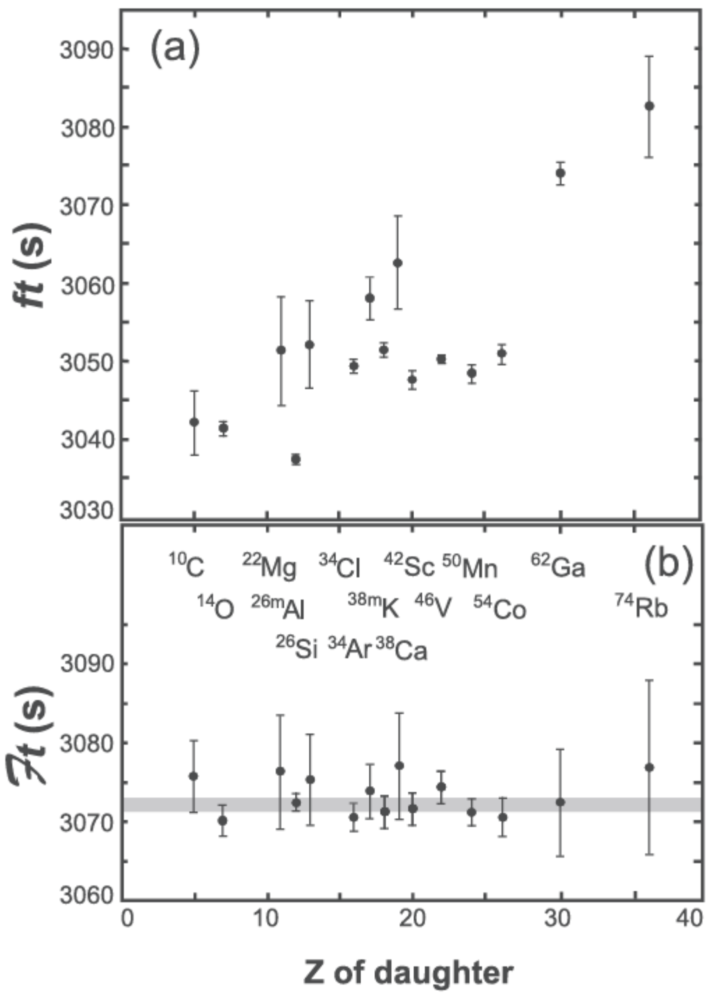
Graciously provided
by Heiko Hergert

We are entering an era in which the precision modelling of strongly-interacting many-body systems is becoming possible



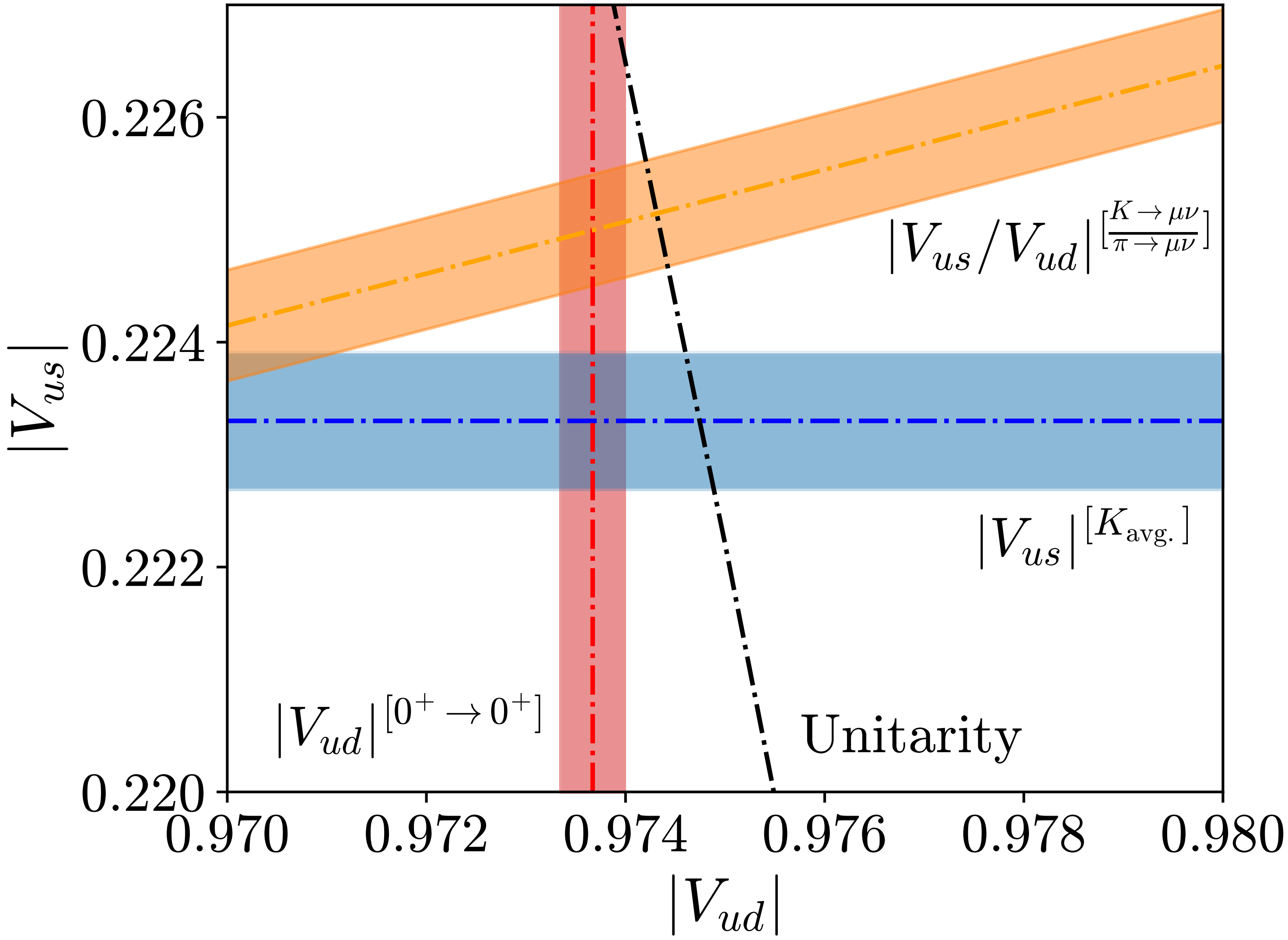


Hardy et al. PRC **102**, 045501 (2020)

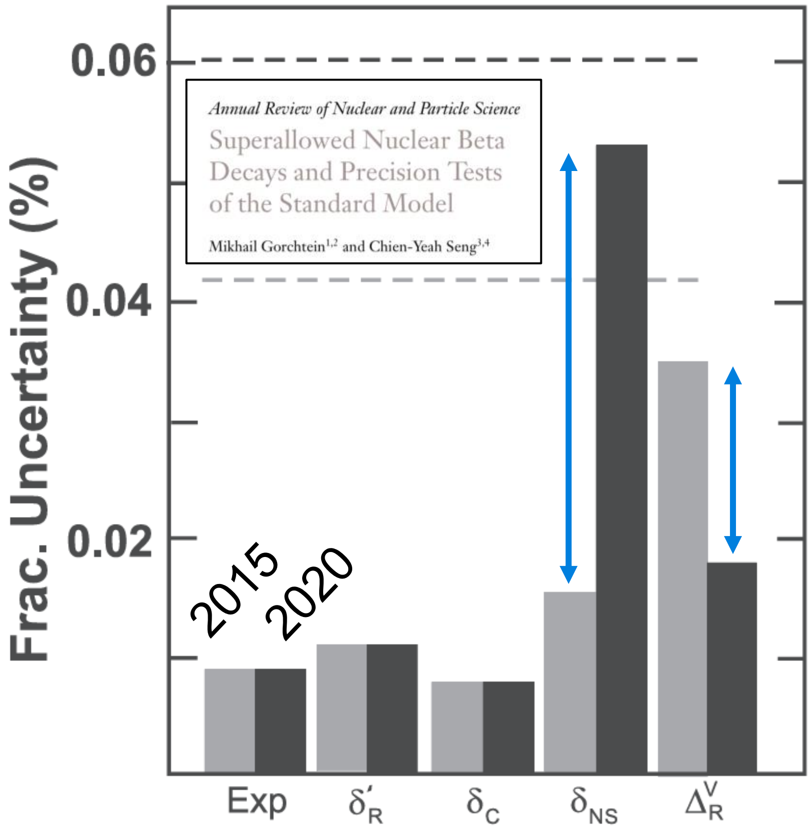


Requires **nuclear theory**

Particle Data Group. PRD **110**, 030001 (2024) and references therein



Hardy et al. PRC **102**, 045501 (2020)

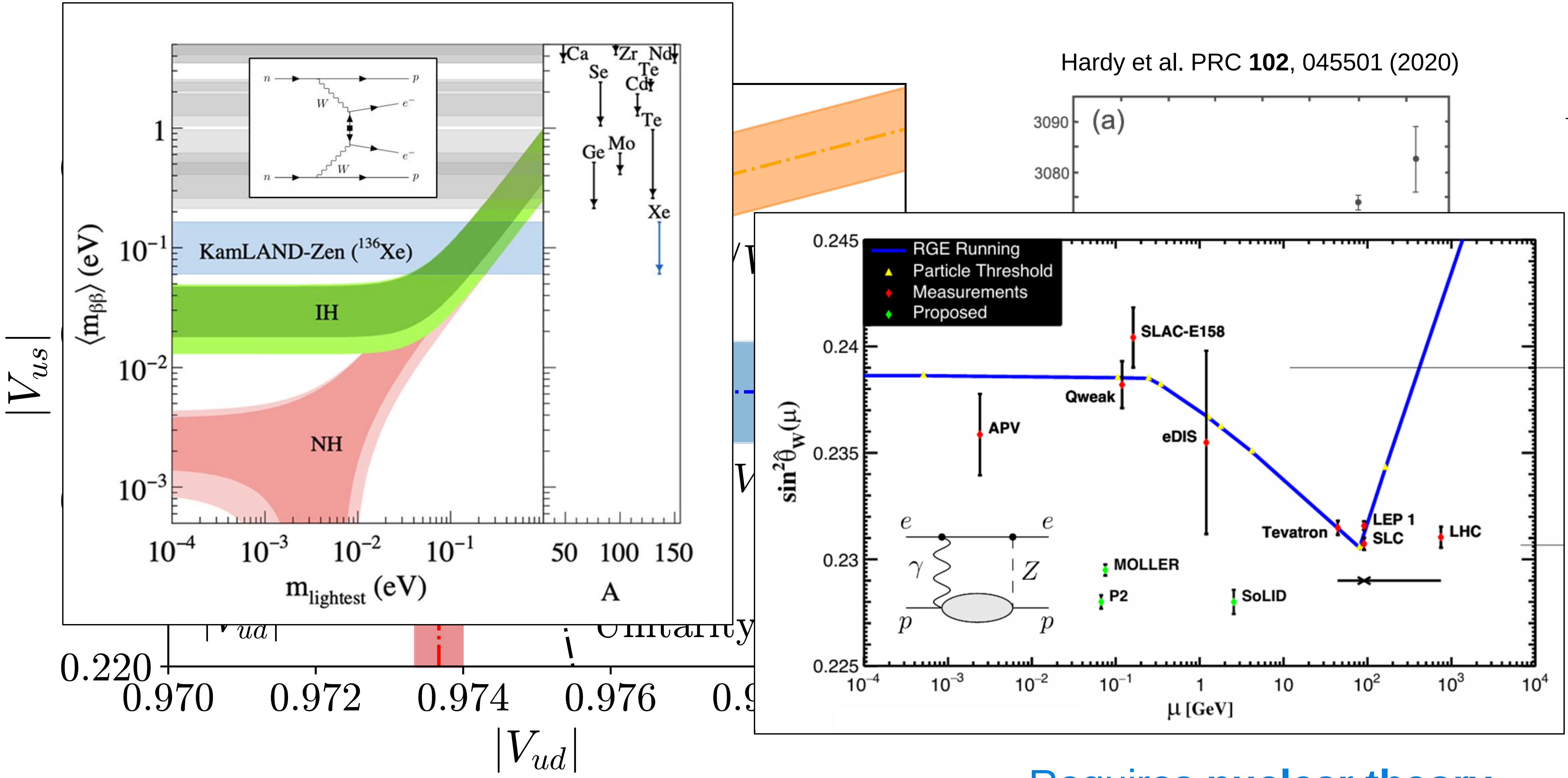


Seng et al. PRL **121**, 241804 (2018)

Requires **nuclear theory**

Particle Data Group. PRD **110**, 030001 (2024) and references therein

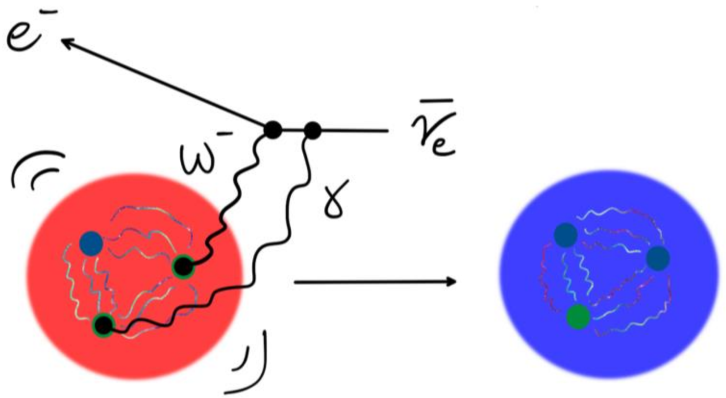
Hardy et al. PRC **102**, 045501 (2020)

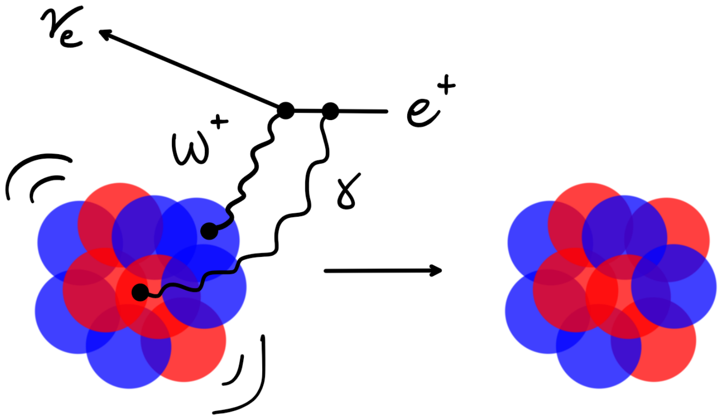


Requires **nuclear theory**

Particle Data Group. PRD **110**, 030001 (2024) and references therein

Nuclear beta decay in the Standard Model





$$|V_{ud}|^2 = \frac{K}{\mathcal{F}t\,(1+\Delta_R^V)}$$

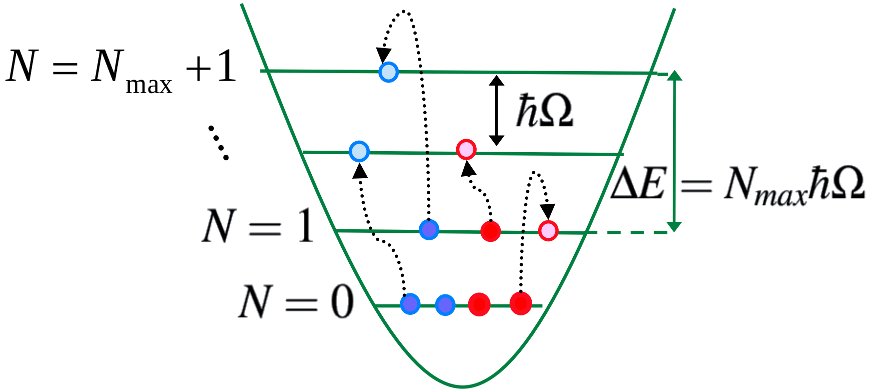
$$\mathcal{F}t = ft(1+\delta_R')(1-\delta_C+\delta_{NS})$$

Accounts for isospin symmetry breaking and electroweak radiative corrections

Particle Data Group. PRD **110**, 030001 (2024)

We use the ab initio no-core shell model, a quasi-exact approach for modelling nuclei as composite structures of nucleons interacting via internucleonic forces

$$H = \frac{1}{A}\sum_{i<j}^{A}\frac{(\vec{p}_i - \vec{p}_j)^2}{2m_N} + \sum_{i<j}^{A} V_{NN}^{(ij)} + \sum_{i<j<k}^{A} V_{NNN}^{(ijk)} + \cdots$$



Barrett et al. PPNP **69** (2013), pp.181-131

Anti-symmetrized products of many-body harmonic oscillator states

$$\left|\Psi_A^{J^\pi T}\right\rangle = \sum_{N=0}^{N_{max}} \sum_{\alpha} c_{N\alpha}^{J^\pi T} \left|\Phi_{N\alpha}^{J^\pi T}\right\rangle$$

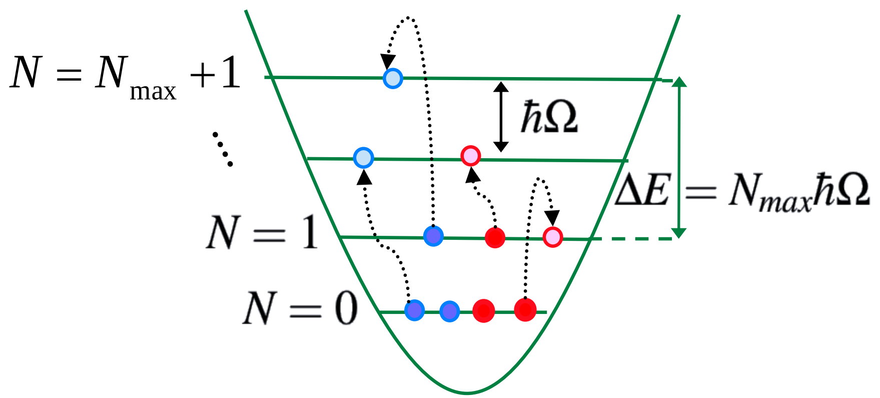
Journal of Research of the National Bureau of Standards Vol. 45, No. 4, October 1950 Research Paper 2133

An Iteration Method for the Solution of the Eigenvalue Problem of Linear Differential and Integral Operators[1]

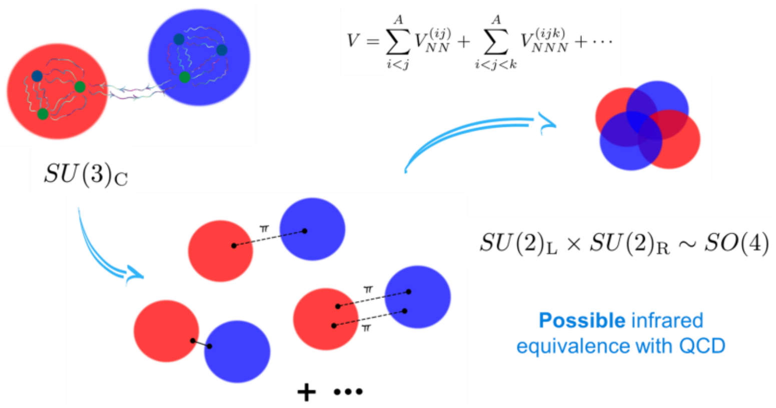
By Cornelius Lanczos

We use the ab initio no-core shell model, a quasi-exact approach for modelling nuclei as composite structures of nucleons interacting via internucleonic forces

No-core Shell Model



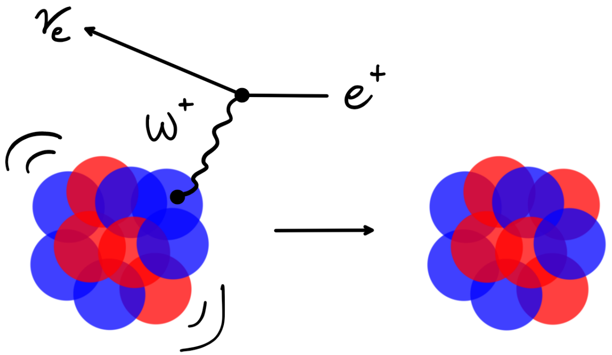
Chiral Effective Field Theory

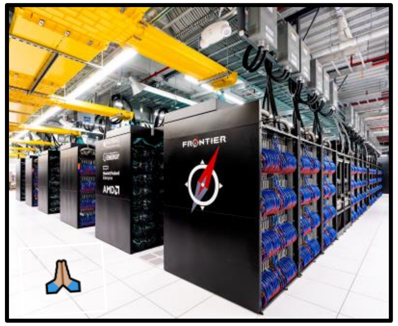


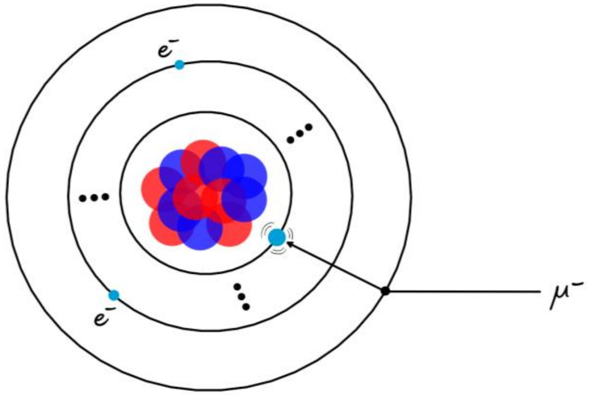
Lanczos Strengths Method

$$(z - H)|\Phi_R\rangle = O|\Phi_i\rangle$$

We use the ab initio no-core shell model, a quasi-exact approach for modelling nuclei as composite structures of nucleons interacting via internucleonic forces







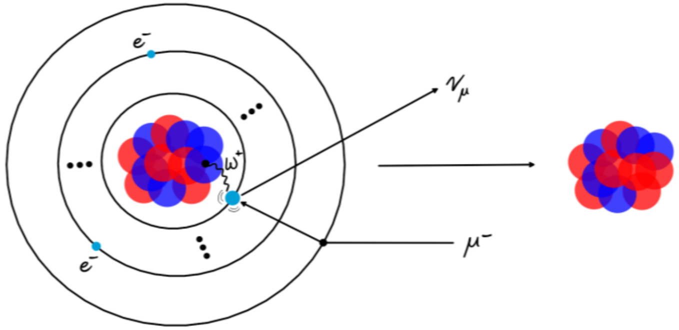
Super-allowed beta decay

Gennari et al.
PRL **134**, 012501 (2025)

PHYSICAL REVIEW LETTERS 134, 012501 (2025)

Ab Initio Strategy for Taming the Nuclear-Structure Dependence of V_{ud} Extractions: The $^{10}C \to {}^{10}B$ Superallowed Transition

Michael Gennari,[1,2] Mehdi Drissi,[1] Mikhail Gorchtein,[3,4] Petr Navrátil,[1,2] and Chien-Yeah Seng[5,6]



Muon capture

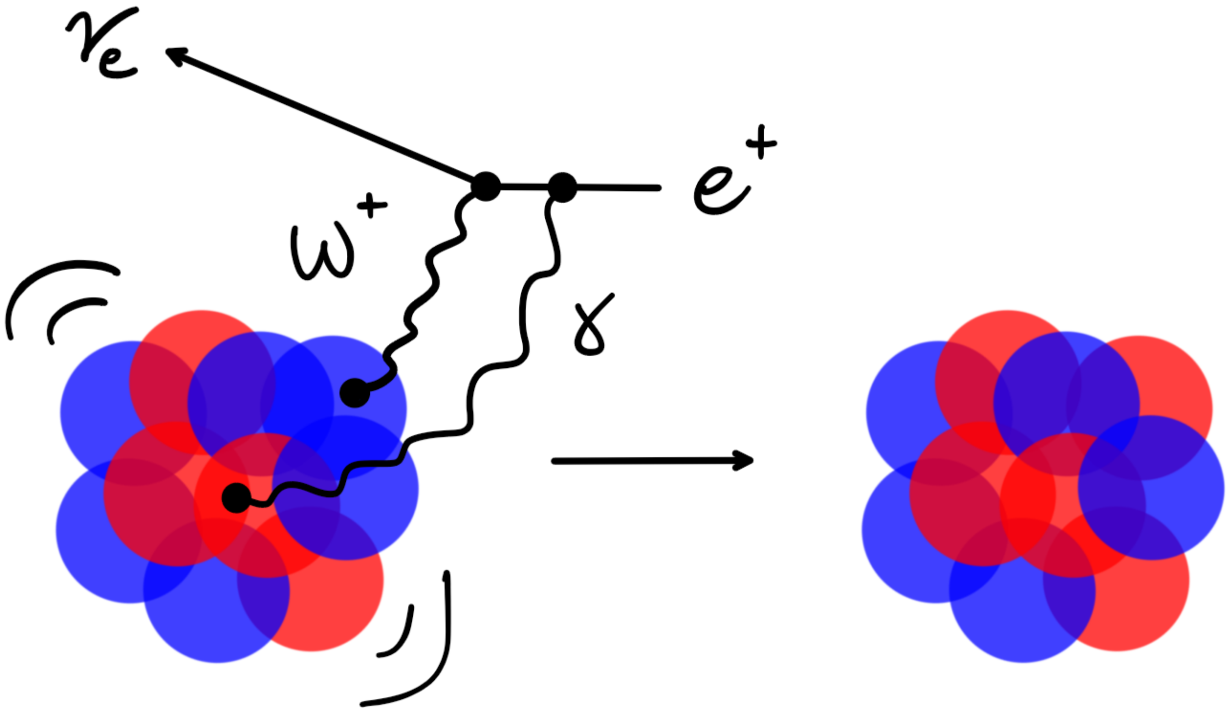
Najera et al. In progress.

Muonic atoms

Drissi et al. In prep.

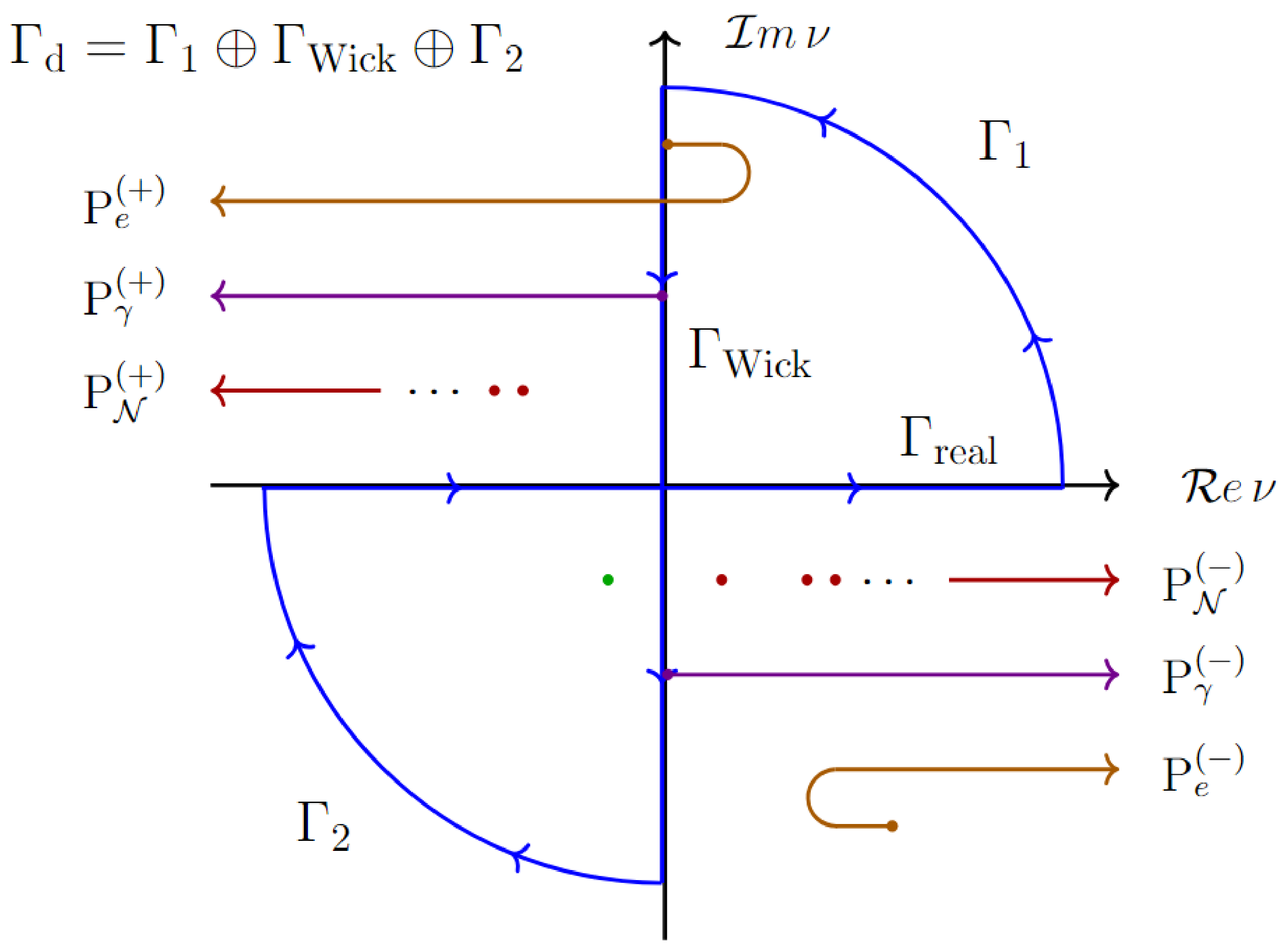
Amplitude and pole structure

$$\mathcal{A}_{fi} = \langle \Phi_f | O_2 \, (z - H)^{-1} \, O_1 | \Phi_i \rangle$$

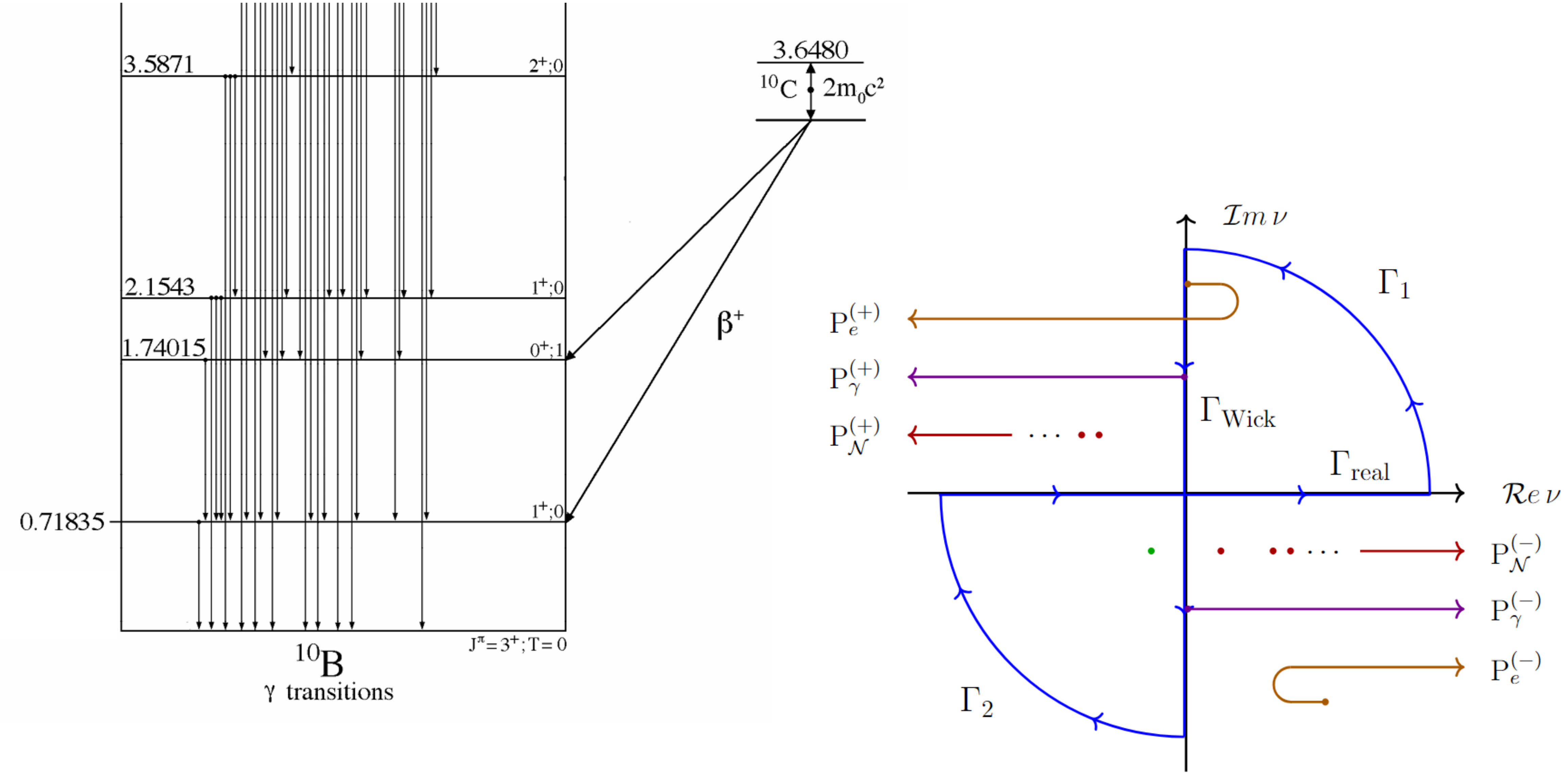


Gennari et al. PRL **134**, 012501

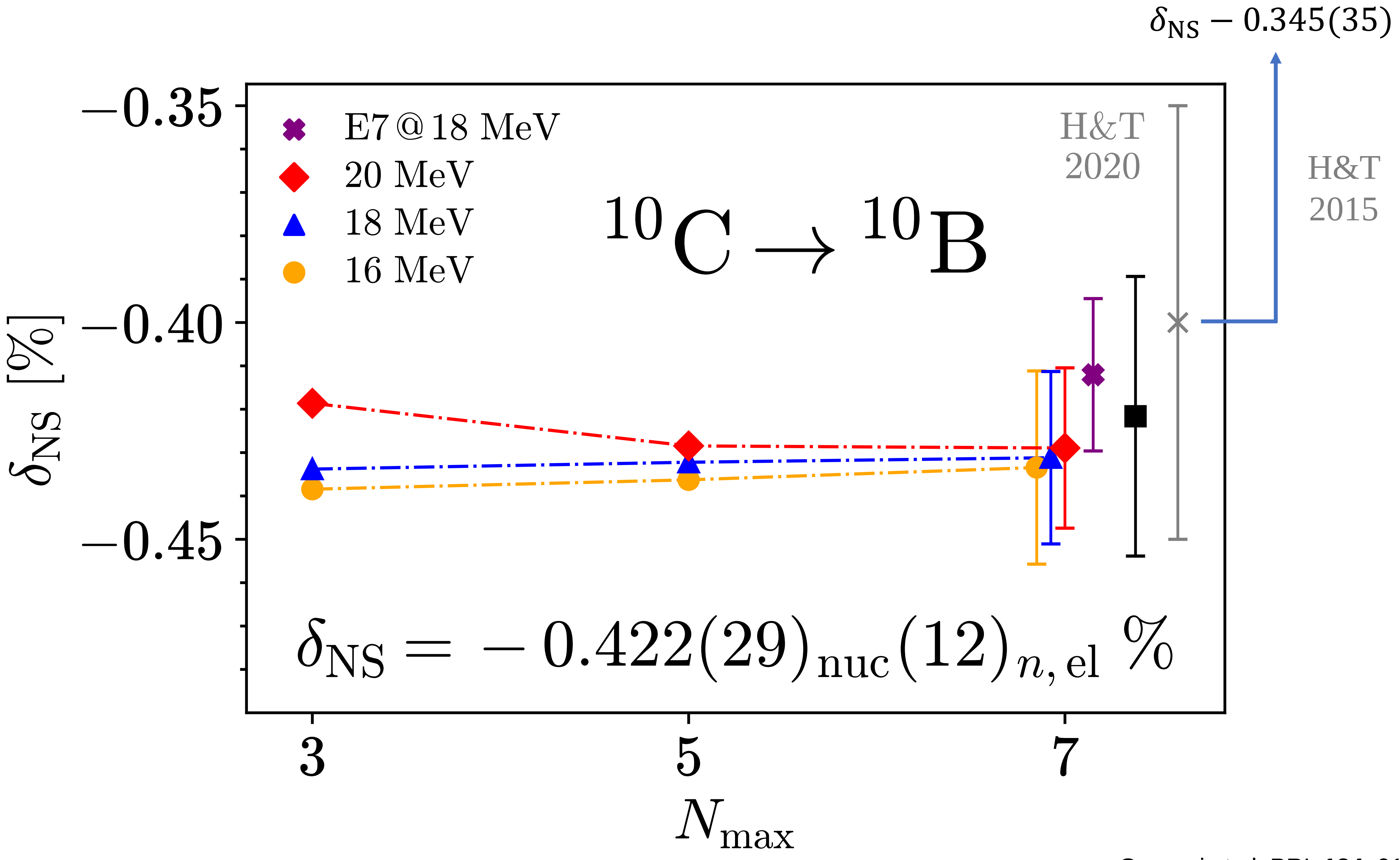
Amplitude and pole structure



Gennari et al. PRL **134**, 012501

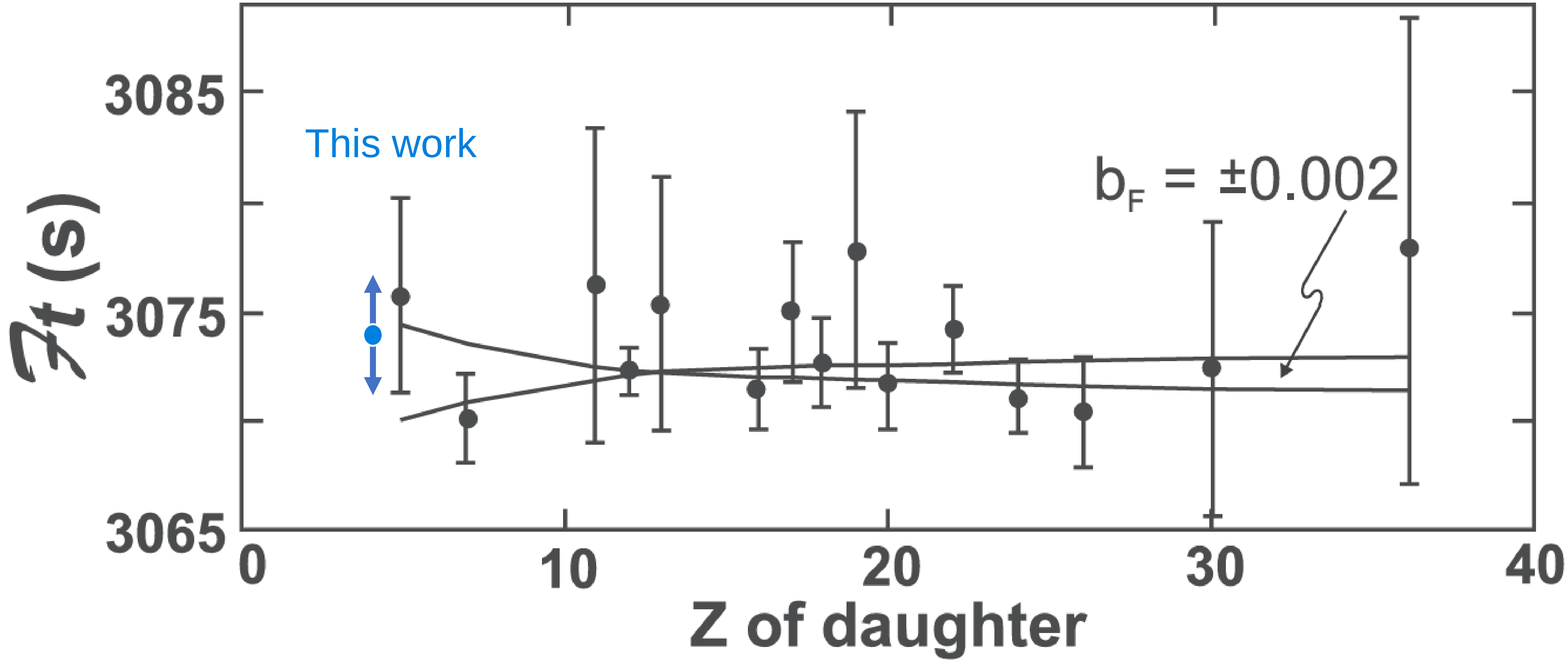


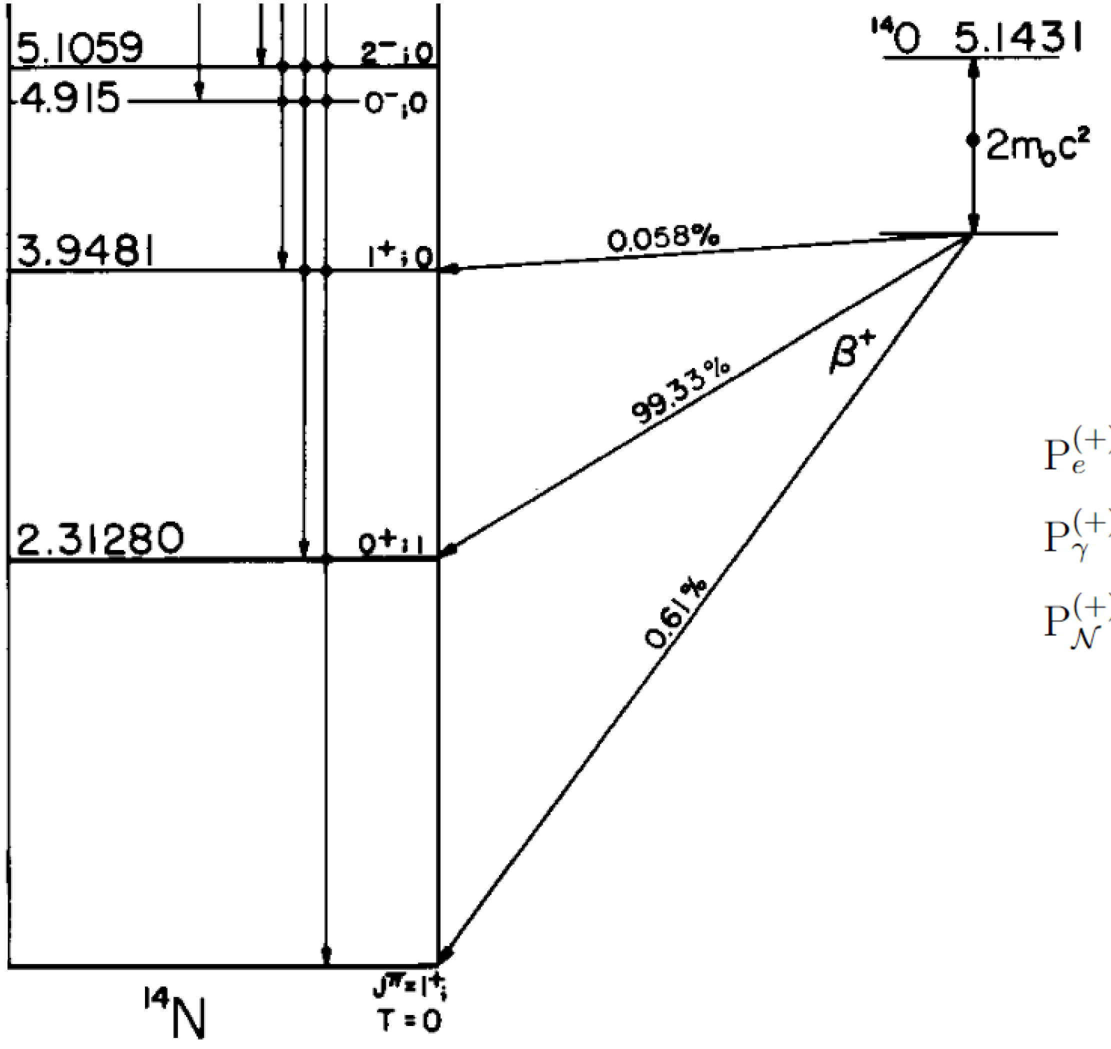
Gennari et al. PRL **134**, 012501

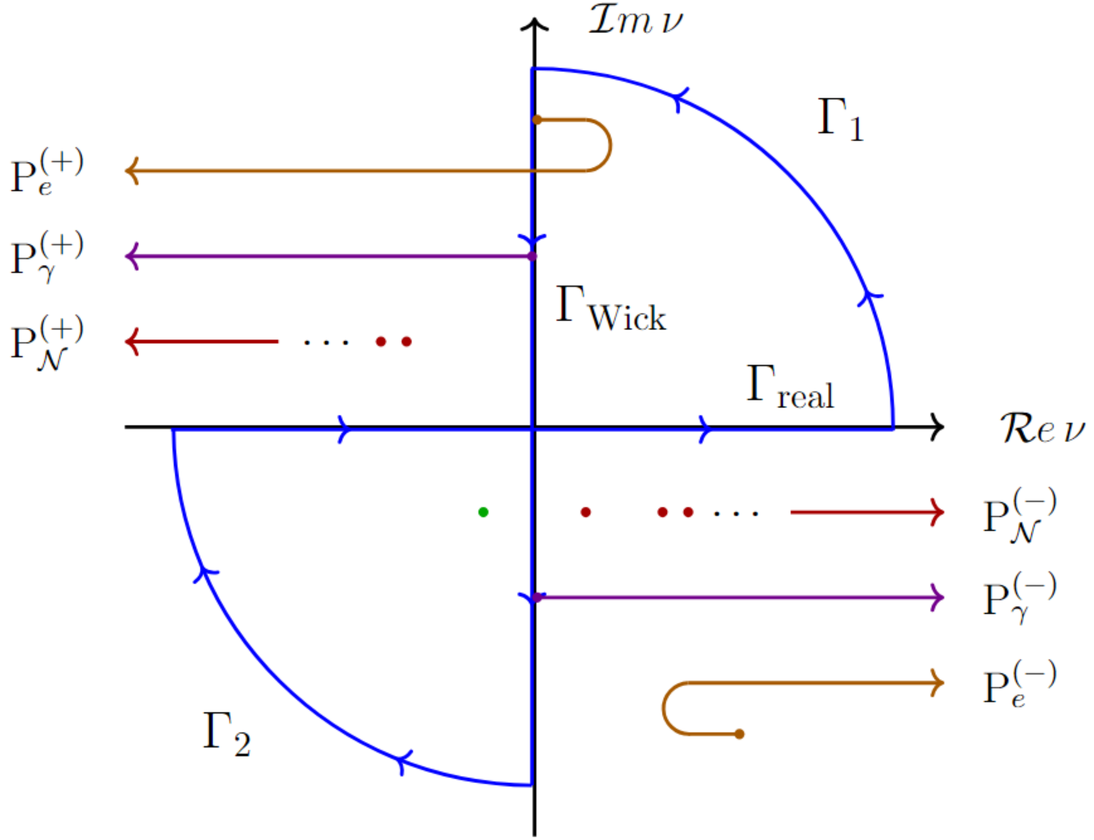


Gennari et al. PRL **134**, 012501

H&T 2020

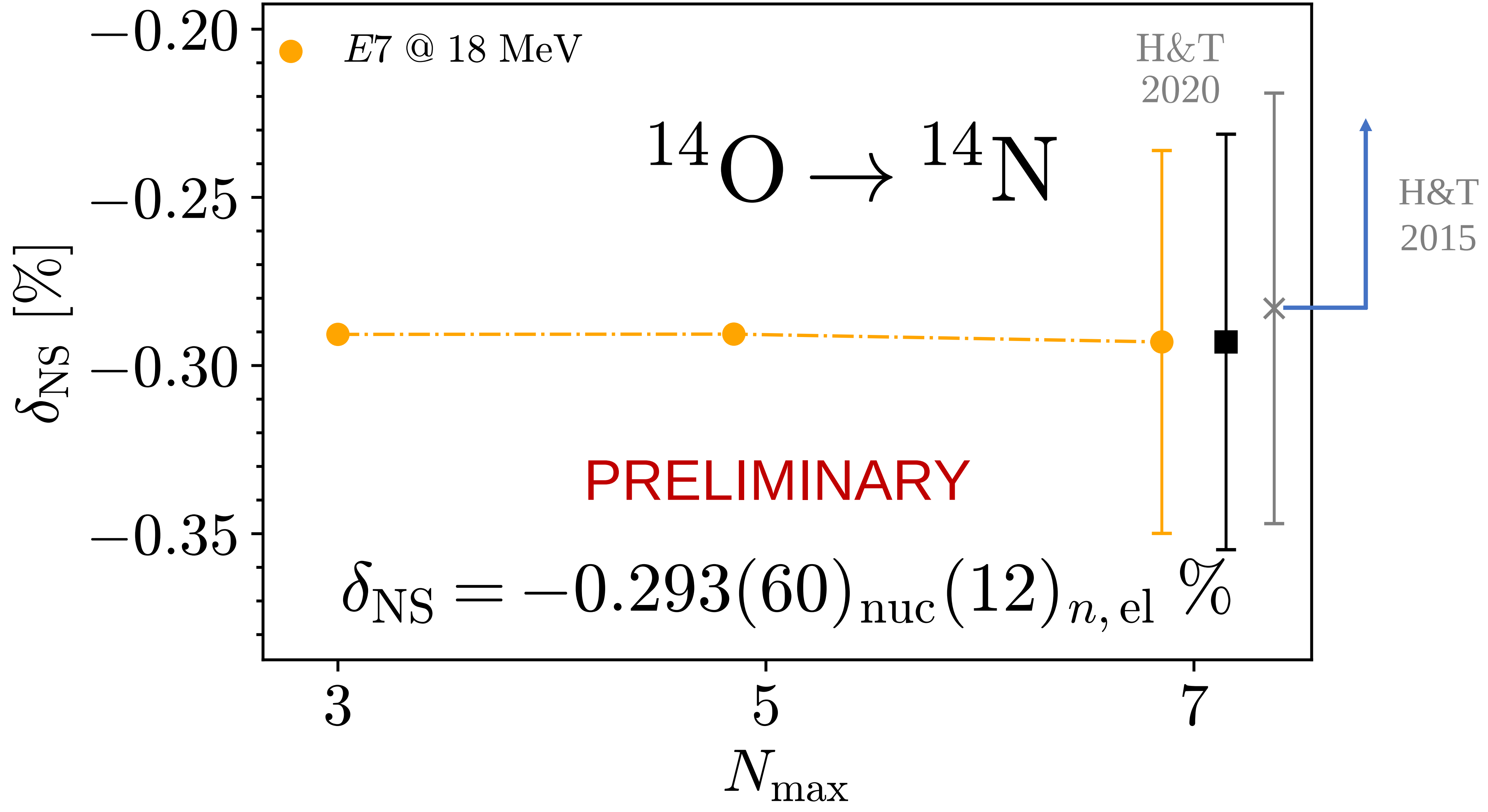

3085
3075
3065
This work
$b_F = \pm 0.002$
0
10
20
30
40
Z of daughter
$\mathcal{F}t$ (s)





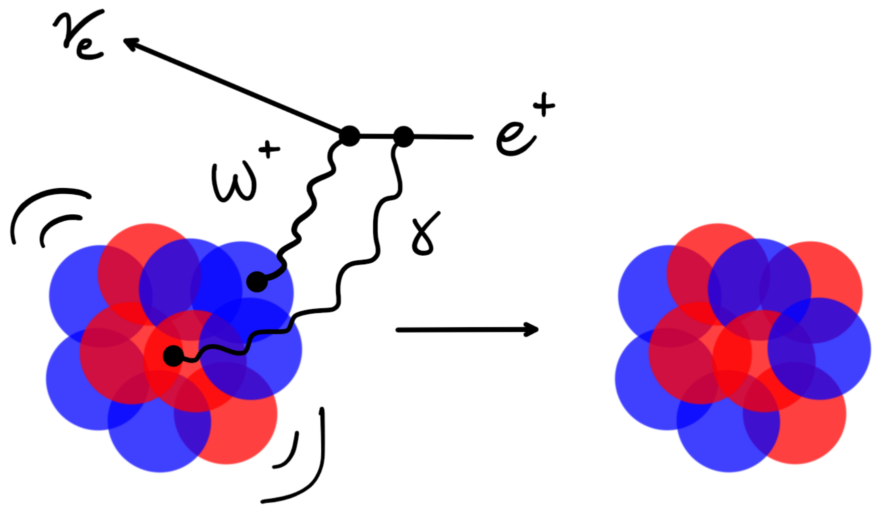
Gennari et al. PRL **134**, 012501

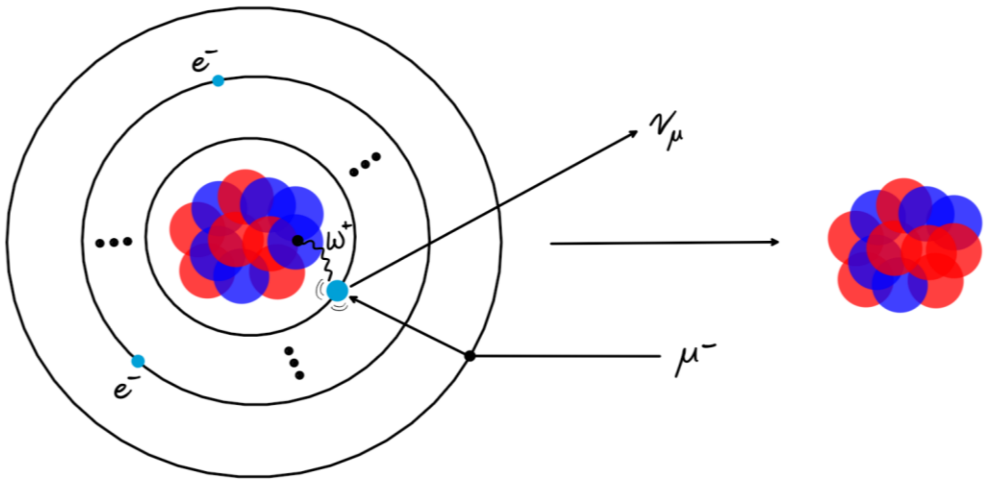
$\delta_{NS} = -0.245(50)$



The future of electroweak theory in nuclei

- **Improved precision for electroweak radiative corrections in nuclei** with the Lanczos Strengths Method coupled to the ab initio NCSM
- Systematic improvements available, e.g., consistent currents in chiral EFT
- Limited by matching of high-energy QCD to low-energy nuclear theory





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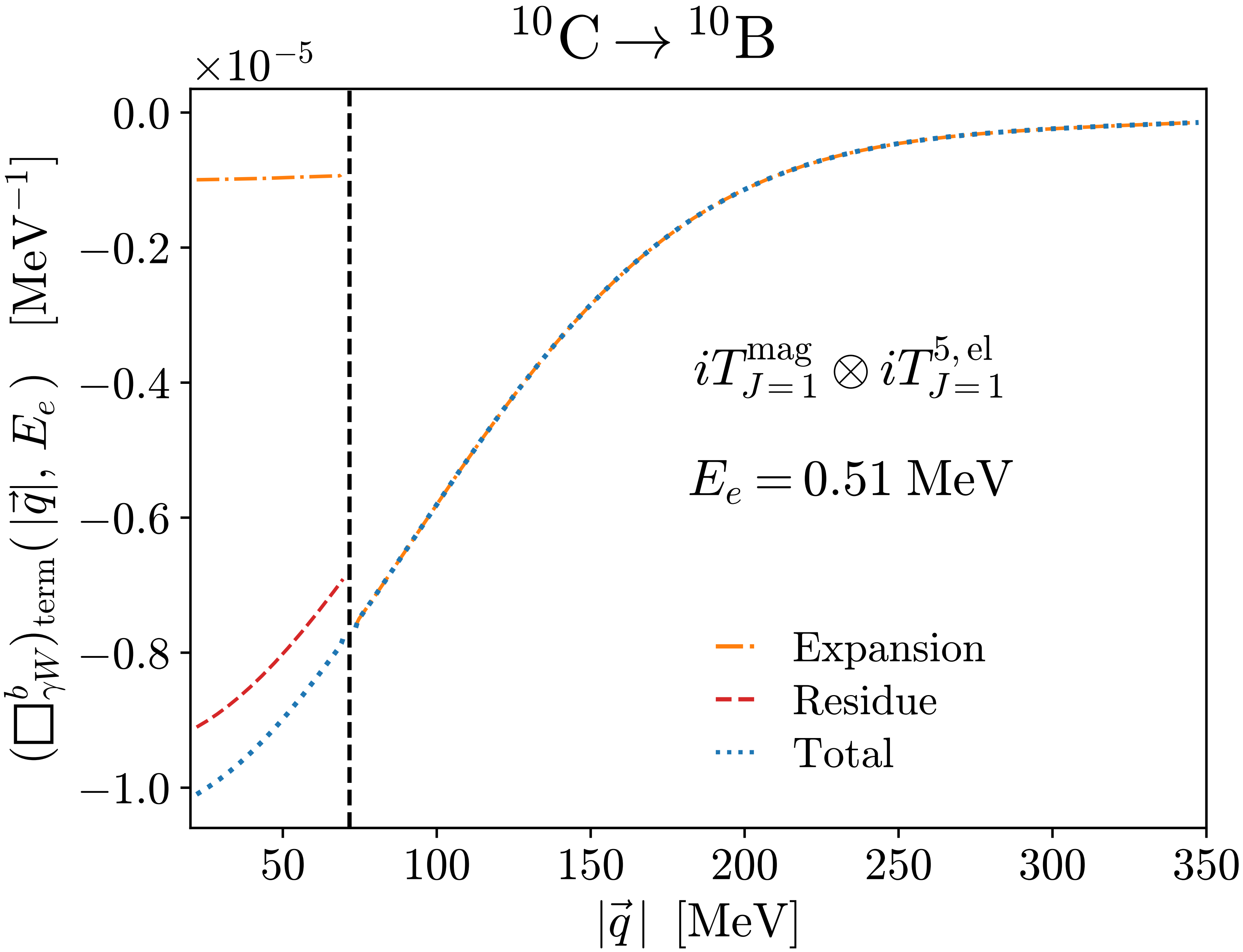
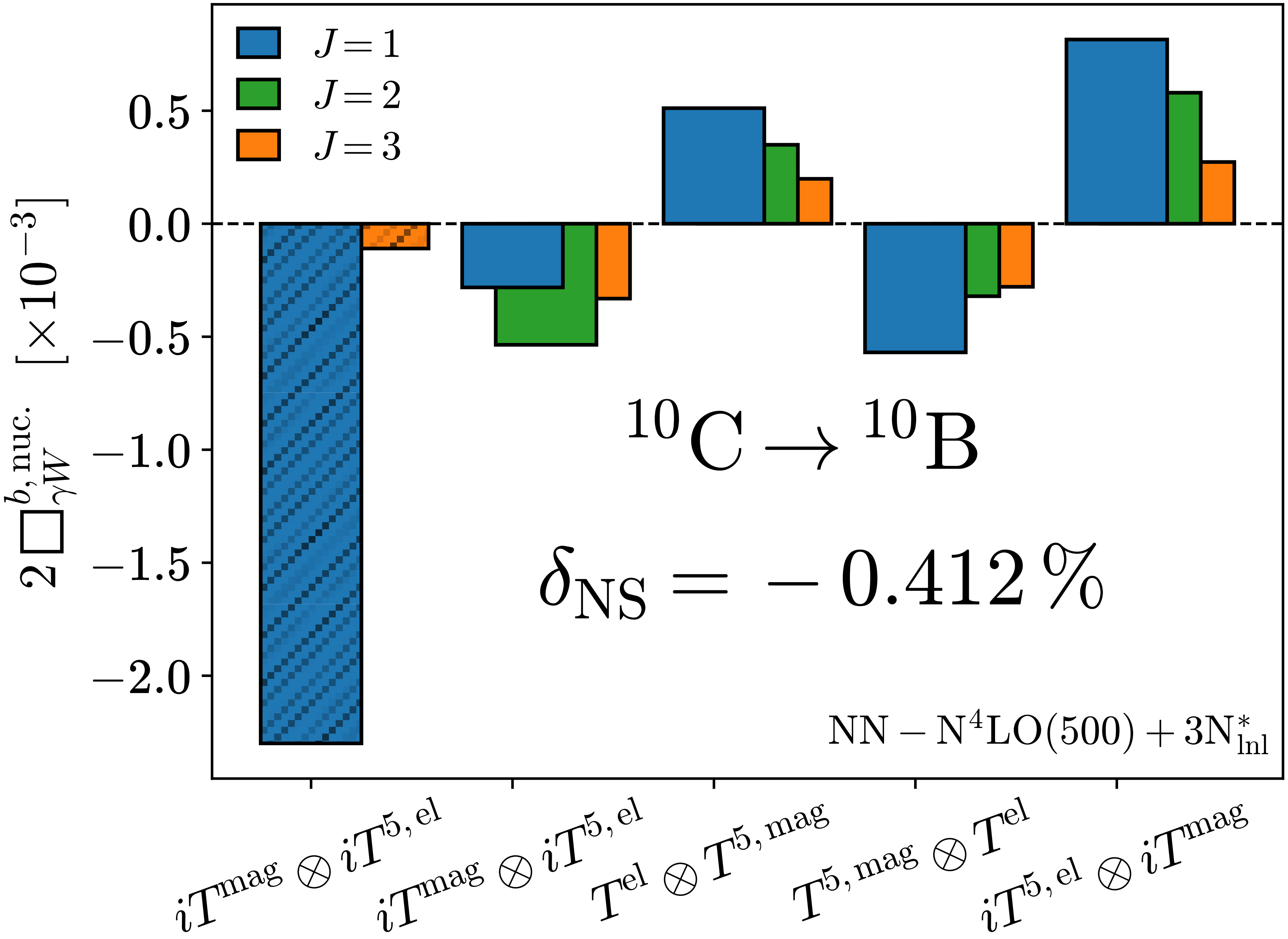
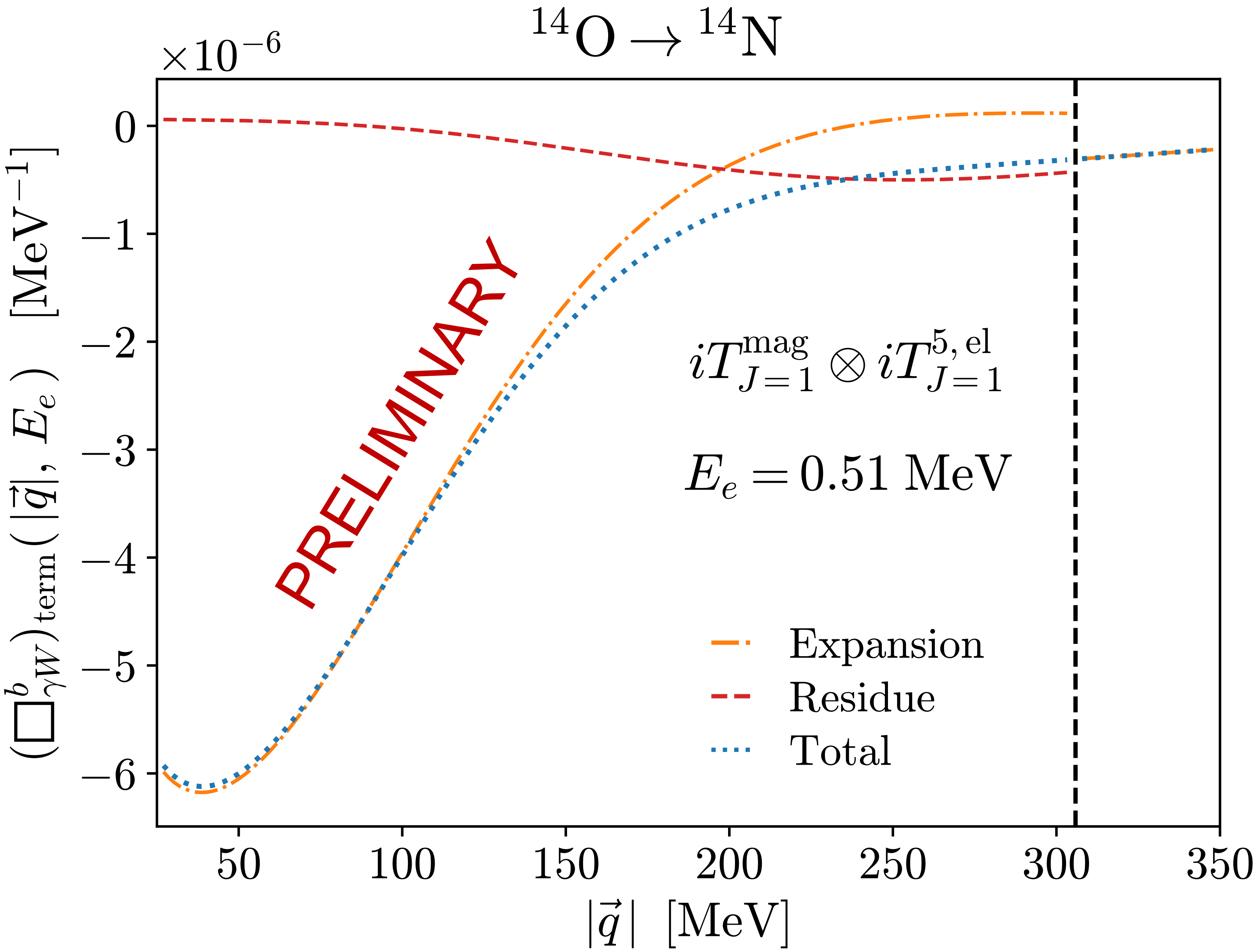


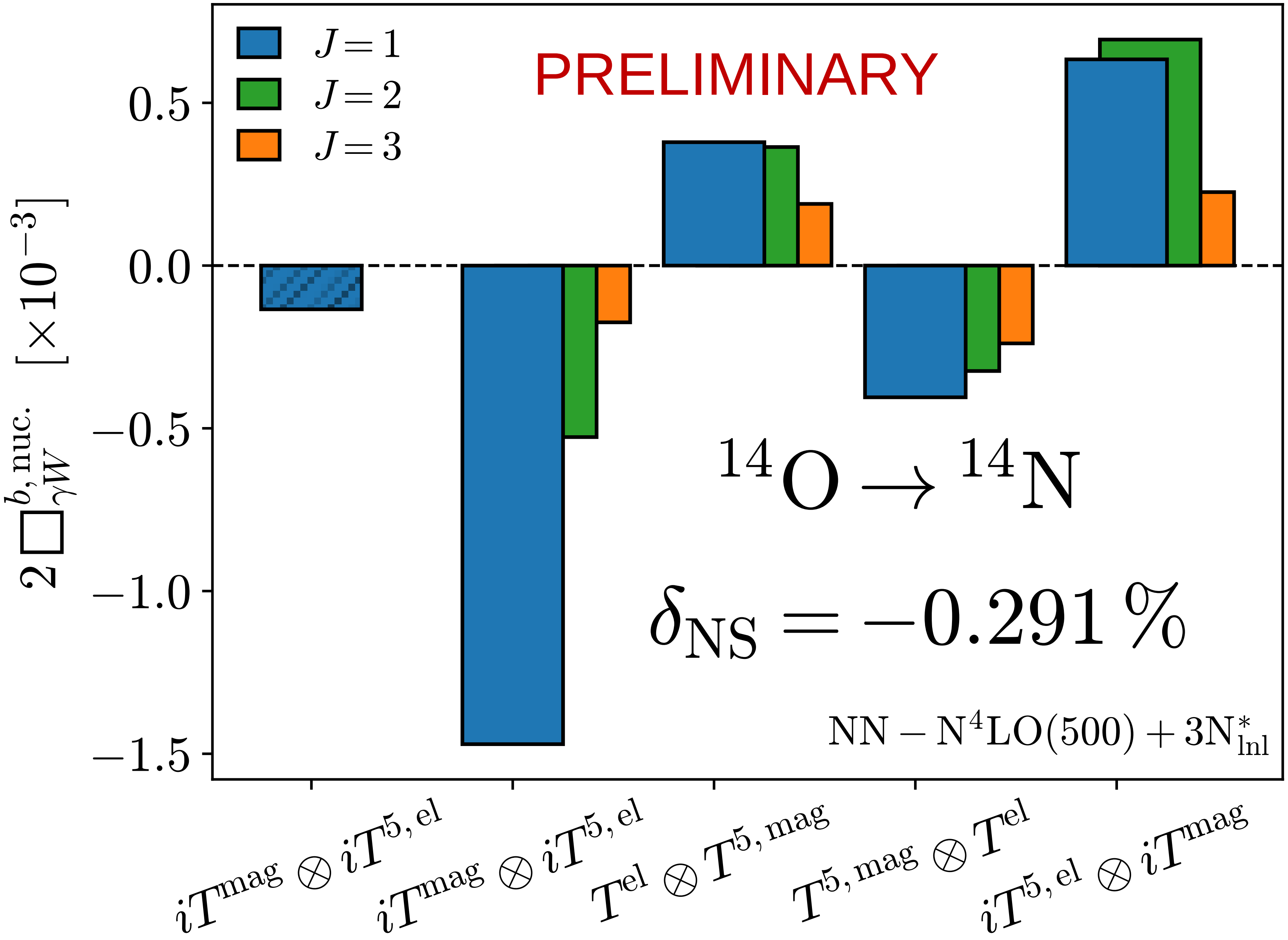
Gennari et al. PRL **134**, 012501



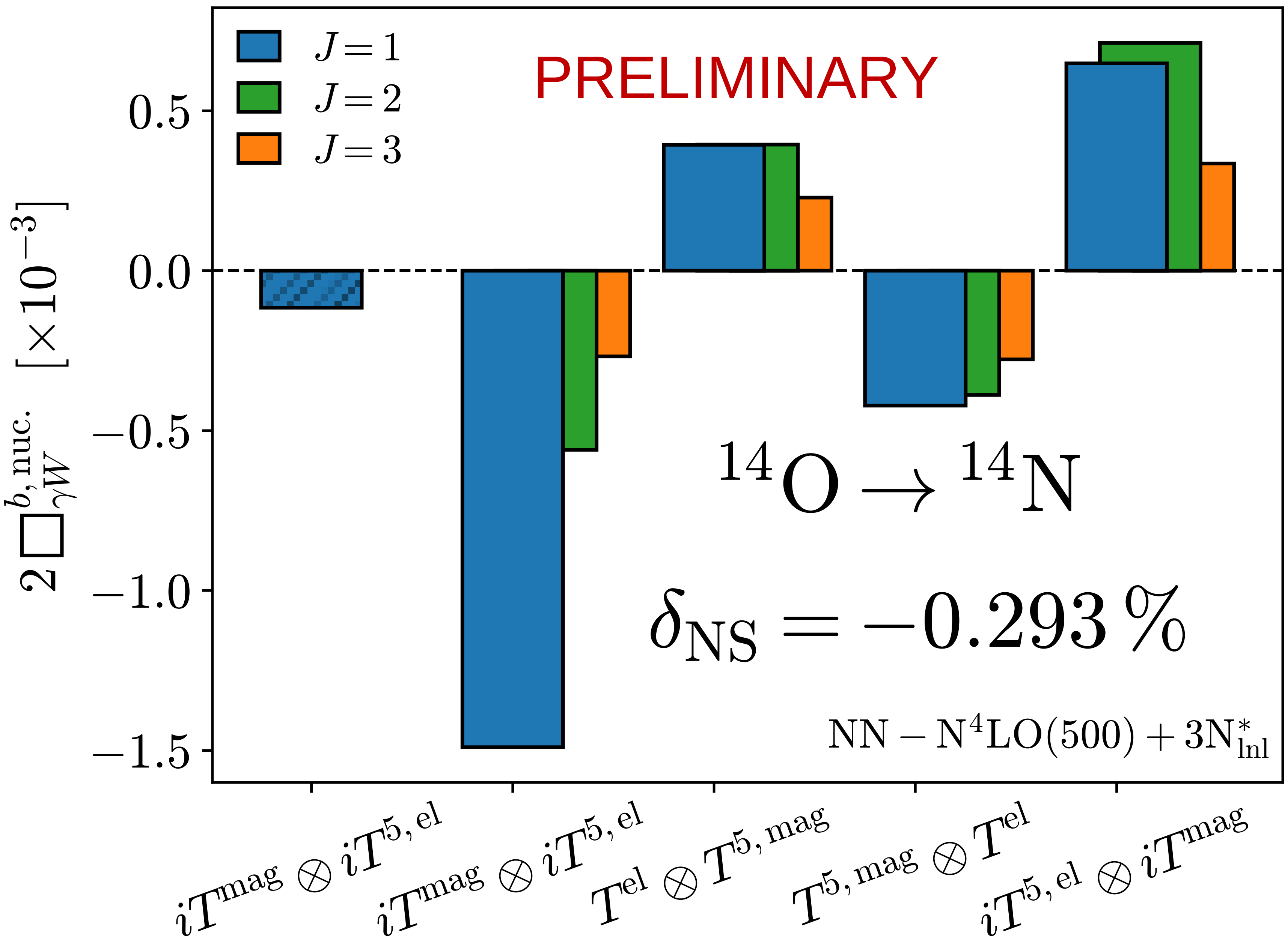
Gennari et al. PRL **134**, 012501



$^{14}\mathrm{O} \rightarrow {}^{14}\mathrm{N}$
$\times 10^{-6}$
$(\Box^{b}_{\gamma W})_{\mathrm{term}}(|\vec{q}|,\ E_e)$ [MeV^{-1}]
0
−1
−2
−3
−4
−5
−6
50
100
150
200
250
300
350
$|\vec{q}|$ [MeV]
PRELIMINARY
$iT^{\mathrm{mag}}_{J=1} \otimes iT^{5,\,\mathrm{el}}_{J=1}$
$E_e = 0.51$ MeV
Expansion
Residue
Total



PRELIMINARY
$J=1$
$J=2$
$J=3$
$2\Box_{\gamma W}^{b,\,\mathrm{nuc.}}\ [\times 10^{-3}]$
0.5
0.0
−0.5
−1.0
−1.5
$^{14}\mathrm{O} \to {}^{14}\mathrm{N}$
$\delta_{\mathrm{NS}} = -0.291\,\%$
$\mathrm{NN-N^4LO(500)+3N^*_{lnl}}$
$iT^{\mathrm{mag}} \otimes iT^{5,\,\mathrm{el}}$
$iT^{\mathrm{mag}} \otimes iT^{5,\,\mathrm{el}}$
$T^{\mathrm{el}} \otimes T^{5,\,\mathrm{mag}}$
$T^{5,\,\mathrm{mag}} \otimes T^{\mathrm{el}}$
$iT^{5,\,\mathrm{el}} \otimes iT^{\mathrm{mag}}$



$J = 1$
$J = 2$
$J = 3$
PRELIMINARY
0.5
0.0
−0.5
−1.0
−1.5
$2\Box_{\gamma W}^{b,\,\mathrm{nuc.}}\ [\times 10^{-3}]$
$^{14}\mathrm{O} \to {}^{14}\mathrm{N}$
$\delta_{\mathrm{NS}} = -0.293\,\%$
$\mathrm{NN-N^4LO(500)+3N^*_{lnl}}$
$iT^{\mathrm{mag}} \otimes iT^{5,\,\mathrm{el}}$
$iT^{\mathrm{mag}} \otimes iT^{5,\,\mathrm{el}}$
$T^{\mathrm{el}} \otimes T^{5,\,\mathrm{mag}}$
$T^{5,\,\mathrm{mag}} \otimes T^{\mathrm{el}}$
$iT^{5,\,\mathrm{el}} \otimes iT^{\mathrm{mag}}$

We use the ab initio no-core shell model, a quasi-exact approach for modelling nuclei as composite structures of nucleons interacting via internucleonic forces

Lanczos Algorithm

$$
\begin{aligned}
H|\eta_0\rangle &= \alpha_0|\eta_0\rangle + \beta_0|\eta_1\rangle \\
H|\eta_1\rangle &= \beta_0|\eta_0\rangle + \alpha_1|\eta_1\rangle + \beta_1|\eta_2\rangle \\
H|\eta_2\rangle &= \beta_1|\eta_1\rangle + \alpha_2|\eta_2\rangle + \beta_2|\eta_3\rangle \\
H|\eta_3\rangle &= \beta_2|\eta_2\rangle + \alpha_3|\eta_3\rangle + \beta_3|\eta_4\rangle \\
&\vdots
\end{aligned}
$$

$$E = P^{-1} H_{\text{Lanczos}} P$$

Lanczos. JRNBS **45** no. 4 (1950)

We use the ab initio no-core shell model, a quasi-exact approach for modelling nuclei as composite structures of nucleons interacting via internucleonic forces

Lanczos Strengths Method

$$\mathcal{A}_{fi} = \langle \Phi_f | O_2 \, (z - H)^{-1} \, O_1 | \Phi_i \rangle = \langle \Phi_f | O_2 | \Phi_\mathrm{R} \rangle$$

$$(z - H) | \Phi_\mathrm{R} \rangle = O | \Phi_i \rangle$$

Method for extracting many-body resolvent amplitudes

Haydock. JPA **7**, 2120 (1974)   Bessis & Villani. JMP **16**, 462 (1975)   Dagotto. RMP **66**, 763 (1994)

We use the ab initio no-core shell model, a quasi-exact approach for modelling nuclei as composite structures of nucleons interacting via internucleonic forces

Lanczos Strengths Method

$$\langle \Phi_n | O_1 | \Phi_i \rangle = \left| \langle \Phi_i | O_1^\dagger O_1 | \Phi_i \rangle \right| \; P_{n0}^\dagger$$

$$\langle \Phi_f | O_2 | \Phi_n \rangle = \left| \langle \Phi_f | O_2^\dagger O_2 | \Phi_f \rangle \right| \sum_m \langle \Phi_f | O_2 | \eta_m \rangle \, P_{mn}$$

Haydock. JPA **7**, 2120 (1974)   Bessis & Villani. JMP **16**, 462 (1975)   Dagotto. RMP **66**, 763 (1994)

Tree-level

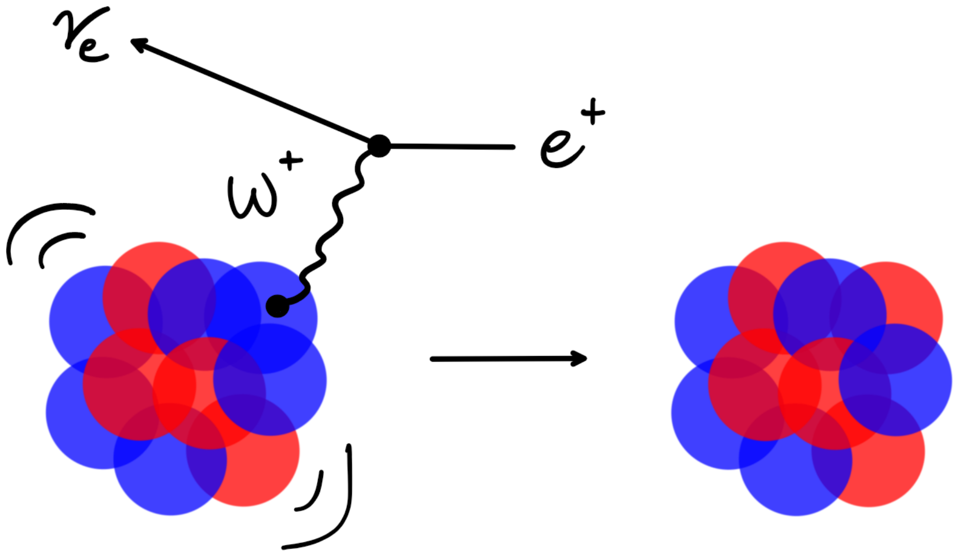
$$\mathcal{M} = \underline{\mathcal{M}_{\text{tree}}} + \delta\mathcal{M}_{\text{one-loop}} + \cdots$$

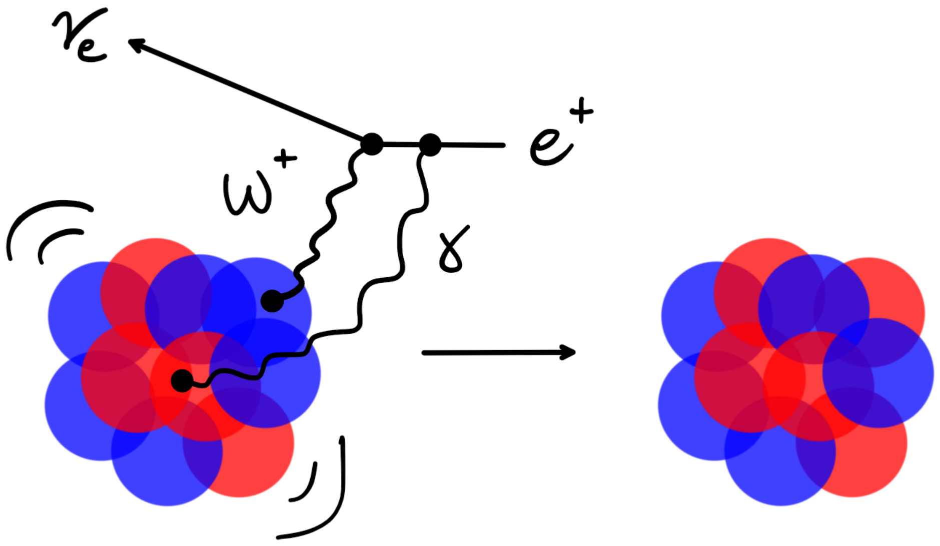
$$\mathcal{M}_{\text{tree}}(p_f, p_i, k_f, k_i) = -\frac{G_F}{\sqrt{2}} L_\lambda(k_f, k_i)\, \underline{\langle \Phi_f\,;\, p_f | J_W^{\dagger\,\lambda}(q) | \Phi_i\,;\, p_i \rangle}$$

Seng & Gorchtein. PRC **107**, 035503 (2023)

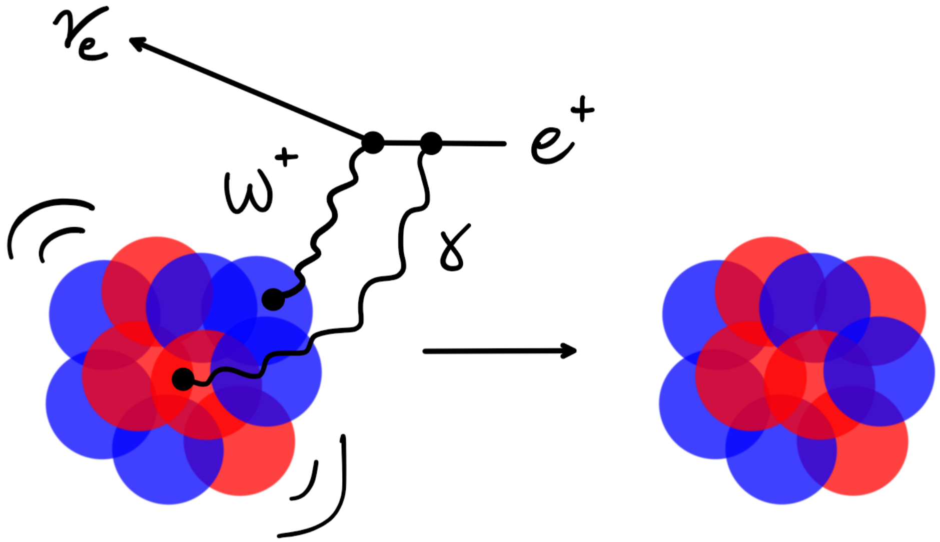
One-loop radiative correction

$$\mathcal{M} = \mathcal{M}_{\text{tree}} + \delta\mathcal{M}_{\text{one-loop}} + \cdots$$





One-loop radiative correction



Convenient [not critical] approximations

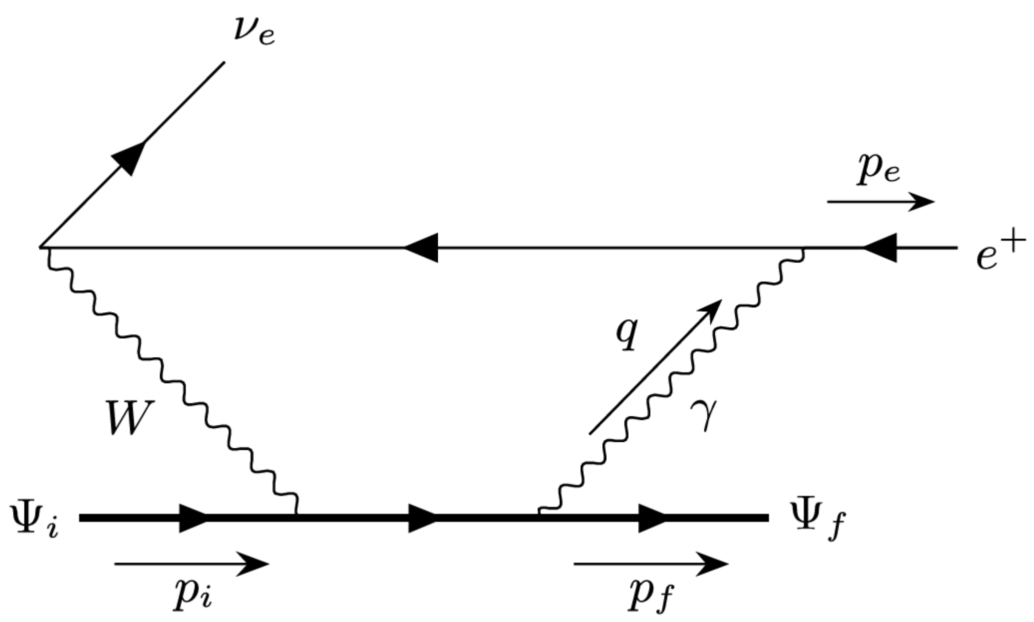
- Forward scattering limit
- Neglect recoil of final nucleus

$$\delta\mathcal{M}_{\text{one-loop}} = \Box_{\gamma W}(E_e)\,\mathcal{M}_{\text{tree}}$$

$$\Box^b_{\gamma W}(E_e) = \frac{e^2}{M}\int\frac{d^4q}{(2\pi)^4}\,R_W(q)\,R_e(q)\,R_\gamma(q)\left(M\frac{p_e\cdot q}{p\cdot p_e} - \frac{q^2}{\nu}\right)\frac{T_3(\nu,|\vec{q}|)}{f_+}$$

Seng & Gorchtein. PRC **107**, 035503 (2023)

One-loop radiative correction



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Seng & Gorchtein. PRC **107**, 035503 (2023)

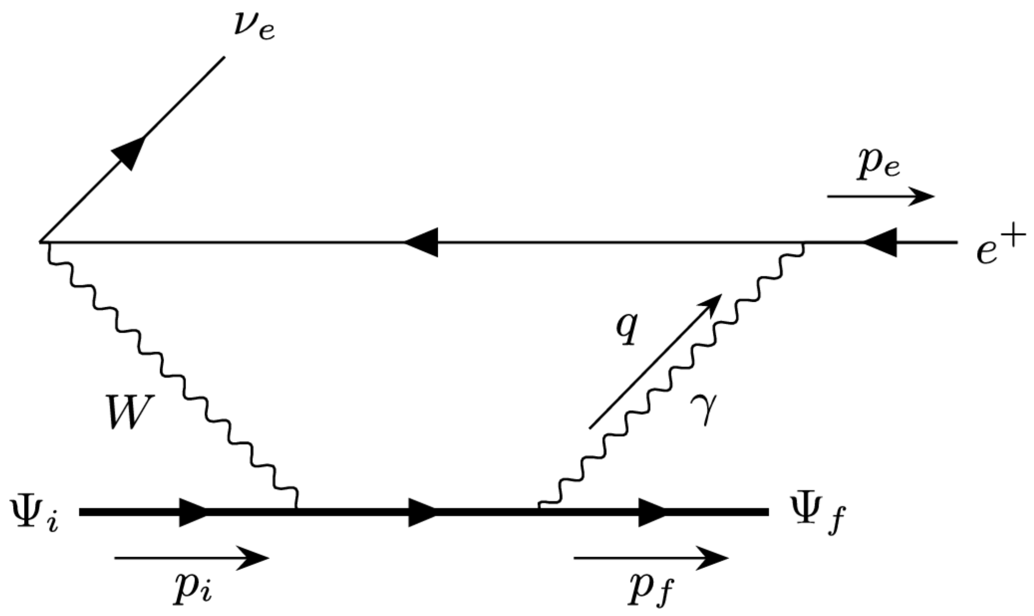
One-loop radiative correction

$$\mathcal{M} = \mathcal{M}_{\text{tree}} + \delta\mathcal{M}_{\text{one-loop}} + \cdots$$

$$\delta\mathcal{M}_{\text{one-loop}} = -i\sqrt{2}\, G_F e^2\, L_\lambda(k_f, k_i) \int \frac{d^4q}{(2\pi)^4} R_W(q)\, R_e(q)\, R_\gamma(q) \times \left[\epsilon^{\mu\nu\alpha\lambda} q_\alpha T_{\mu\nu}(p_f, p_i; q)\right]$$

Seng & Gorchtein. PRC **107**, 035503 (2023)

One-loop radiative correction



Convenient [not critical] approximations

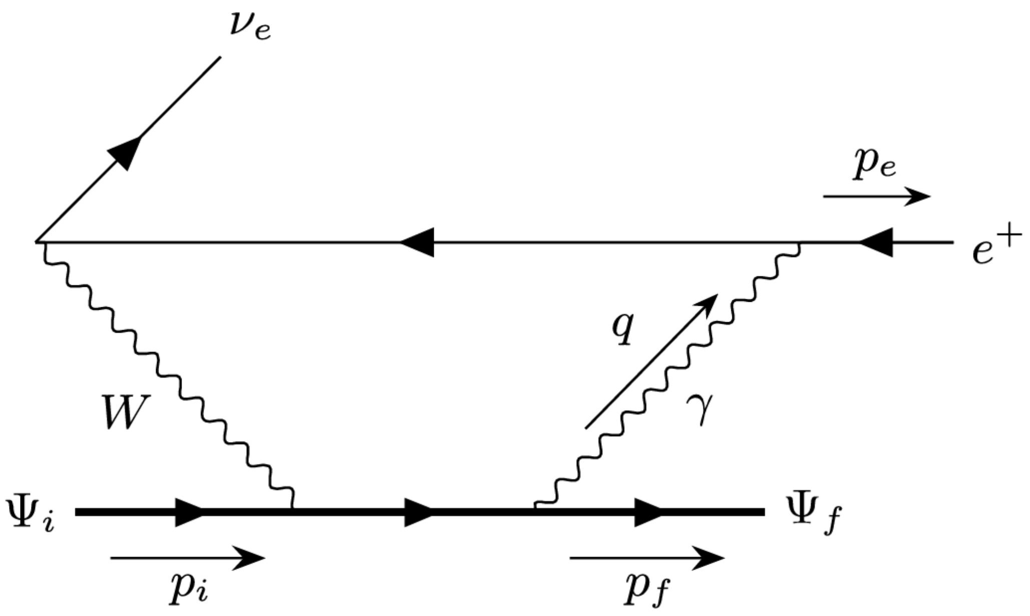
- Forward scattering limit
- Neglect recoil of final nucleus

$$\delta\mathcal{M}_{\text{one-loop}} = \Box_{\gamma W}(E_e)\,\mathcal{M}_{\text{tree}}$$

$$T^{\mu\nu}(p_f, p_i; q) = \langle \Phi_f\,;\,p_f | \left\{ \frac{1}{2}\int d^4x\; e^{iq\cdot x}\, \mathrm{T}\left[J^{\mu}_{\text{em}}(x)\, J^{\nu}_{W}(0)^{\dagger} \right] \right\} | \Phi_i\,;\,p_i \rangle$$

Seng & Gorchtein. PRC **107**, 035503 (2023)

One-loop radiative correction



Convenient [not critical] approximations

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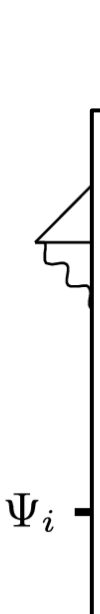
$$\Box^b_{\gamma W}(E_e) = \frac{e^2}{M}\int \frac{d^4q}{(2\pi)^4}\, R_W(q)\, R_e(q)\, R_\gamma(q)\left(M\frac{p_e\cdot q}{p\cdot p_e} - \frac{q^2}{\nu}\right)\frac{T_3(\nu,|\vec{q}|)}{f_+}$$

Seng & Gorchtein. PRC **107**, 035503 (2023)

One-loop radiative correction



No resolution for Compton amplitude above pion threshold,
thus $\delta_{\rm NS}$ matched with the free nucleon Born contribution **only**

$$\delta_{\rm NS} = 2\left\{\left(\Box^{b,\rm nuc}_{\gamma W}\right)_{\rm a.i.} - \left(\Box^{b,n}_{\gamma W}\right)_{\rm el} + \delta\left(\Box^{b,n}_{\gamma W}\right)_{\rm sh}\right\}$$

Gennari et al. PRL **134**, 012501

$$\Box^b_{\gamma W}(E_e) = \frac{e^2}{M}\int \frac{d^4q}{(2\pi)^4}\, R_W(q)\, R_e(q)\, R_\gamma(q)\left(M\frac{p_e\cdot q}{p\cdot p_e} - \frac{q^2}{\nu}\right)\frac{T_3(\nu,|\vec{q}|)}{f_+}$$

Seng & Gorchtein. PRC **107**, 035503 (2023)

Wick rotation and electron energy expansion

$$\Box^b_{\gamma W}(E_e) = \left(\Box^b_{\gamma W}\right)_{\mathrm{Wick}}(E_e) + \left(\Box^b_{\gamma W}\right)_{\mathrm{Res},e}(E_e) + \left(\Box^b_{\gamma W}\right)_{\mathrm{Res},T_3}(E_e)$$

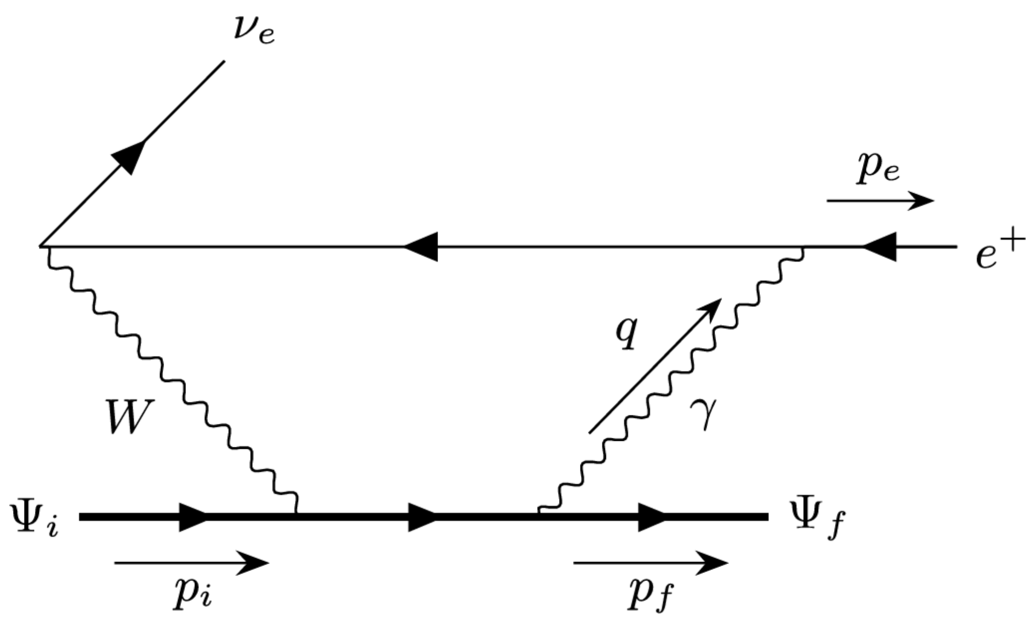
Wick rotated box diagram and electron residue contribution are regular as $E_e \to 0$

Nuclear residue contribution is **singular**

$$\Box^b_{\gamma W}(E_e) = \boxminus_0 + E_e \boxminus_1 + \left(\Box^b_{\gamma W}\right)_{\mathrm{Res},T_3}(E_e) + \mathcal{O}(E_e^2)$$

Gennari et al. PRL **134**, 012501

Deriving the non-relativistic Compton amplitude



$$J(\vec{r}) = \int \frac{d^3r}{(2\pi)^3}\, e^{i\vec{q}\cdot\vec{r}} J(\vec{q})$$

$$J^\mu(t,\vec{x}) = e^{-iHt} J^\mu(0,\vec{x})\, e^{iHt}$$

$$T^{\mu\nu}(p_f,p_i;q) = -\frac{i}{2}\langle\Phi_f\,;\,p_f|\,J^\mu_{\rm em}(-\vec{q})(z_f - H)^{-1} J^{\dagger\,\nu}_{\rm W}(\vec{q})|\Phi_i\,;\,p_i\rangle$$
$$-\frac{i}{2}\langle\Phi_f\,;\,p_f|\,J^{\dagger\,\nu}_{\rm W}(-\vec{q})(z_i - H)^{-1} J^\mu_{\rm em}(\vec{q})|\Phi_i\,;\,p_i\rangle$$

Deriving the non-relativistic Compton amplitude

$$M_{JM}(q) := \int d^3r \; \mathcal{M}_{JM}(q,\vec{r})\, \rho(\vec{r})$$

$$L_{JM}(q) := \int d^3r \; \frac{i}{q}\left(\vec{\nabla}\mathcal{M}_{JM}(q,\vec{r})\right) \cdot \vec{J}(\vec{r})$$

$$T^{\mathrm{el}}_{JM}(q) := \int d^3r \; \frac{1}{q}\left(\vec{\nabla} \times \vec{\mathcal{M}}^{M}_{JJ}(q,\vec{r})\right) \cdot \vec{J}(\vec{r})$$

$$T^{\mathrm{mag}}_{JM}(q) := \int d^3r \; \vec{\mathcal{M}}^{M}_{JJ}(q,\vec{r}) \cdot \vec{J}(\vec{r})$$

Donnelly et al. ADNDT **23**, pp. 103-176 (1979)