A Generalized Uncertainty Quantum Black hole

Saeed Rastgoo

UNIVERSITY OF ALBERTA

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with: F. Fragomeno, D. Gingrich, S. Hergott, E. Vienneau

Introduction

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Must derive interior GUP metric, then extend to full spacetime: $t \leftrightarrow r$

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We derive, for the first time, the full metric of a spherically symmetric static quantum black hole in GUP

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Needs applying an *improved scheme* from LQG

Interior of BH in AB variables

Classical metric of the interior in AB variables:

$$ds^2 = -\frac{N(p_b, p_c)}{t^2}dt^2 + \frac{p_b^2}{L_0^2 p_c}dr^2 + p_c\left(d\theta^2 + \sin^2(\theta)d\phi^2\right)$$

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Classical Hamiltonian (constraint) of the interior in AB variables:

$$H = -\frac{N\left(p_b, p_c\right)}{2G\gamma^2}\left[\left(b^2 + \gamma^2\right)\frac{p_b}{\sqrt{p_c}} + 2bc\sqrt{p_c}\right]$$

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with the algebra

$$\{b, p_b\} = 2G\gamma, \qquad \{c, p_c\} = G\gamma$$

GUP-Modified Solution

1. Modify the algebra as

$$\{b, p_b\} = 2G\gamma\left(1 + \beta_b b^2\right), \qquad \{c, p_c\} = G\gamma\left(1 + \beta_c c^2\right)$$

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4. Analytically extend the metric $t \leftrightarrow r$ to obtain full spacetime GUP-modified effective metric $g_{\mu\nu}^{\text{GUP}}$

Asymptotic Issues of GUP-Modified metric

Classical limits are fine:

$$\lim_{\beta_b, \beta_c \to 0} g_{\mu\nu}^{\text{GUP}} = g_{\mu\nu}^{\text{Schw}}$$

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And similar to [Ashtekar, Olmedo, Singh, PRD 98, 126003 (2018)] Kretschmann falls off as

$$K_{\text{GUP}}(r\to\infty) \propto \frac{1}{r^4} \text{ and not } \frac{1}{r^6}$$

Improved Scheme in General

In BHs in LQG, and in LQC sometimes these issues exist

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$$\beta \to \bar{\beta}\left(p\right)$$

Improved Scheme: Our Model

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In our model it becomes

$$\beta_b \to \bar{\beta}_b = \frac{\beta_b L_0^4}{p_b^2}, \qquad \beta_c \to \bar{\beta}_c = \frac{\beta_c L_0^4}{p_c^2}$$

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which means

$$\{b, p_b\} = 2G\gamma \left(1 + \frac{\beta_b L_0^4}{p_b^2} b^2\right), \qquad \{c, p_c\} = G\gamma \left(1 + \frac{\beta_c L_0^4}{p_c^2} c^2\right)$$

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1. Rework the interior again: Solve the EoM of the interior

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GUP-Modified Improved Metric

Result: first GUP BH metric derived until now

$$g_{00}^{\text{GUP-Imp}} = -\left(1+\frac{Q_b}{r^2}\right)\left(1+\frac{Q_c R_s^2}{4r^8}\right)^{-1/4}\left(1-\frac{R_s}{\sqrt{r^2+Q_b}}\right)$$

$$g_{11}^{\text{GUP-Imp}} = \left(1+\frac{Q_c R_s^2}{4r^8}\right)^{1/4}\left(1-\frac{R_s}{\sqrt{r^2+Q_b}}\right)^{-1}$$

$$g_{22}^{\text{GUP-Imp}} = r^2\left(1+\frac{Q_c R_s^2}{4r^8}\right)^{1/4}$$

Fragomeno, Gingrich, Hergott, SR, Vienneau,

On arXiv next week!

where now our dimensionful quantum parameters are

$$Q_b = -\text{sgn}(\beta_b)|\beta_b|\gamma^2 L_0^2, \qquad Q_c = -\text{sgn}(\beta_c)|\beta_c|\gamma^2 L_0^6$$

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Reality of the metric on $r \in [0, \infty)$ dictates

$$Q_b > 0 \Rightarrow \text{sgn}(\beta_b) = -1, \qquad Q_c > 0 \Rightarrow \text{sgn}(\beta_c) = -1$$

Correct Classical/Asymptotic Limits

Now, the classical limit is fine

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The asymptotic expansion and limit are as expected

$$g_{00}\big|_{r\to\infty} = -1 + \frac{R_s}{r} - \frac{Q_b}{r^2} + \mathcal{O}\left(\frac{1}{r}\right)^3,$$

$$g_{11}\big|_{r\to\infty} = 1 + \frac{R_s}{r} + \frac{R_s^2}{r^2} + \frac{R_s}{2r^3}\left(2R_s^2 - Q_b\right) + \mathcal{O}\left(\frac{1}{r}\right)^4,$$

$$g_{22}\big|_{r\to\infty} = r^2$$

Correct Classical/Asymptotic Limits

Also, there is a minimum radius of the 2-spheres

$$\sqrt{g_{22}^{\text{GUP-Imp}}}\Big|_{r=0} = \left(r^8 + \frac{Q_c R_s^2}{4} \right)^{1/8} \Bigg|_{r=0} = \left(\frac{Q_c R_s^2}{4} \right)^{1/8}$$

Correct Classical/Asymptotic Limits

Also, there is a minimum radius of the 2-spheres

$$\sqrt{g_{22}^{\text{GUP-Imp}}}\Big|_{r=0} = \left(r^8 + \frac{Q_c R_s^2}{4}\right)^{1/8}\Bigg|_{r=0} = \left(\frac{Q_c R_s^2}{4}\right)^{1/8}$$

but there is no bounce (e.g., a $1/r$ behavior): remnant

Horizon

The horizon $g^{11}(r_H) = 0$ or $g_{00}(r_H) = 0$ is at

$$r_H = R_s \sqrt{1 - \frac{Q_b}{R_s^2}} = R_s - \frac{1}{2}\frac{Q_b}{R_s} + \mathcal{O}\left(\frac{Q_b^2}{R_s^3}\right)$$

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and based on M and Q_b

$$\begin{cases} R_s > Q_b \Rightarrow M > \frac{\sqrt{Q_b}}{2G}, & \text{Black hole} \\ R_s = Q_b \Rightarrow M = \frac{\sqrt{Q_b}}{2G}, & \text{Min mass (universal), remnant} \\ R_s < Q_b \Rightarrow M < \frac{\sqrt{Q_b}}{2G}, & \text{Not allowed} \end{cases}$$

Kretschmann and Singularity

The Kretschmann scalar is

$$K = \frac{\cdots}{\sqrt{r^8 + \frac{1}{4} Q_c R_s^2}} + \frac{\cdots}{\left(r^2 + Q_b\right)^5 \left(r^8 + \frac{1}{4} Q_c R_s^2\right)^{9/2}}$$

which is everywhere regular

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which is everywhere regular, particularly at the origin

$$\lim_{r \to 0^+} K = K\left(r = 0\right) = \frac{8}{R_s \sqrt{Q_c}}.$$

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Its asymptotic expansion and limits are precisely what they should be

$$K\Big|_{r\to\infty} = \frac{12 R_s^2}{r^6} + \mathcal{O}\left(\frac{1}{r}\right)^7$$

Masses

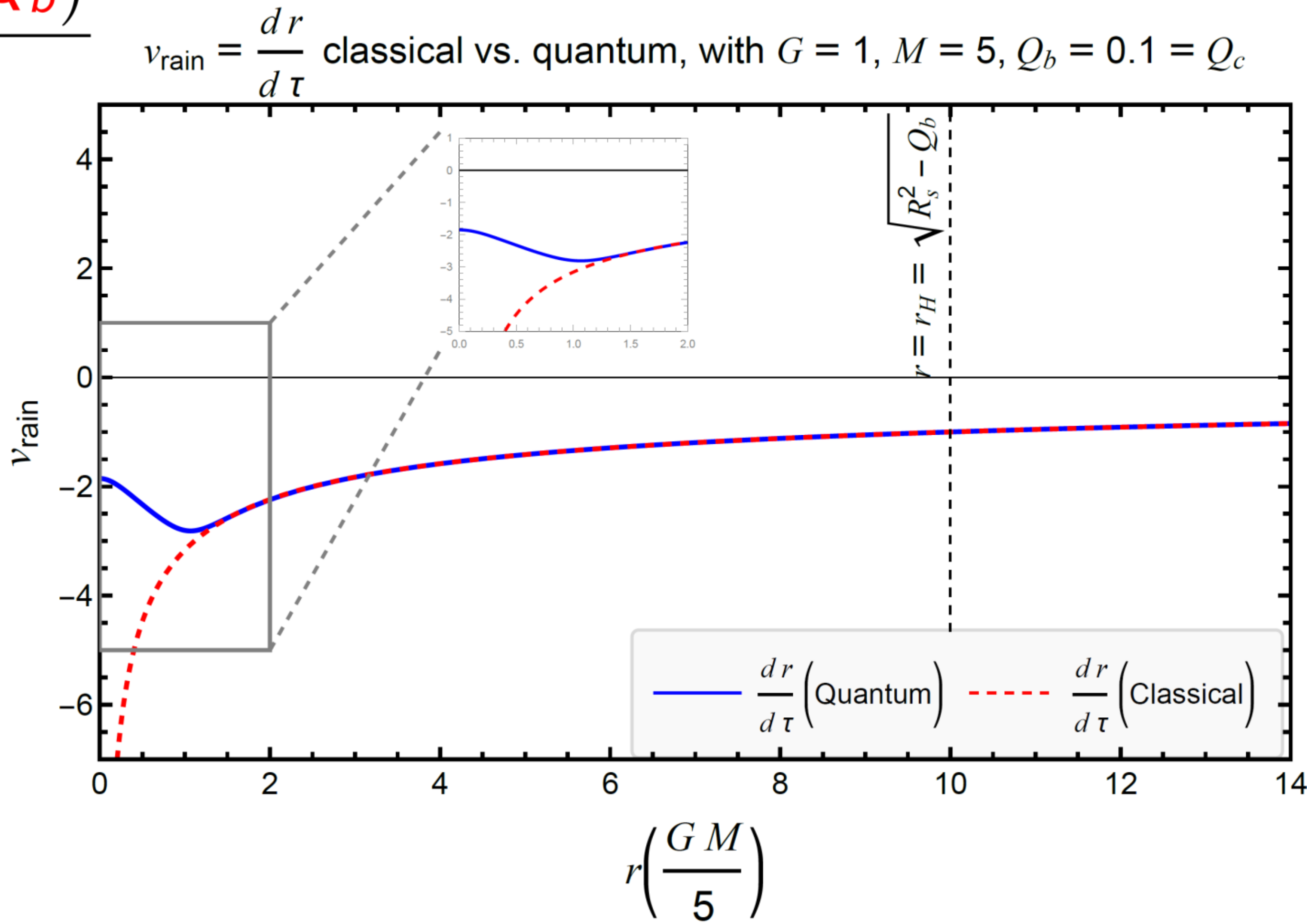
The ADM , Kumar, and Misner-Sharp-Hernandez (MSH) masses match and are equal to the parameter M in the metric

$$\lim_{r\to\infty} M_{\mathrm{ADM}}(r) = \lim_{r\to\infty} M_{\mathrm{MSH}}(r) = \lim_{r\to\infty} M_{\mathrm{K}}(r) = M$$

PG Coordinates and Infalling Observer

In PG coordinates, the velocity of the infalling observer remains finite

$$v_{\text{rain}} = -\sqrt{1 + \frac{\sqrt{r^2 + Q_b}\left(R_s - \sqrt{r^2 + Q_b}\right)}{\left(r^8 + \frac{1}{4} Q_c R_s^2\right)^{1/4}}}$$



Expansion and Raychaudhuri Equation

Null expansion is everywhere regular

$$\theta_{\pm} = \frac{8r^7}{4r^8 + Q_c R_s^2}\left(\pm 1 - \sqrt{1 + \frac{\sqrt{2}\sqrt{Q_b + r^2}\left(R_s - \sqrt{Q_b + r^2}\right)}{\sqrt[4]{4r^8 + Q_c R_s^2}}}\right)$$

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Both expansions vanish at the origin

$$\theta_{\pm}\left(r = 0\right) = 0$$

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Notice that θ_+ changes sign at the quantum horizon

$$\theta_+(r_H) = 0$$

$$\theta_-(r_H) = -\frac{16\left(R_s^2 - Q_b\right)^{7/2}}{4\left(R_s^2 - Q_b\right)^4 + Q_c R_s^2}$$

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has correct classical limit

$$\theta_\pm = \frac{2}{r}\left(\pm 1 - \sqrt{\frac{R_s}{r}}\right)$$

Expansion and Raychaudhuri Equation

Null Raychaudhuri equation is everywhere regular

$$\frac{d\theta_\pm}{d\lambda} = \frac{\cdots}{\left(4r^8 + Q_c R_s^2\right)^2}$$

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and it vanishes at the origin

$$\frac{d\theta_\pm}{d\lambda}(r = 0) = 0$$

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$$\frac{d\theta_\pm}{d\lambda} = \frac{2}{r^3}\left(\pm 2\sqrt{rR_s} - r - R_s\right)$$

Expansion and Raychaudhuri Equation

- Null expansion,
- Raychaudhuri equation,
- Kretschmann scalar

all are everywhere regular

Singularity resolution!

Summary

- We derived the first full spacetime metric of a GUP-inspired BH
- The usual analytic extension of interior metric does not work
- We borrowed the well-known improved scheme from LQG
- Extension of this improved GUP interior metric works perfectly
  - All the correct classical and asymptotic expansions and limits
  - Regular: no singularity
  - One horizon, remnant
- Future: study stability, add matter, phenomenology, …