Quantum Resource Theories and Beyond

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Quantum revolution

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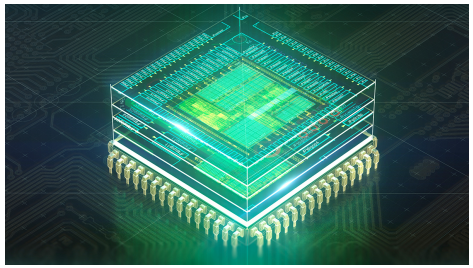
- quantum computing



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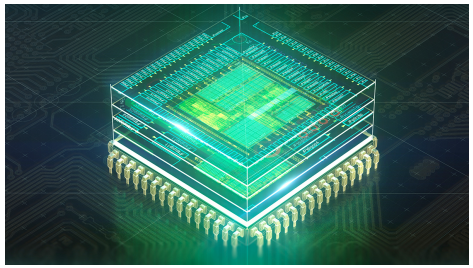

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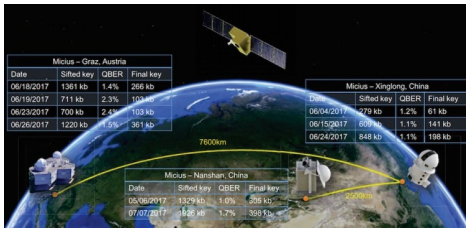

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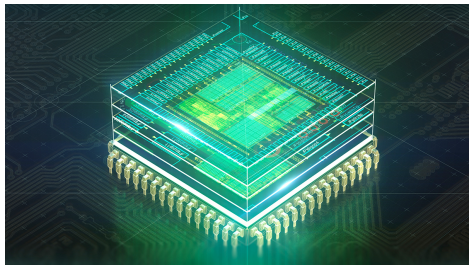

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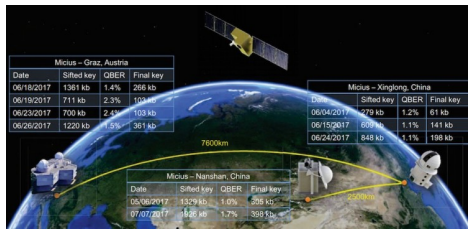

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- Unifying theme in quantum information: quantum is a resource.

This idea is made mathematically rigorous with resource theories.

Outline

Section 1

Resource theories

What is a resource?

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Oil as a resource

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It becomes more valuable if there isn't much of it.

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- If this happens, ρ is more valuable than σ...
- Indeed, we can reach a larger set of states from it.

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- complex numbers in quantum theory [Wu et al. (CMS)].

Section 2

Beyond quantum: GPTs

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States of A are processes from the trivial system to A.

Resource theories as subcategories [Coecke et al.]

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Equilibrium state

$$\chi = \int \psi \, \mathrm{d}\psi,$$

ψ *pure*, i.e. with maximal information.

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We get that nature should be (almost) quantum even from resource theories!

Section 3

Beyond quantum: discrete dynamical systems

Discrete dynamical systems

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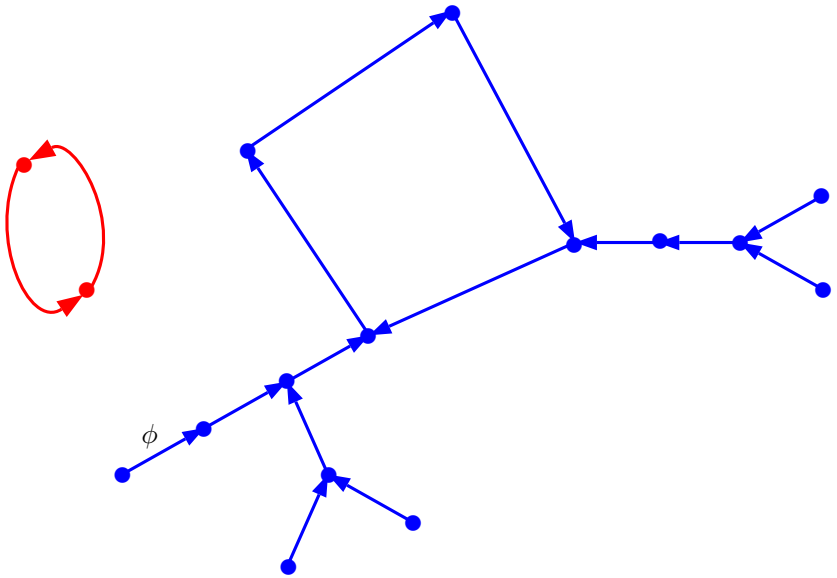
An external influence is any stochastic map $f : \mathfrak{S} \to \mathfrak{S}$, with $\mathfrak{S}$ simplex of probability vectors on S.

Features of a (finite) discrete dynamical system



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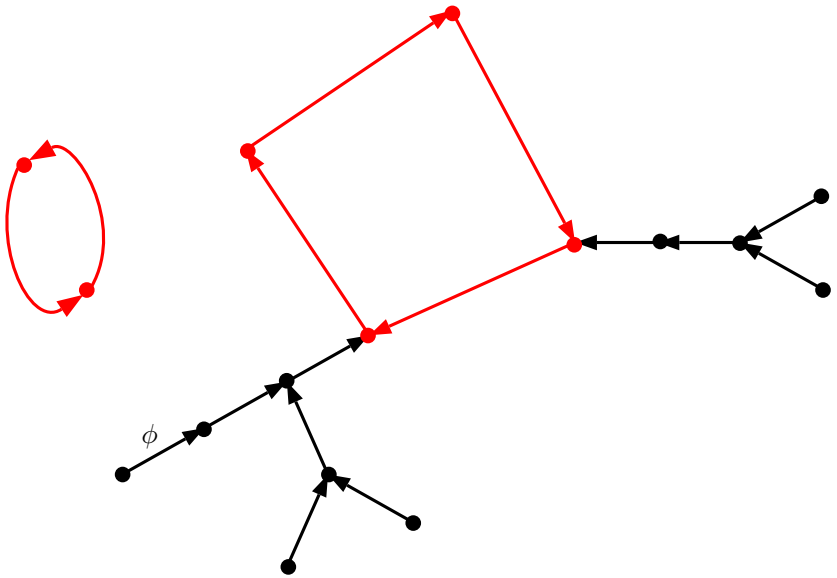

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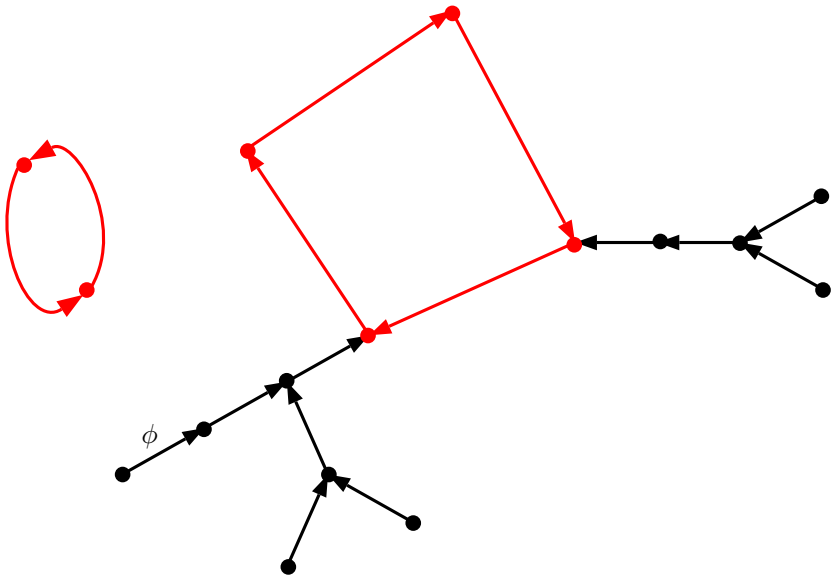

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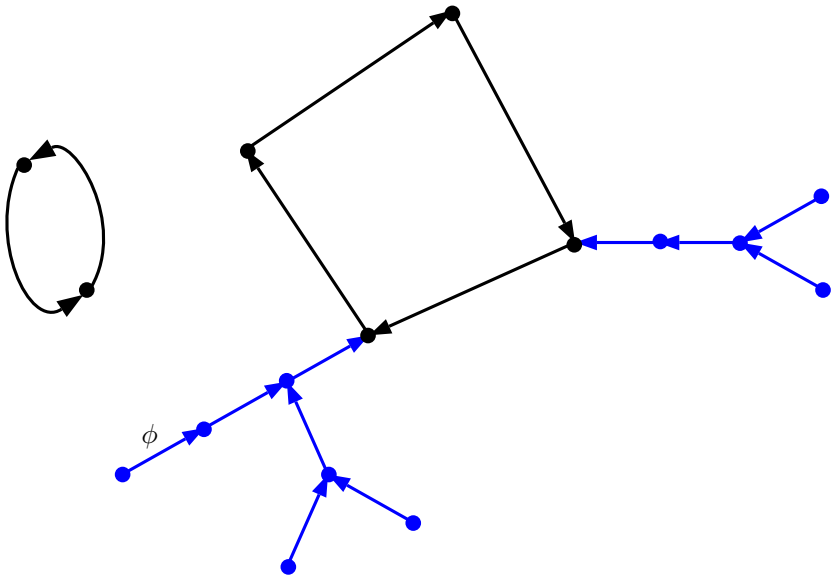

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- There may be transient states too.

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The idea is that the dynamical system has time to adapt to the external influence.

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Stochasticity is a resource: it activates transitions.

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- e.g. why the world is quantum from a thermodynamic angle.
- Resource theories are interdisciplinary!
- For the first time, we applied them to dynamical systems.

References

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