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Ab initio studies on muon capture in light nuclei

Lotta Jokiniemi (she/her)
TRIUMF, Theory Department
CAP Congress
28/05/2024

Arthur B. McDonald
Canadian Astroparticle Physics Research Institute

Discovery, accelerated

Collaboration

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J. Kotila

Lawrence Livermore
National Laboratory

K. Kravvaris

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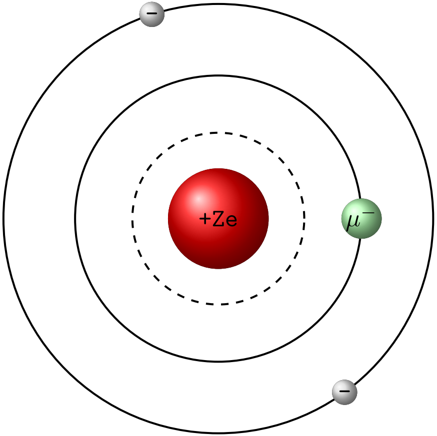
Outline

Discovery, accelerated

Ordinary Muon Capture (OMC)

- A muon can replace an electron in an atom, forming a *muonic atom*



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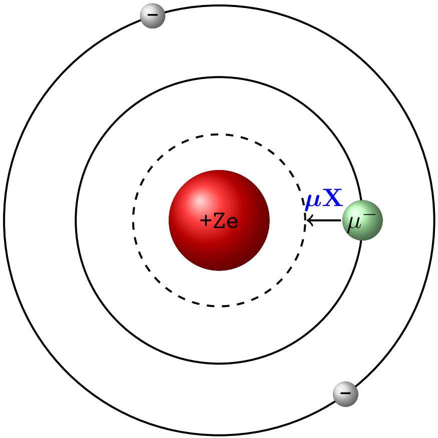

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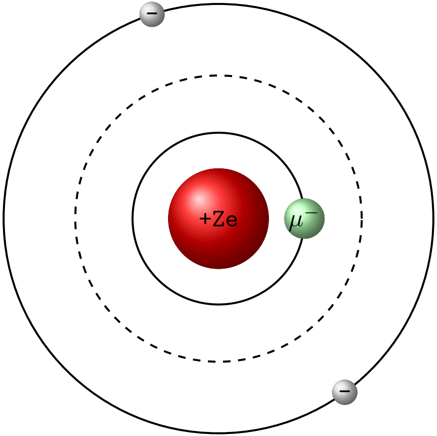

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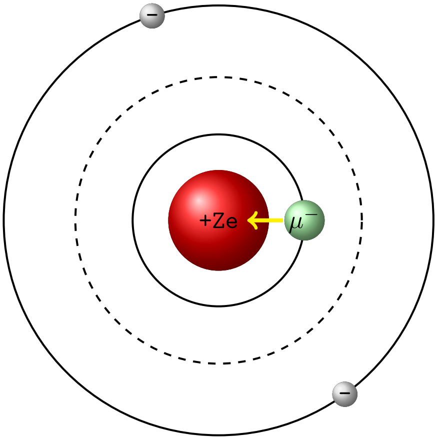

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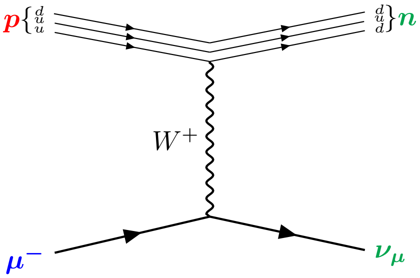

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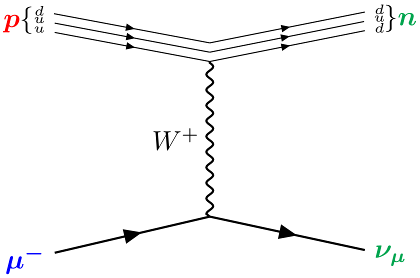

Ordinary = non-radiative

(Radiative muon capture (RMC):
$$\mu^- + {}^{A}_{Z}\mathrm{X}(J_i^{\pi_i}) \rightarrow \nu_\mu + {}^{A}_{Z-1}\mathrm{Y}(J_f^{\pi_f}) + \gamma$$)

Discovery, accelerated

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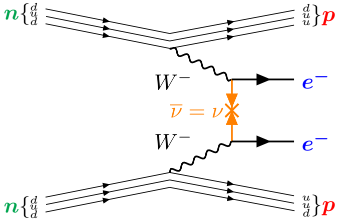
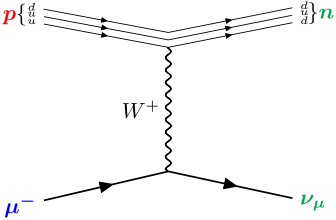
$0\nu\beta\beta$ Decay vs. Muon Capture



$$^{A}_{Z}\mathrm{X}(J_i^{\pi_i}) \rightarrow \,^{A}_{Z+2}\mathrm{X}'(J_f^{\pi_f}) + 2e^-$$



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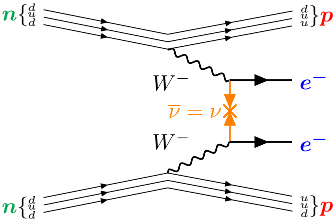

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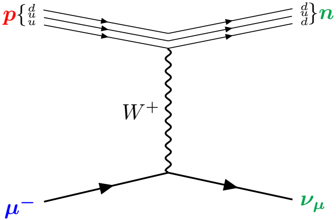

$$\boxed{\mu^- + {}^{A}_{Z}\mathrm{X}(J_i^{\pi_i}) \rightarrow \nu_\mu + {}^{A}_{Z-1}\mathrm{Y}(J_f^{\pi_f})}$$

Both involve hadronic current:

$$j^{\alpha\dagger} = \bar{\Psi}\left[g_{\mathrm{V}}(q^2)\gamma^\alpha + i g_{\mathrm{M}}(q^2)\frac{\sigma^{\alpha\beta}}{2m_p}q_\beta - g_{\mathrm{A}}(q^2)\gamma^\alpha\gamma_5 - g_{\mathrm{P}}(q^2)q^\alpha\gamma_5\right]\Psi$$

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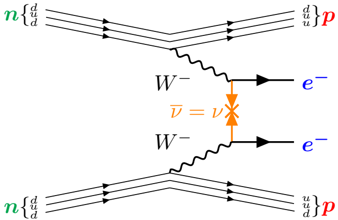
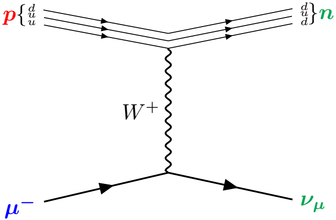


$$ {}^{A}_{Z}\mathrm{X}(J_i^{\pi_i}) \rightarrow {}_{Z+2}^{A}\mathrm{X}'(J_f^{\pi_f}) + 2e^- $$

- $q \approx 1/|\mathbf{r}_1 - \mathbf{r}_2| \approx 100 - 200$ **MeV**



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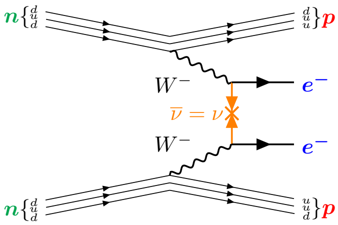

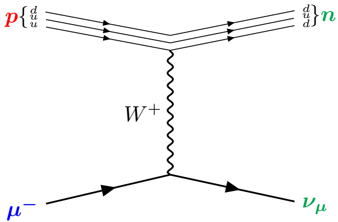

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- $q \approx 1/|\mathbf{r}_1 - \mathbf{r}_2| \approx 100 - 200$ **MeV**
- $q \approx m_\mu + E_i - E_f \approx 100$ **MeV**

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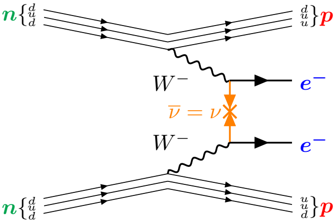

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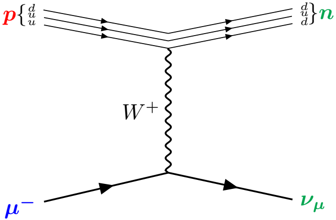
- **Yet hypothetical**



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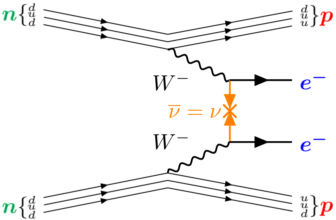

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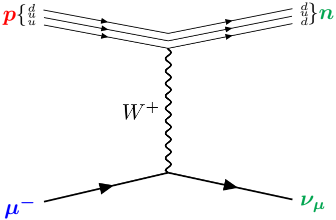

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- $q \approx m_\mu + E_i - E_f \approx 100$ **MeV**
- **Has been measured!**

<u>Both involve hadronic current:</u>

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Outline

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Muon-Capture Theory

- Interaction Hamiltonian → capture rate:

$$W(J_i \rightarrow J_f) = \frac{1}{2J_i+1}\left(1 - \frac{q}{m_\mu + AM}\right) q^2 \sum_{\kappa u} \left| g_\mathrm{V} M_\mathrm{V}(\kappa, u) + g_\mathrm{M} M_\mathrm{M}(\ldots) + g_\mathrm{A} M_\mathrm{A}(\ldots) + g_\mathrm{P} M_\mathrm{P}(\ldots) \right|^2$$

PHYSICAL REVIEW VOLUME 118, NUMBER 2 APRIL 15, 1960

Theory of Allowed and Forbidden Transitions in Muon Capture Reactions*

MASATO MORITA
Columbia University, New York, New York

AND

AKIHIKO FUJII†
Brookhaven National Laboratory, Upton, Long Island, New York

(Received November 9, 1959)

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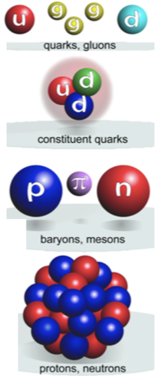
+ Translationally invariant **nuclear wave functions** from no-core shell model
+ Approximate **two-body currents** via normal-ordering

What is *ab initio*?

Ideally:

Quantum Chromodynamics (QCD)



Genuine *Ab Initio*

Figure courtesy of P. Navrátil

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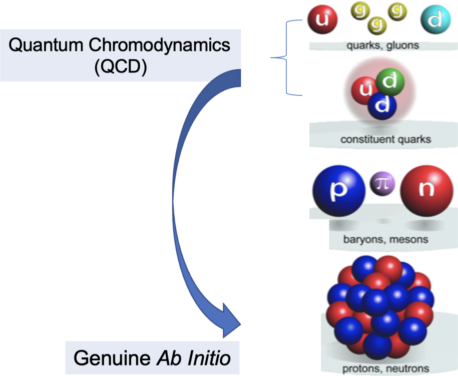


Currently:

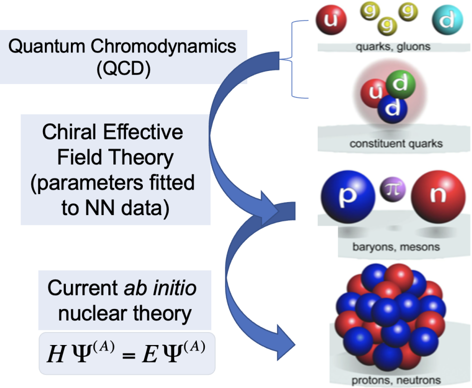
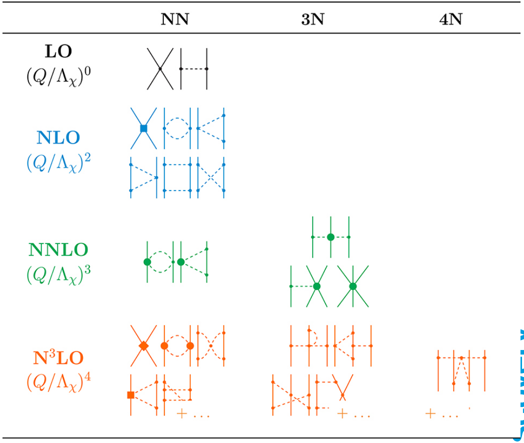


Figure courtesy of P. Navrátil

Discovery, accelerated

Nuclear Forces from Chiral Effective Field Theory (χEFT)

- Expansion organized in terms of expansion parameter (Q/Λ_χ)



H. Hergert, Front. Phys. 8 (2020)

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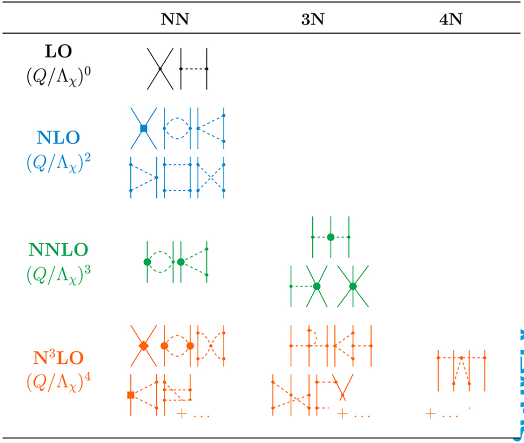

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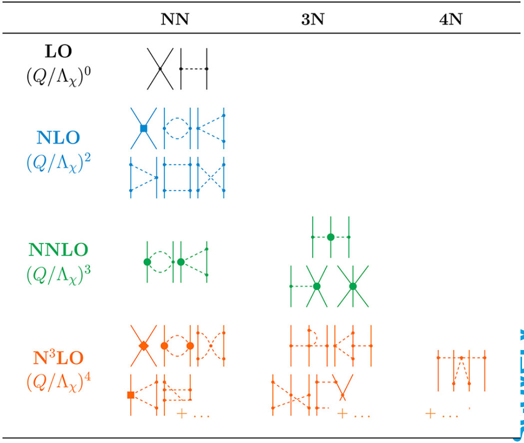

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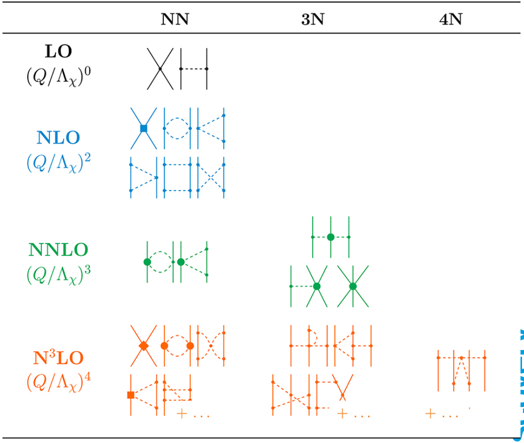

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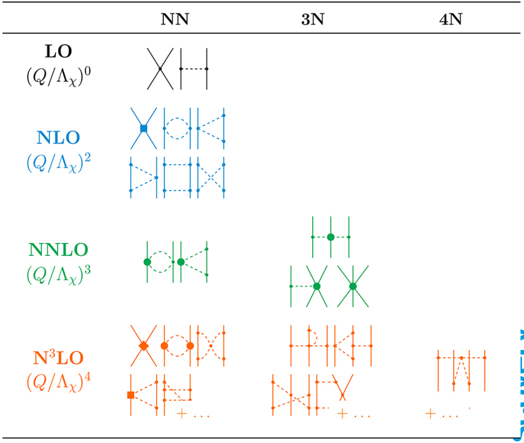

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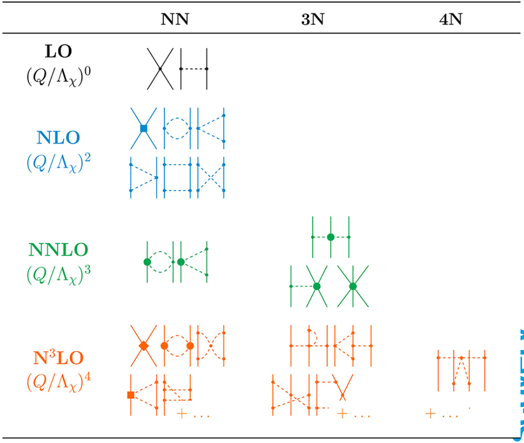
  - Ideally, solved from QCD
  - Currently, fitted to data



H. Hergert, Front. Phys. 8 (2020)

Ab initio No-Core Shell Model (NCSM)

- Solve nuclear many-body problem

$$H^{(A)}\Psi^{(A)}(\mathbf{r}_1,\mathbf{r}_2,...,\mathbf{r}_A) = E^{(A)}\Psi^{(A)}(\mathbf{r}_1,\mathbf{r}_2,...,\mathbf{r}_A)$$

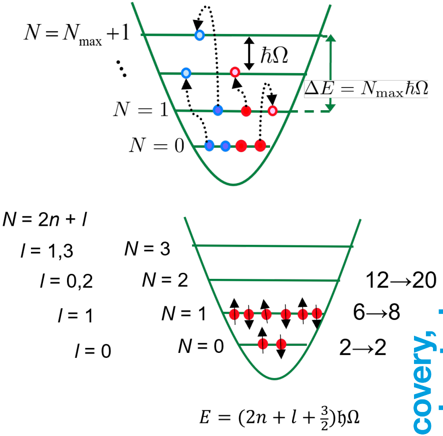


Figure courtesy of P. Navrátil

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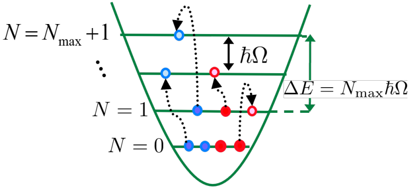
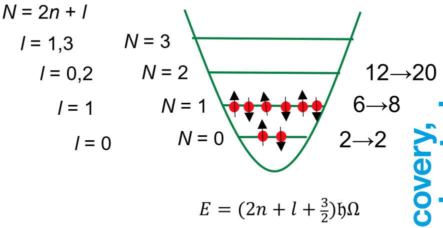
- **Two- (NN)** and **three-nucleon (3N)** forces from χEFT

$$H^{(A)} = \sum_{i=1}^{A} \frac{p_i^2}{2m} + \sum_{i<j=1}^{A} V^{NN}(\mathbf{r}_i - \mathbf{r}_j) + \sum_{i<j<k=1}^{A} V_{ijk}^{3N}$$





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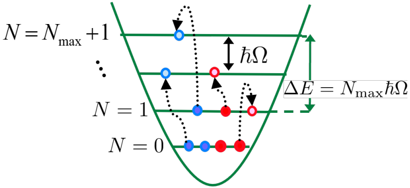
- Expansion in harmonic oscillator (HO) basis

$$\Psi^{(A)} = \sum_{N=0}^{N_{\max}}\sum_{j} c_{Nj}\Phi^{\mathrm{HO}}_{Nj}(\mathbf{r}_1,\mathbf{r}_2,...,\mathbf{r}_A)$$



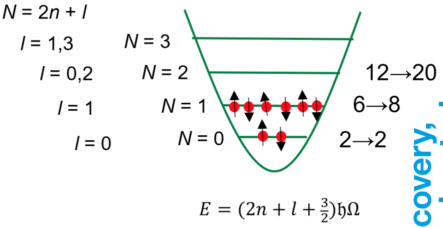


Figure courtesy of P. Navrátil

Discovery, accelerated

Dependency on the Harmonic-Oscillator Frequency

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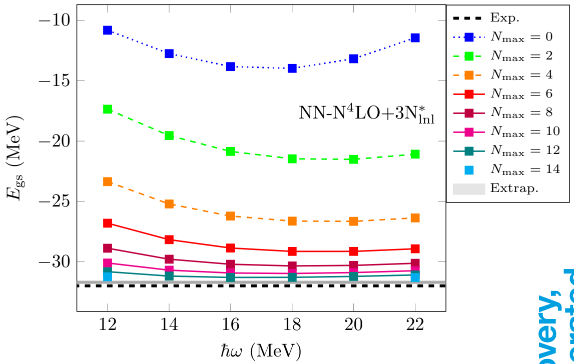
- The expansion depends on the HO frequency because of the $N_{\max}$ truncation
  - **Increasing $N_{\max}$ leads towards convergenced results**

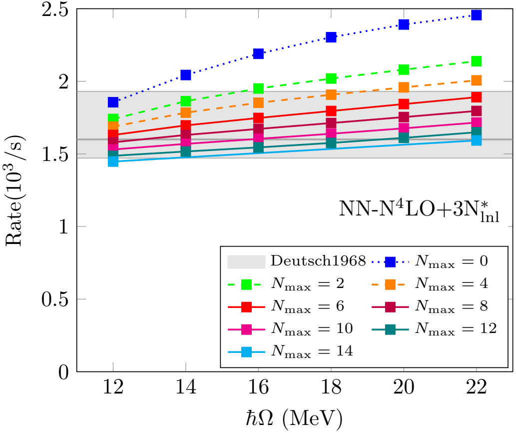
Ground-state energy of ^{6}Li



LJ, Navrátil, Kotila, Kravvaris, arXiv:2403.05776

Harmonic-Oscillator Frequency Dependence of Muon Capture

$^{6}\mathrm{Li}(1^{+}_{\mathrm{gs}}) + \mu^{-} \rightarrow {}^{6}\mathrm{He}(0^{+}_{\mathrm{gs}}) + \nu_{\mu}$



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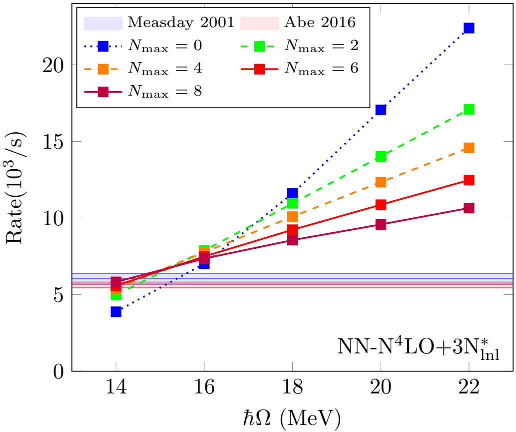

LJ, Navrátil, Kotila and Kravvaris, arXiv:2403.05776 (accepted to PRC)

Discovery, accelerated

TRIUMF

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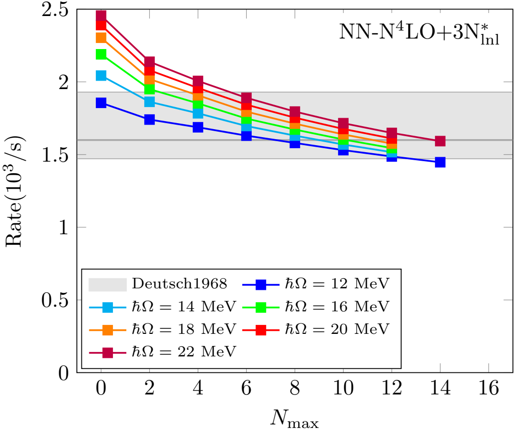

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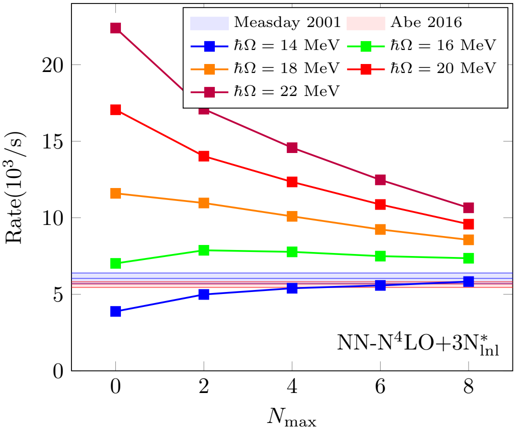


LJ, Navrátil, Kotila and Kravvaris, arXiv:2403.05776 (accepted to PRC)

Discovery, accelerated

Outline

TRIUMF

Outline

Introduction

Muon Capture from No-Core Shell Model

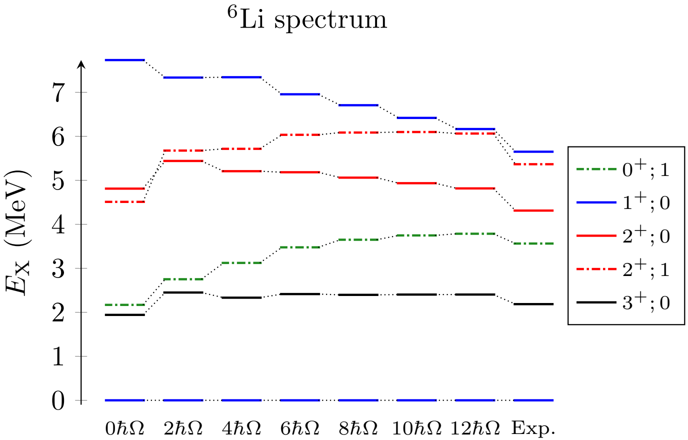
Results

- Muon capture on ^{6}Li
- Muon capture on ^{12}C
- Muon capture on ^{16}O

Summary

Discovery, accelerated

Energy spectrum of ^{6}Li



LJ, Navrátil, Kotila, Kravvaris, arXiv:2403.05776 (accepted to PRC)

Capture Rates to the Ground State of ^{6}He

- NCSM slightly underestimating experiment

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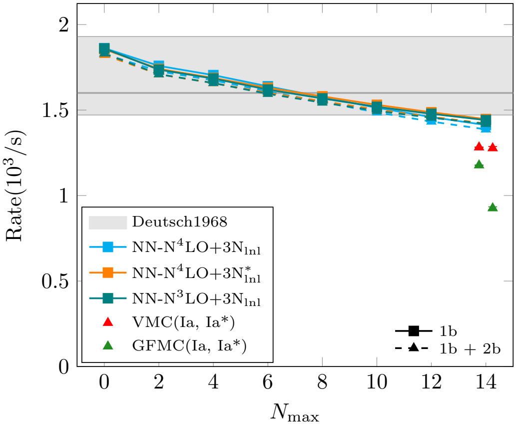

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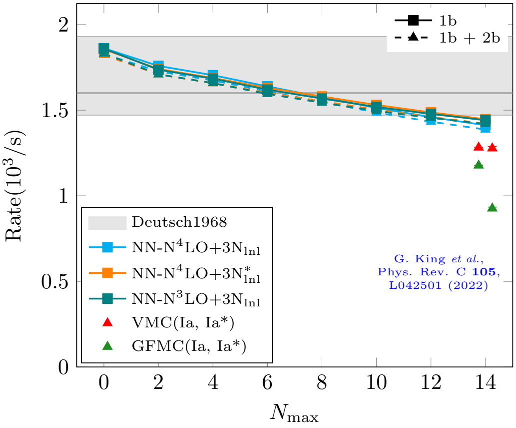
King *et al.*, Phys. Rev. C **105**, L042501 (2022)



LJ, Navrátil, Kotila, Kravvaris, arXiv:2403.05776 (accepted to PRC)

Discovery, accelerated

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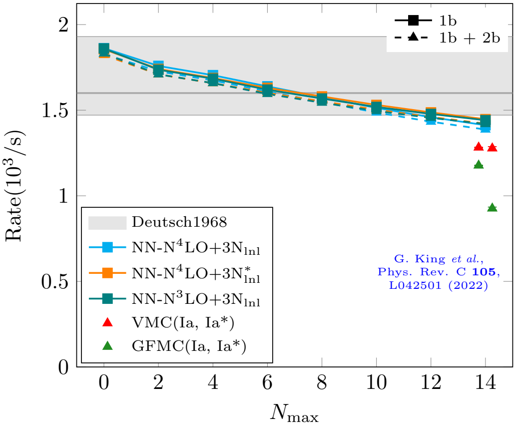

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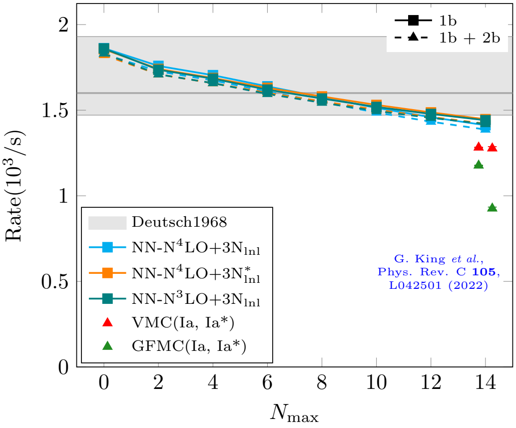
King *et al.*, Phys. Rev. C **105**, L042501 (2022)

  - Slow convergence likely due to cluster-structure
  - NCSM with continuum (NCSMC) might give better results?

$$^6\mathrm{Li}(1^+_{\mathrm{gs}}) + \mu^- \to {}^6\mathrm{He}(0^+_{\mathrm{gs}}) + \nu_\mu$$



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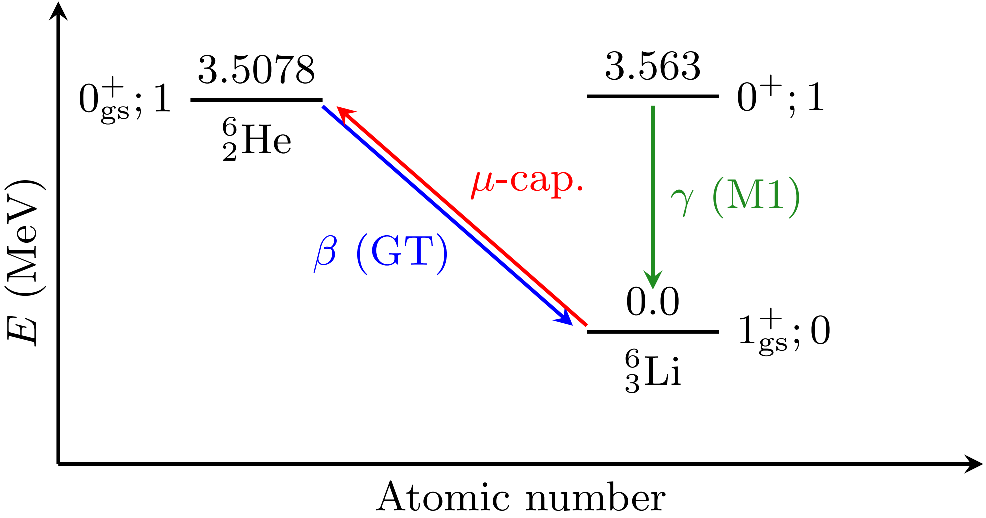
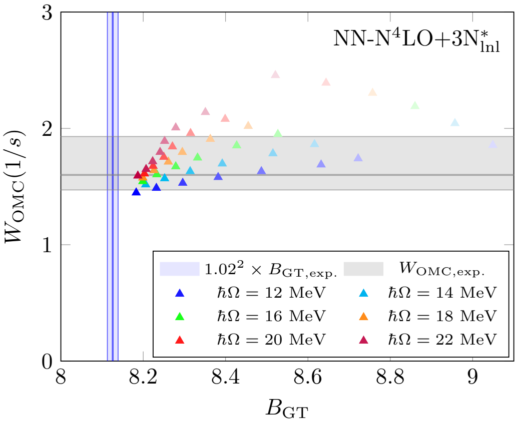
Correlations with Other Observables



Discovery, accelerated

Correlations with Other Observables

GT β decay:



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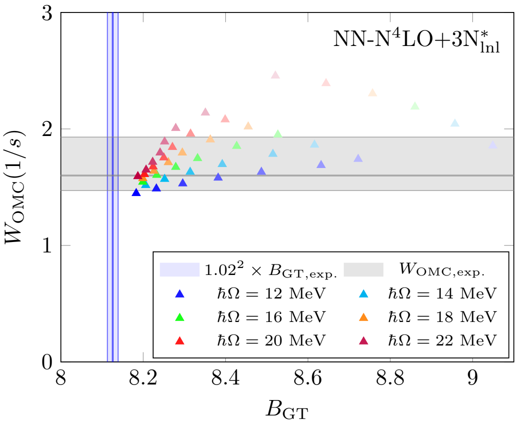

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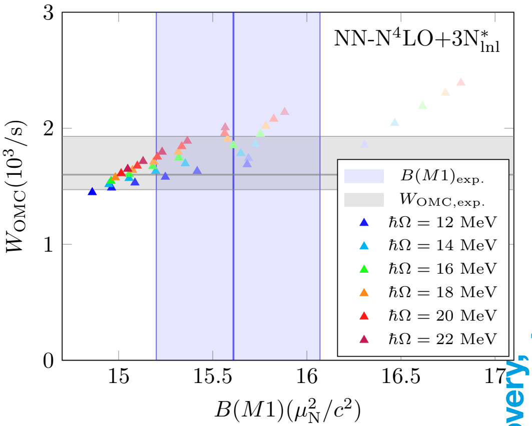

LJ, Navrátil, Kotila and Kravvaris, arXiv:2403.05776 (accepted to PRC)

Discovery, accelerated

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Outline

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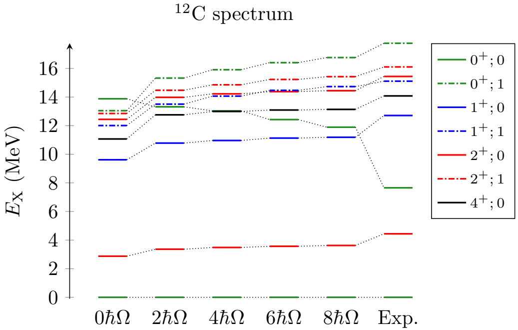
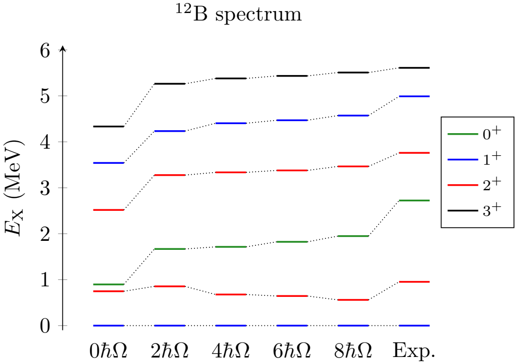
Energy spectra of ^{12}C and ^{12}B





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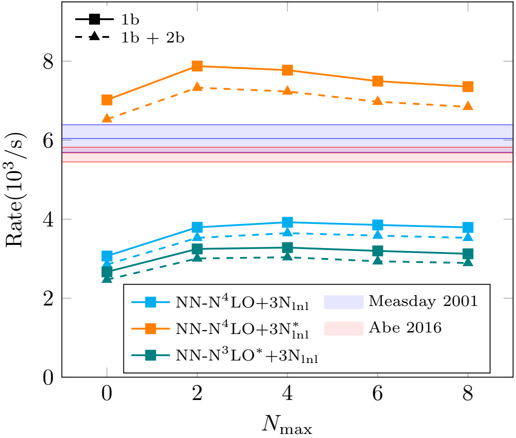
Discovery, accelerated

Capture Rates to the Ground State of ^{12}B

$$^{12}\mathrm{C}(0^+_{\mathrm{gs}}) + \mu^- \rightarrow \ ^{12}\mathrm{B}(1^+_{\mathrm{gs}}) + \nu_\mu$$

- Significant interaction dependence



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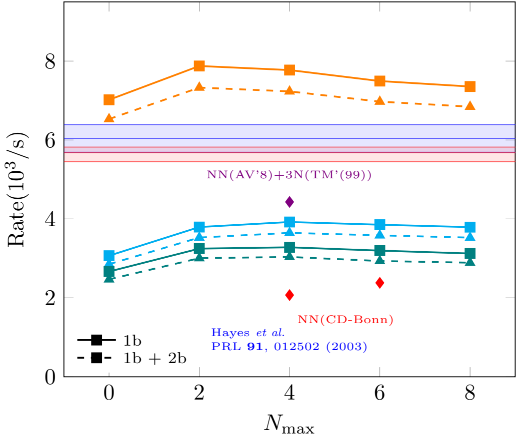
  - The **NN-N^{4}LO+3N$^*_{\mathrm{lnl}}$** interaction with the additional spin-orbit term most consistent with experiment



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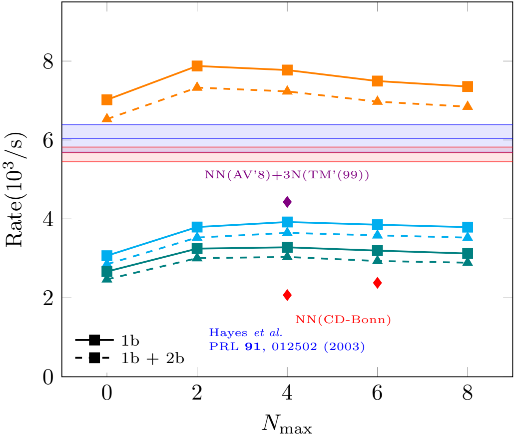
- The results can be compared against earlier NCSM calculations with phenomenological interactions

Hayes *et al.*, *Phys. Rev. Lett.* **91**, 012502 (2003)



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Discovery, accelerated

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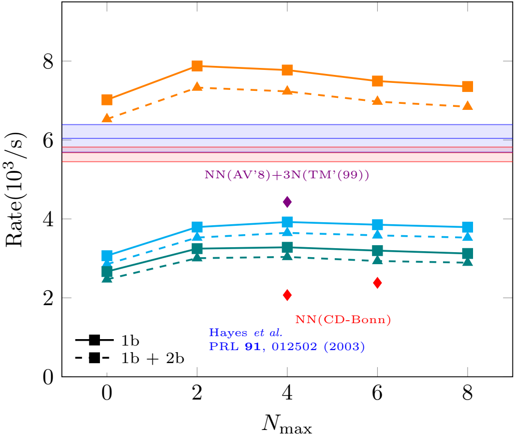
  Hayes *et al.*, *Phys. Rev. Lett.* **91**, 012502 (2003)
- 3-body forces essential to reproduce the measured rate



LJ, Navrátil, Kotila, Kravvaris, arXiv:2403.05776 (accepted to PRC)

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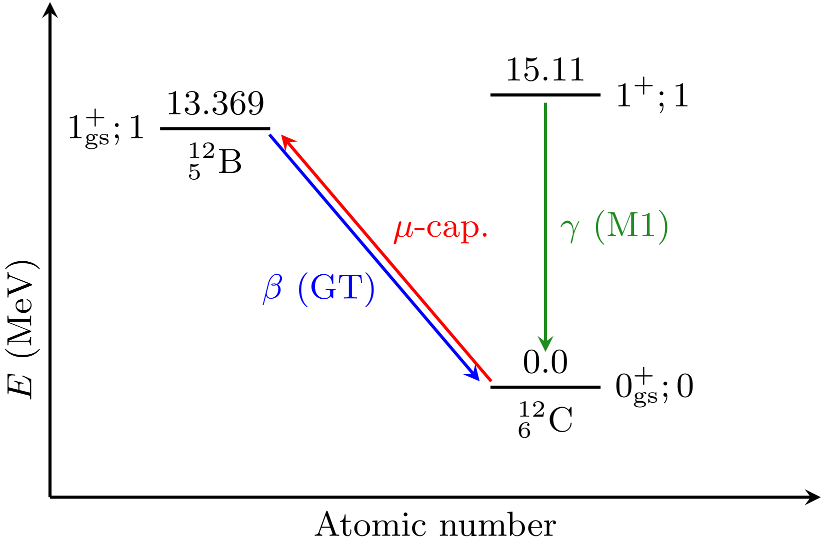
Correlations with Other Observables



Discovery, accelerated

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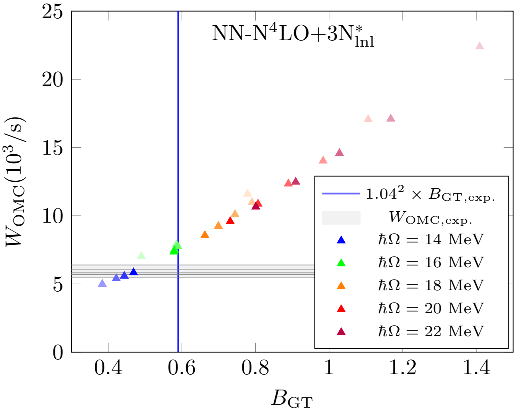

LJ, Navrátil, Kotila and Kravvaris, arXiv:2403.95776

Discovery, accelerated

TRIUMF

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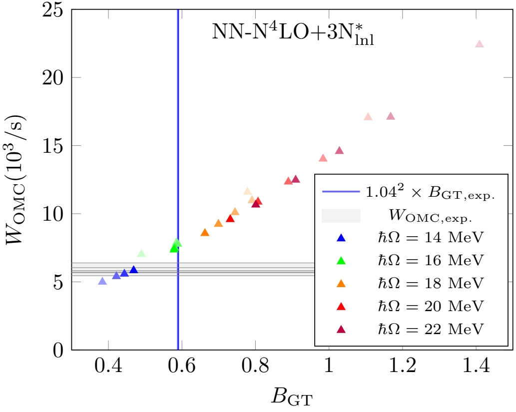

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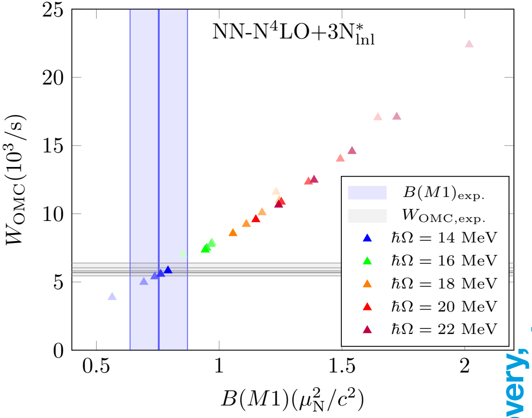

LJ, Navrátil, Kotila and Kravvaris, arXiv:2403.95776

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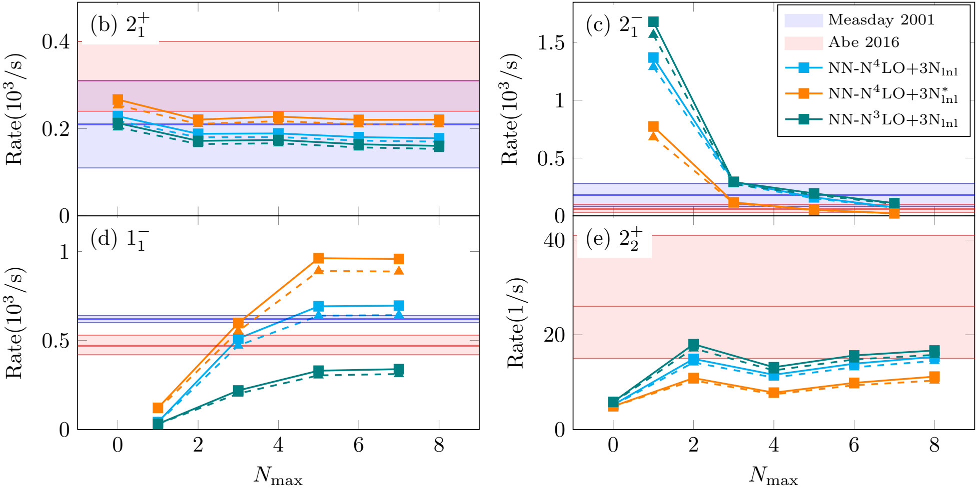
Capture Rates to Low-Lying States in ^{12}B



LJ, Navrátil, Kotila, Kravvaris, arXiv:2403.05776 (accepted to PRC)

Discovery, accelerated

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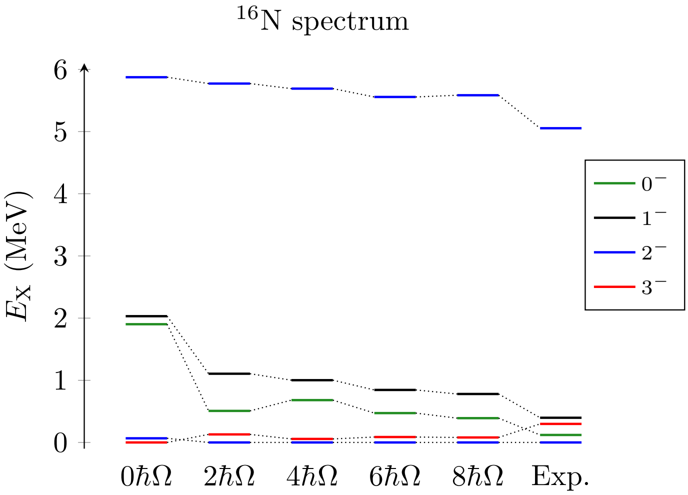
Outline

Discovery, accelerated

Energy spectra of ^{16}N



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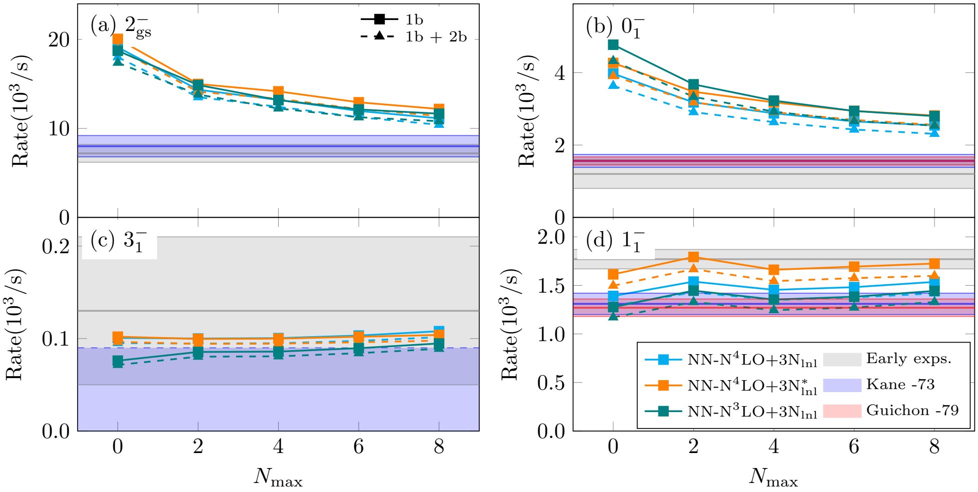

LJ, Navrátil, Kotila, Kravvaris, arXiv:2403.05776 (accepted to PRC)

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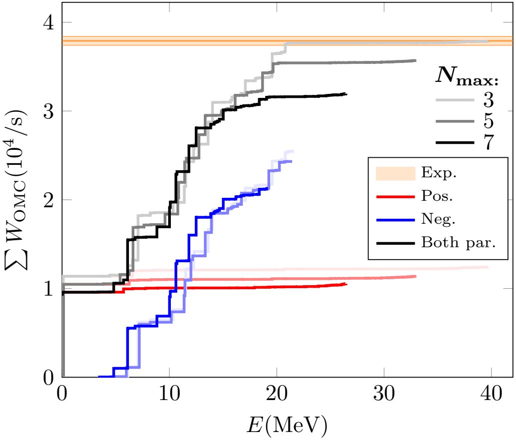
Total Muon-Capture Rates in ^{12}B and ^{16}N

$$\mu^- + {}^{12}\mathrm{C}(0^+_{\mathrm{gs}}) \rightarrow \nu_\mu + {}^{12}\mathrm{B}(J^\pi_k)$$

- Rates obtained summing over ~ 50 final states of each parity



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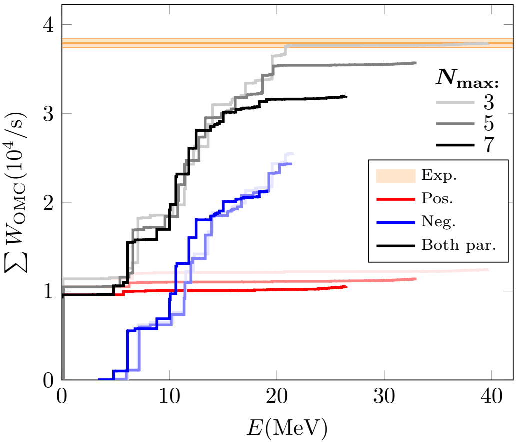

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Discovery, accelerated

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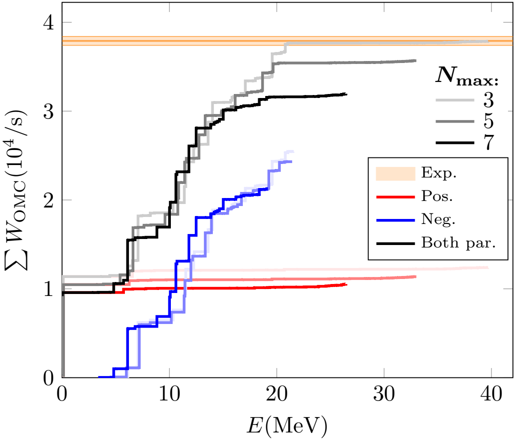
- Better estimation with the Lanczos strength function method ongoing



LJ, Navrátil, Kotila, Kravvaris, arXiv:2403.05776 (accepted to PRC)

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Outline

Discovery, accelerated

Summary

- *Ab initio* muon-capture studies could shed light on nuclear electroweak currents at finite momentum exchange regime

Discovery, accelerated

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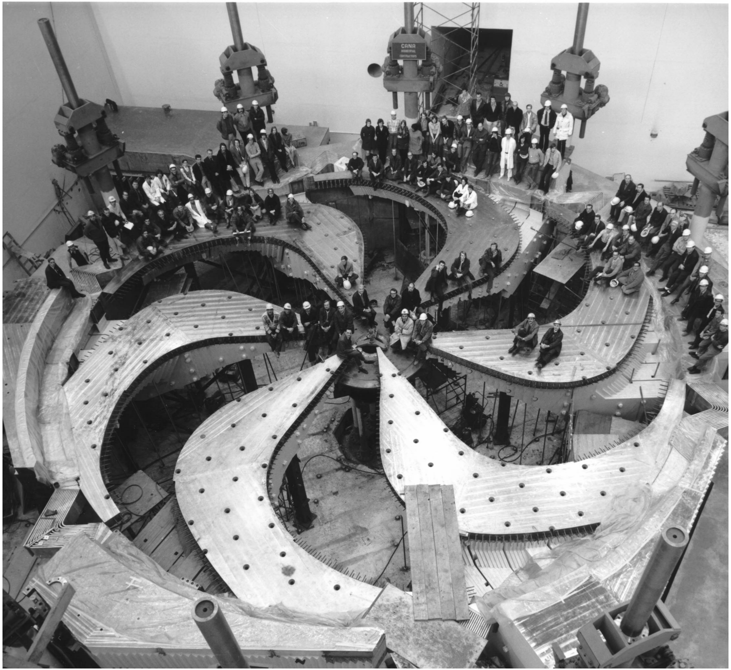
Summary

- *Ab initio* muon-capture studies could shed light on nuclear electroweak currents at finite momentum exchange regime
- No-core shell-model describes well partial muon-capture rates in light nuclei ^{6}He, ^{12}B and ^{16}N
- Calculation of total capture rates currently in progress in NCSM

TRIUMF

Thank you
Merci



Discovery, accelerated

OMC operators

- Rates written in terms of reduced one-body matrix elements:

$$(\Psi_f||\sum_{s=1}^{A}\hat{O}_{kwux}(\mathbf{r}_s,\mathbf{p}_s)||\Psi_i) = -\frac{1}{\sqrt{2u+1}}\sum_{pn}(n||\hat{O}_{kwux}(\mathbf{r}_s,\mathbf{p}_s)||p)(\Psi_f||[a_n^\dagger\tilde{a}_p]_u||\Psi_i)$$

NME	$\hat{O}_{kwux}(\mathbf{r}_s,\mathbf{p}_s)$
$\mathcal{M}[0\,w\,u]$	$j_w(qr_s)G_{-1}(r_s)\mathcal{Y}_{0wu}^{M_f-M_i}(\hat{\mathbf{r}}_s)\delta_{wu}$
$\mathcal{M}[1\,w\,u]$	$j_w(qr_s)G_{-1}(r_s)\mathcal{Y}_{1wu}^{M_f-M_i}(\hat{\mathbf{r}}_s,\boldsymbol{\sigma}_s)$
$\mathcal{M}[0\,w\,u\,\pm]$	$[j_w(qr_s)G_{-1}(r_s)\mp\frac{1}{q}j_{w\mp1}(qr_s)\frac{d}{dr_s}G_{-1}(r_s)]\mathcal{Y}_{0wu}^{M_f-M_i}(\hat{\mathbf{r}}_s)\delta_{wu}$
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$\mathcal{M}[0\,w\,u\,p]$	$ij_w(qr_s)G_{-1}(r_s)\mathcal{Y}_{0wu}^{M_f-M_i}(\hat{\mathbf{r}}_s)\boldsymbol{\sigma}_s\cdot\mathbf{p}_s\delta_{wu}$
$\mathcal{M}[1\,w\,u\,p]$	$ij_w(qr_s)G_{-1}(r_s)\mathcal{Y}_{1wu}^{M_f-M_i}(\hat{\mathbf{r}}_s,\mathbf{p}_s)$

Morita, Fujii, *Phys. Rev.* **118**, 606 (1960)

Bound-Muon Wave Functions

- Expand the muon wave function in terms of spherical spinors

$$\psi_\mu(\kappa,\mu;\mathbf{r}) = \psi_{\kappa\mu}^{(\mu)} = \begin{bmatrix} -iF_\kappa(r)\chi_{-\kappa\mu} \\ G_\kappa(r)\chi_{\kappa\mu} \end{bmatrix},$$

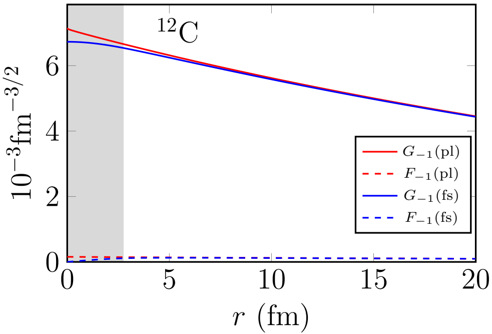
where $\kappa = -j(j+1) + l(l+1) - \frac{1}{4}$
($\kappa = -1$ for the $1s_{1/2}$ orbit)

- Solve the Dirac equations in the Coulomb potential $V(r)$:

$$\begin{cases} \frac{\mathrm{d}}{\mathrm{d}r}G_{-1} + \frac{1}{r}G_{-1} = \frac{1}{\hbar c}(mc^2 - E + V(r))F_{-1} \\ \frac{\mathrm{d}}{\mathrm{d}r}F_{-1} - \frac{1}{r}F_{-1} = \frac{1}{\hbar c}(mc^2 + E - V(r))G_{-1} \end{cases}$$

$$\rightarrow (J_f || \textstyle\sum_{s=1}^{A} G_{-1}(r_s)\mathcal{O}_s(q_s, r_s, \boldsymbol{\sigma}_s) || J_i)$$

pl = pointlike
fs = finite size nucleus



LJ, Navrátil, Kotila, Kravvaris, arXiv:2403.05776

TRIUMF

Translationally invariant wave function

- We are not interested in the motion of the center of mass (CM) of the HO potential but only the intrinsic motion
- Translationally invariant wave functions can be achieved in two ways:
  - Working with $A-1$ Jacobi coordinates $\boldsymbol{\xi}_s = -\sqrt{A/(A-1)}(\mathbf{r}_s - \mathbf{R}_{\mathrm{CM}})$:

$$\Psi^A = \sum_{N=0}^{N_{\max}} \sum_i c_{Ni} \Phi^{\mathrm{HO}}_{Ni}(\boldsymbol{\xi}_1, \boldsymbol{\xi}_2, ..., \boldsymbol{\xi}_{A-1})$$

  - Working with A single-particle coordinates and separating the center-of-mass motion:

$$\Psi^A_{\mathrm{SD}} = \sum_{N=0}^{N_{\max}} \sum_i c^{\mathrm{SD}}_{Nj} \Phi^{\mathrm{HO}}_{\mathrm{SD}\ Nj}(\mathbf{r}_1, \mathbf{r}_2, ..., \mathbf{r}_A) = \Psi^A \Psi_{\mathrm{CM}}(\mathbf{R}_{\mathrm{CM}})$$

Discovery, accelerated

Translationally Invariant Operators

- Operators depend on coordinates $\mathbf{r_s}$ and $\mathbf{p_s}$ w.r.t. the center of mass (CM) of the HO potential
- We remove CM contamination as:

Navrátil, *Phys. Rev. C* **104**, 064322 (2021)

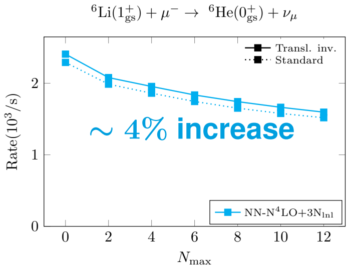
$$(\Psi_f || \sum_{s=1}^{A} \hat{O}_s(\mathbf{r}_s - \mathbf{R}_{\mathrm{CM}}, \mathbf{p}_s - \mathbf{P}) || \Psi_i)$$

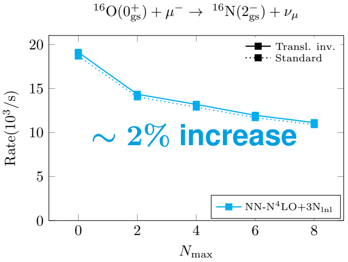
$$= \sum_{pnp'n'} (n' || \hat{O}_s \left(-\sqrt{\frac{A-1}{A}} \boldsymbol{\xi}_s, -\sqrt{\frac{A-1}{A}} \boldsymbol{\pi}_s \right) || p')$$

$$\times (\boldsymbol{M^u})^{-1}_{\boldsymbol{n'p',np}} \frac{-1}{\sqrt{2u+1}} (\Psi_f || [a^\dagger_n \tilde{a}_p]_u || \Psi_i) \ ,$$

where

$$\boldsymbol{\xi}_s = -\sqrt{A/(A-1)}(\mathbf{r}_s - \mathbf{R}_{\mathrm{CM}}) \ ; \ \boldsymbol{\pi}_s = -\sqrt{A/(A-1)}(\mathbf{p}_s - \mathbf{P})$$





LJ, Navrátil, Kotila and Kravvaris, arXiv:2403.05776

Discovery, accelerated

Axial-Vector Two-Body Currents (2BCs)

- One-body (1b) axial-vector currents given by

$$\mathbf{J}^3_{i,1\mathrm{b}} = \frac{\tau^3_i}{2}\left(g_\mathrm{A}\boldsymbol{\sigma}_i - \frac{g_\mathrm{P}}{2m_\mathrm{N}}\mathbf{q}\cdot\boldsymbol{\sigma}_i\right) ,$$

where $g_\mathrm{P} = (2m_\mathrm{N}q/(q^2+m_\pi^2))g_\mathrm{A}$

- Additional **pion-exchange, pion-pole, and contact** two-body (2b) currents

Hoferichter, Klos, Schwenk *Phys. Lett. B* **746**, 410 (2015)

$$\begin{aligned}\mathbf{J}^3_{12} = &-\frac{g_\mathrm{A}}{2F_\pi^2}[\tau_1\times\tau_2]^3\left[c_4\left(1-\frac{\mathbf{q}}{\mathbf{q}^2+M_\pi}\mathbf{q}\cdot\right)(\boldsymbol{\sigma}_1\times\mathbf{k}_2)+\frac{c_6}{4}(\boldsymbol{\sigma}_1\times\mathbf{q})+i\frac{\mathbf{p}_1+\mathbf{p}_1'}{4m_\mathrm{N}}\right]\frac{\boldsymbol{\sigma}_2\cdot\mathbf{k}_2}{M_\pi^2+k_2^2}\\ &-\frac{g_\mathrm{A}}{F_\pi^2}\tau_2^3\left[c_3\left(1-\frac{\mathbf{q}}{\mathbf{q}^2+M_\pi}\mathbf{q}\cdot\right)\mathbf{k}_2+2c_1M_\pi^2\frac{\mathbf{q}}{\mathbf{q}^2+M_\pi^2}\right]\frac{\boldsymbol{\sigma}_2\cdot\mathbf{k}_2}{M_\pi^2+k_2^2}\\ &-d_1\tau_1^3\left(1-\frac{\mathbf{q}}{\mathbf{q}^2+M_\pi^2}\mathbf{q}\cdot\right)\boldsymbol{\sigma}_1+(1\leftrightarrow 2)-d_2(\tau_1\times\tau_2)^3(\boldsymbol{\sigma}_1\times\boldsymbol{\sigma}_2)\left(1-\cdot\mathbf{q}\frac{\mathbf{q}}{\mathbf{q}^2+M_\pi^2}\right)\end{aligned}$$

where $\mathbf{k}_i = \mathbf{p}_i' - \mathbf{p}_i$ and $\mathbf{q} = -\mathbf{k}_1 - \mathbf{k}_2$

TRIUMF

Axial-Vector Two-Body Currents (2BCs)

- Approximate 2BCs by normal-ordering w.r.t. spin-isospin–symmetric reference state with $\rho = 2k_{\rm F}^3/(3\pi^2)$:

Hoferichter, Menéndez, Schwenk, *Phys. Rev. D* **102**,074018 (2020)

$$\mathbf{J}^{\rm eff}_{i,2b} = \sum_j (1 - P_{ij}) \mathbf{J}^3_{ij}$$

$$\rightarrow \boxed{\mathbf{J}^{\rm eff}_{i,2b} = g_{\rm A} \frac{\tau_i^3}{2} \left[\delta a(\mathbf{q}^2) \boldsymbol{\sigma}_i + \frac{\delta a^P(\mathbf{q}^2)}{\mathbf{q}^2} (\mathbf{q} \cdot \boldsymbol{\sigma}_i) \mathbf{q} \right]},$$

where

$$\delta_a(\mathbf{q}^2) = -\frac{\rho}{F_\pi^2} \left[\frac{c_4}{3} [3 I_2^\sigma(\rho, \mathbf{q}) - I_1^\sigma(\rho, |\mathbf{q}|)] - \frac{1}{3}\left(c_3 - \frac{1}{4m_{\rm N}}\right) I_1^\sigma(\rho, |\mathbf{q}|) - \frac{c_6}{12} I_{c6}(\rho, |\mathbf{q}|) - \frac{c_D}{4 g_A \Lambda_\chi} \right],$$

$$\delta_a^P(\mathbf{q}^2) = \frac{\rho}{F_\pi^2} \left[-2(c_3 - 2c_1) \frac{m_\pi^2 \mathbf{q}^2}{(m_\pi^2 + \mathbf{q}^2)^2} + \frac{1}{3}\left(c_3 + c_4 - \frac{1}{4m_{\rm N}}\right) I^P(\rho, |\mathbf{q}|) - \left(\frac{c_6}{12} - \frac{2}{3}\frac{c_1 m_\pi^2}{m_\pi^2 + \mathbf{q}^2}\right) I_{c6}(\rho, |\mathbf{q}|) \right.$$
$$\left. - \frac{\mathbf{q}^2}{m_\pi^2 + \mathbf{q}^2} \left(\frac{c_3}{3} [I_1^\sigma(\rho, |\mathbf{q}|) + I^P(\rho, |\mathbf{q}|)] + \frac{c_4}{3} [I_1^\sigma(\rho, |\mathbf{q}|) + I^P(\rho, |\mathbf{q}|) - 3 I_2^\sigma(\rho, |\mathbf{q}|)] \right) - \frac{c_D}{4 g_A \Lambda_\chi} \frac{\mathbf{q}^2}{m_\pi^2 + \mathbf{q}^2} \right]$$

TRIUMF

Axial-Vector Two-Body Currents (2BCs)

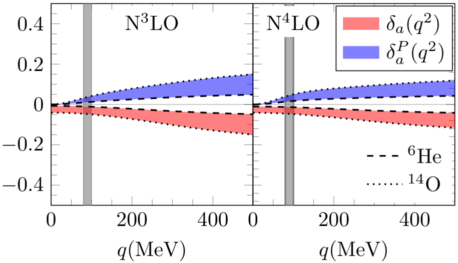
- One-body currents

$$\mathbf{J}^3_{i,1\mathrm{b}} = \tau_i^- \left(g_\mathrm{A}(q^2)\boldsymbol{\sigma}_i - \frac{g_\mathrm{P}(q^2)}{2m_\mathrm{N}} \mathbf{q} \cdot \boldsymbol{\sigma}_i \right)$$

+ two-body currents

$$\mathbf{J}^\mathrm{eff}_{i,2\mathrm{b}} = g_\mathrm{A}\tau_i^- \left[\delta a(\mathbf{q}^2)\boldsymbol{\sigma}_i + \frac{\delta a^P(\mathbf{q}^2)}{\mathbf{q}^2}(\mathbf{q}\cdot\boldsymbol{\sigma}_i)\mathbf{q} \right]$$

*Hoferichter, Klos, Schwenk Phys. Lett. B **746**, 410 (2015)*



LJ, Navrátil, Kotila, Kravvaris, arXiv:2403.05776

- Two-body currents approximated by

$$\begin{cases} g_\mathrm{A}(q^2, 2\mathrm{b}) \to g_\mathrm{A}(q^2) + g_\mathrm{A}\boldsymbol{\delta_a(q^2)}, \\ g_\mathrm{P}(q^2, 2\mathrm{b}) \to g_\mathrm{P}(q^2) - \frac{2m_\mathrm{N}g_\mathrm{A}}{q}\boldsymbol{\delta_a^P(q^2)} \end{cases}$$

Discovery, accelerated