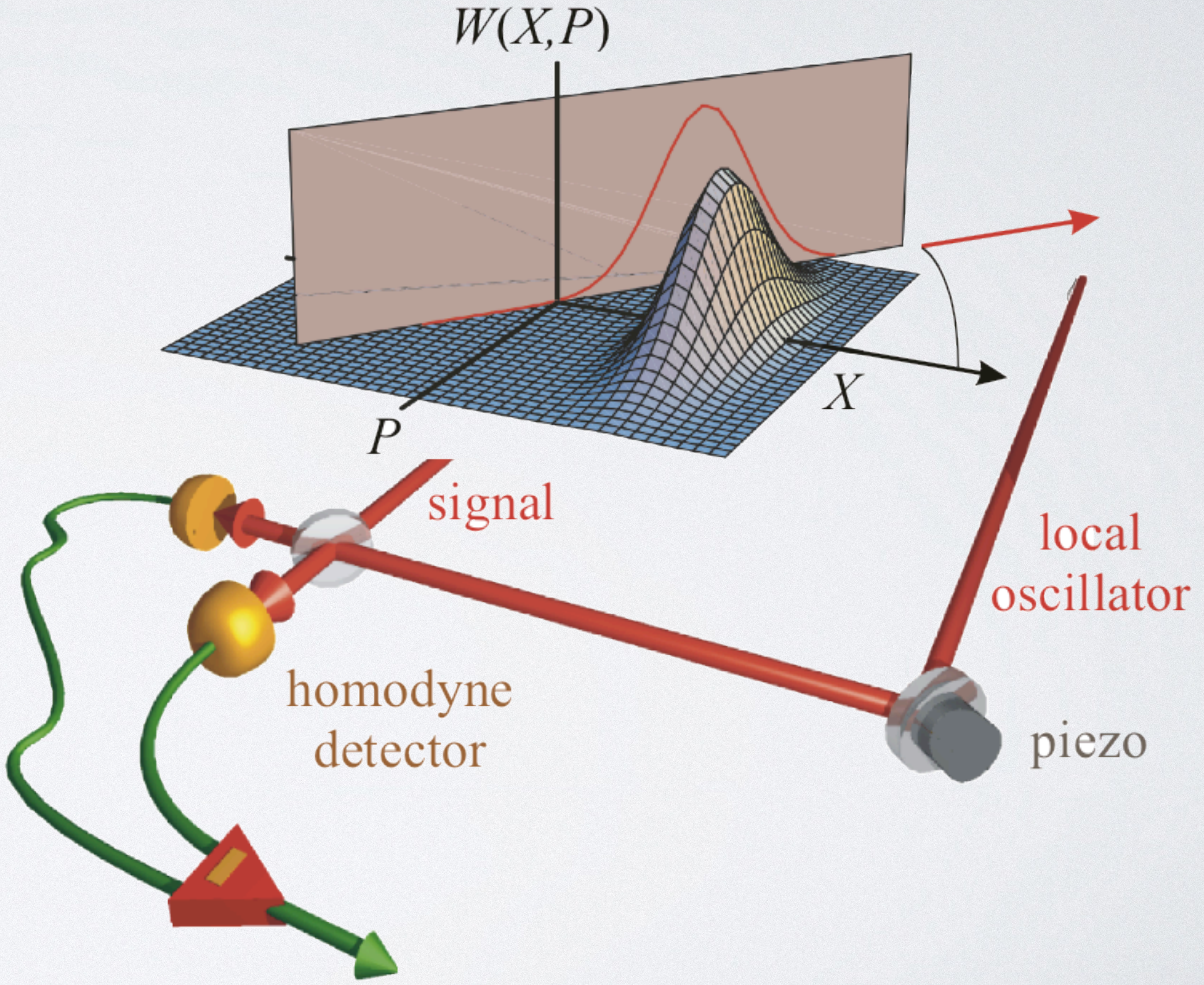
QUANTUM STATE ENGINEERING

Using Collective Spin Excitations

Andrew MacRae, University of Victoria
June 20, 2023

EXPERIMENTAL QUANTUM OPTICS

Quantum state tomography



QUANTUM OPTICS

Basic Definitions

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- Hamiltonian of single mode of electromagnetic field:

$$\hat{H} = \hbar\omega\left(\hat{a}^\dagger\hat{a} + \frac{1}{2}\right) = \hbar\omega\left(\hat{n} + \frac{1}{2}\right)$$

QUANTUM OPTICS

$$\hat{a} = \frac{\hat{q} + i\hat{p}}{\sqrt{2}} \quad \hat{a}^\dagger = \frac{\hat{q} - i\hat{p}}{\sqrt{2}}$$

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- Hamiltonian of single mode of electromagnetic field:
  - Identical to quantum harmonic oscillator

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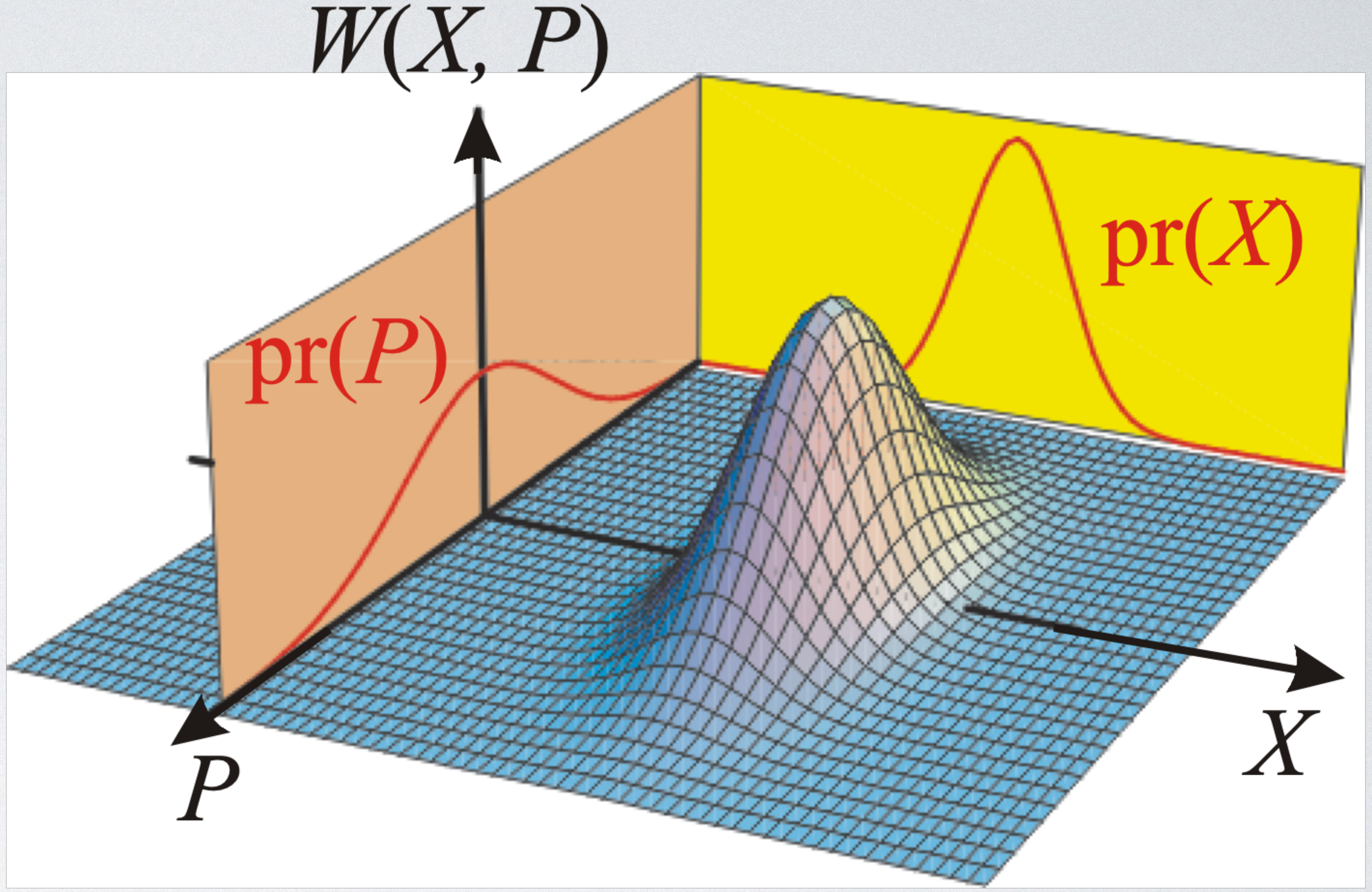
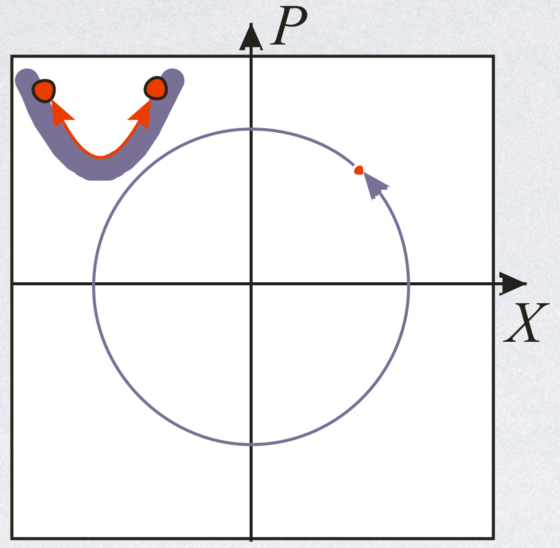
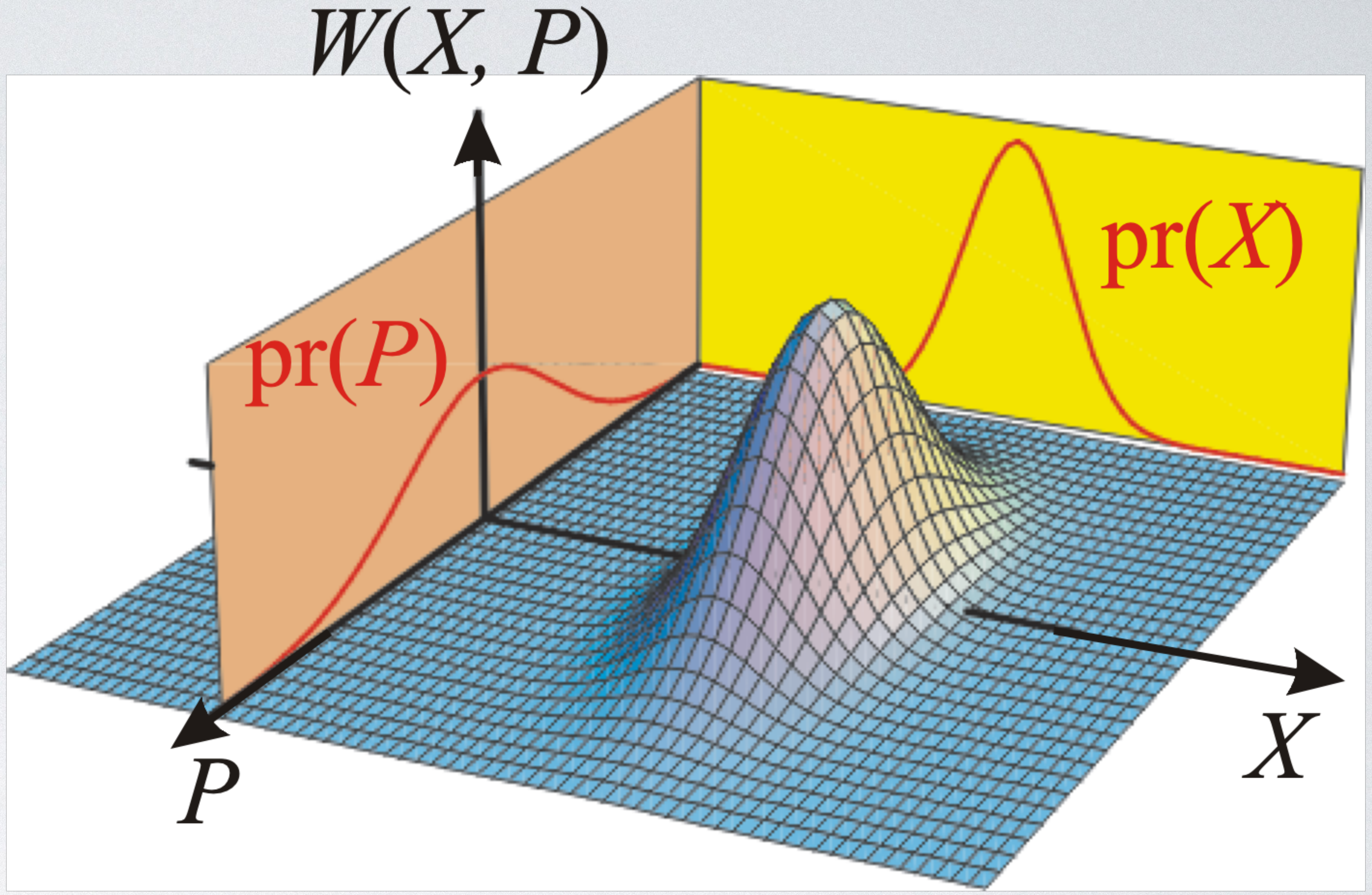
CLASSICAL STATE TOMOGRAPHY



CLASSICAL STATE TOMOGRAPHY

- Harmonic oscillator: phase space description





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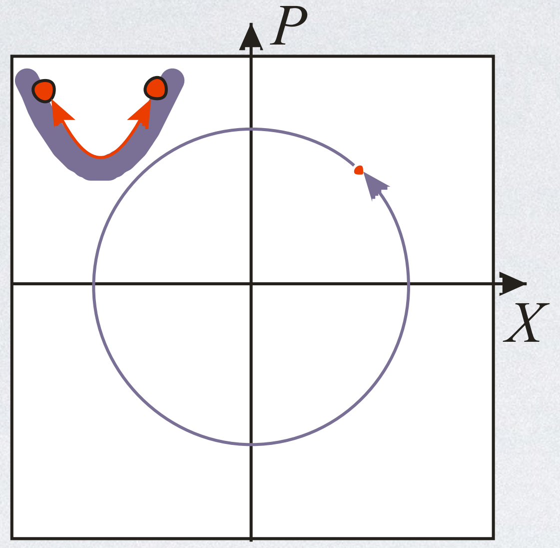
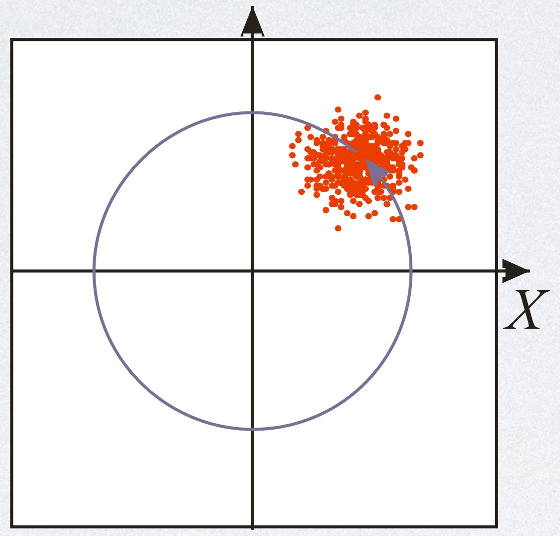
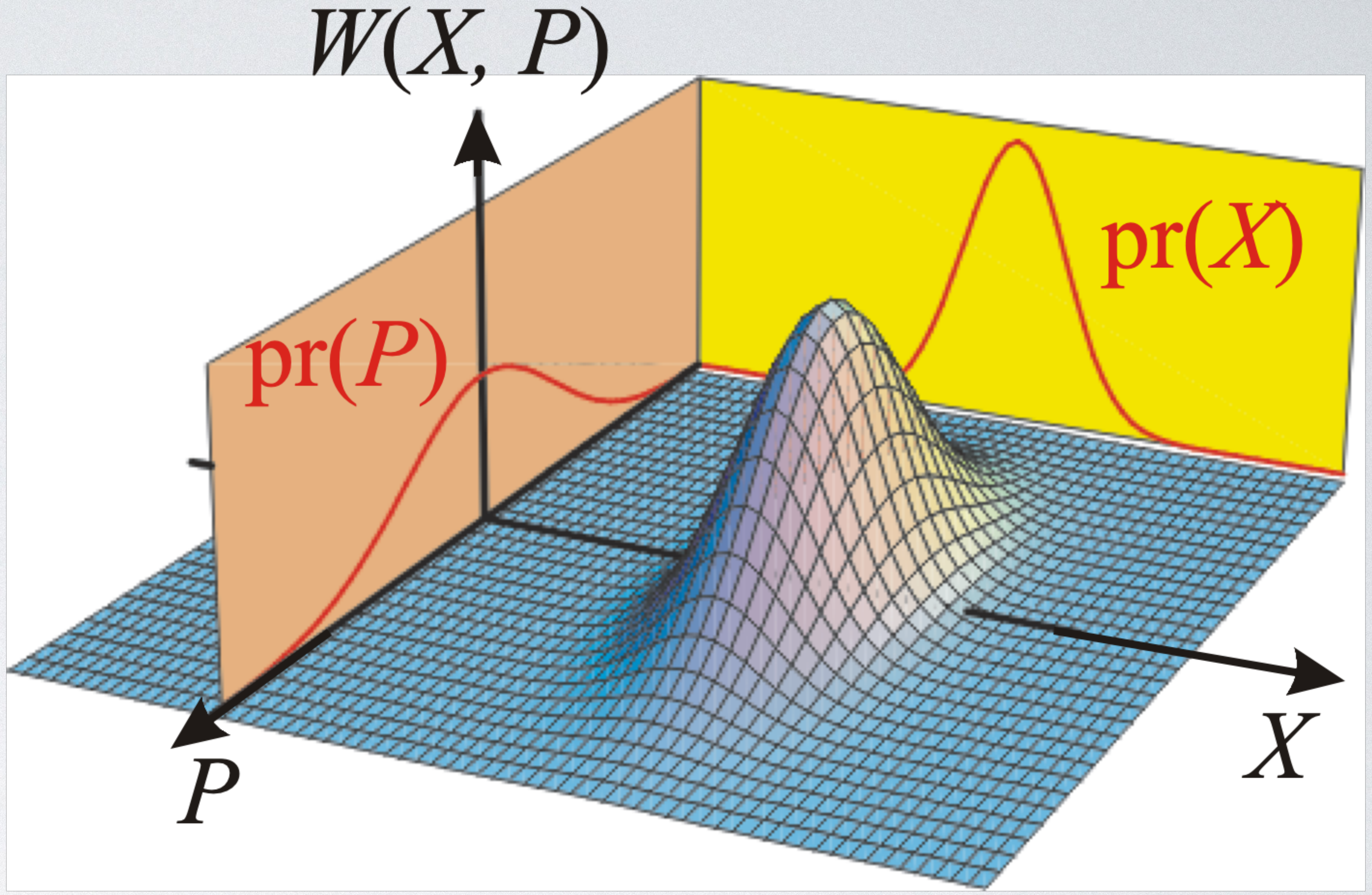
- Harmonic oscillator: phase space description
- Ensemble of harmonic oscillators: phase space probability density: $W(X, P)$







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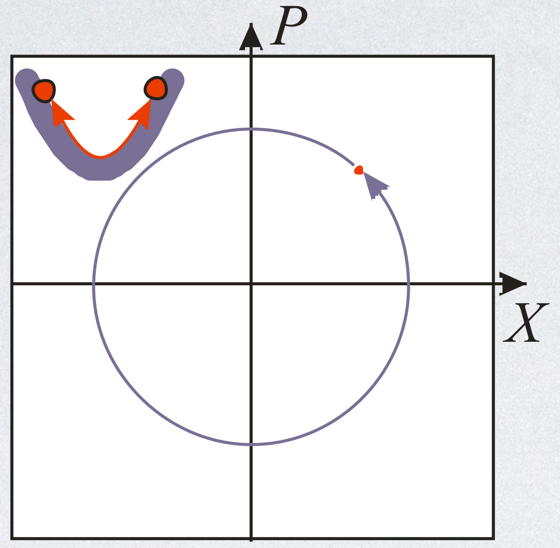

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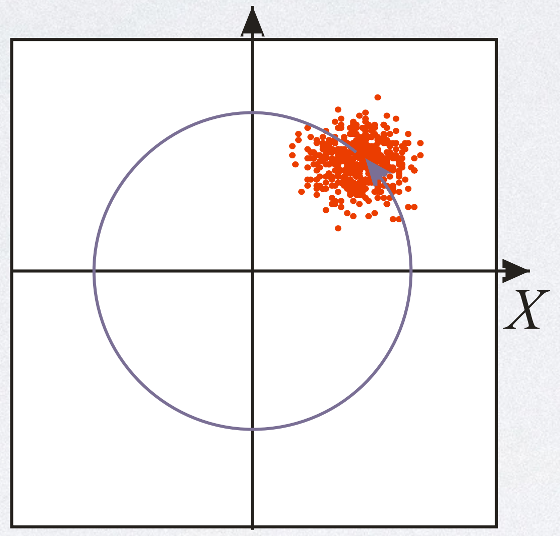

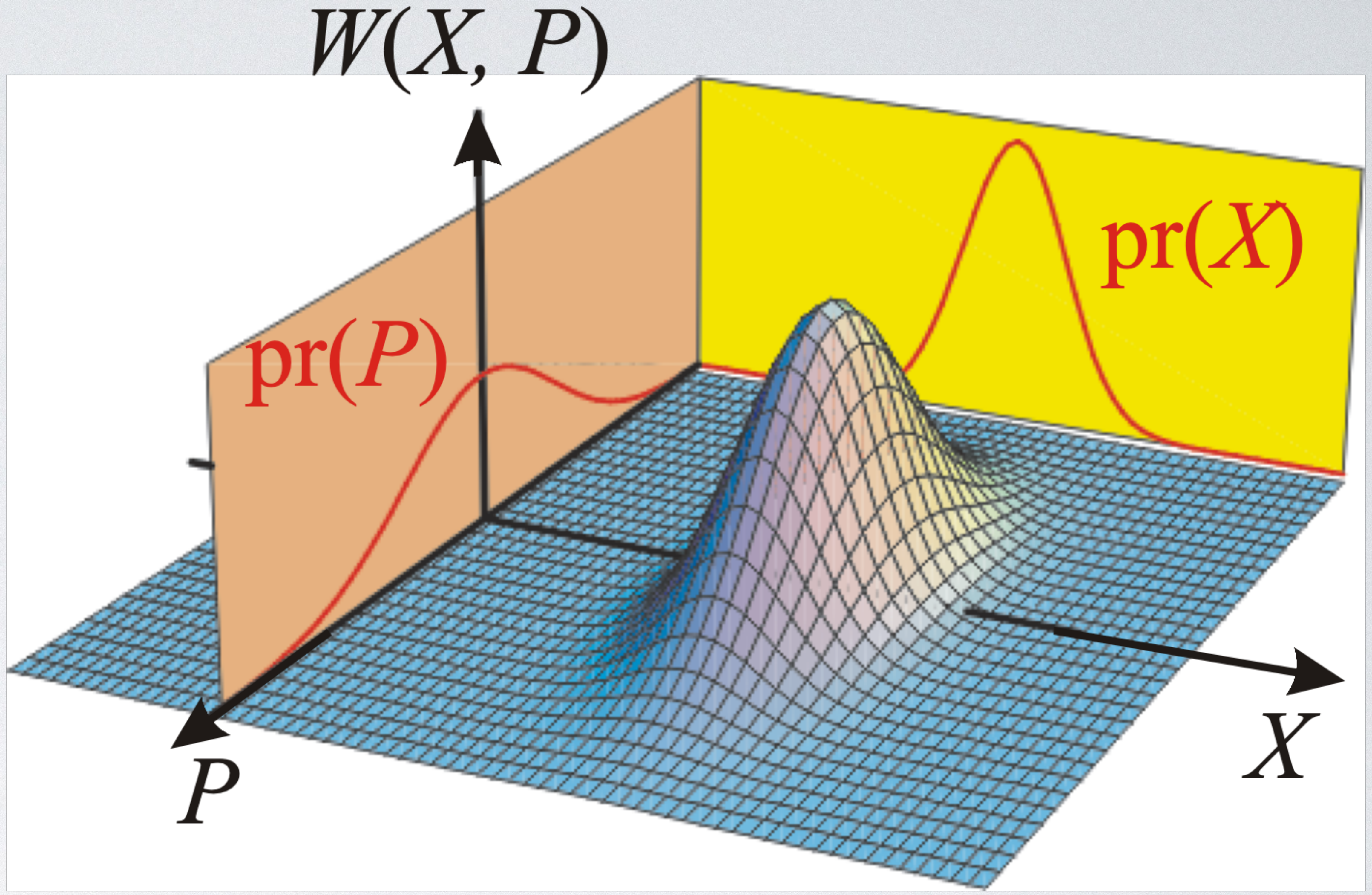

- Repeated measurement of X: **marginal distribution** $pr(X)$

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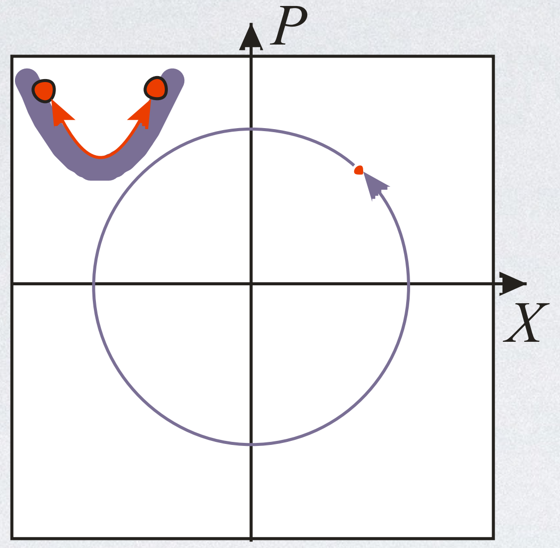

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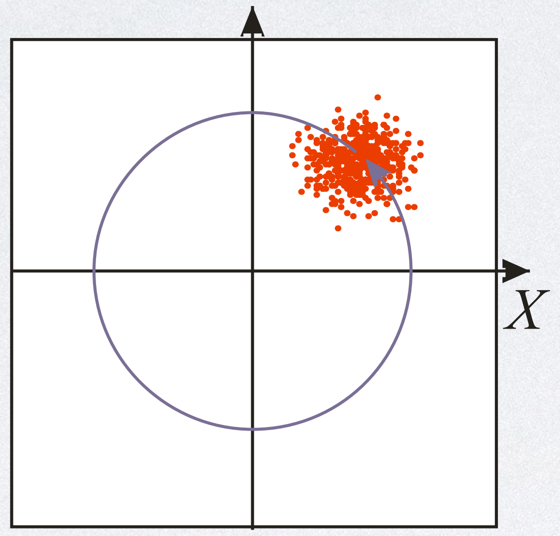
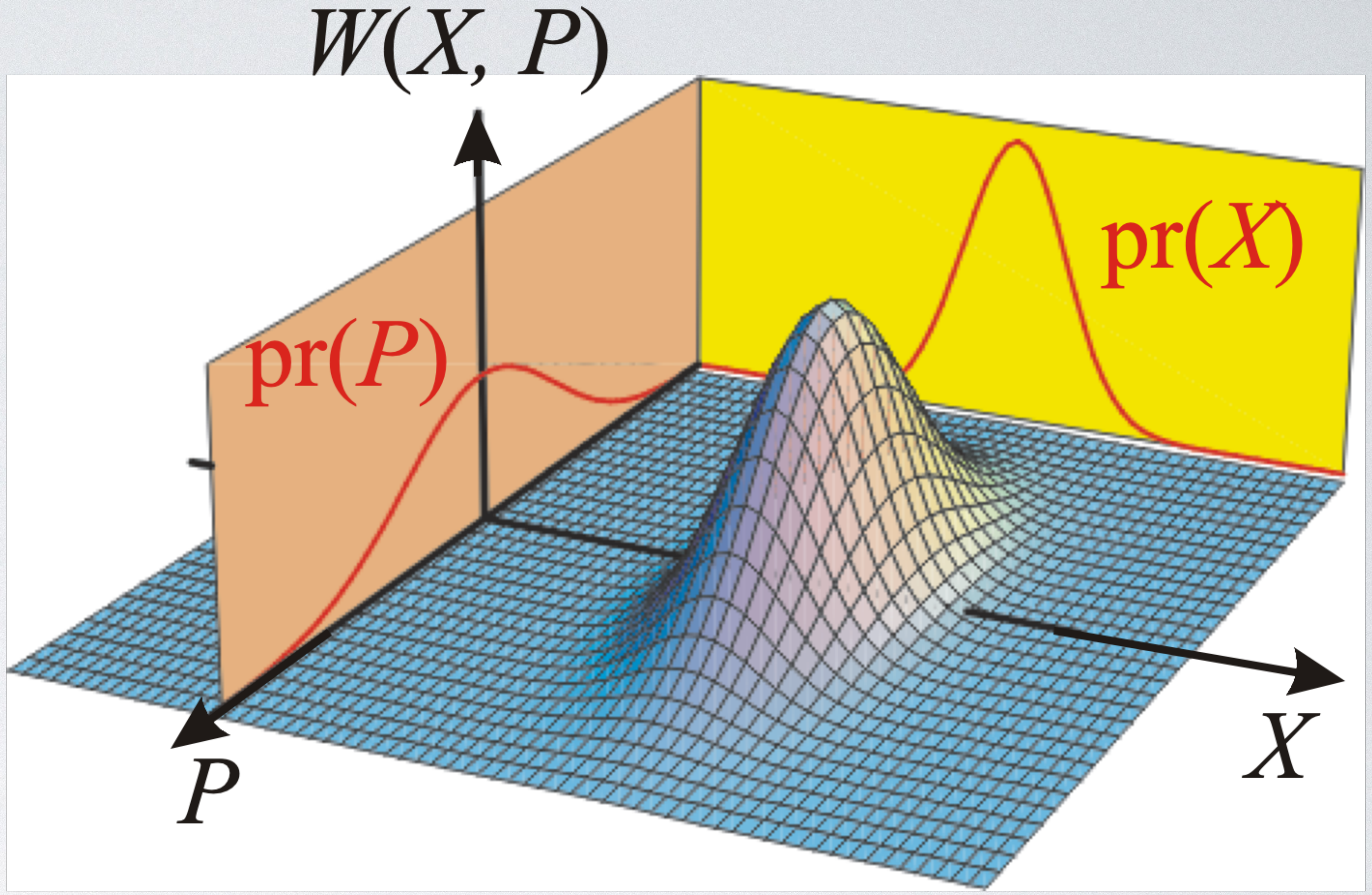
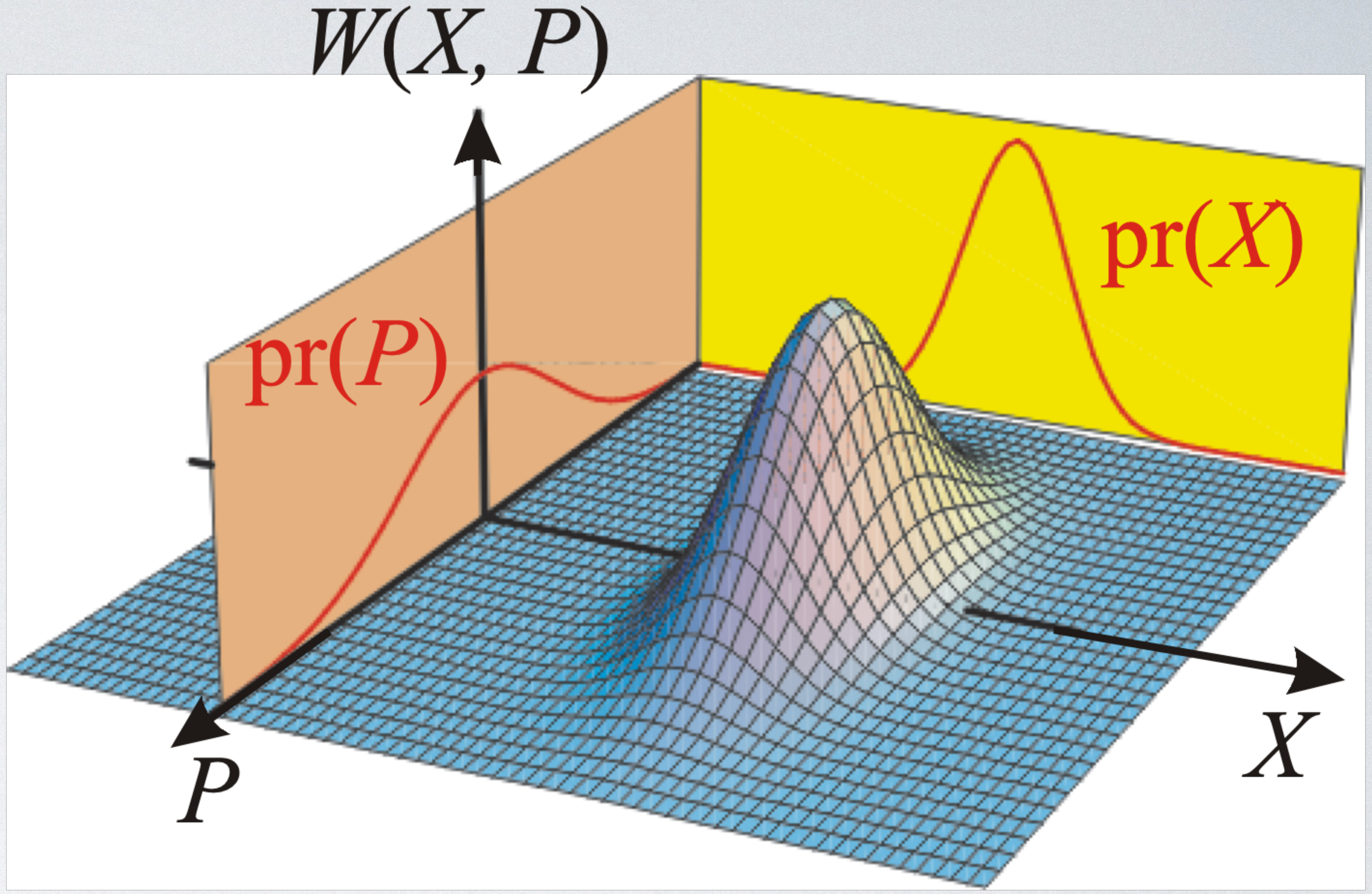




- Repeated measurement of X: **marginal distribution** $pr(X)$

- Relate marginal to density: $$pr(X) = \int_{-\infty}^{\infty} W(X,P)dP \qquad pr(P) = \int_{-\infty}^{\infty} W(X,P)dX$$

QUANTUM STATE TOMOGRAPHY



QUANTUM STATE TOMOGRAPHY

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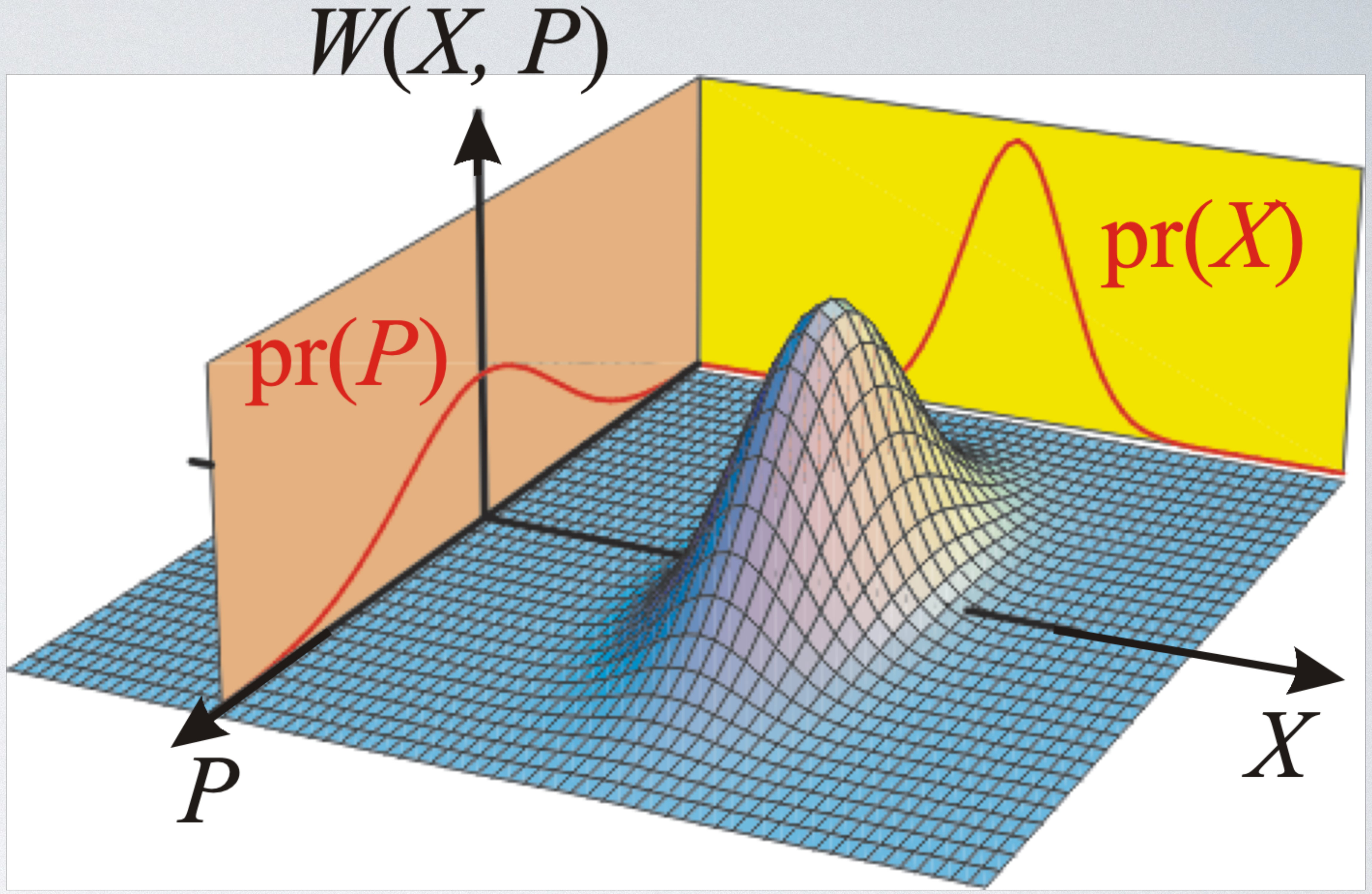
  Can't have "point in phase space"



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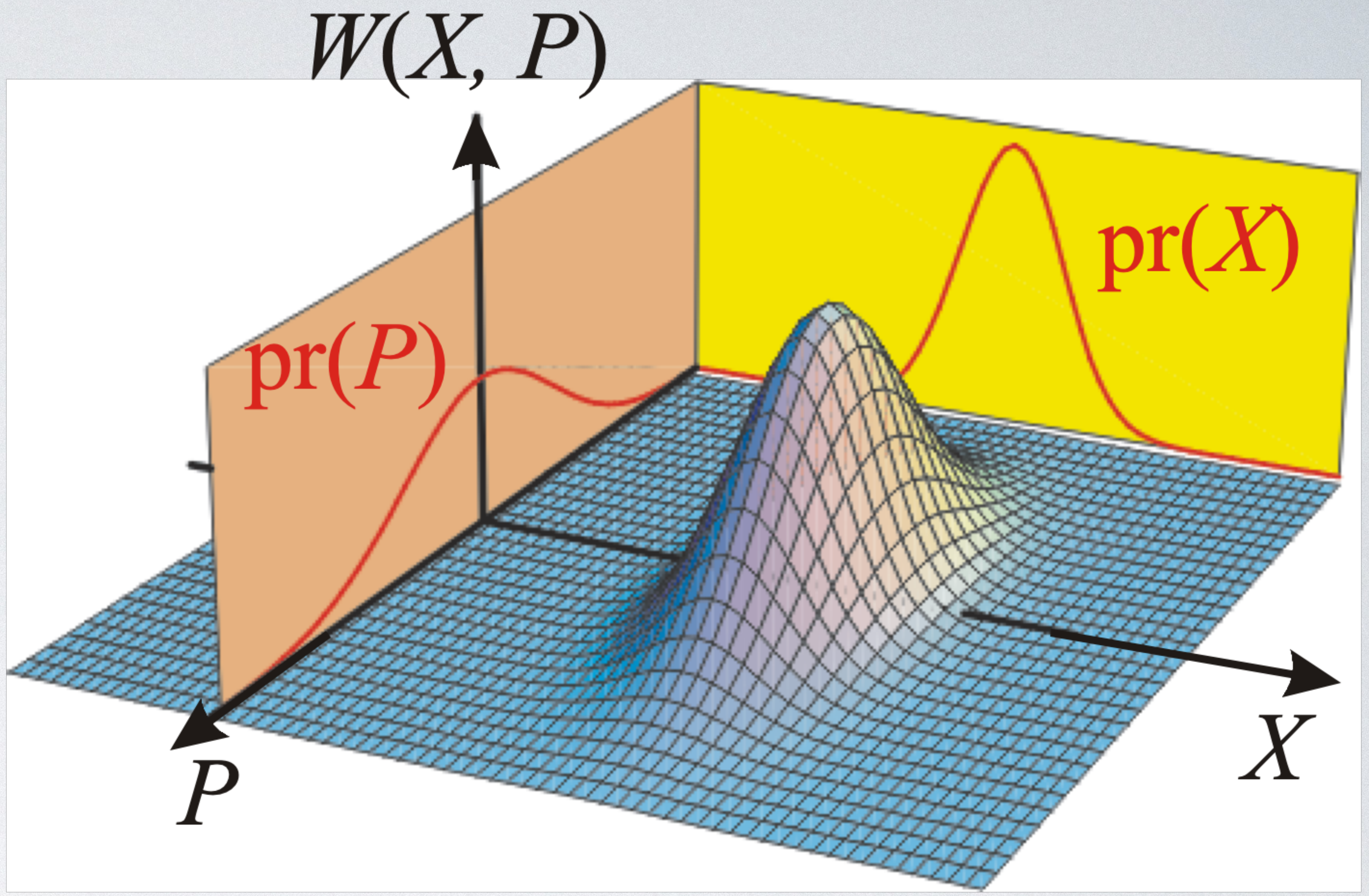

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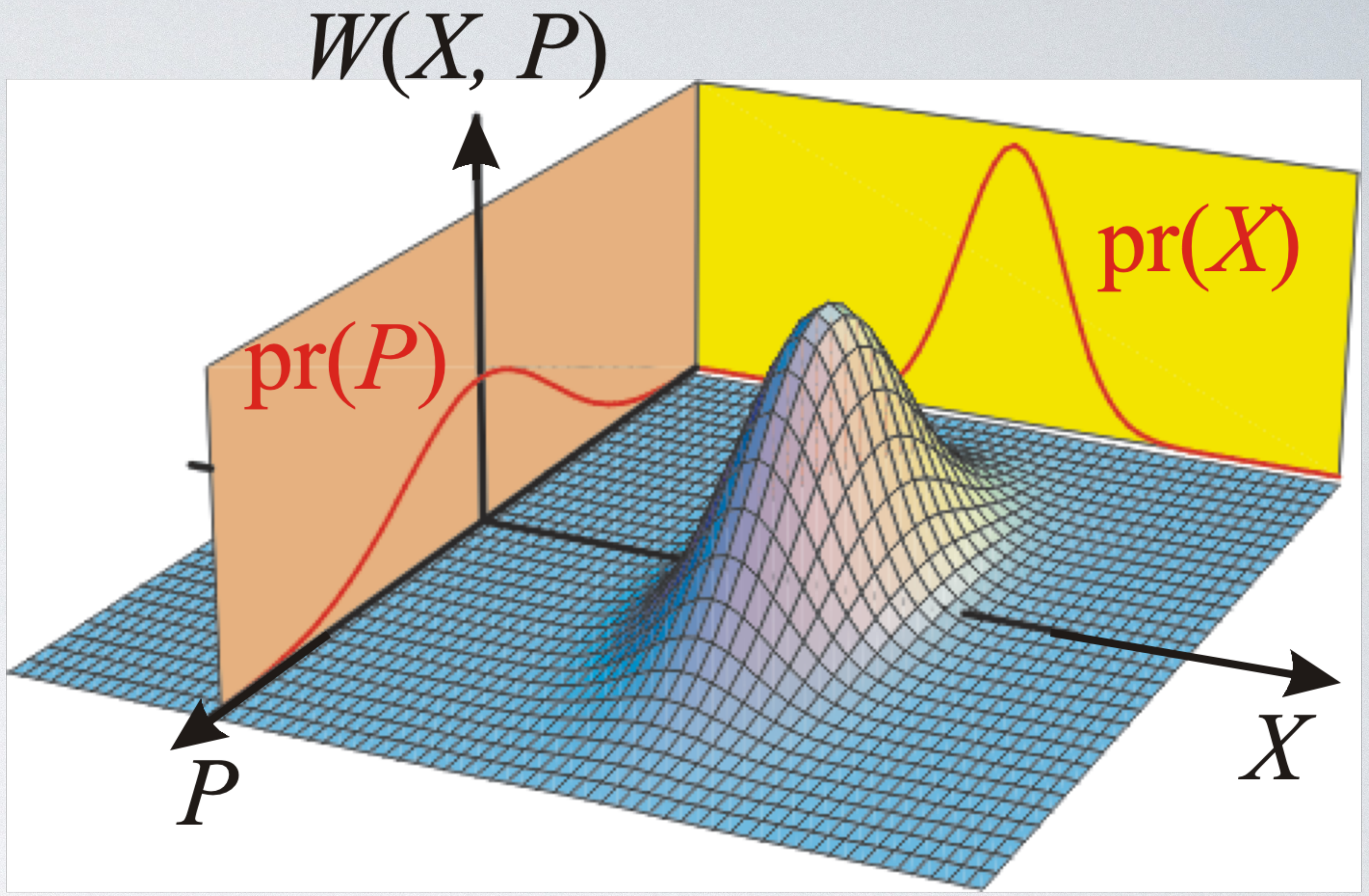

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- Wigner function: $W_\rho(X,P) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{ipq}\langle X - \frac{q}{2} | \hat{\rho} | X + \frac{q}{2}\rangle dq$

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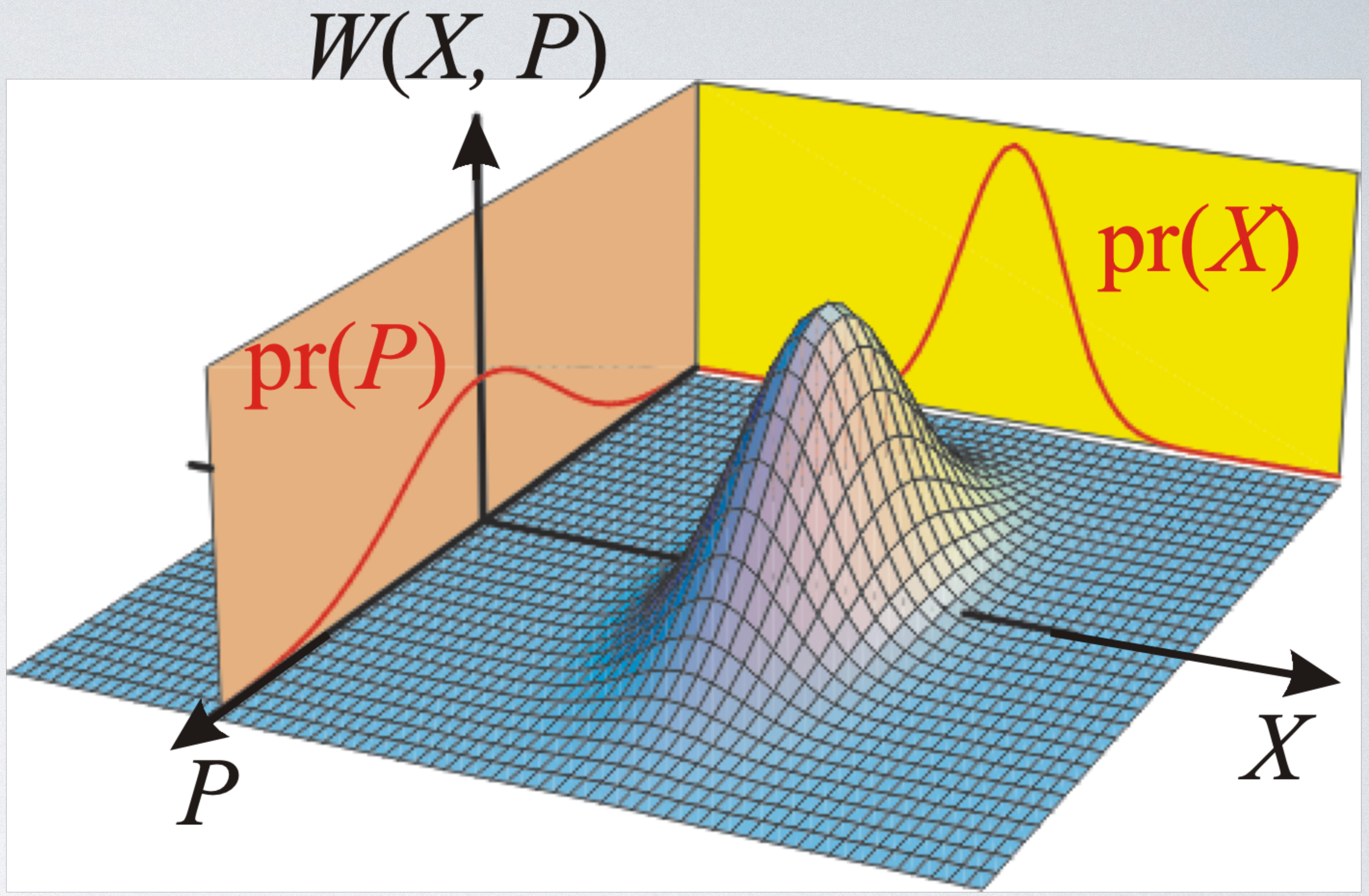

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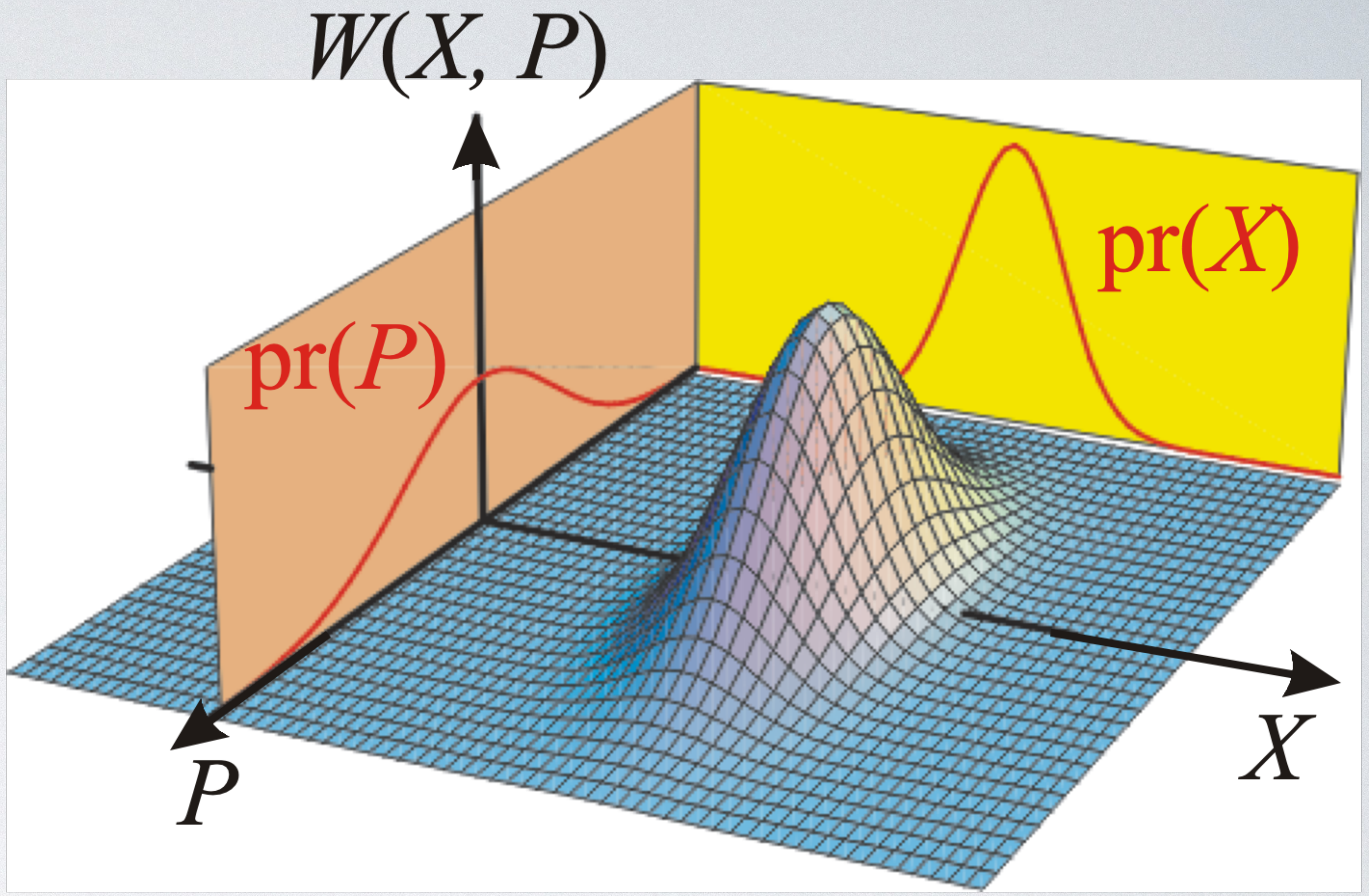

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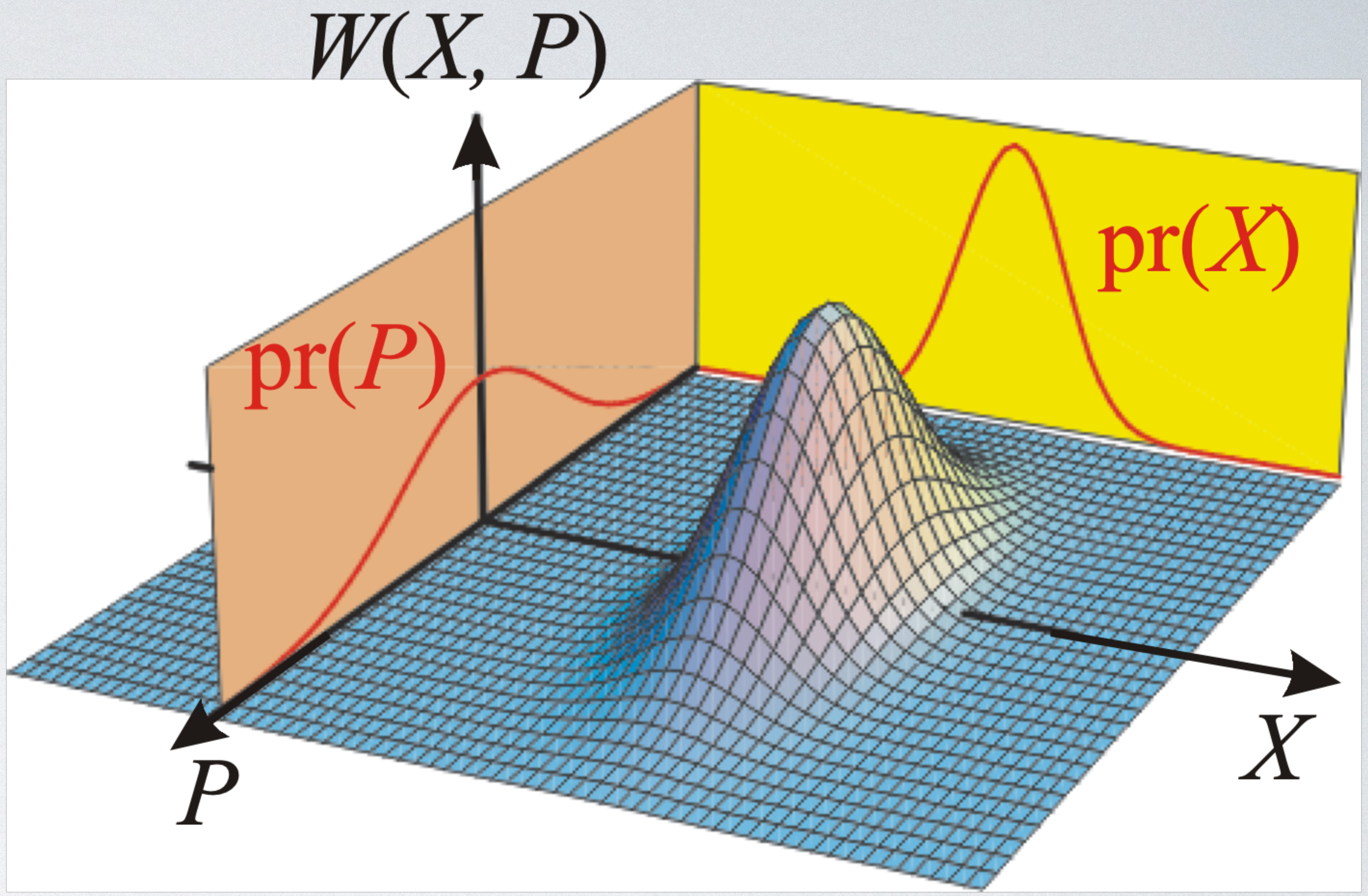

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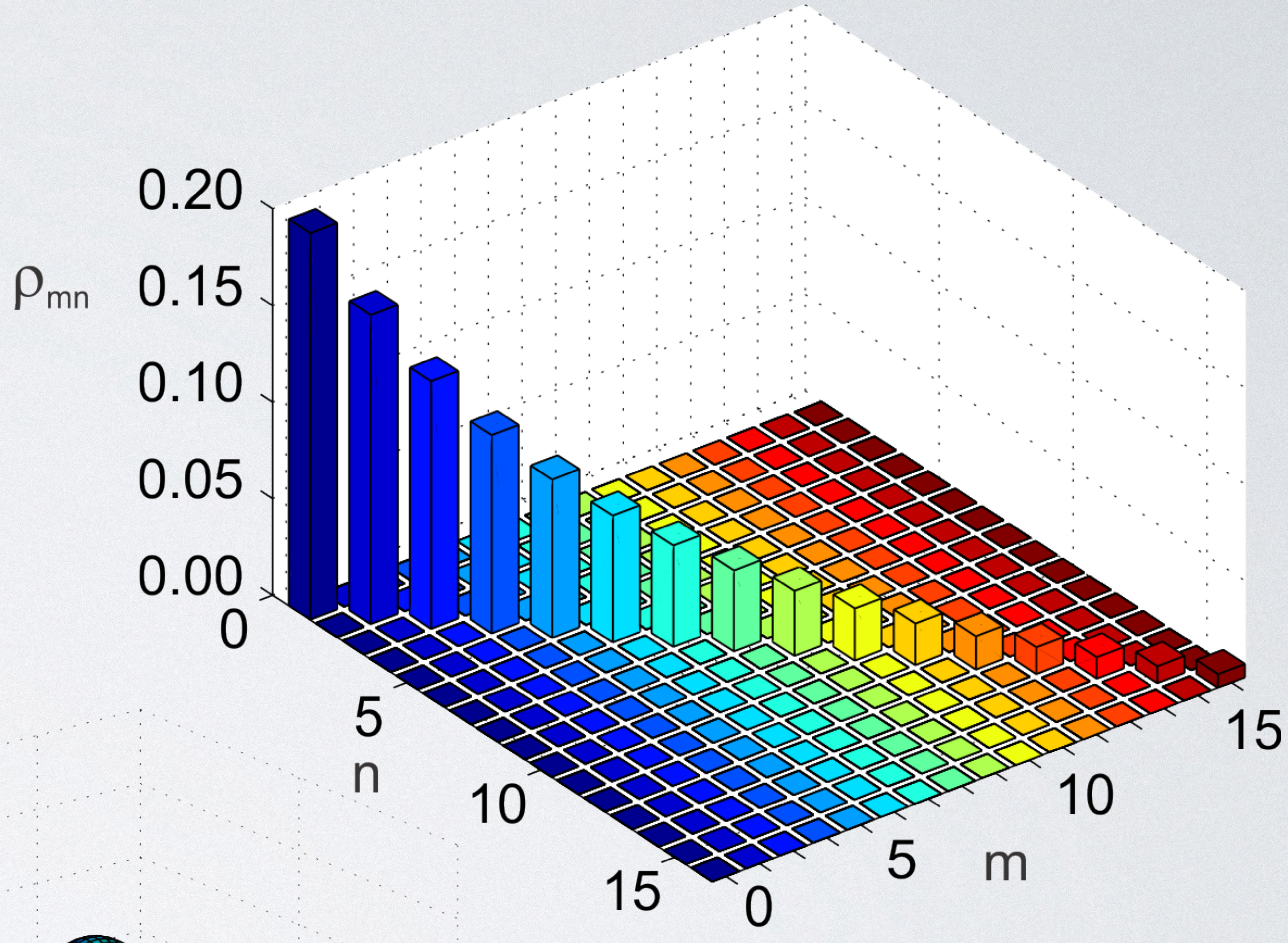
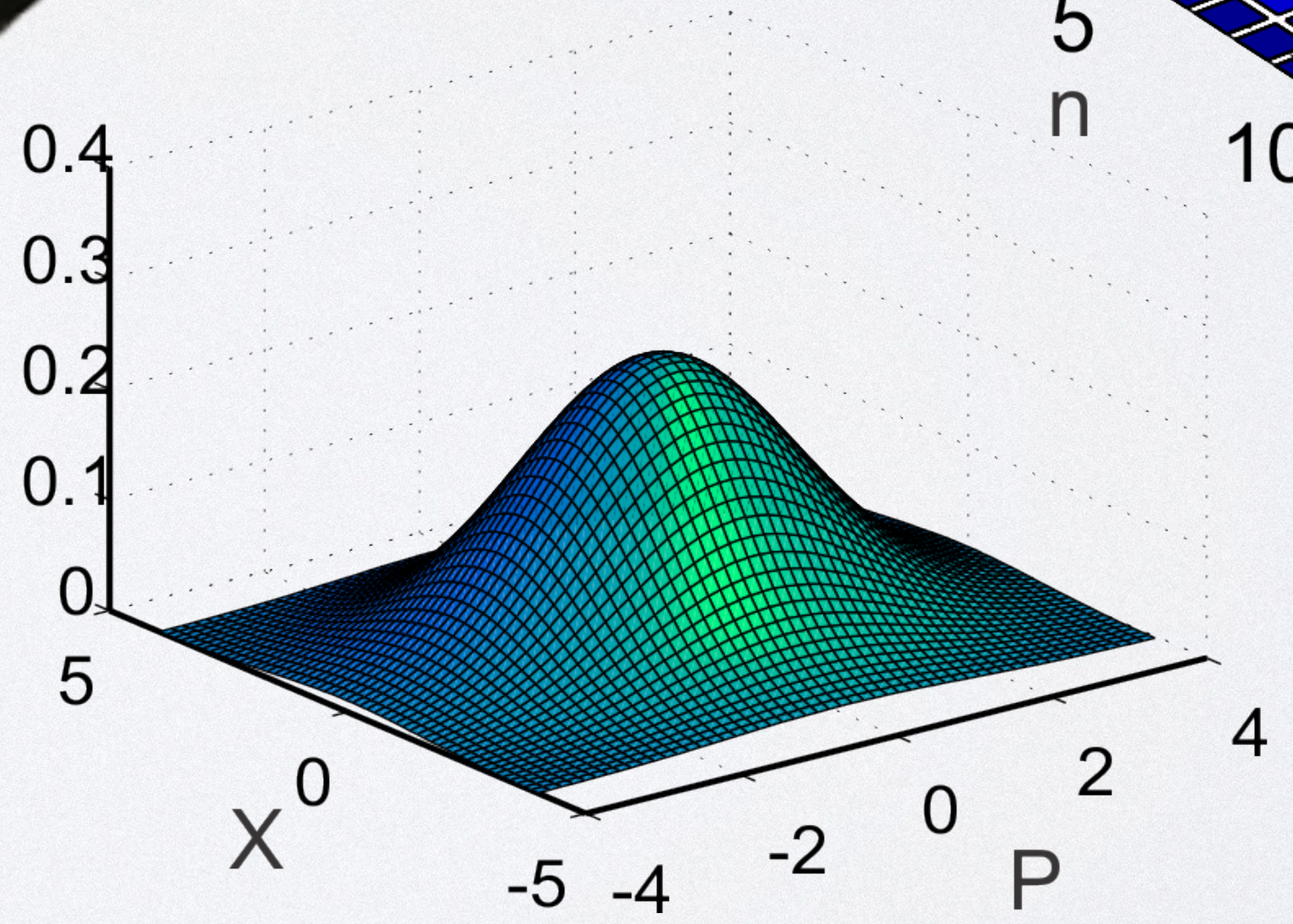
  - Not a probability - can be negative!

BUFFET OF QUANTUM OPTICAL STATES

Thermal State



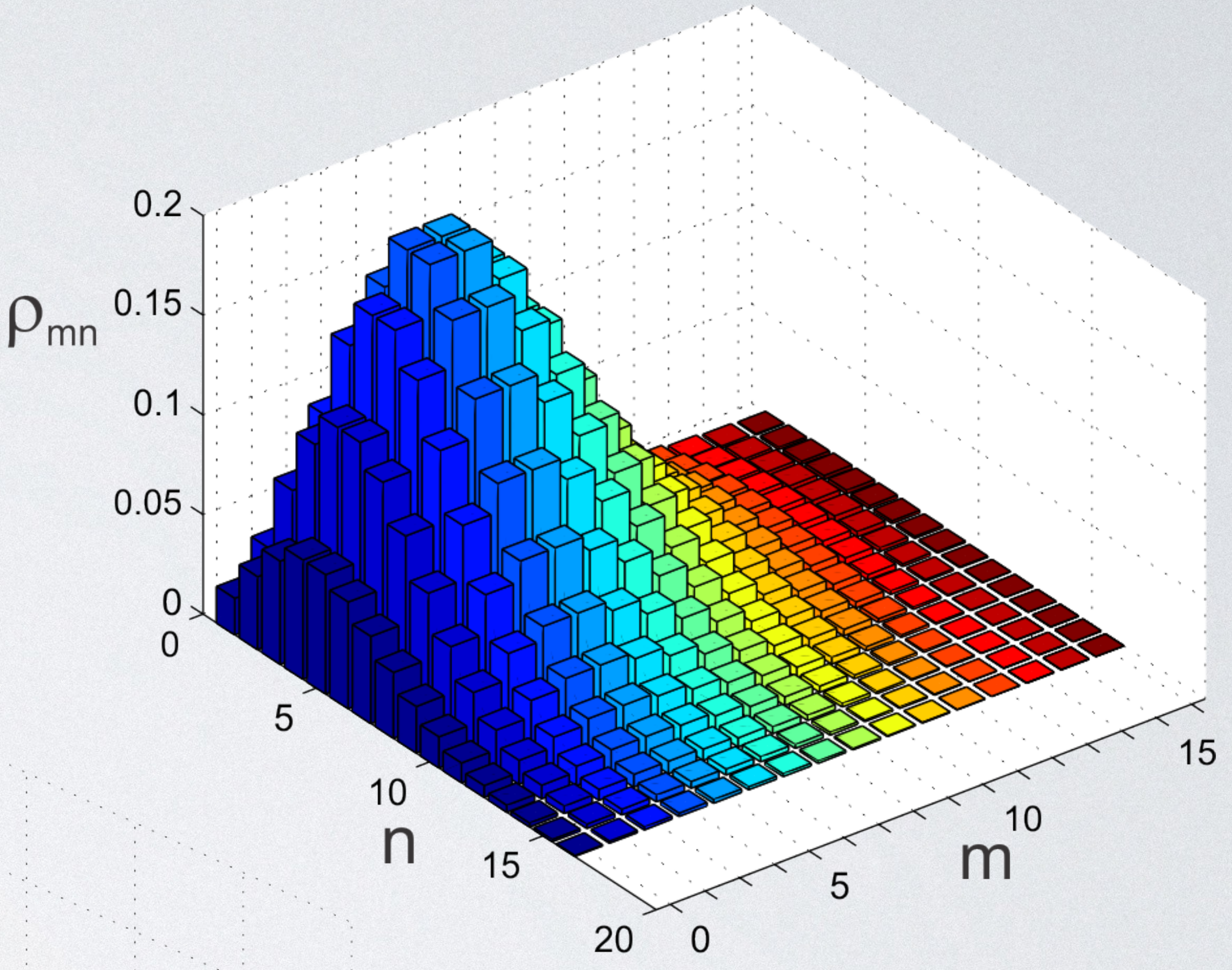
- Incoherent mixture of photons
- $$\hat{\rho} = \sum_n \gamma^n |n\rangle\langle n|$$

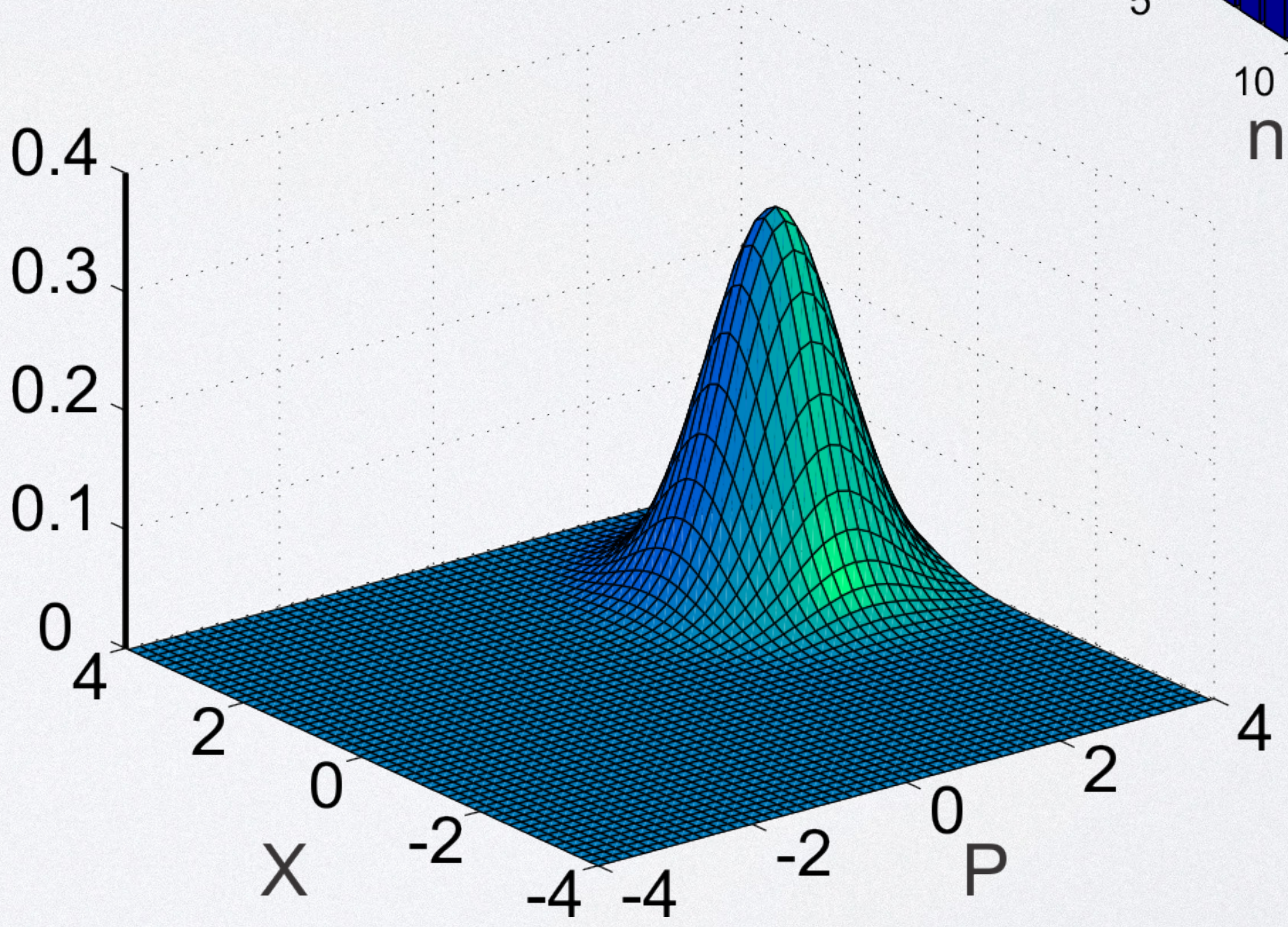


BUFFET OF QUANTUM OPTICAL STATES

Coherent State

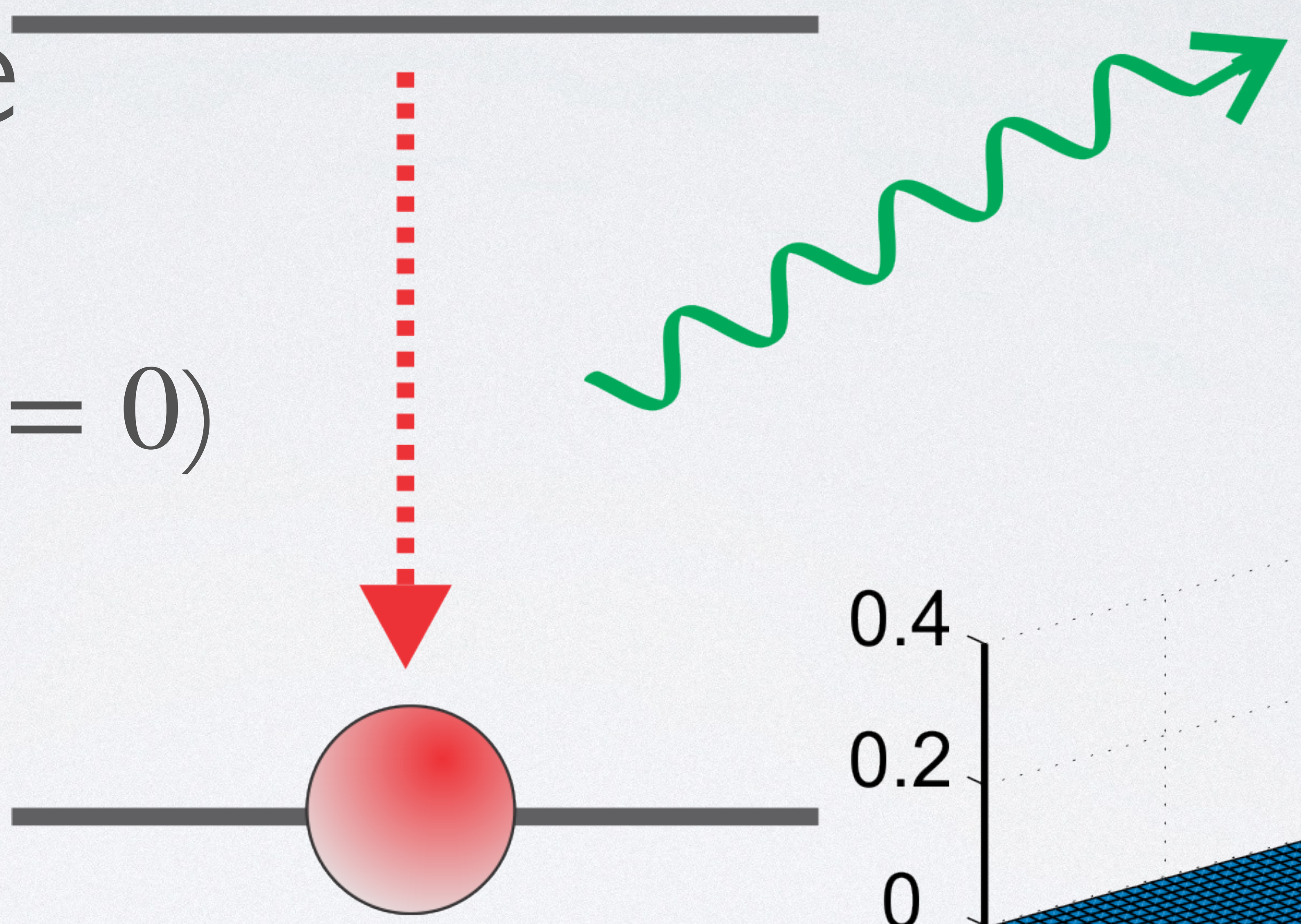
- Coherent mixture of photons
- Canonical classical state
- $$|\alpha\rangle = e^{-\frac{1}{2}|\alpha|^2} \sum_n \frac{\alpha^2}{\sqrt{n!}} |n\rangle$$

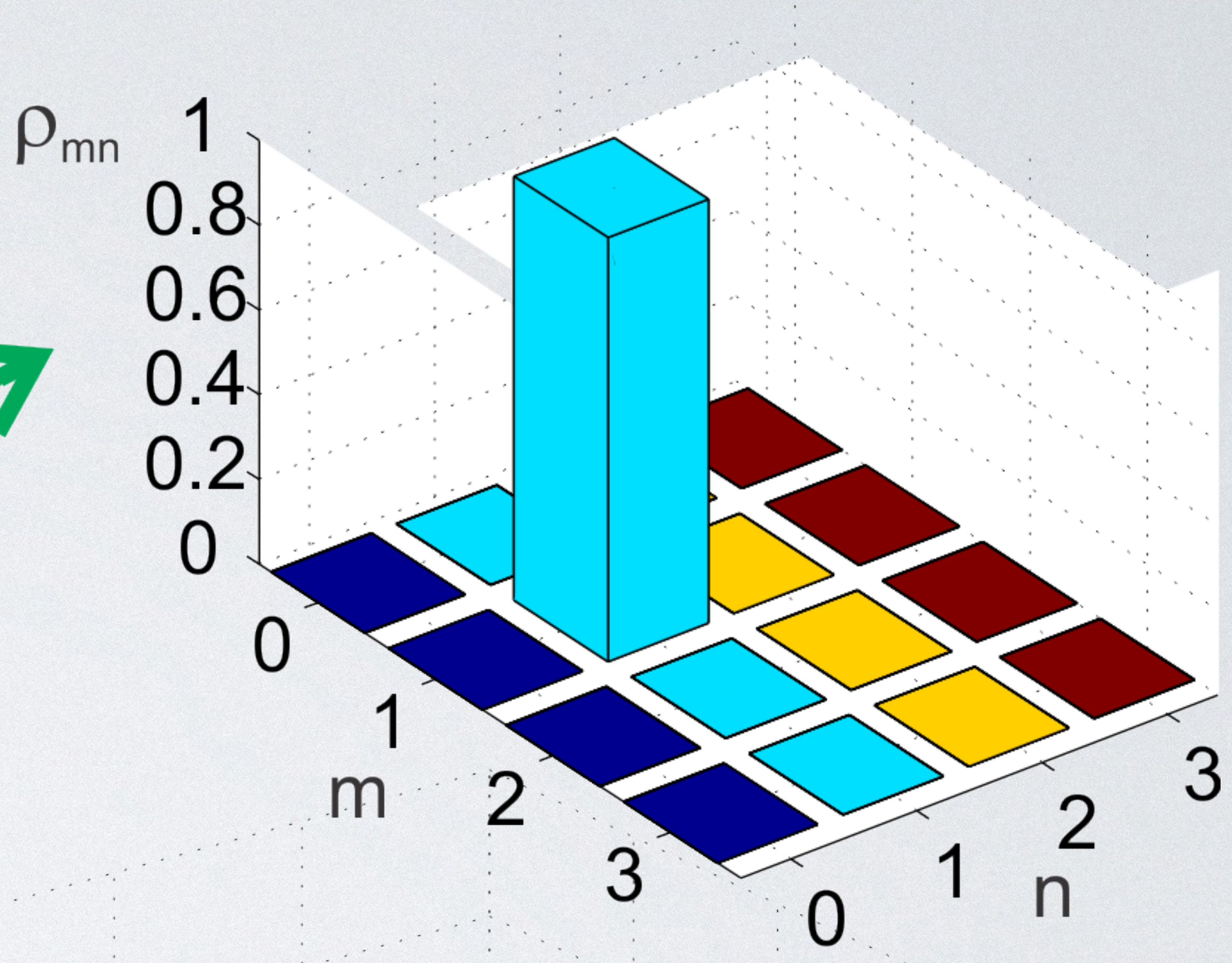




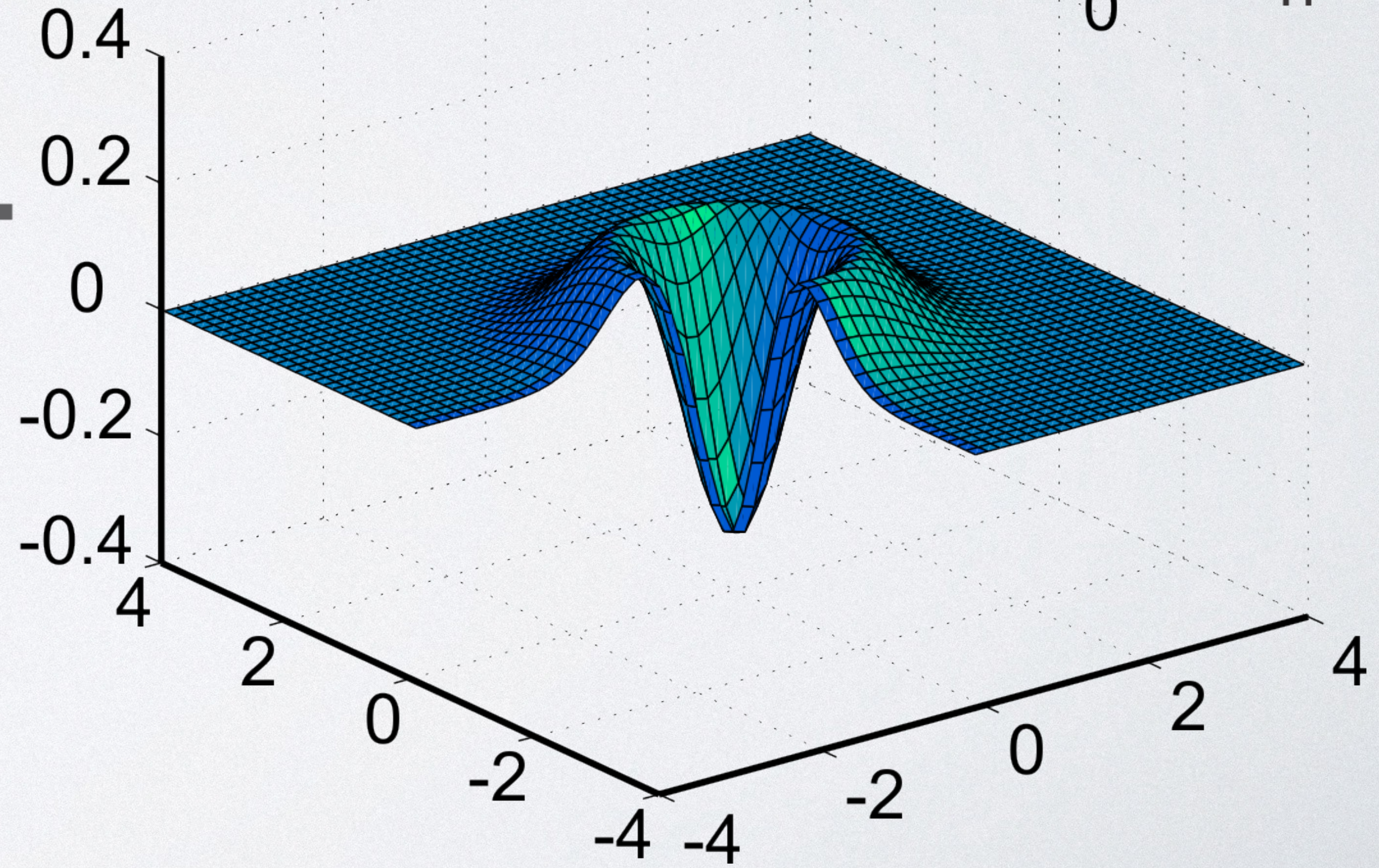
BUFFET OF QUANTUM OPTICAL STATES

Single Photon Fock state





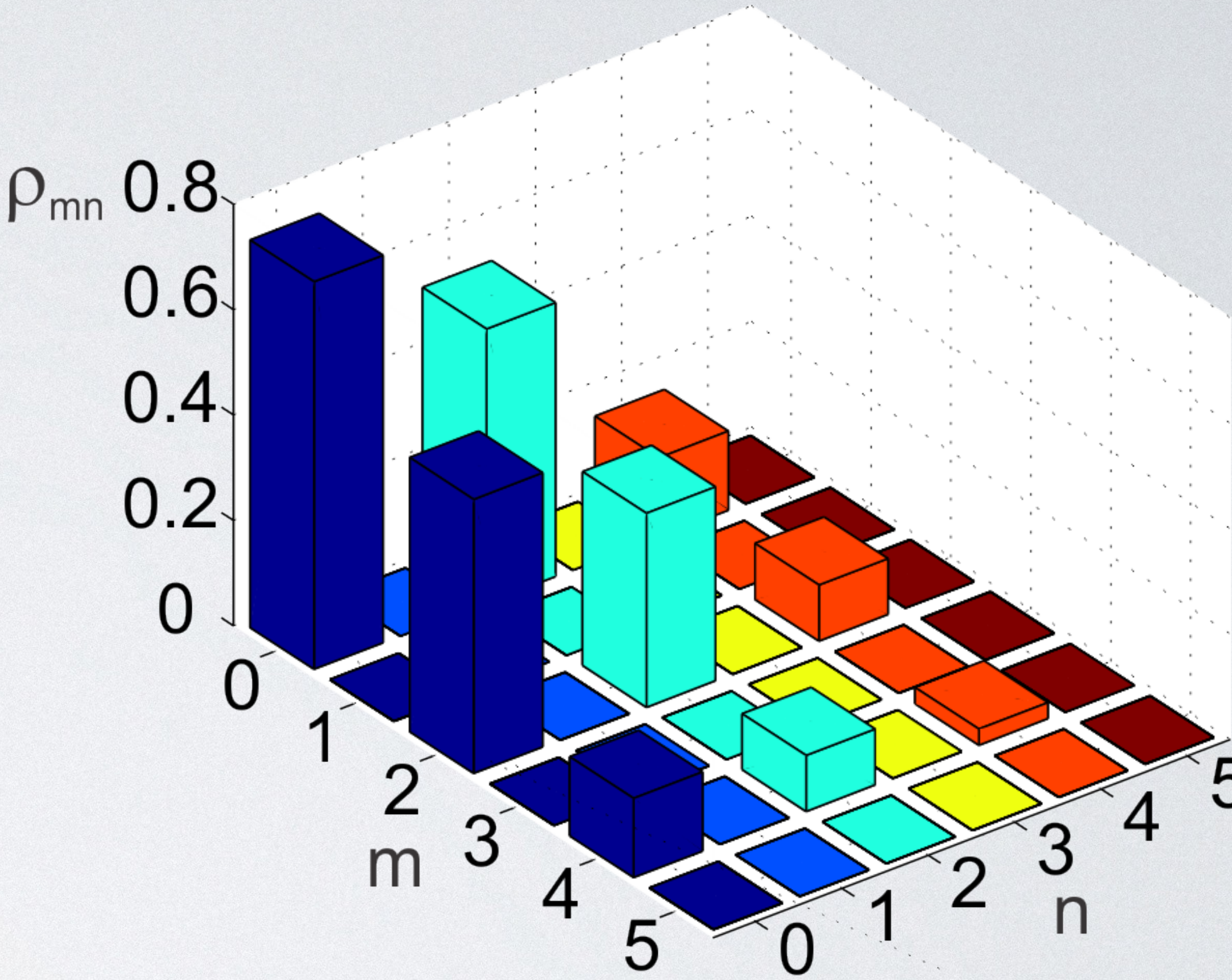
- Highly nonclassical ($\langle X \rangle = \langle P \rangle = 0$)
- Negative Wigner Function
- $\hat{\rho} = |1\rangle\langle 1|$

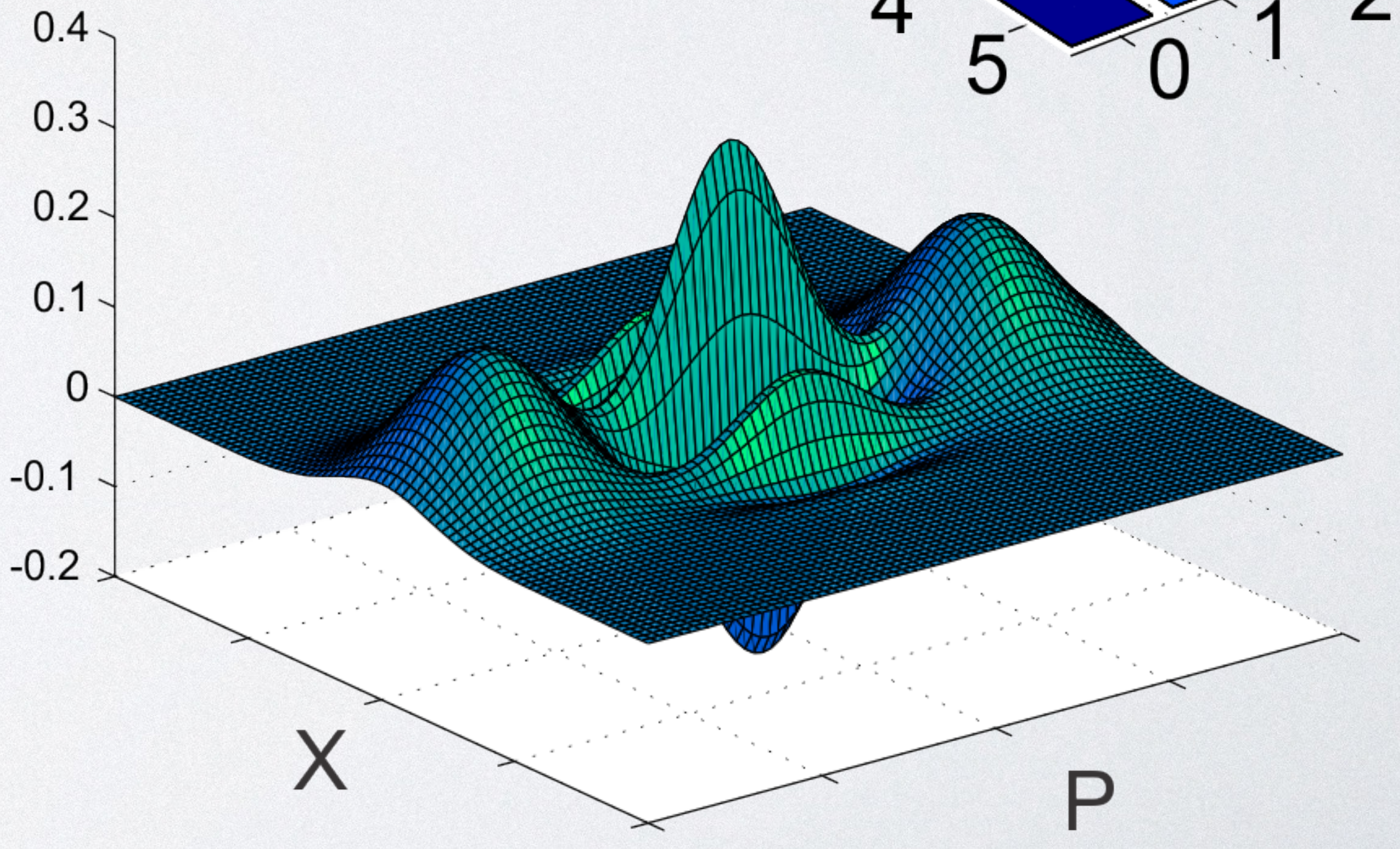


BUFFET OF QUANTUM OPTICAL STATES

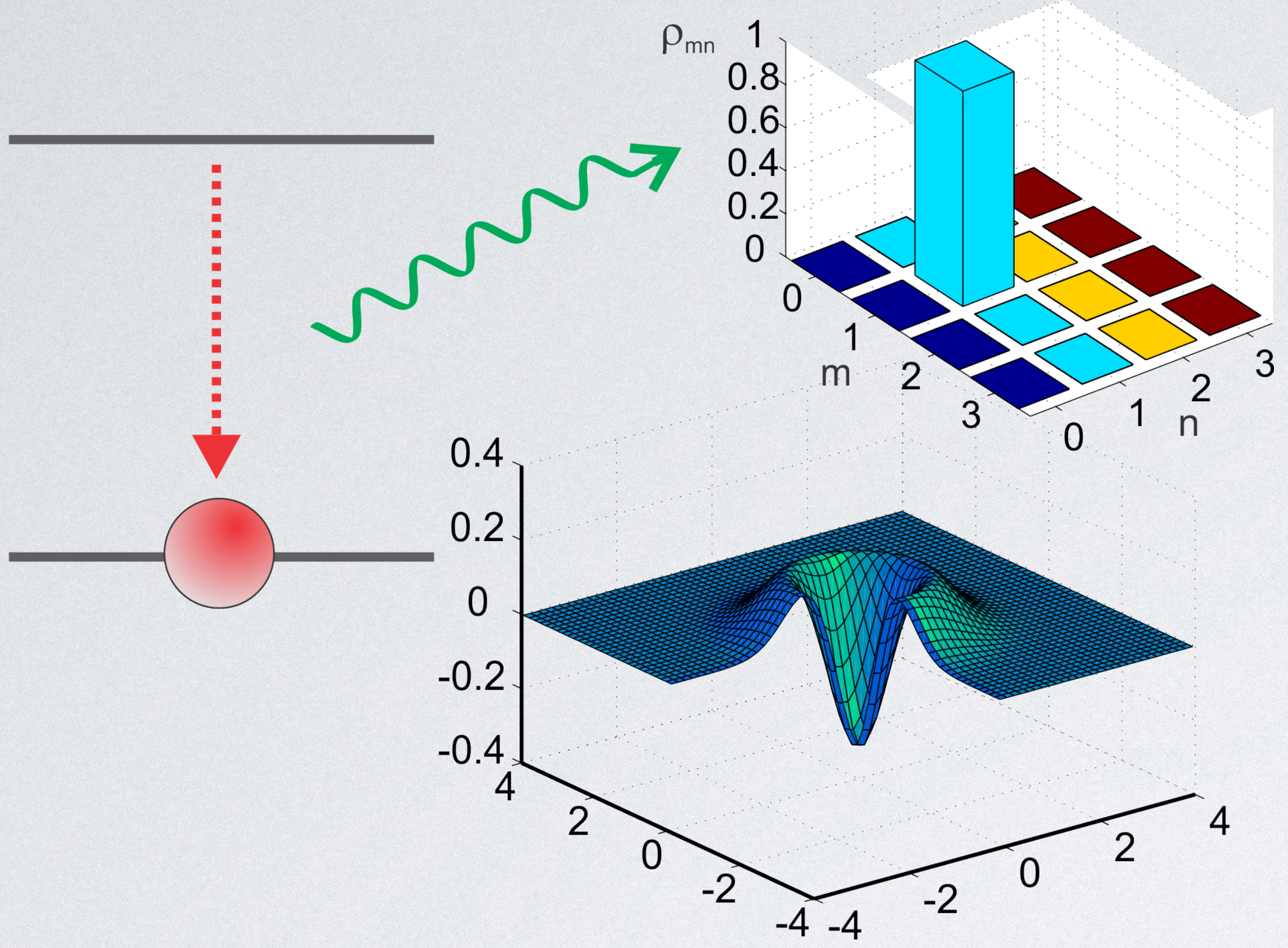
Schrödinger cat state

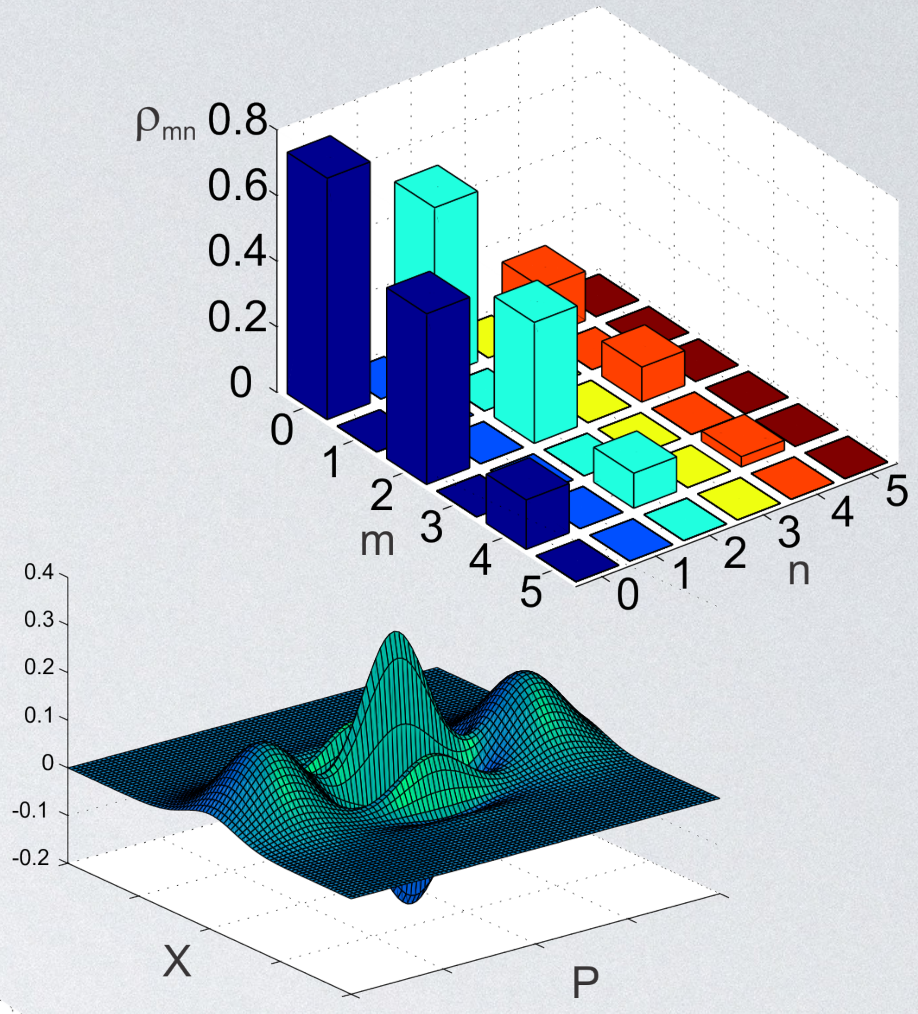
- Superposition of two coherent states
- $$\hat{\psi} = \frac{|\alpha\rangle + |-\alpha\rangle}{\sqrt{2}}$$

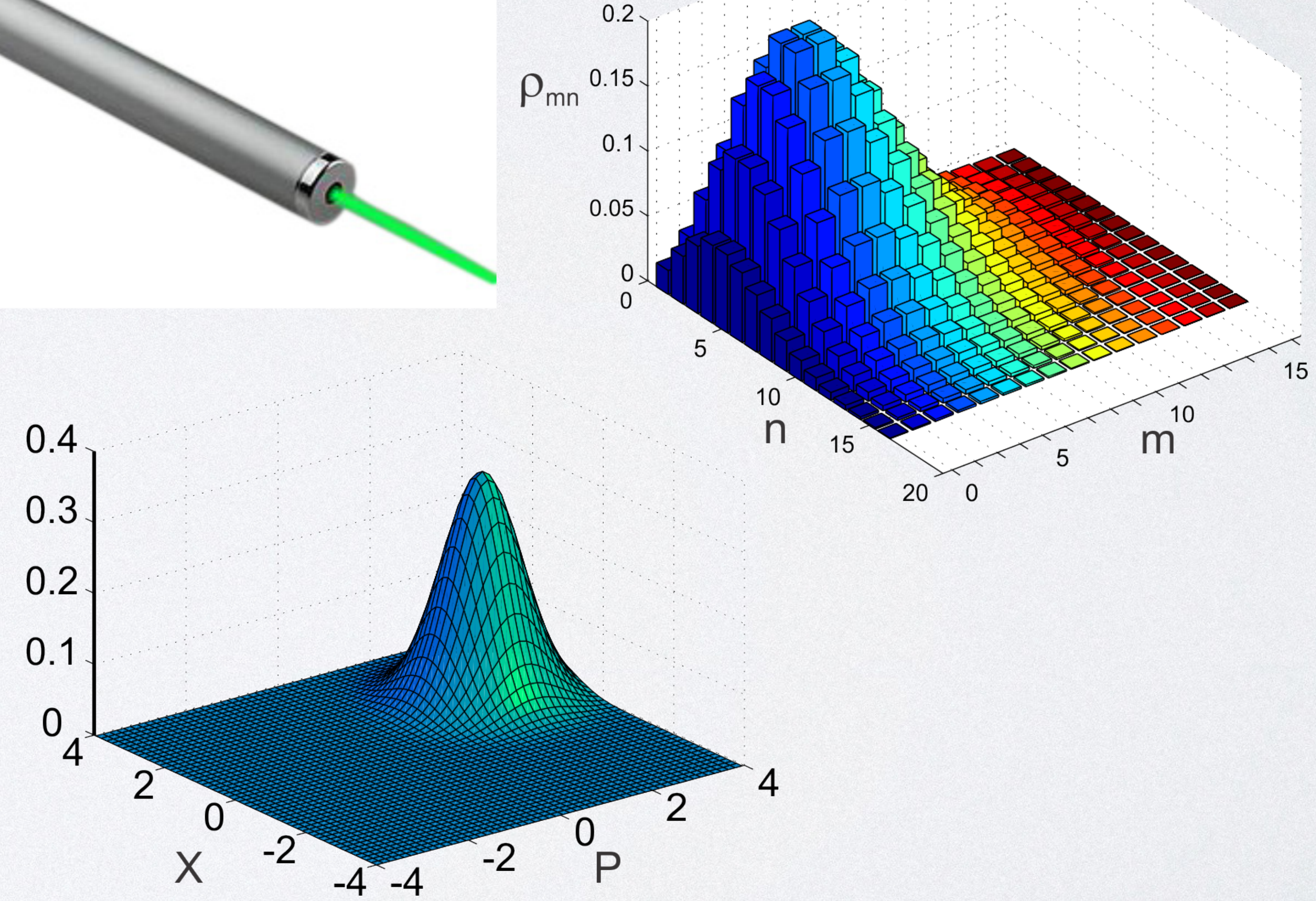


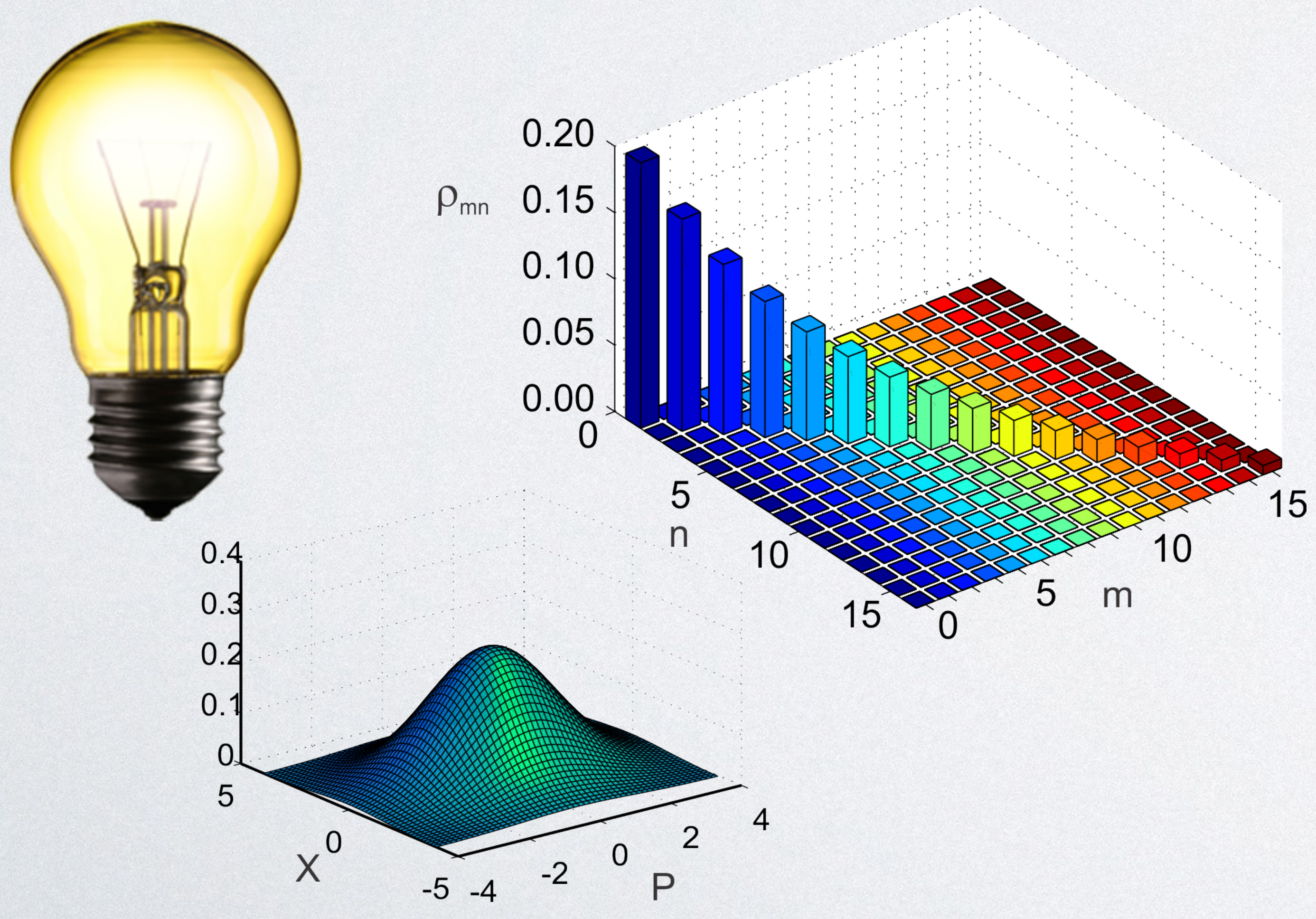


GOAL: FULL OPTICAL QUANTUM STATE ENGINEERING

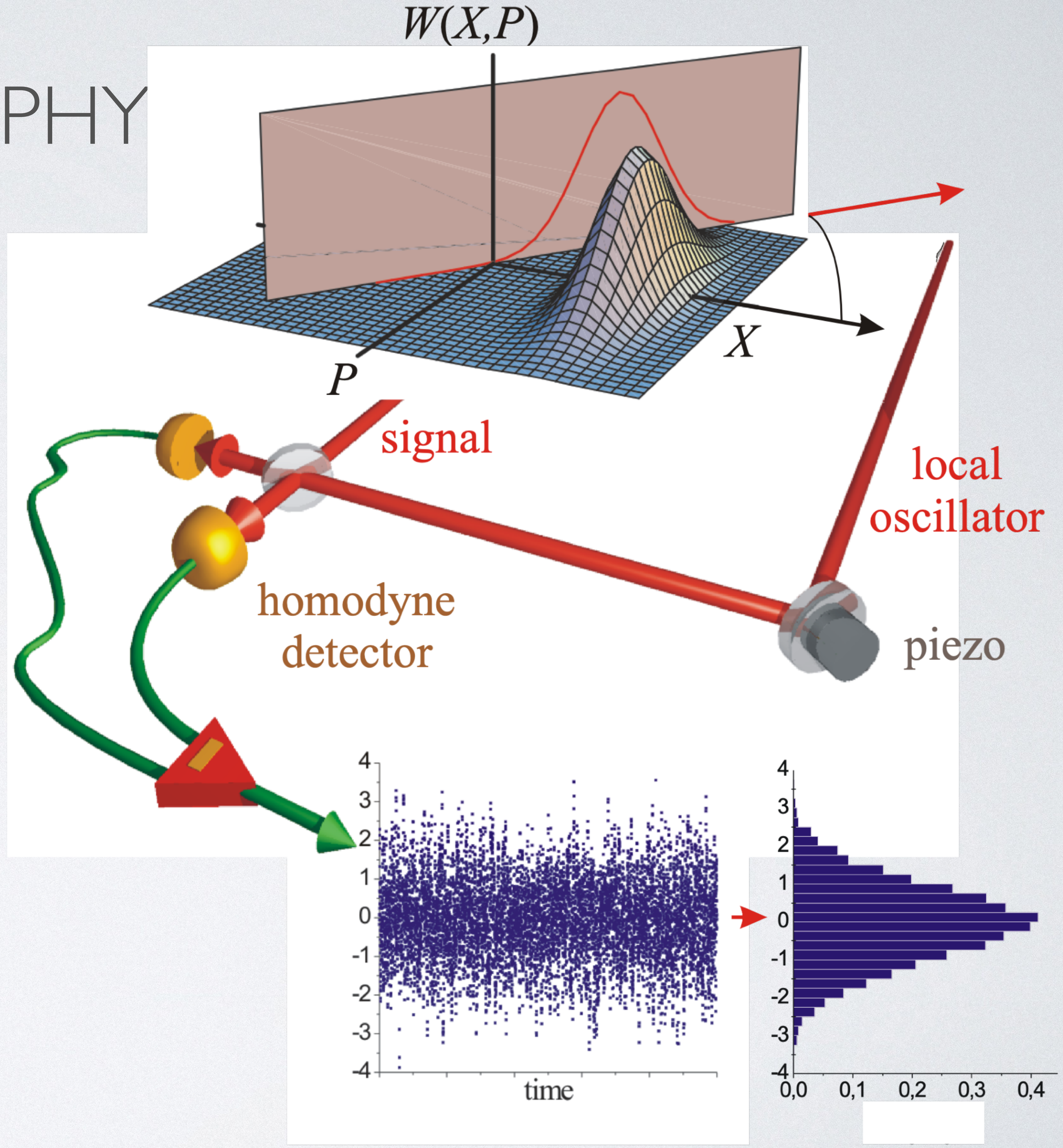








KEY TOOL:
OPTICAL HOMODYNE TOMOGRAPHY



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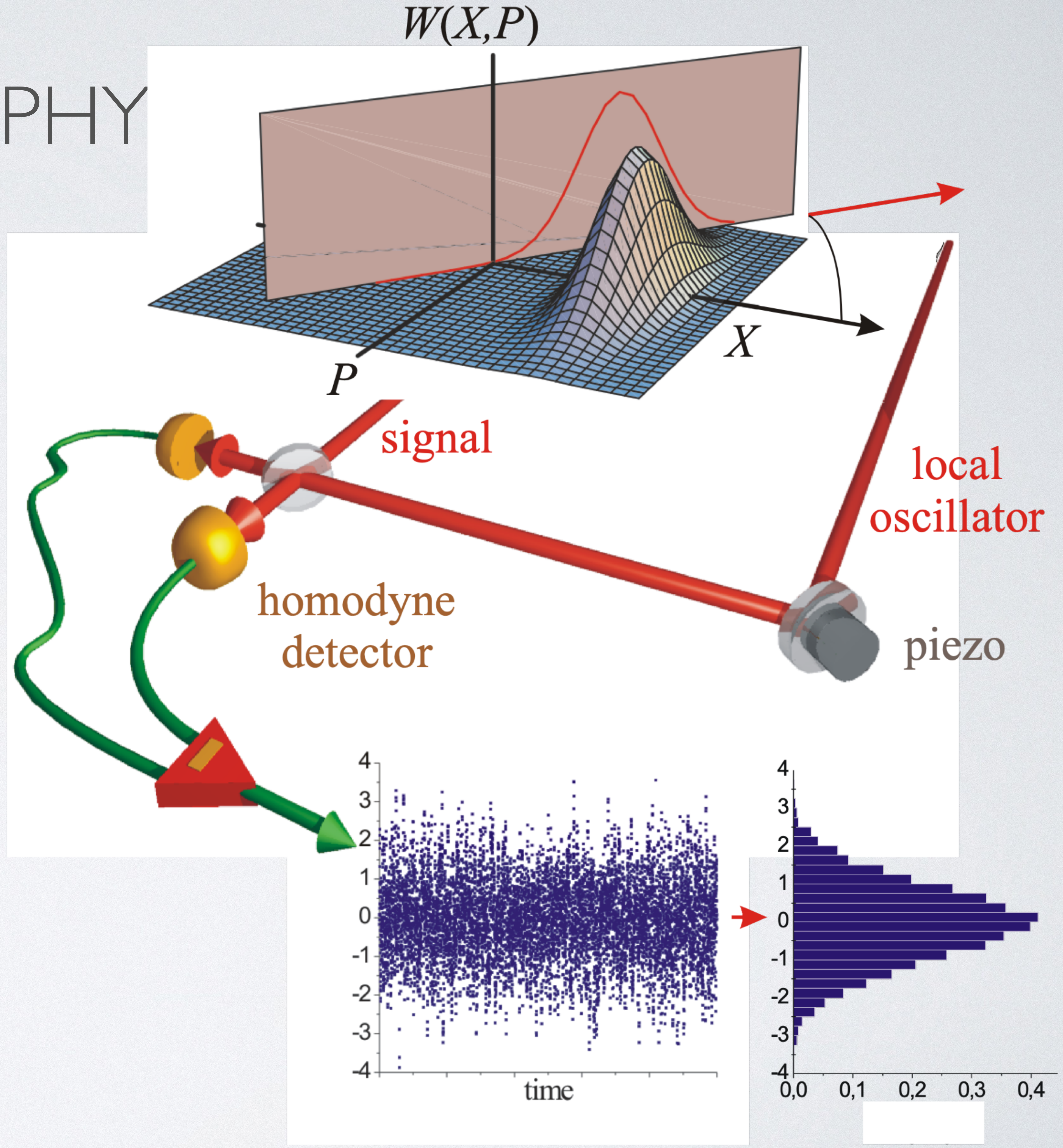
- How to obtain $W(X, P)$ (or $\hat{\rho}$)



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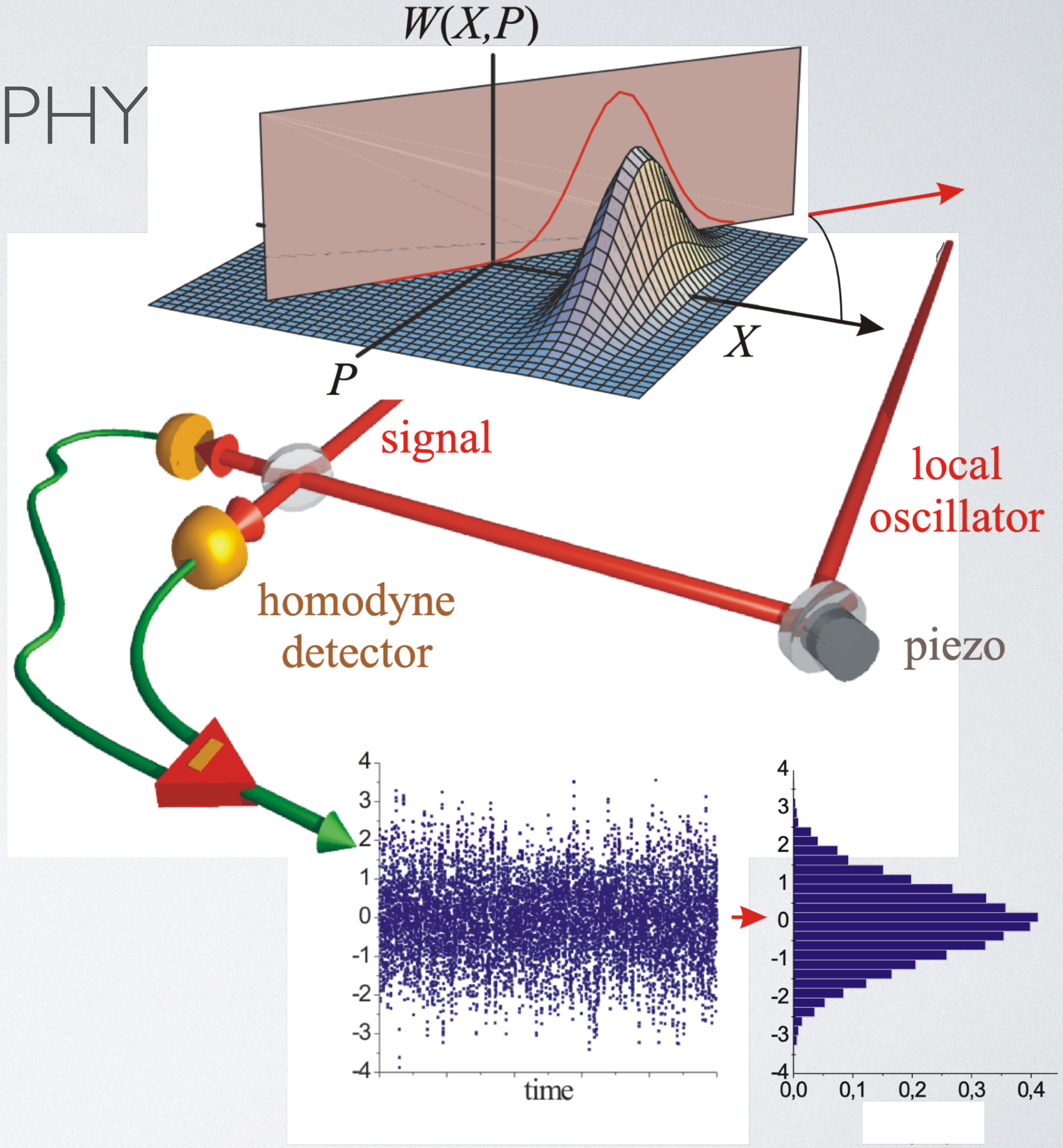

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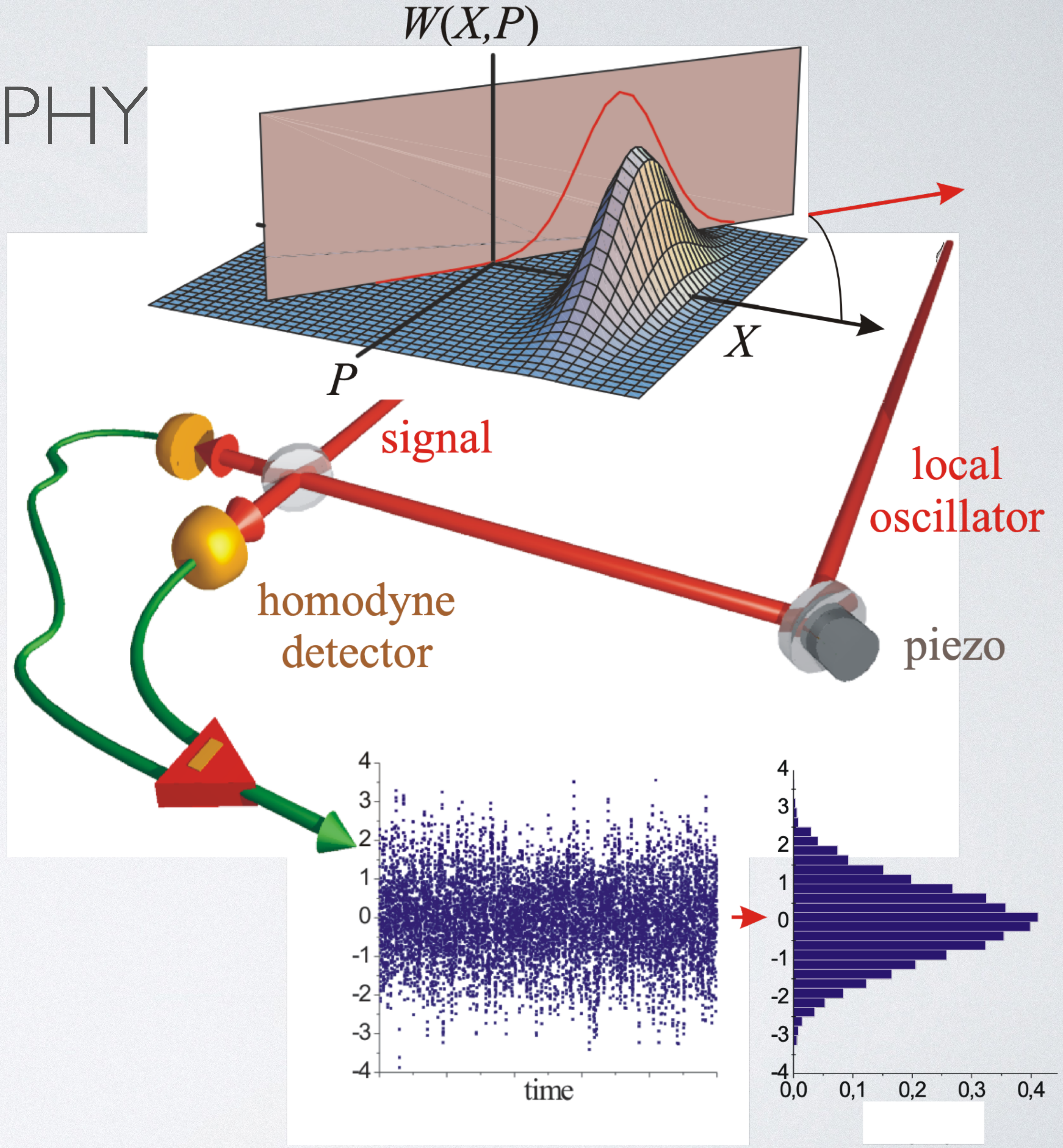

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- Strategy: measure marginals and reconstruct (c.f. medical tomography)

- How to measure $\hat{X}$?
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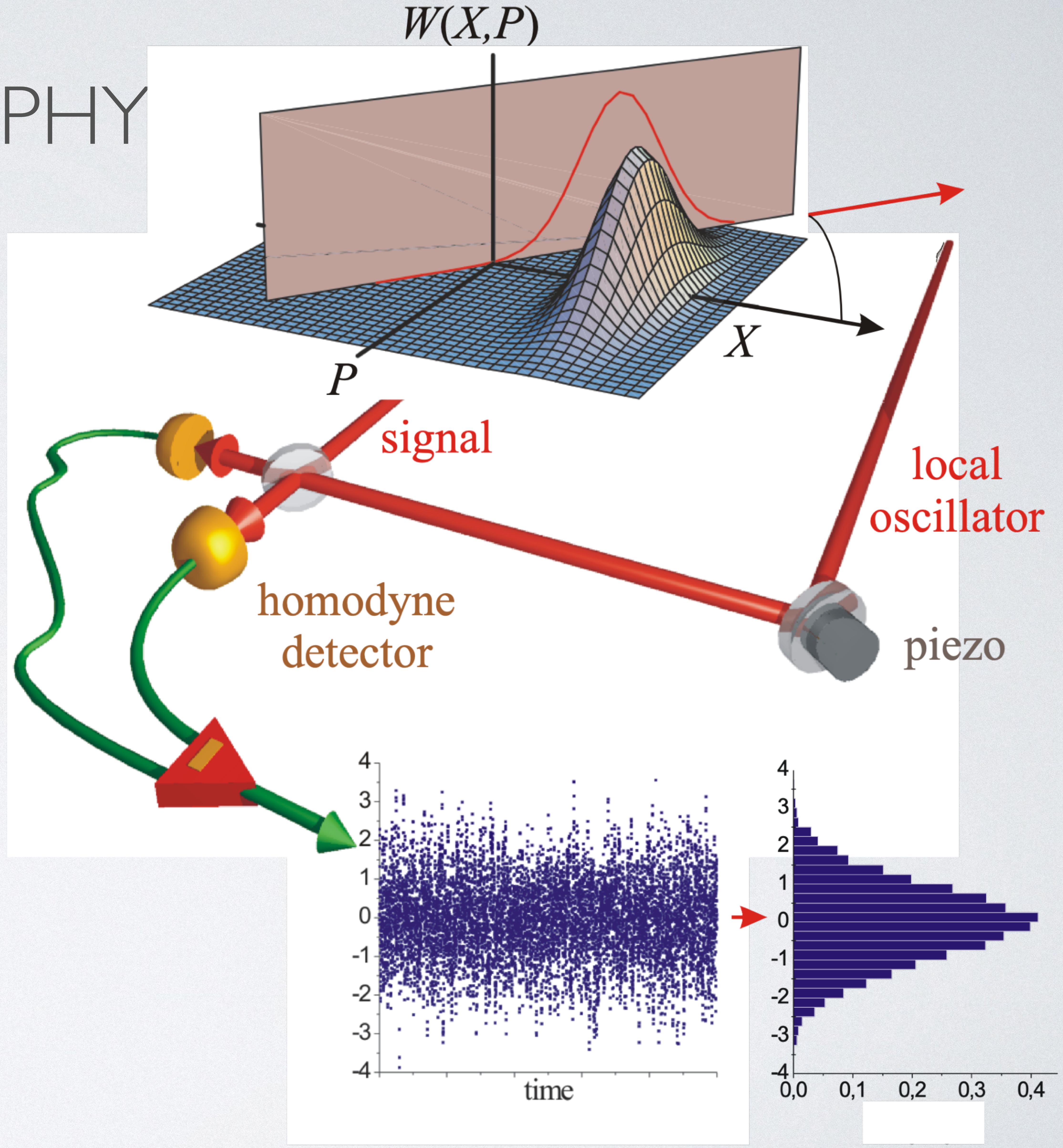
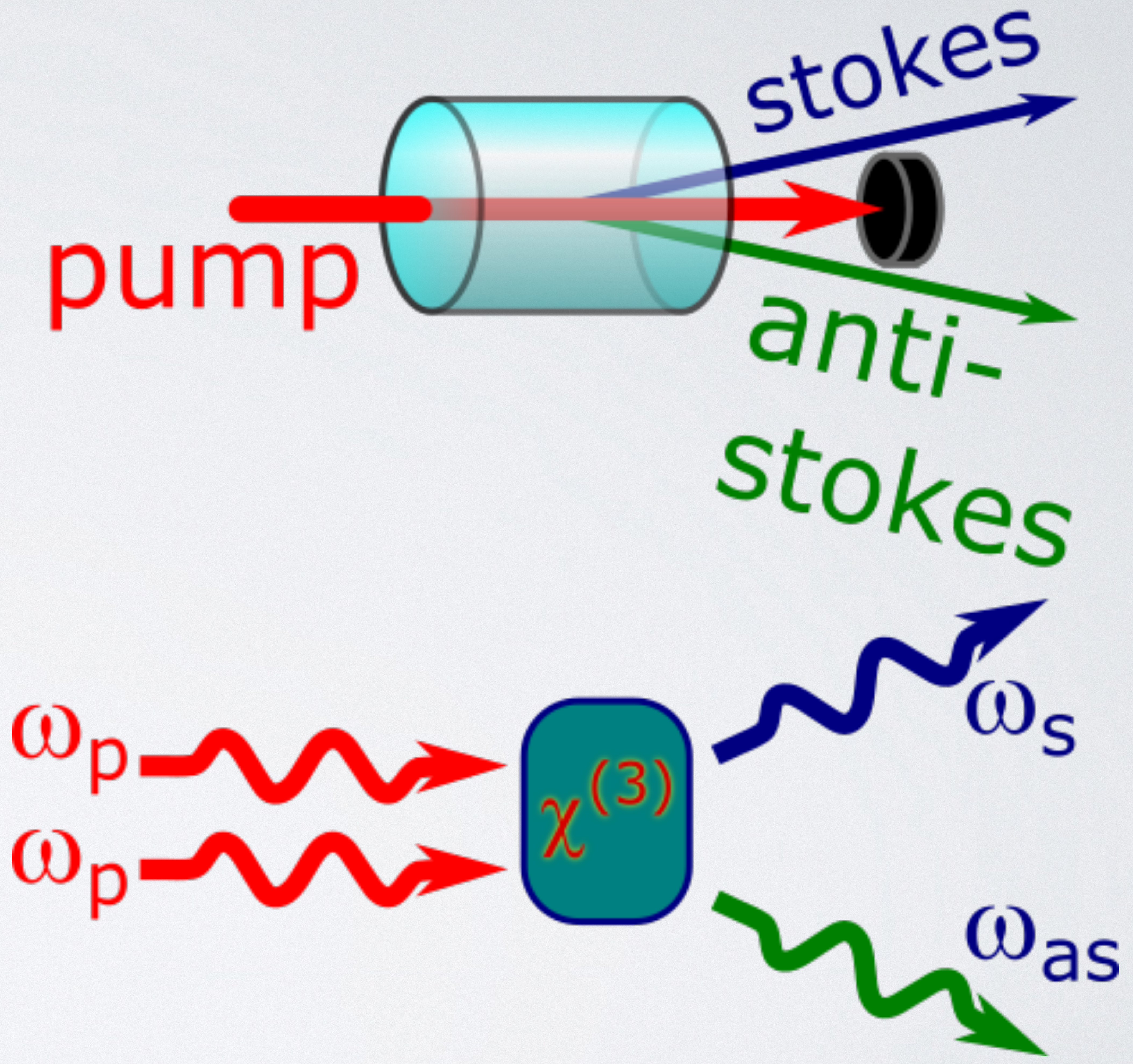
- Reconstruct a complex temporal mode via:

$$X_\psi = \int \psi(t) i(t) dt$$



ATOMIC SOURCE OF NONCLASSICAL LIGHT

Four Wave Mixing in Rb Vapour



4WM SOURCE

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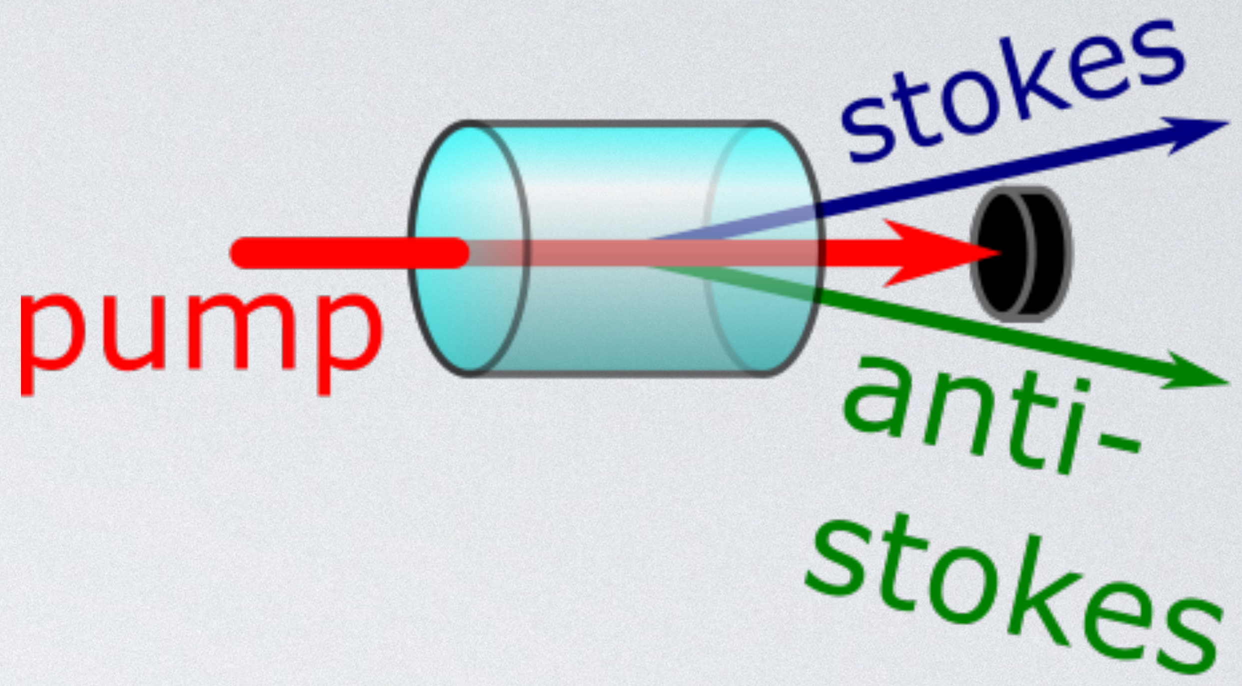

- Strong **pump** creates correlated **stokes** and **anti-stokes** fields

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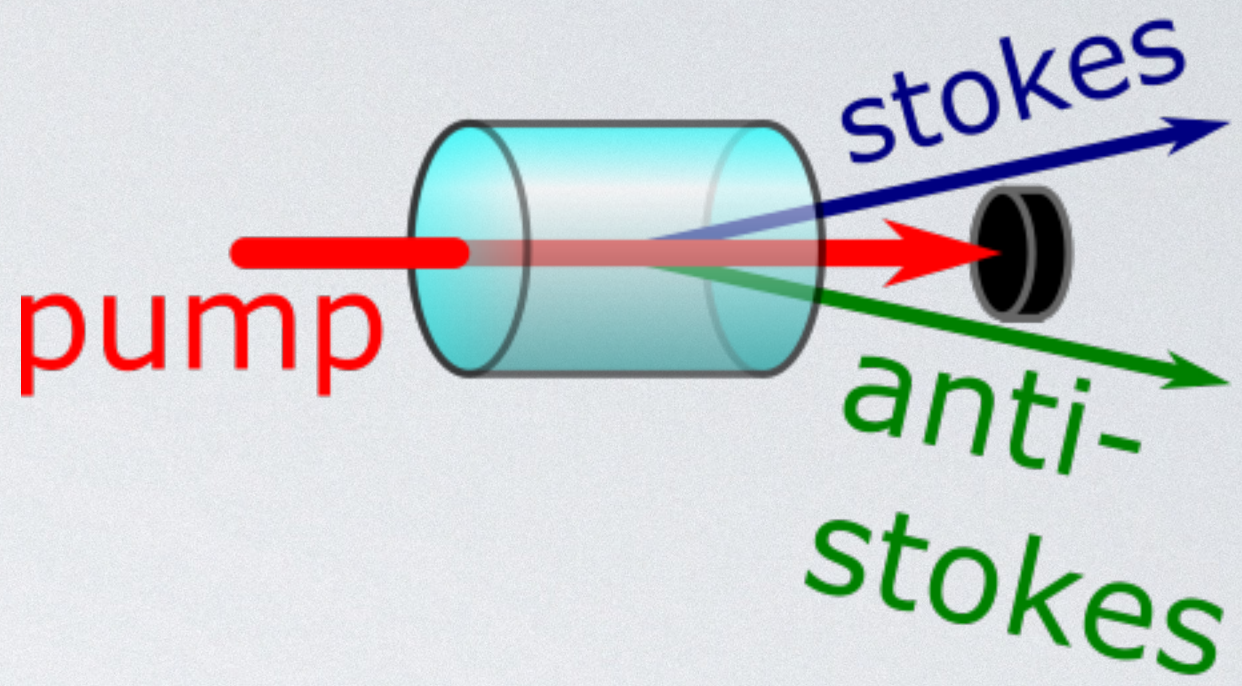

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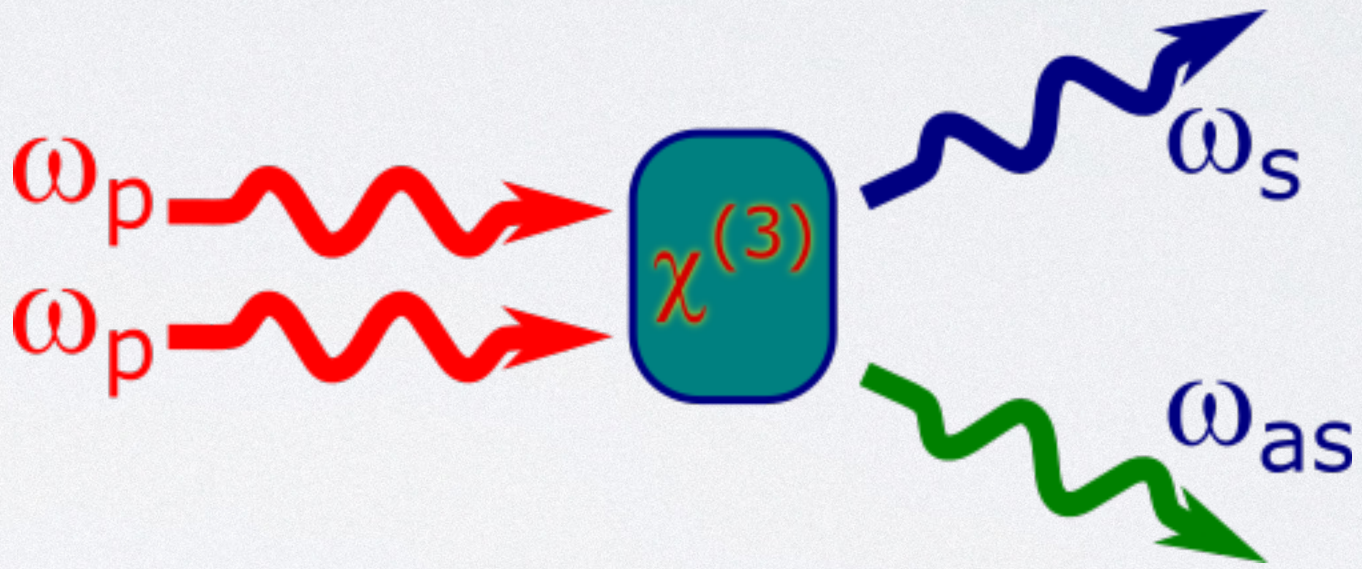

- Third order nonlinear process
  - Energy: $2\omega_p = \omega_s + \omega_i$
  - Momentum: $2\vec{k}_p = \vec{k}_s + \vec{k}_i$

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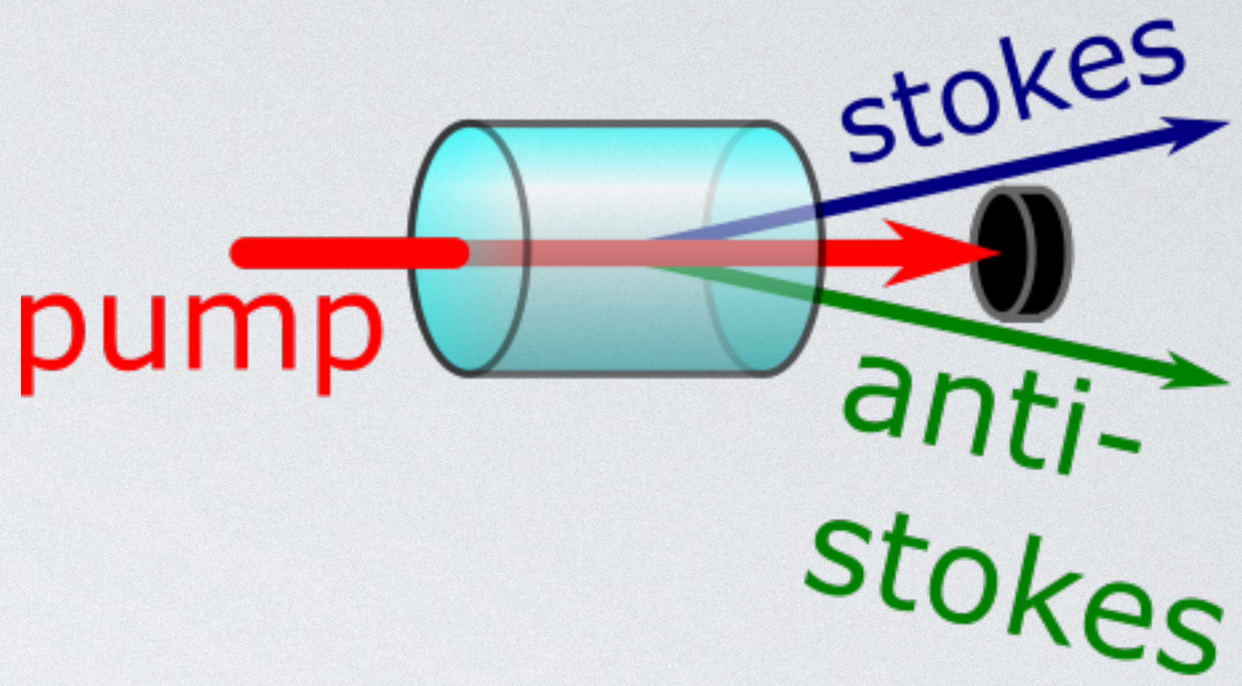

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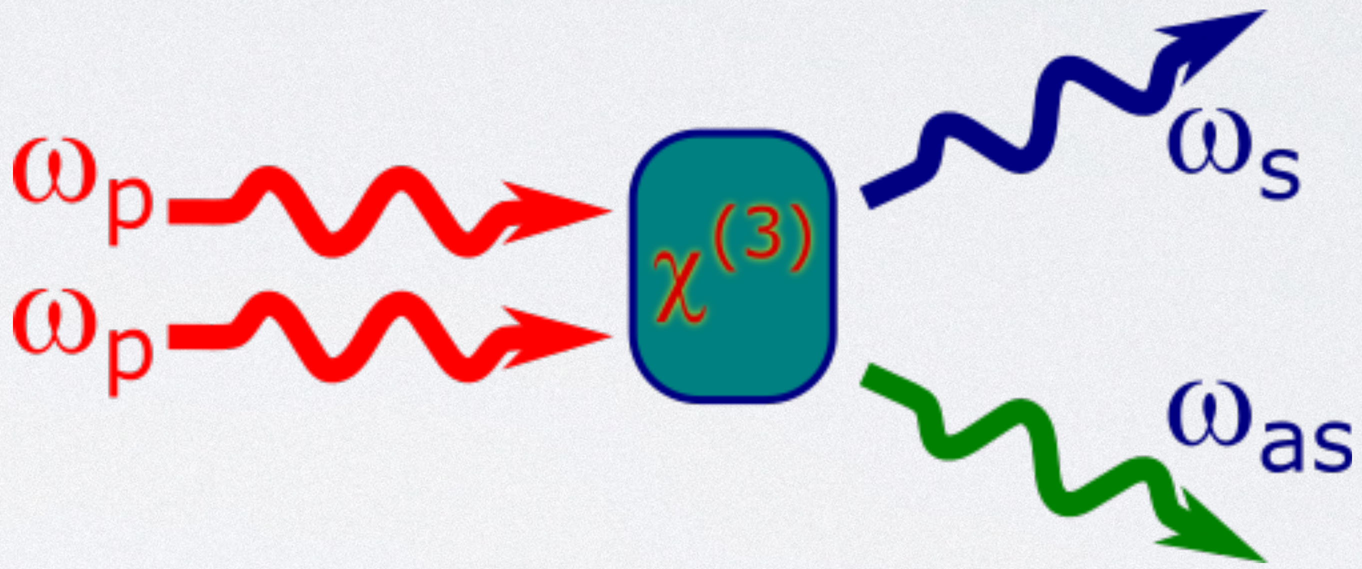
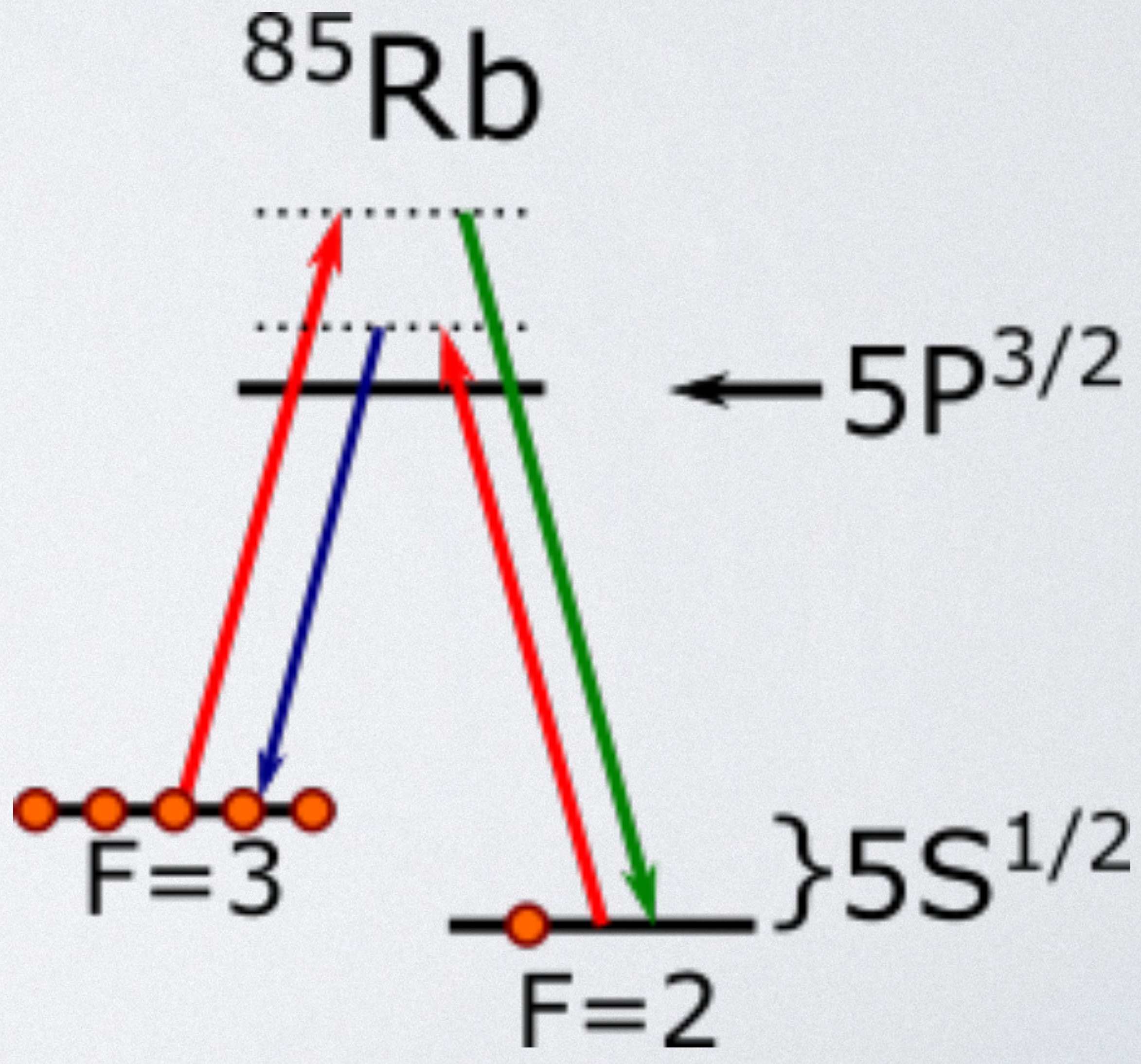


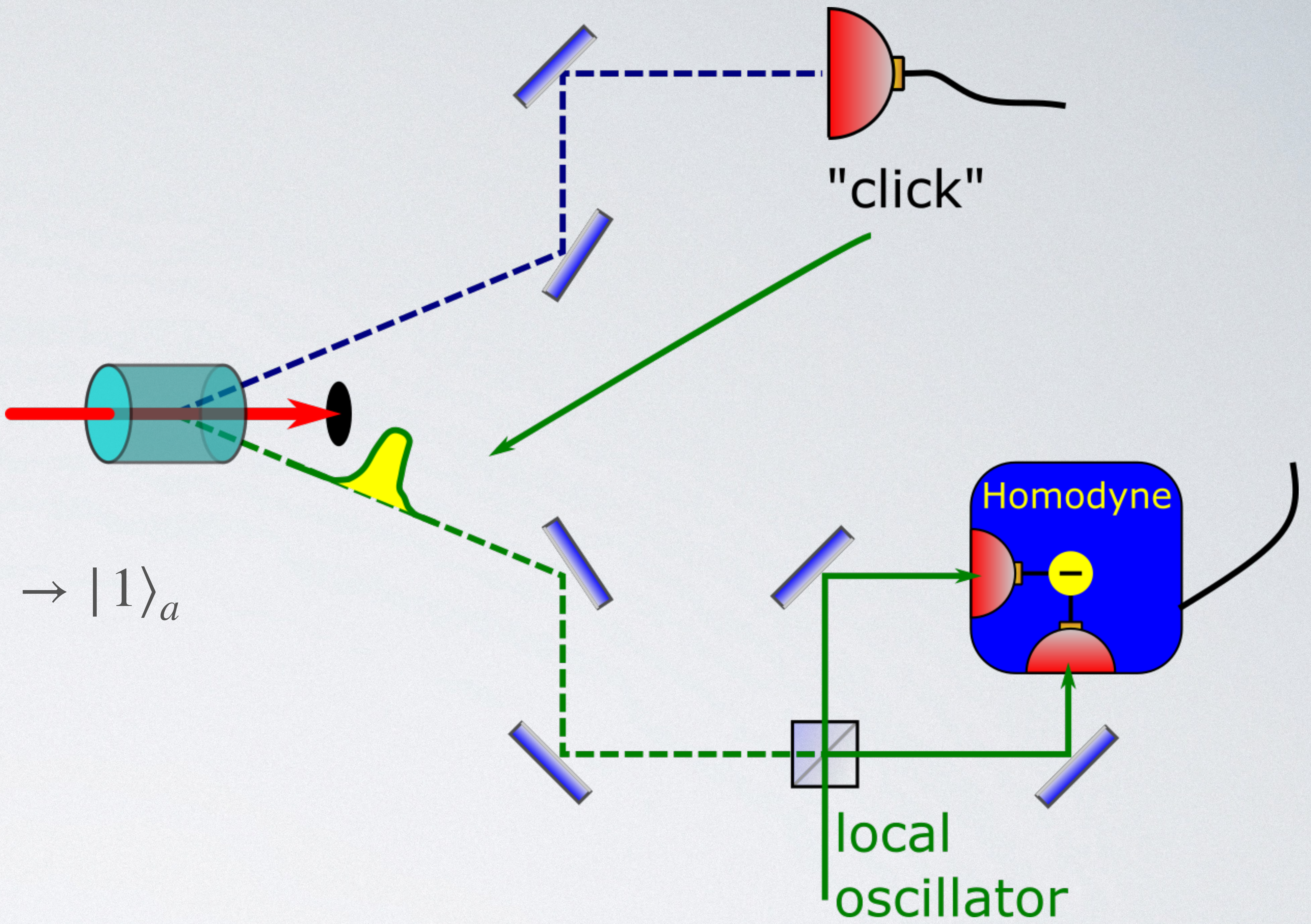


4WM SOURCE

Narrowband Single Photons

- Condition on Stokes photon detection:
$$\langle 1|_s \left(c_0|0\rangle_s|0\rangle_a + c_1|1\rangle_s|1\rangle_a + c_2|2\rangle_s|2\rangle_a + \ldots\right) \rightarrow |1\rangle_a$$

- **Heralded single photon!**



MacRae et al PRL (2012)

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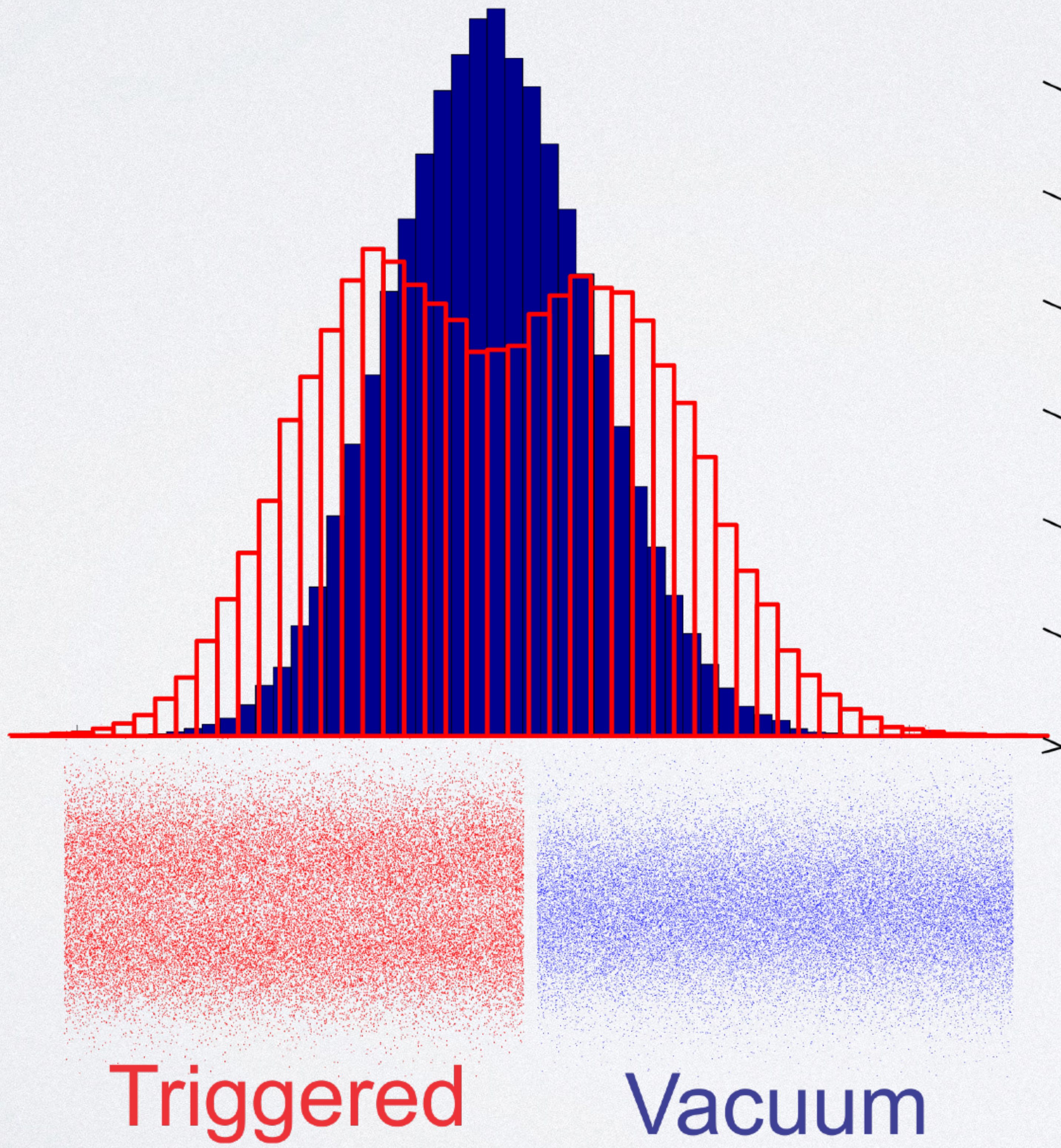
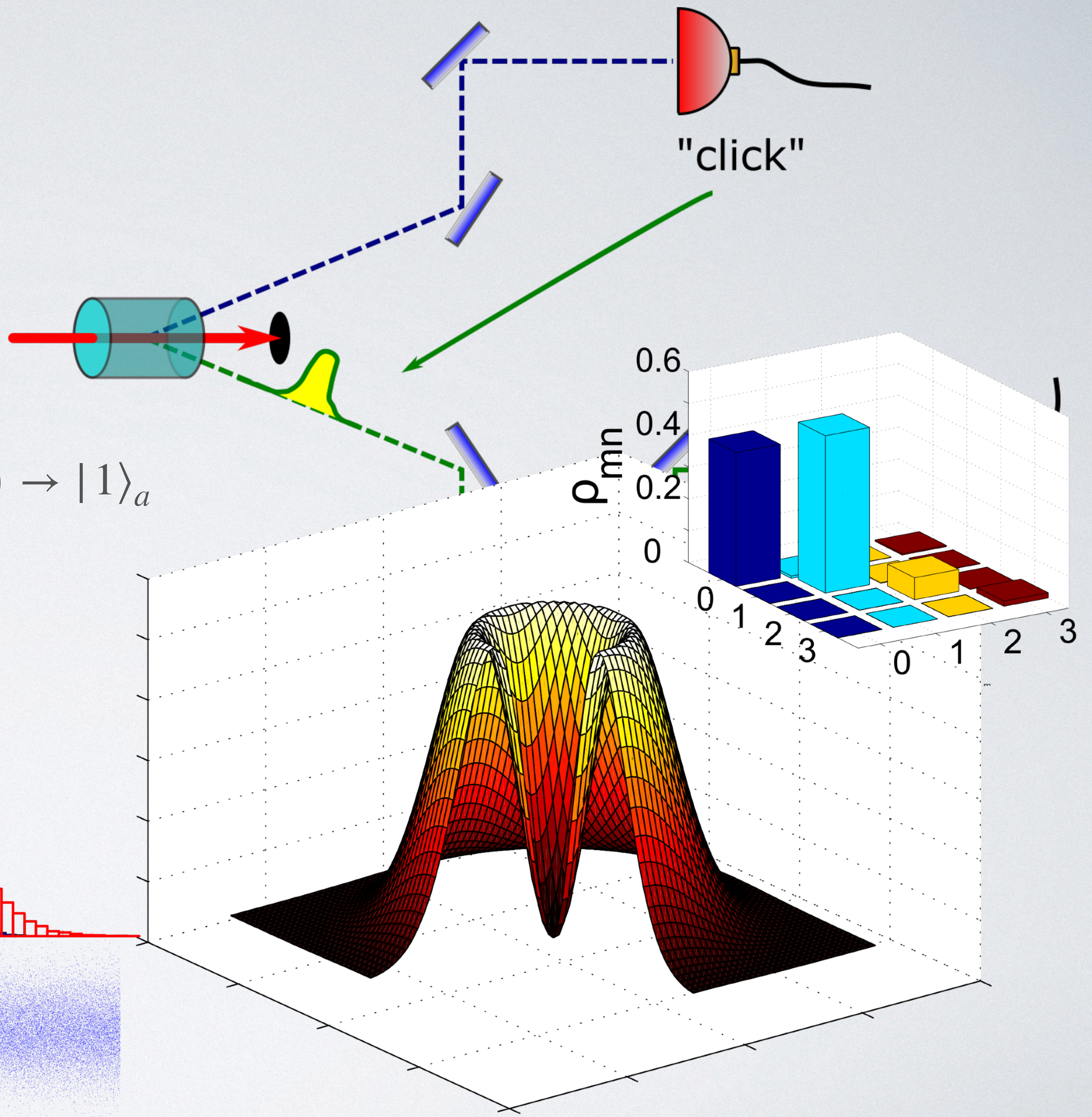
- **Heralded single photon!**





MacRae et al PRL (2012)

4WM SOURCE

Temporal Wavefunction

Z. Qin, et al LSA (2015)

4WM SOURCE

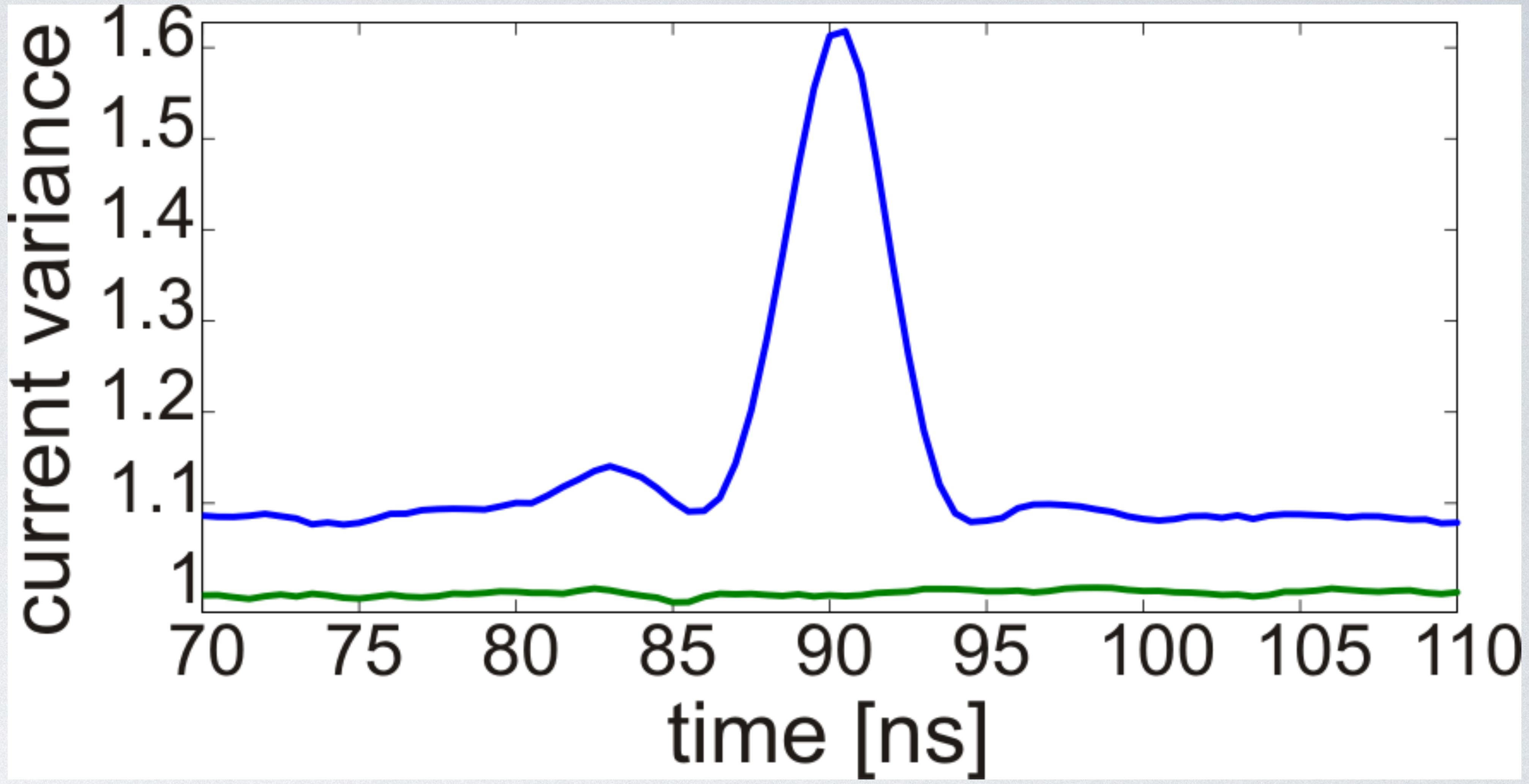
Temporal Wavefunction

- **Narrowband** single photon: $\Delta\omega \approx 2\pi \times 10$MHz

Z. Qin, et al LSA (2015)

4WM SOURCE

Temporal Wavefunction



- **Narrowband** single photon: $\Delta\omega \approx 2\pi \times 10\text{MHz}$
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Z. Qin, et al LSA (2015)

4WM SOURCE

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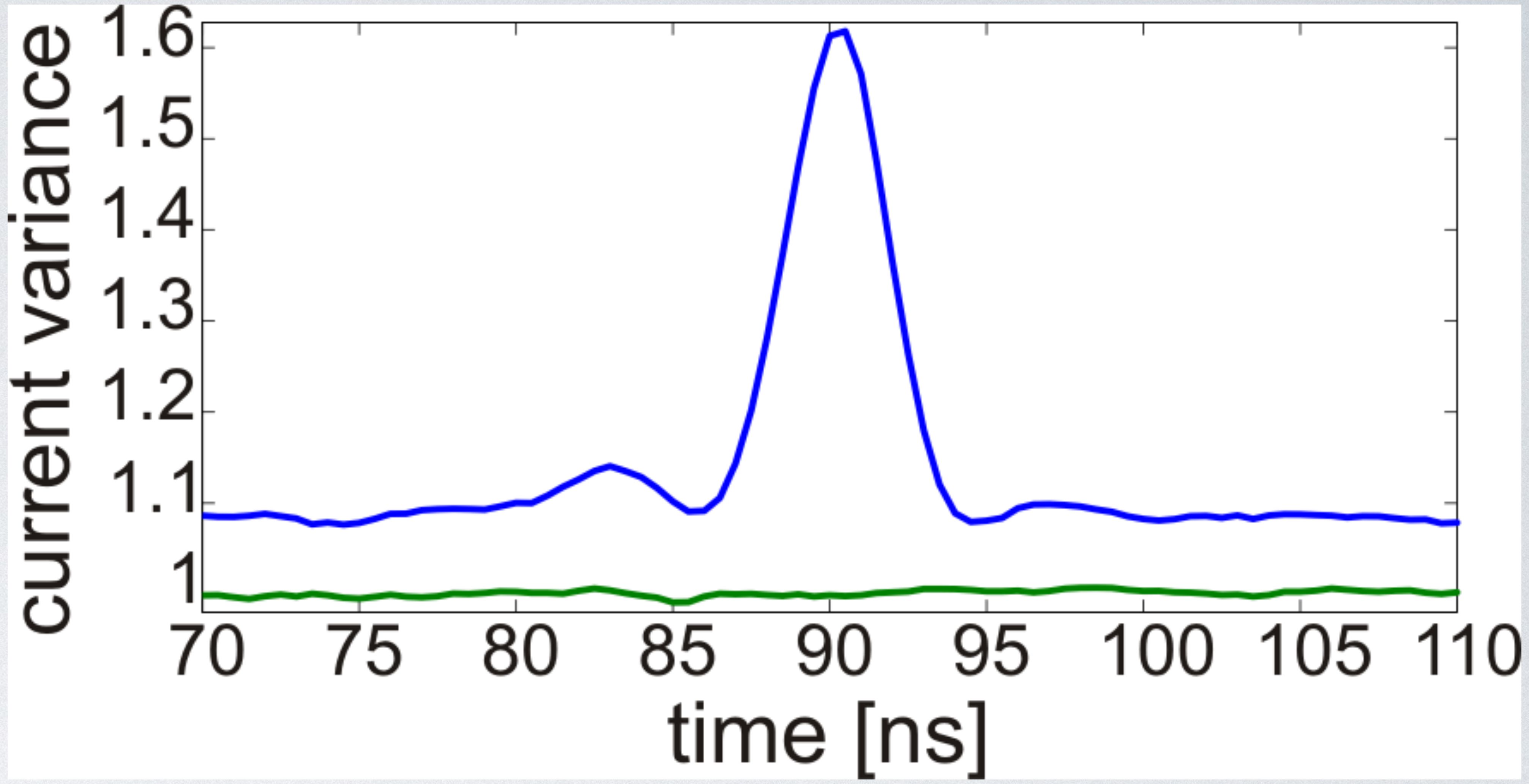

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Z. Qin, et al LSA (2015)

4WM SOURCE

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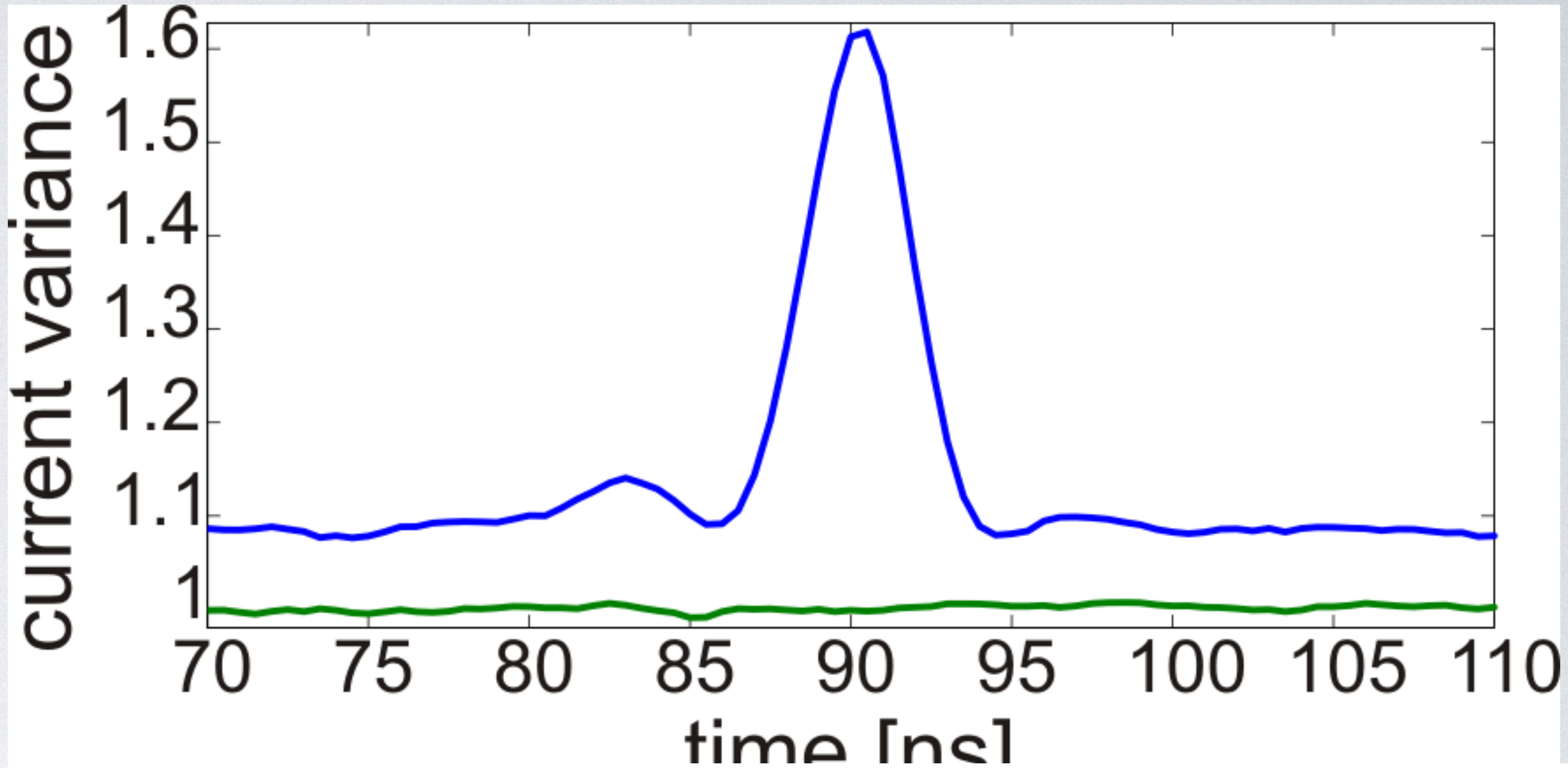
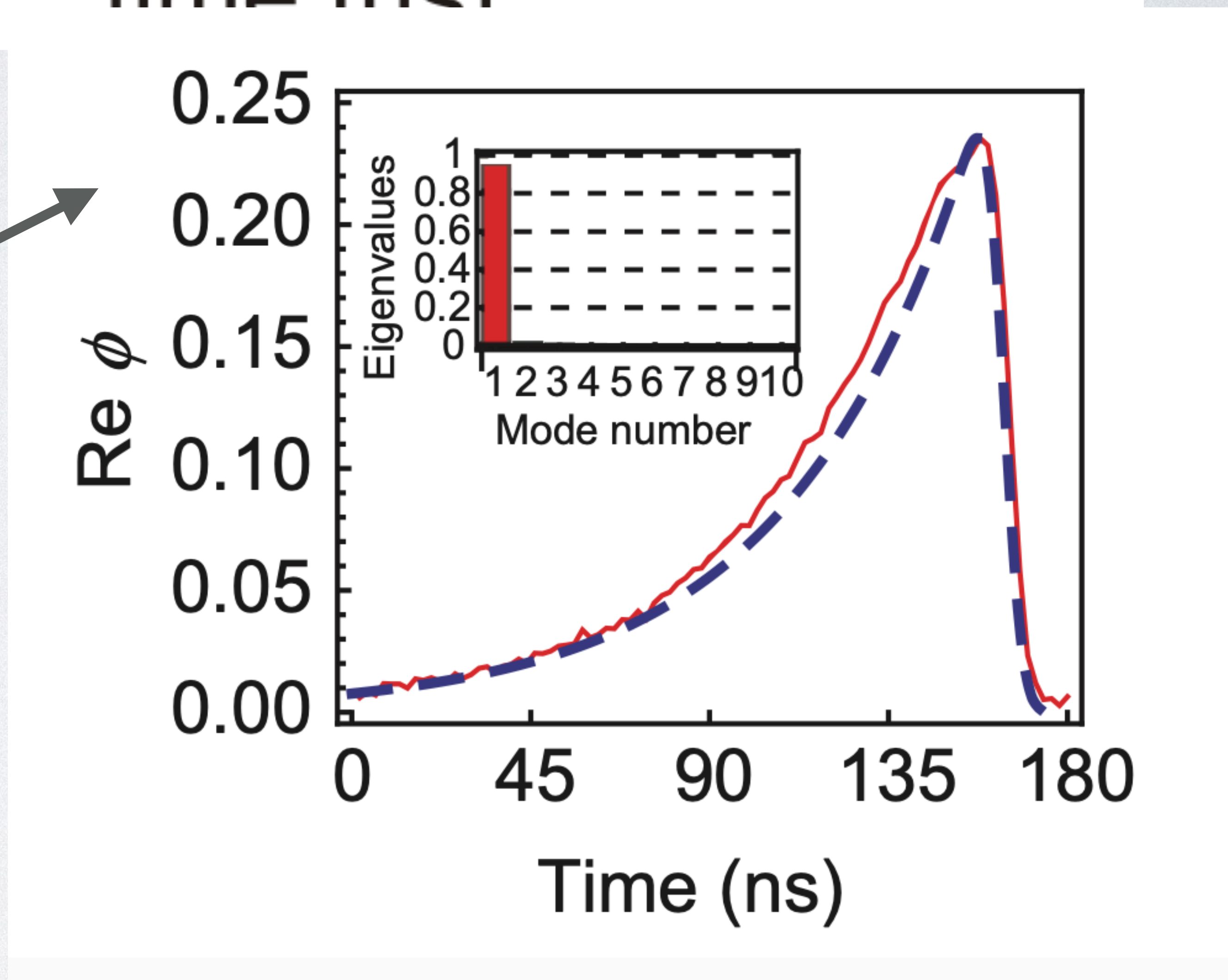


After cavity



Z. Qin, et al LSA (2015)

4WM SOURCE

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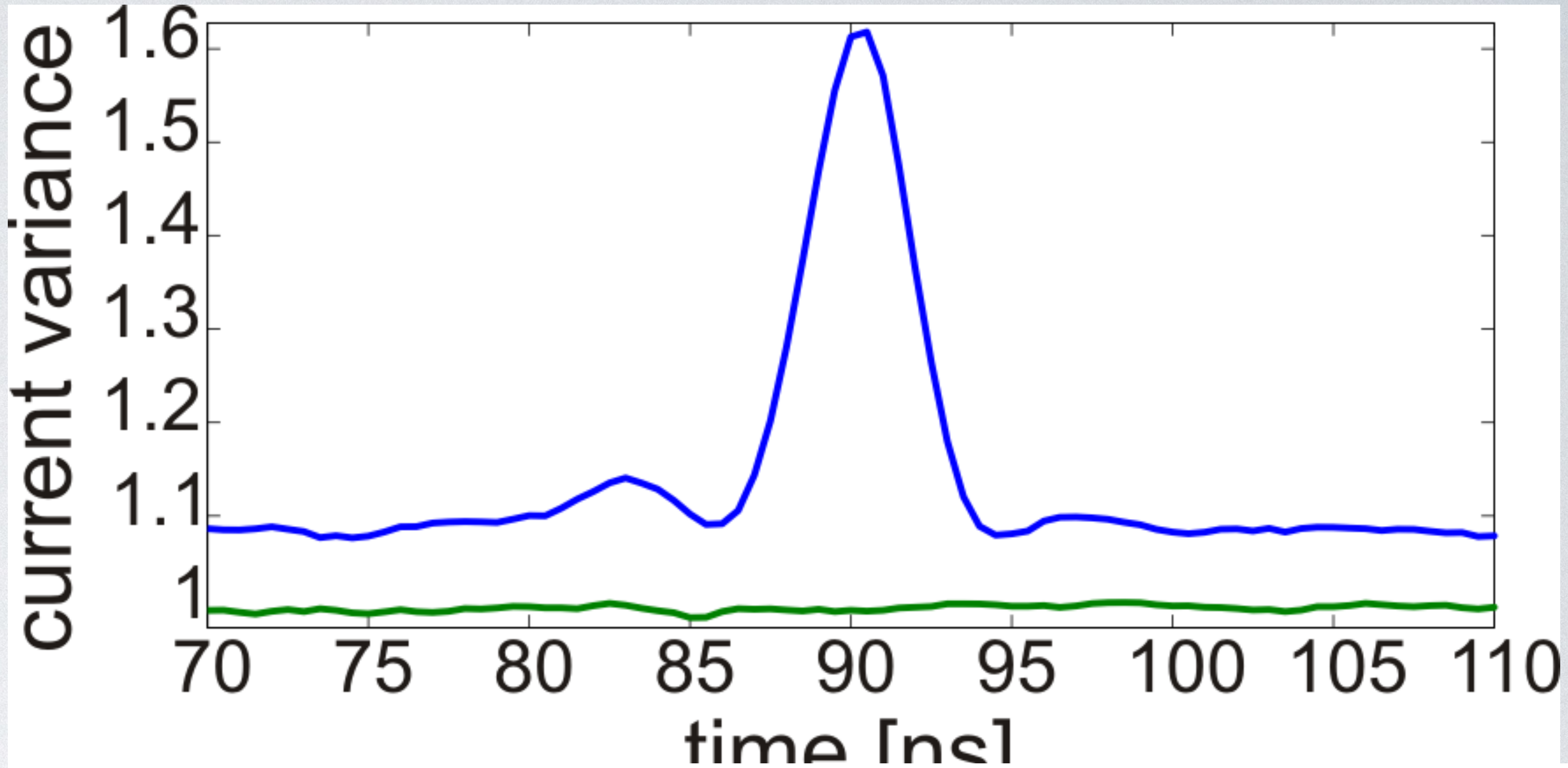

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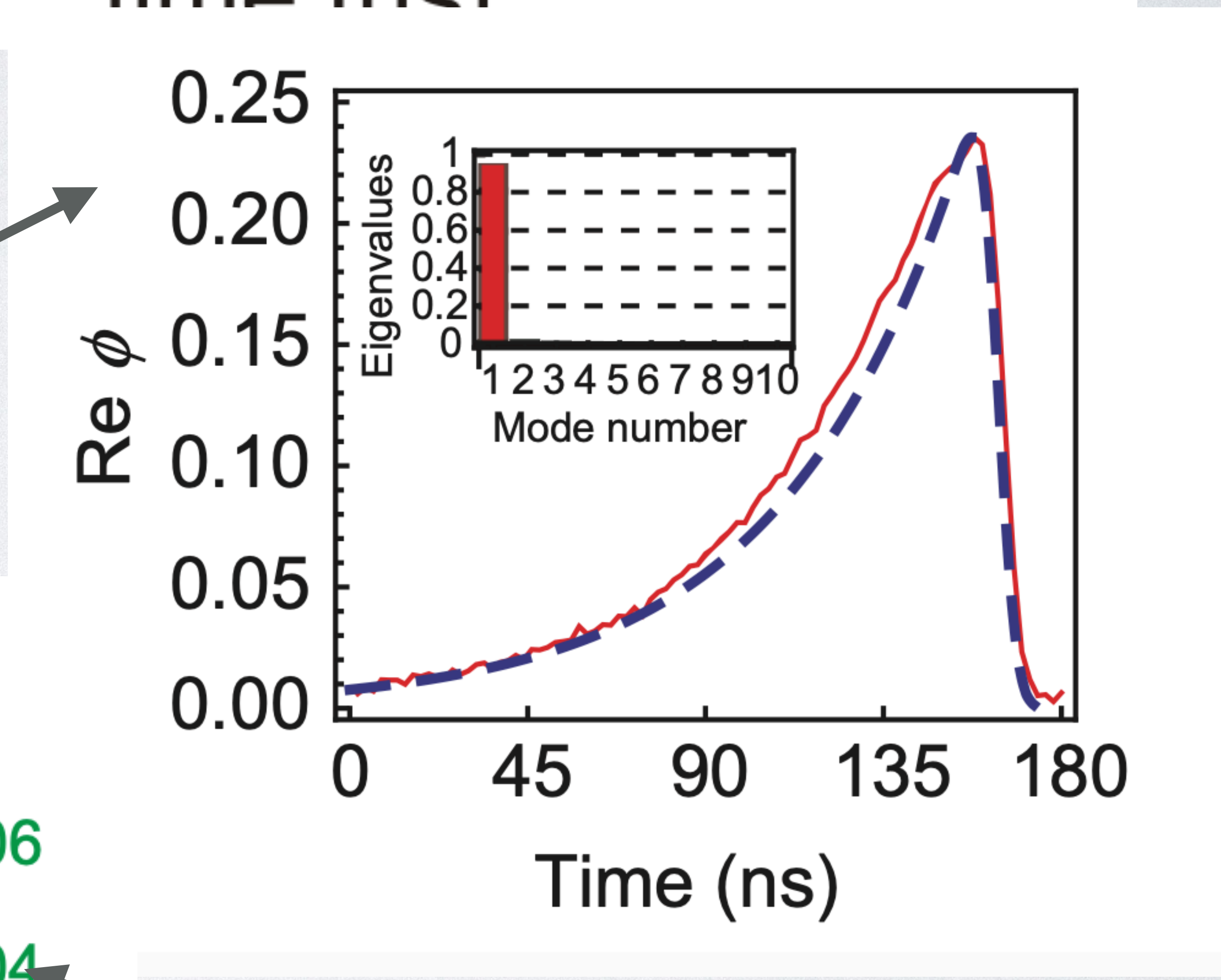
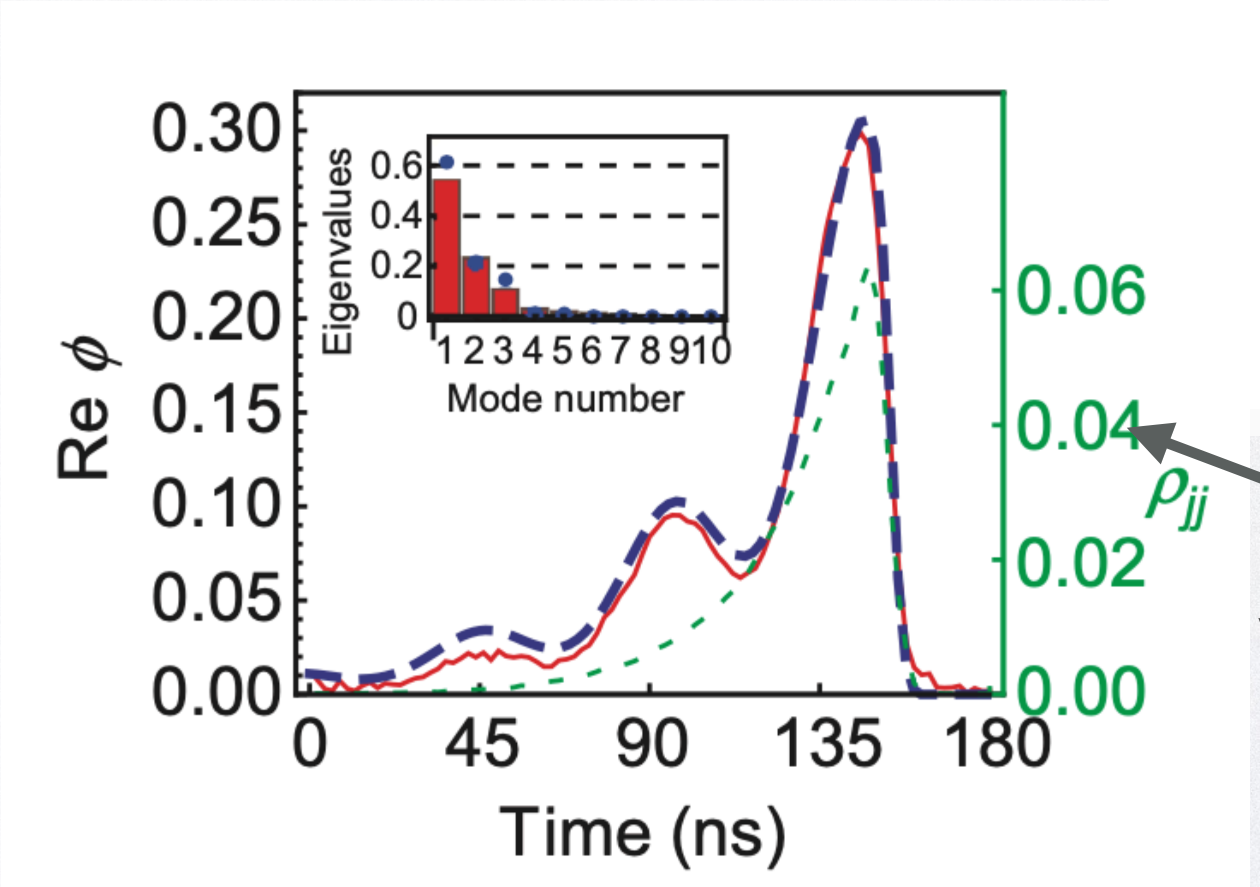




With phase-modulation

Z. Qin, et al LSA (2015)

4WM SOURCE

Toward full quantum state engineering

Brannan et al. Opt Lett (2014)

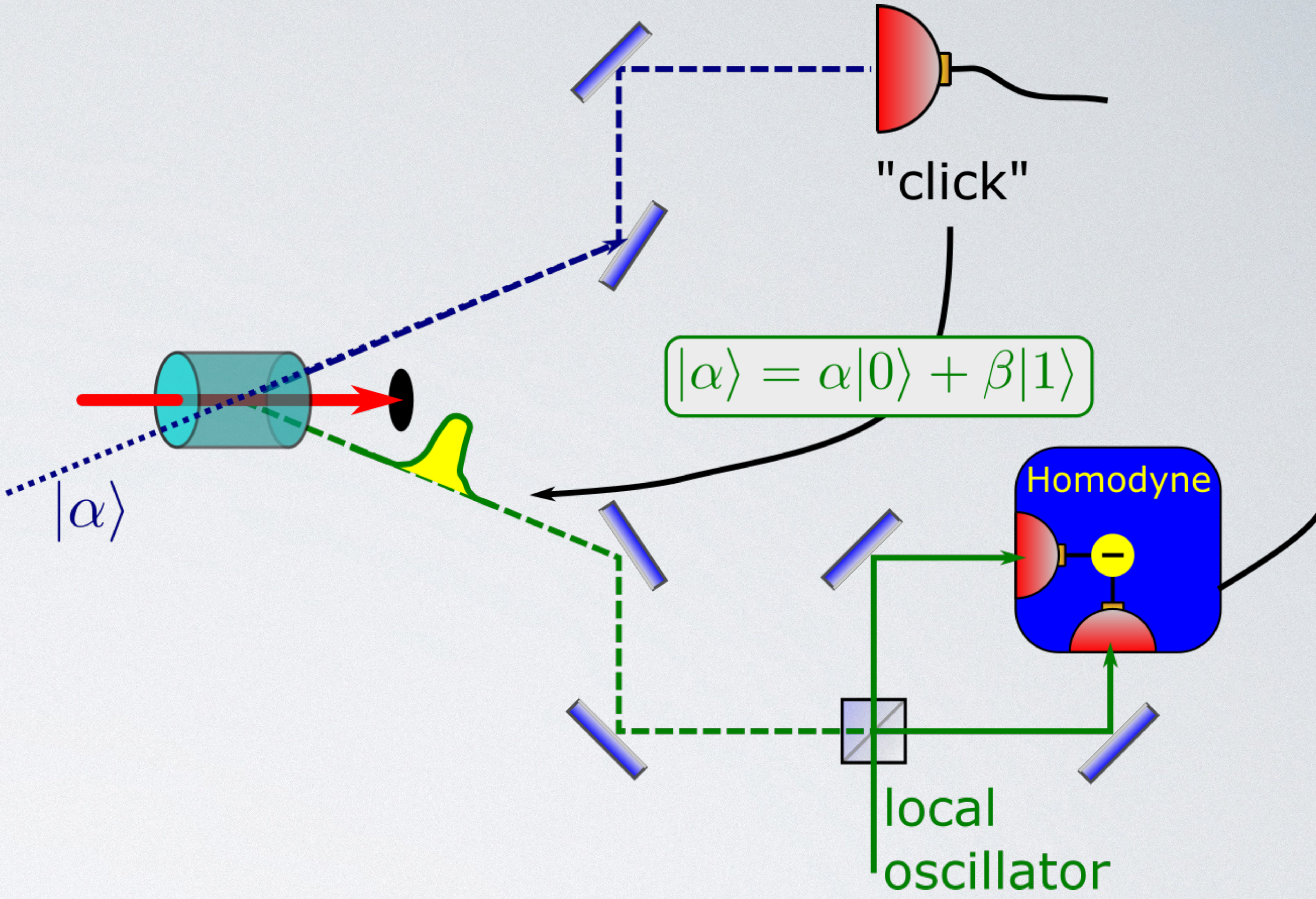
4WM SOURCE

Toward full quantum state engineering

- Seed process with weak coherent state:

$$|\alpha\rangle \approx |0\rangle_s + \alpha|1\rangle_s$$

$$|\Psi\rangle = |0_s0_a\rangle + \alpha|1_s1_a\rangle - i\gamma/\hbar|1_s0_a\rangle$$



Brannan et al. Opt Lett (2014)

4WM SOURCE

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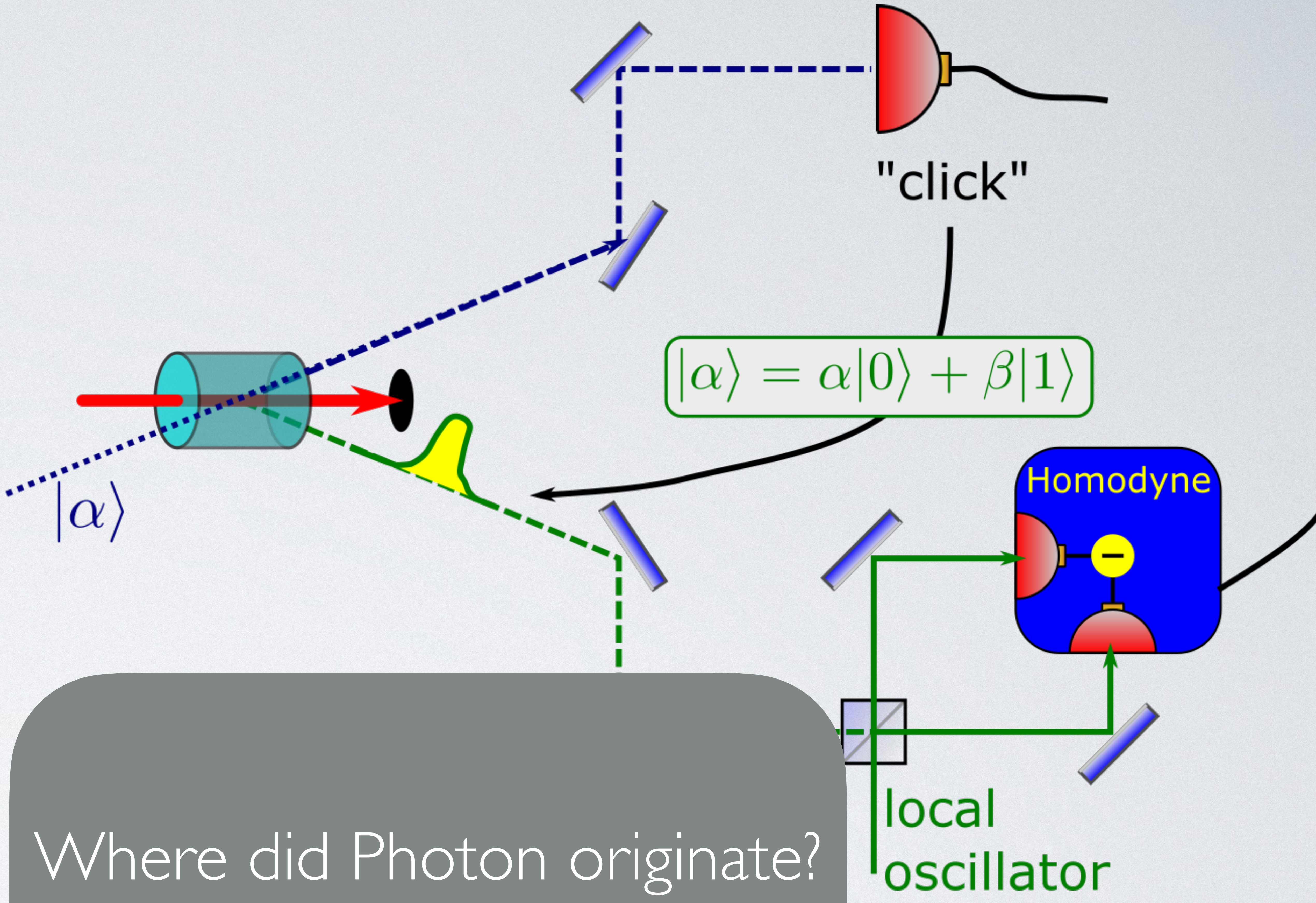
- Condition on signal channel photon:

$$\langle 1_s|\Psi\rangle = \alpha|0\rangle - \frac{i\gamma}{\hbar}|1\rangle$$



Where did Photon originate?

From $|\alpha\rangle$? Then $|\psi\rangle = |0\rangle$

From 4WM? Then $|\psi\rangle = |1\rangle$

Brannan et al. Opt Lett (2014)

4WM SOURCE

Toward full quantum state engineering

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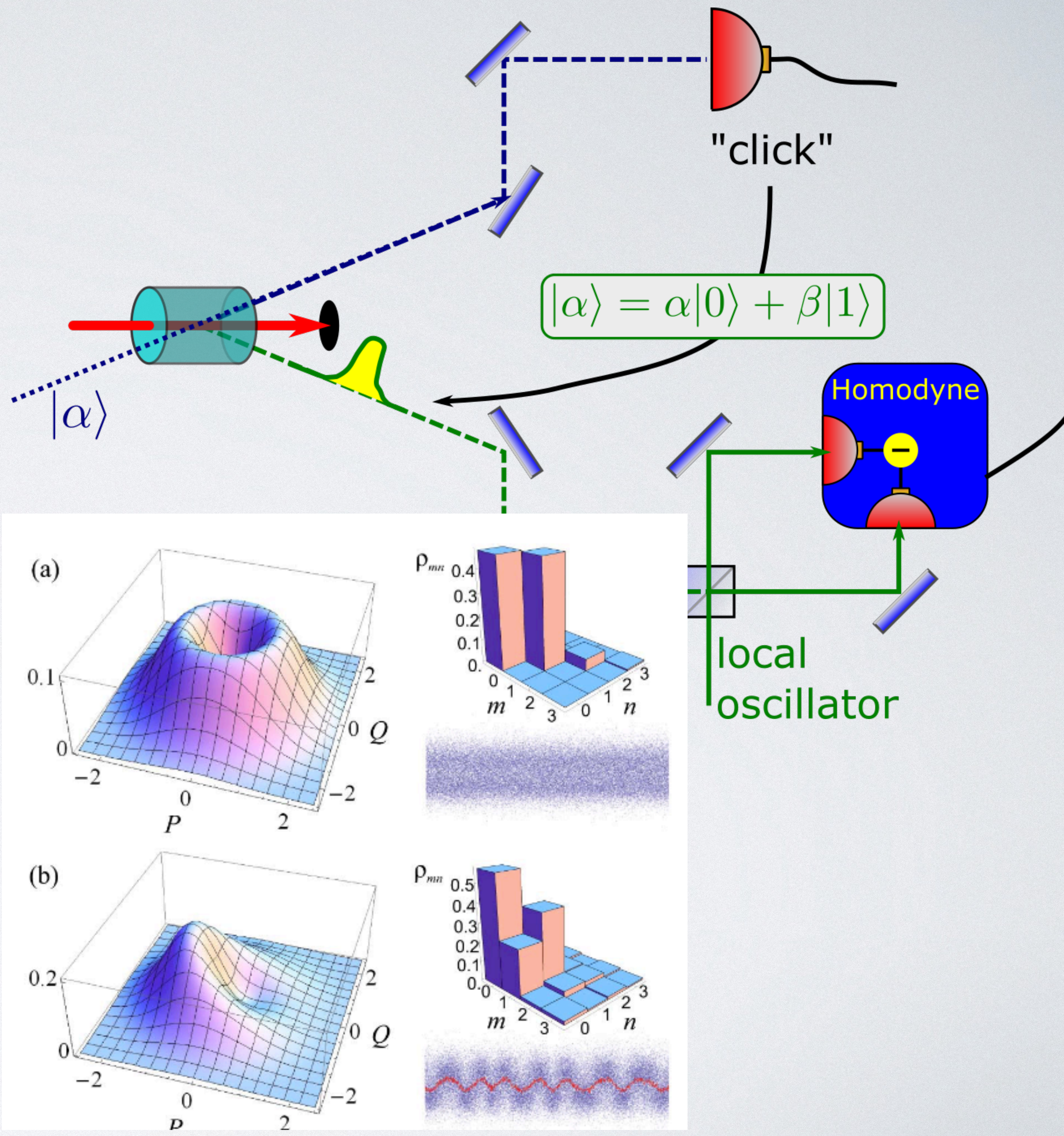
- α and γ experimentally accessible



Brannan et al. Opt Lett (2014)

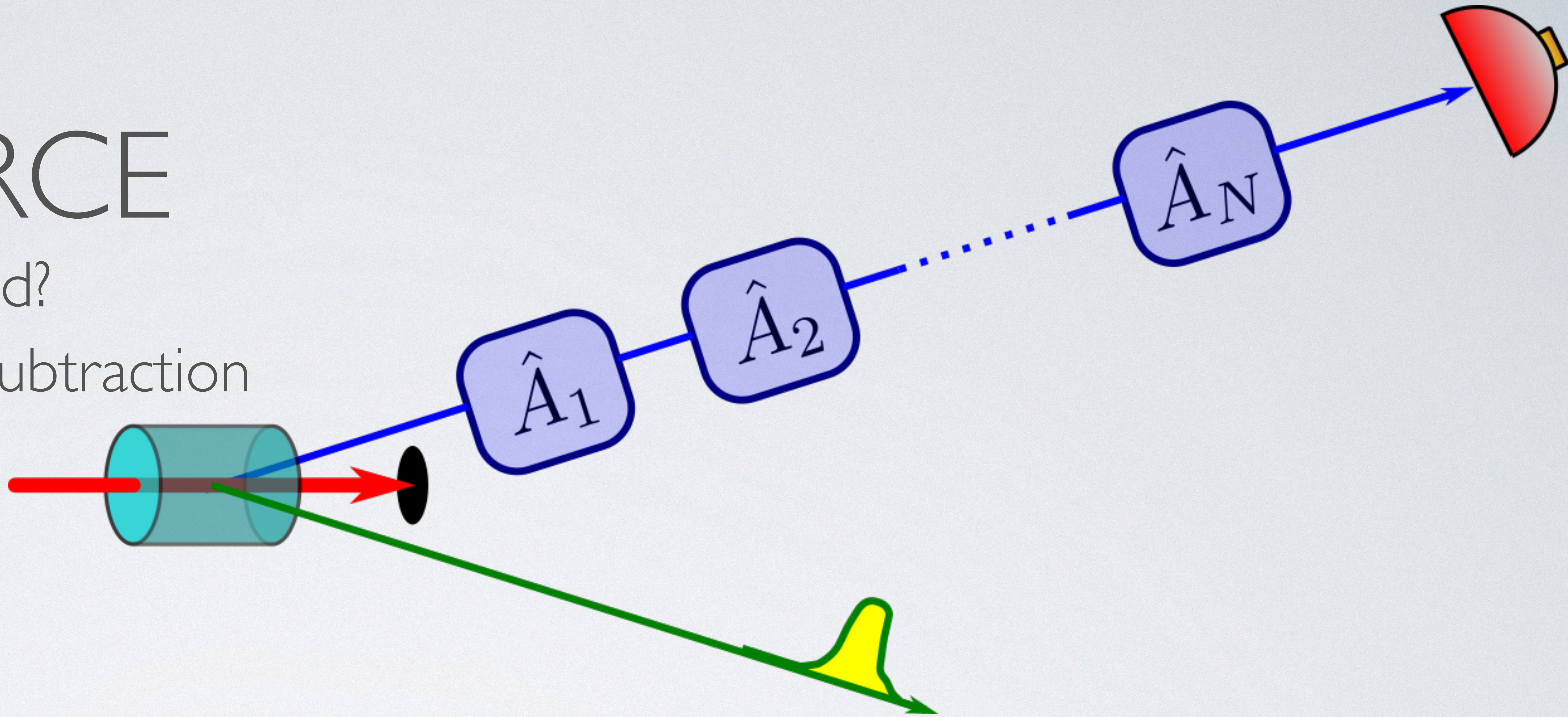
4WM SOURCE

Can this be extended?

4WM SOURCE

Can this be extended?

- **YES!** Use multiple photon subtraction



4WM SOURCE

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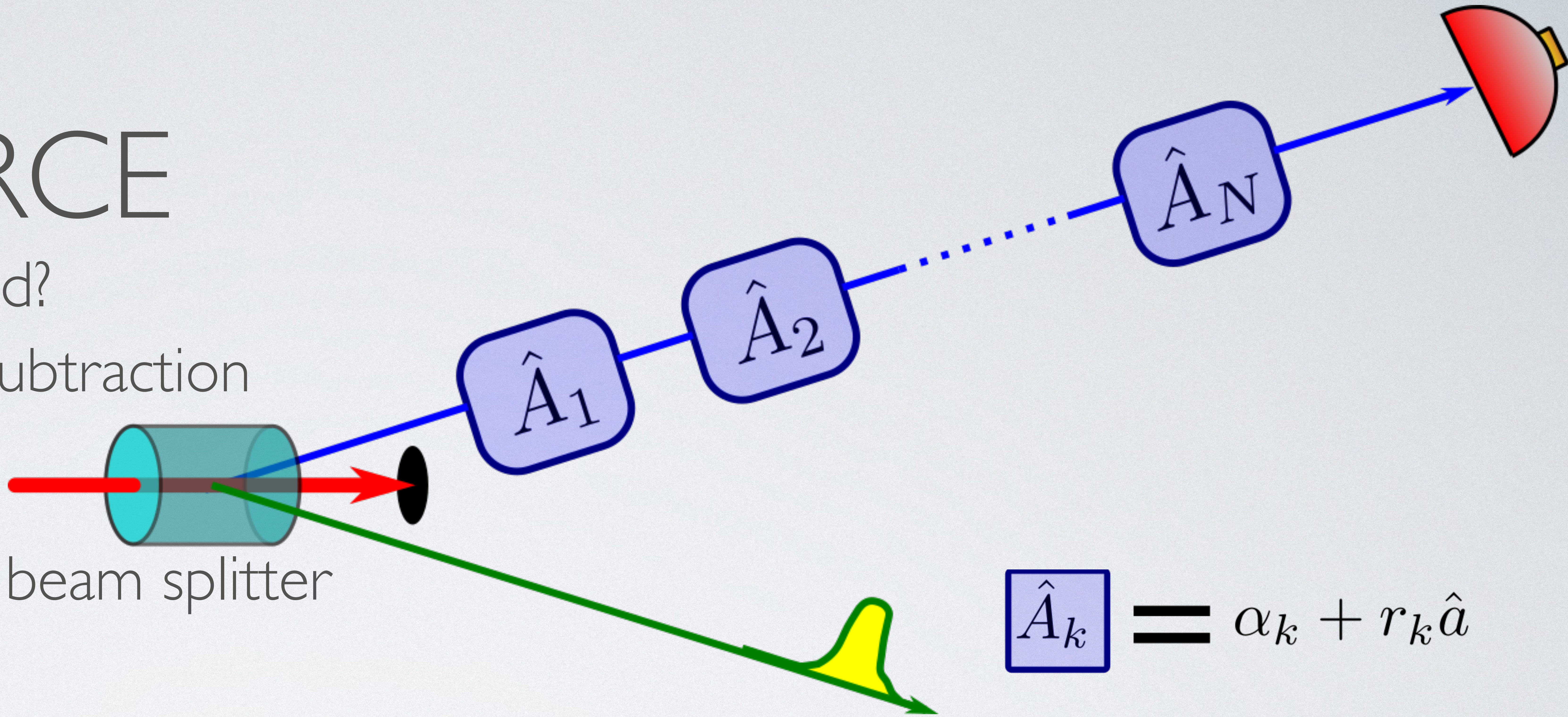
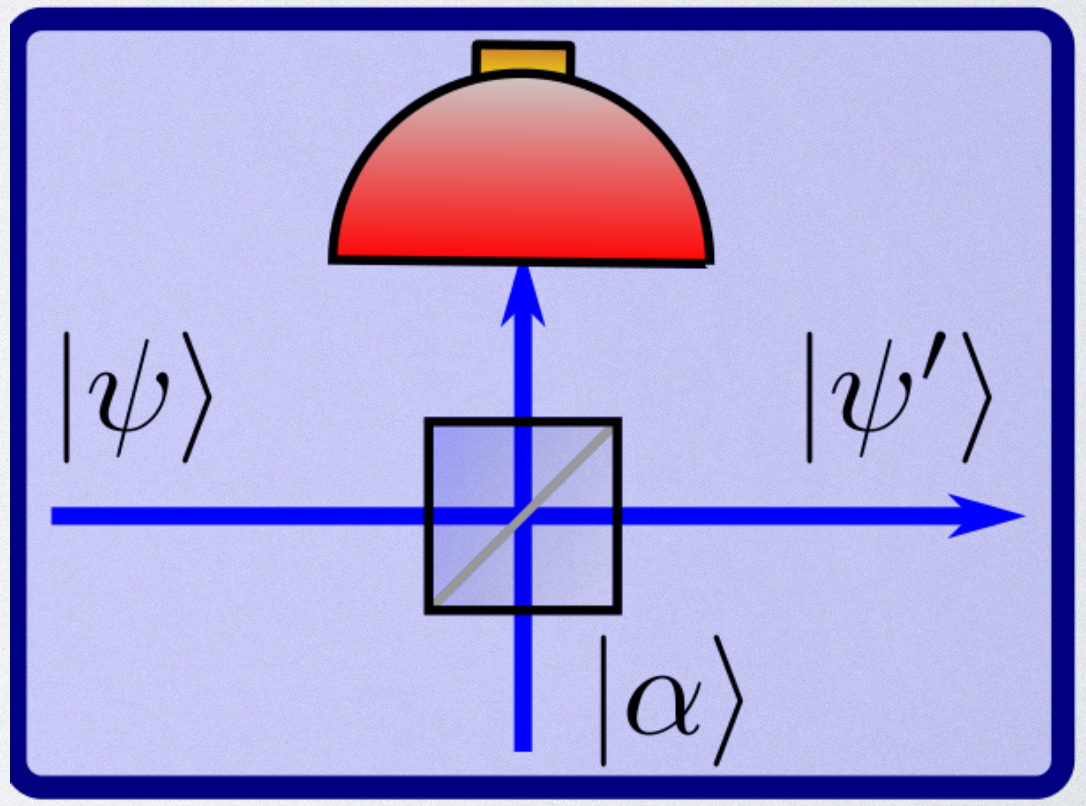


- Each box is a low-reflectivity beam splitter with coherent state at input:



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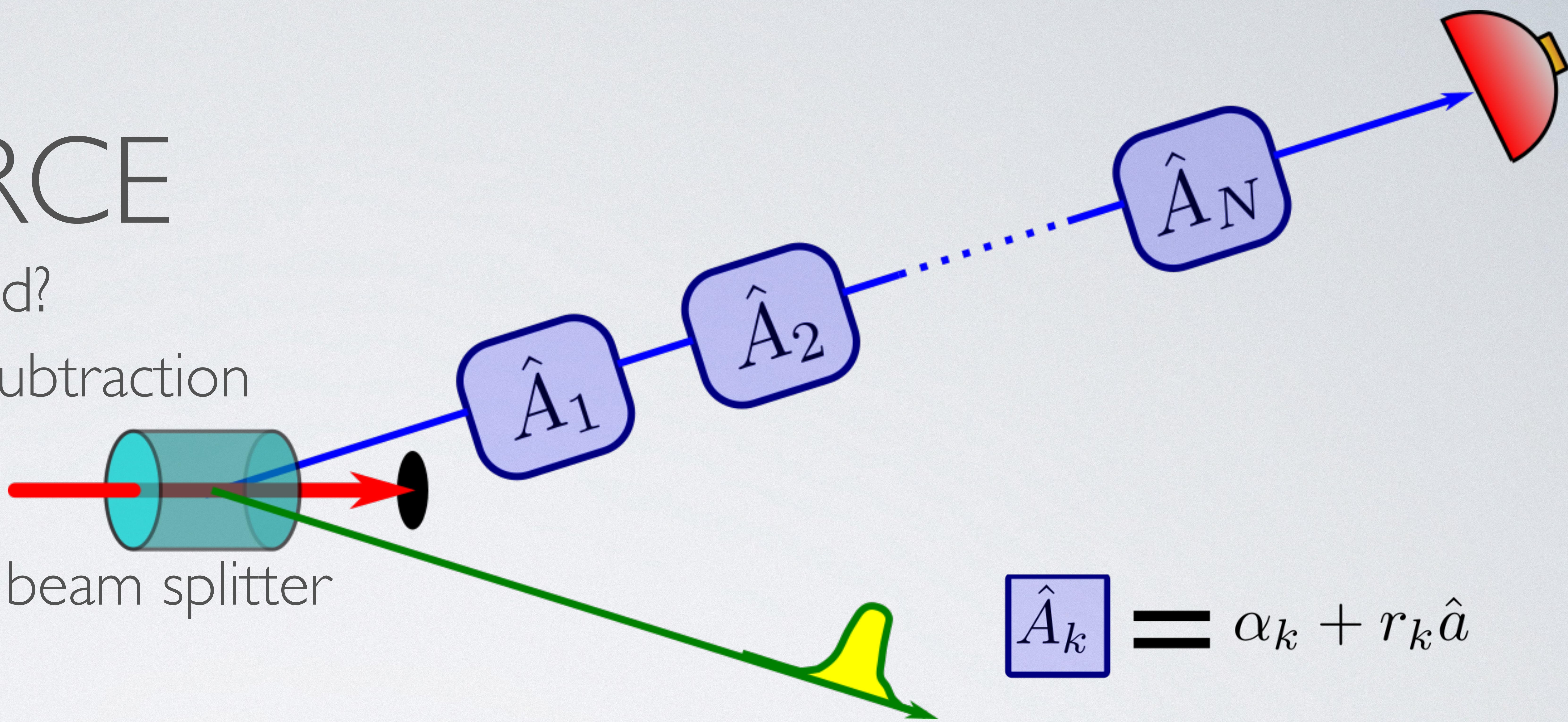
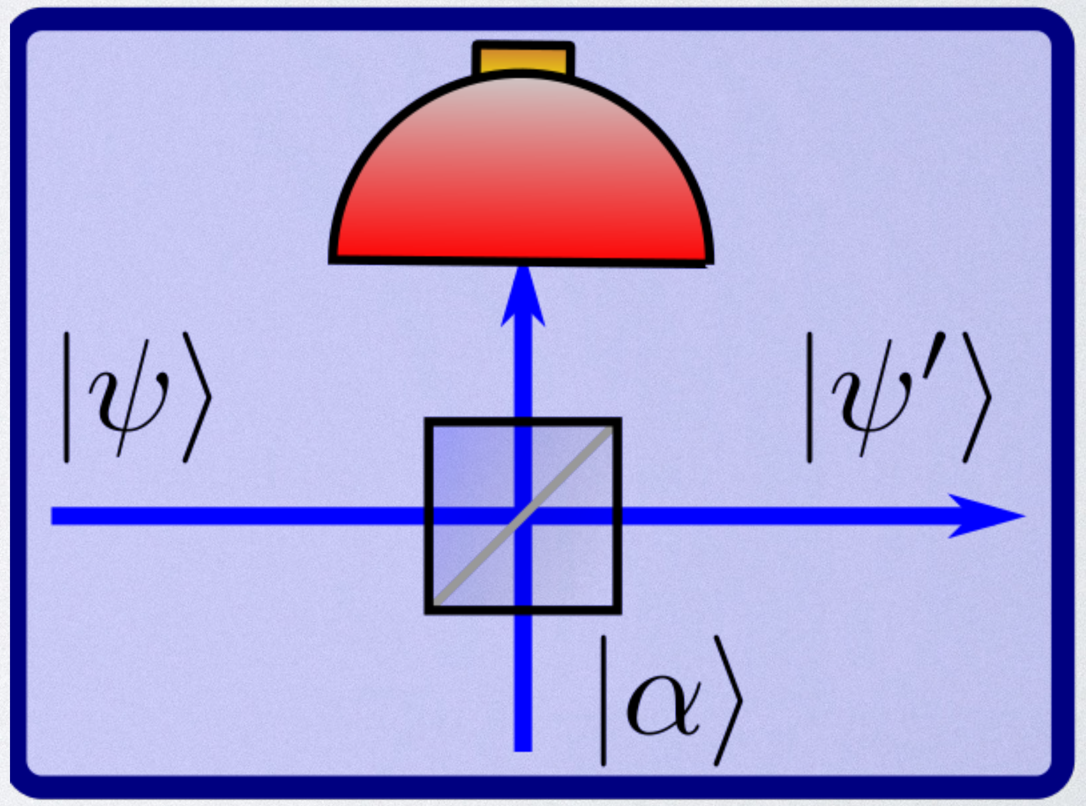


- Each box is a low-reflectivity beam splitter with coherent state at input:

$$\hat{A}_k = \alpha_k + r_k \hat{a}$$



- Condition at $|0\rangle$ at output:

$$|\psi > = \langle 0| \prod_{n=0}^{N} \left(\alpha_n + r_n \hat{a}_n\right) \hat{H}_{4WM} |0_s 0_a\rangle$$

4WM SOURCE

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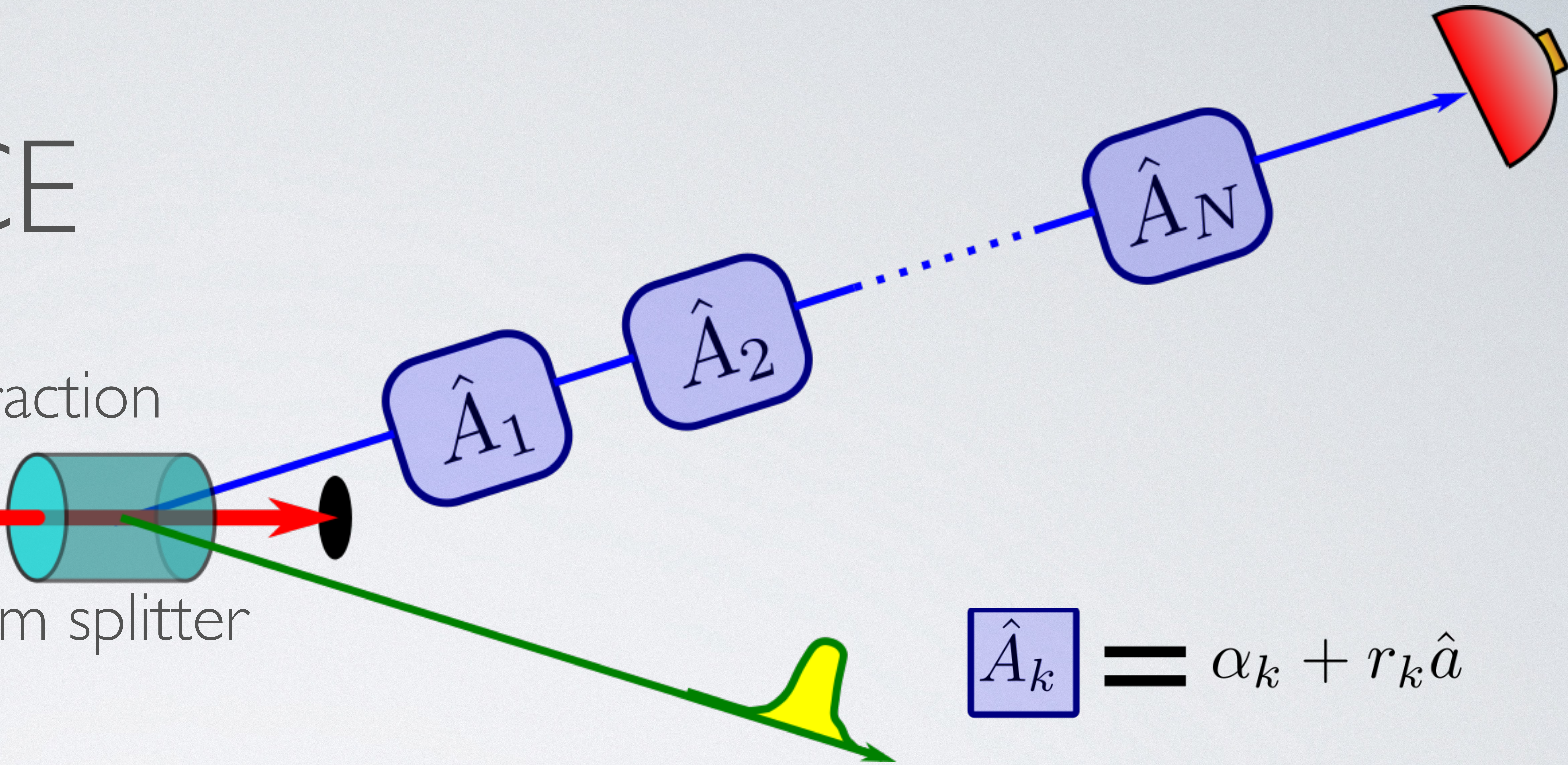

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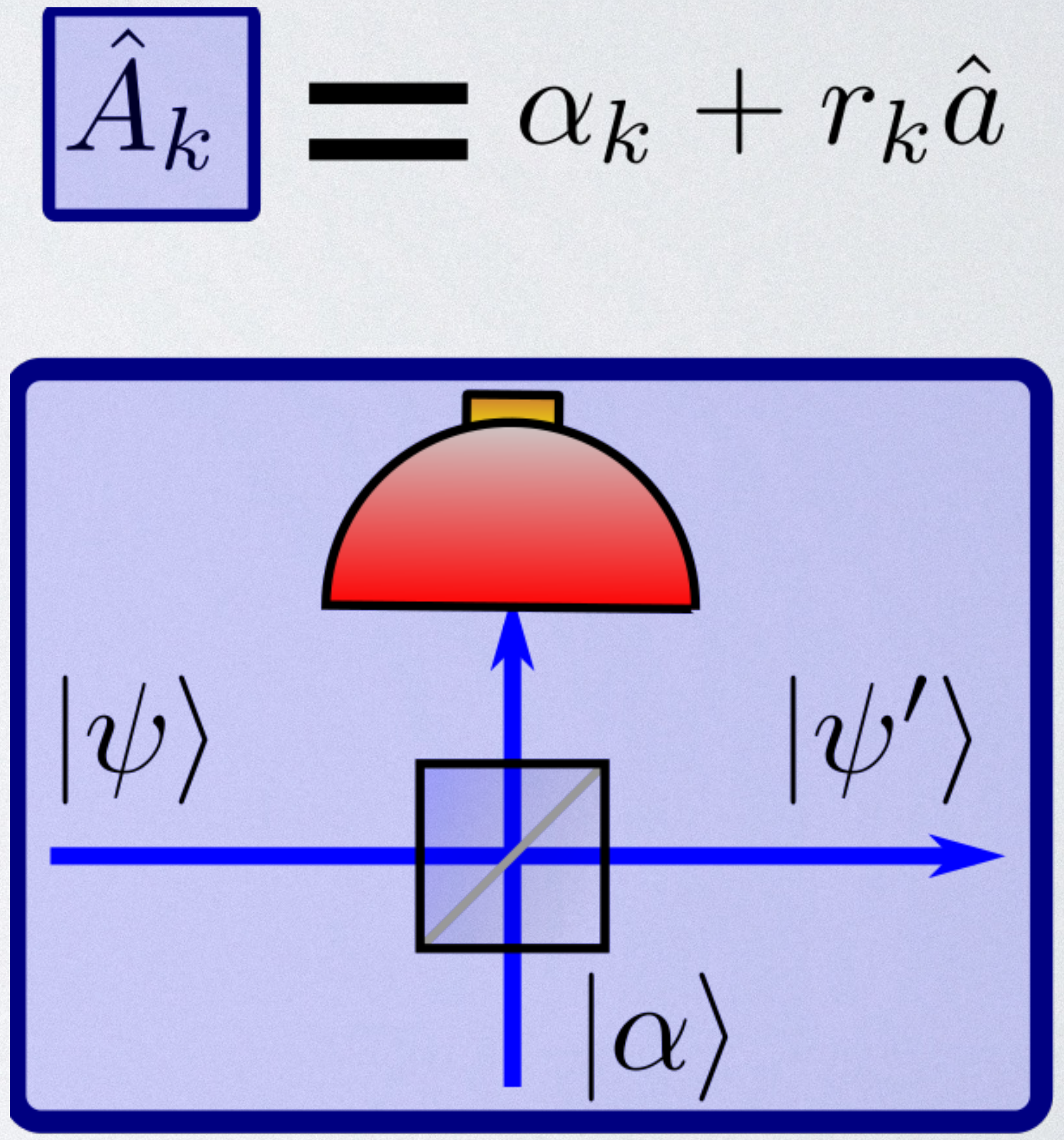

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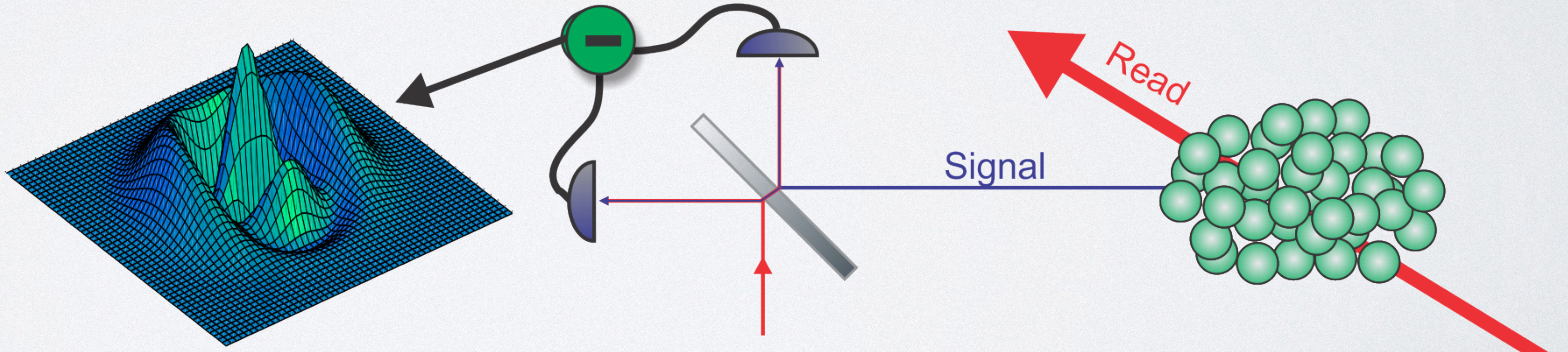
$$= \langle 0_s | \sum_{n=0}^{N} b_n \hat{a}_i^n | n_a n_s \rangle = \sum_{n=0}^{N} c_n | n_a \rangle$$

ENGINEERING THE ATOMIC STATE

Collective Spin Excitations

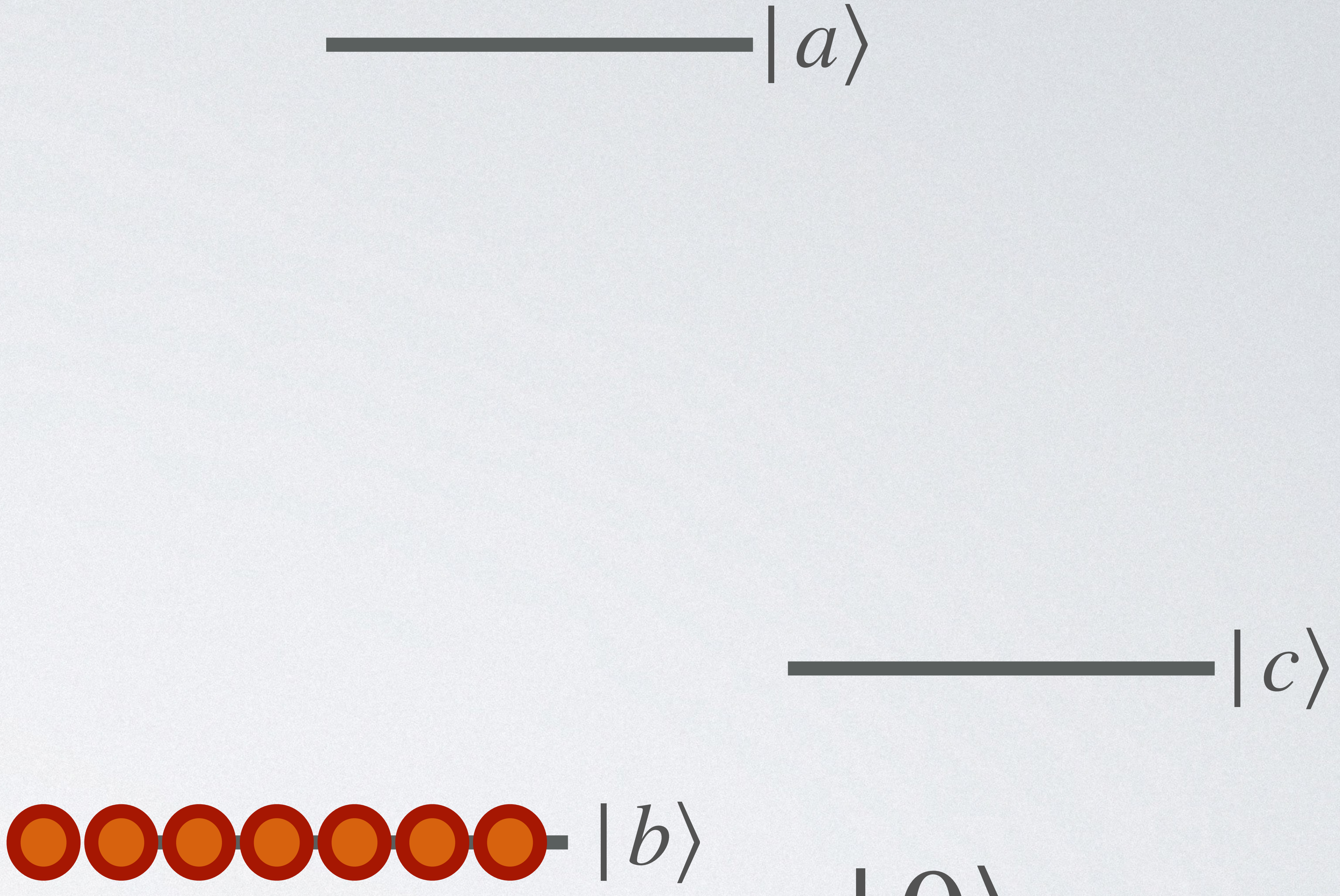
ENGINEERING THE ATOMIC STATE

Collective Spin Excitations



COLLECTIVE SPIN EXCITATIONS

DLCZ protocol

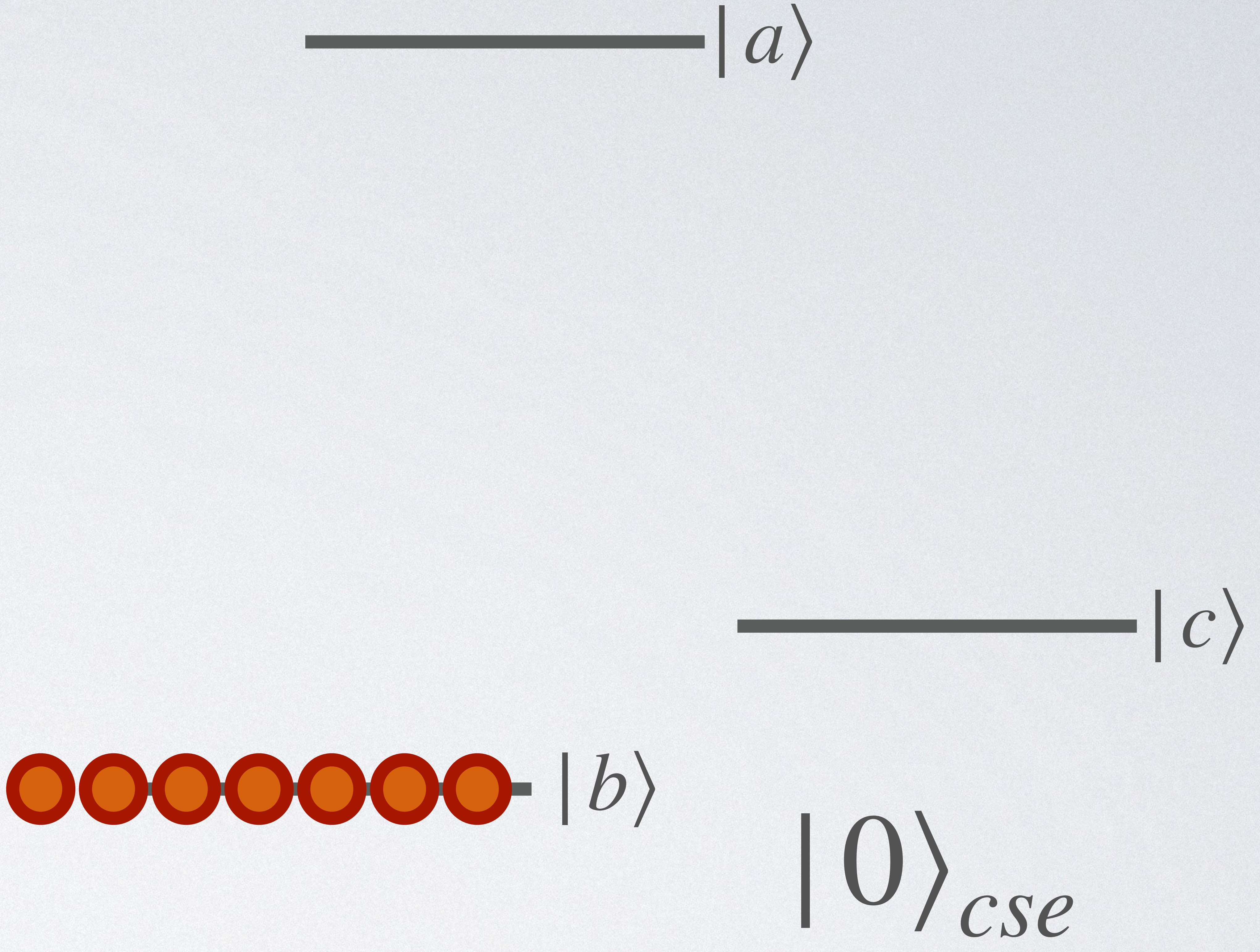


$$|0\rangle_{cse}$$

COLLECTIVE SPIN EXCITATIONS

DLCZ protocol

- Lambda system: all atoms in state $|b\rangle$



COLLECTIVE SPIN EXCITATIONS

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- Lambda system: all atoms in state $|b\rangle$
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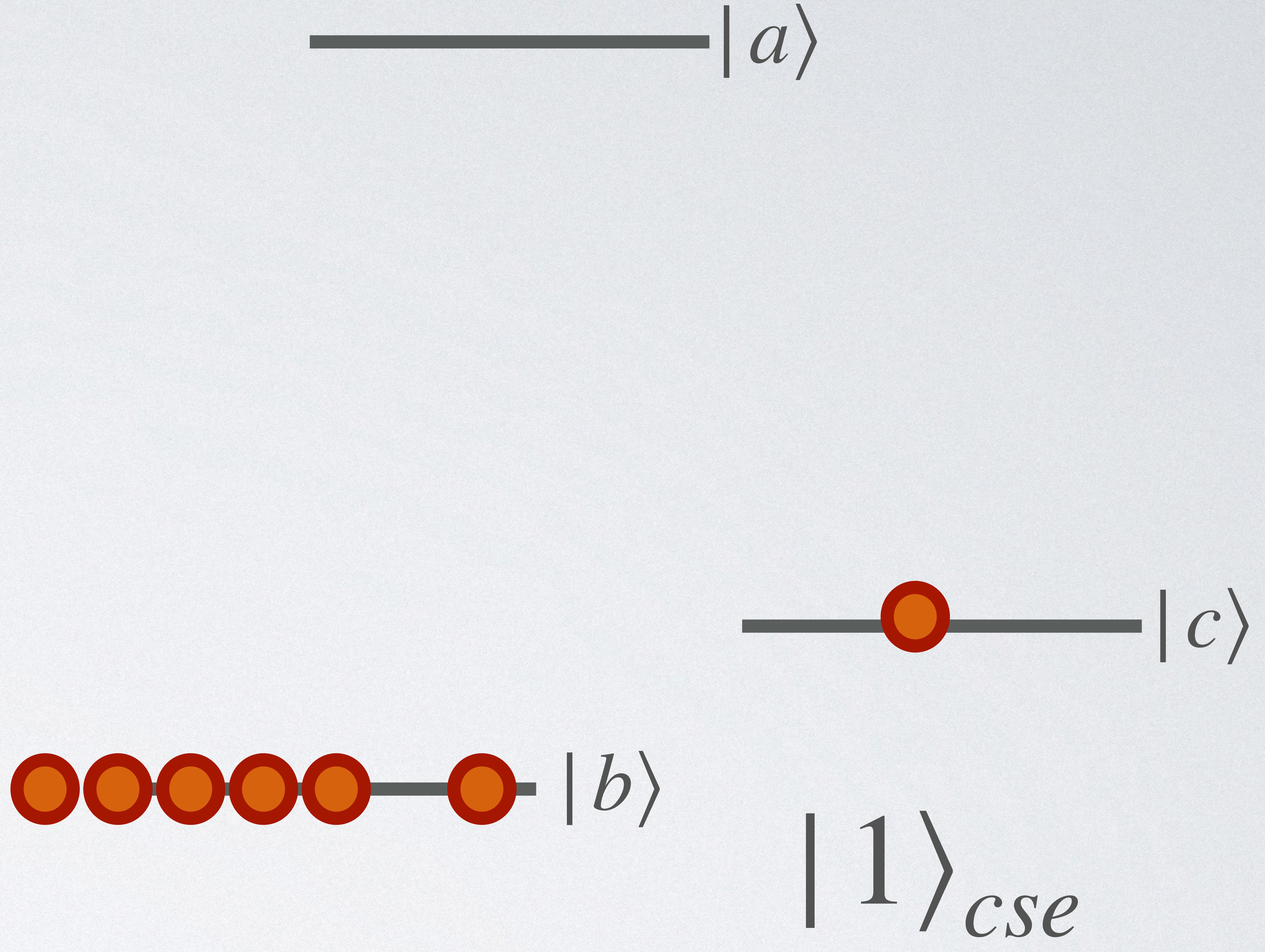
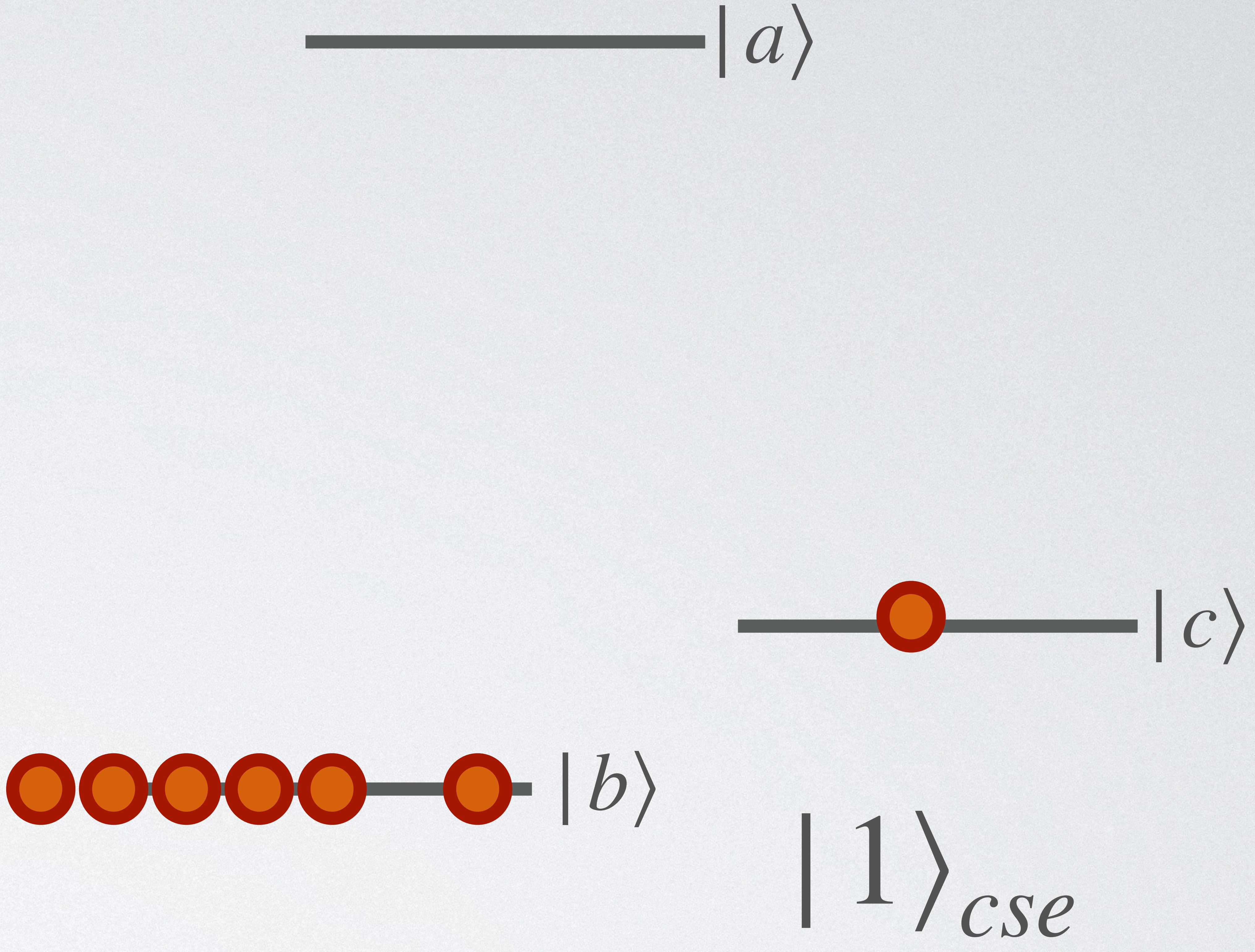


COLLECTIVE SPIN EXCITATIONS

DLCZ protocol

- Lambda system: all atoms in state $|b\rangle$
- Pump scatters single photon $|b\rangle \to |c\rangle$
- Indistinguishability:

$$|1\rangle_{cse} = e^{ikr_1}|cb\ldots b\rangle + e^{ikr_2}|bc\ldots b\rangle + \ldots + e^{ikr_N}|bb\ldots c\rangle$$



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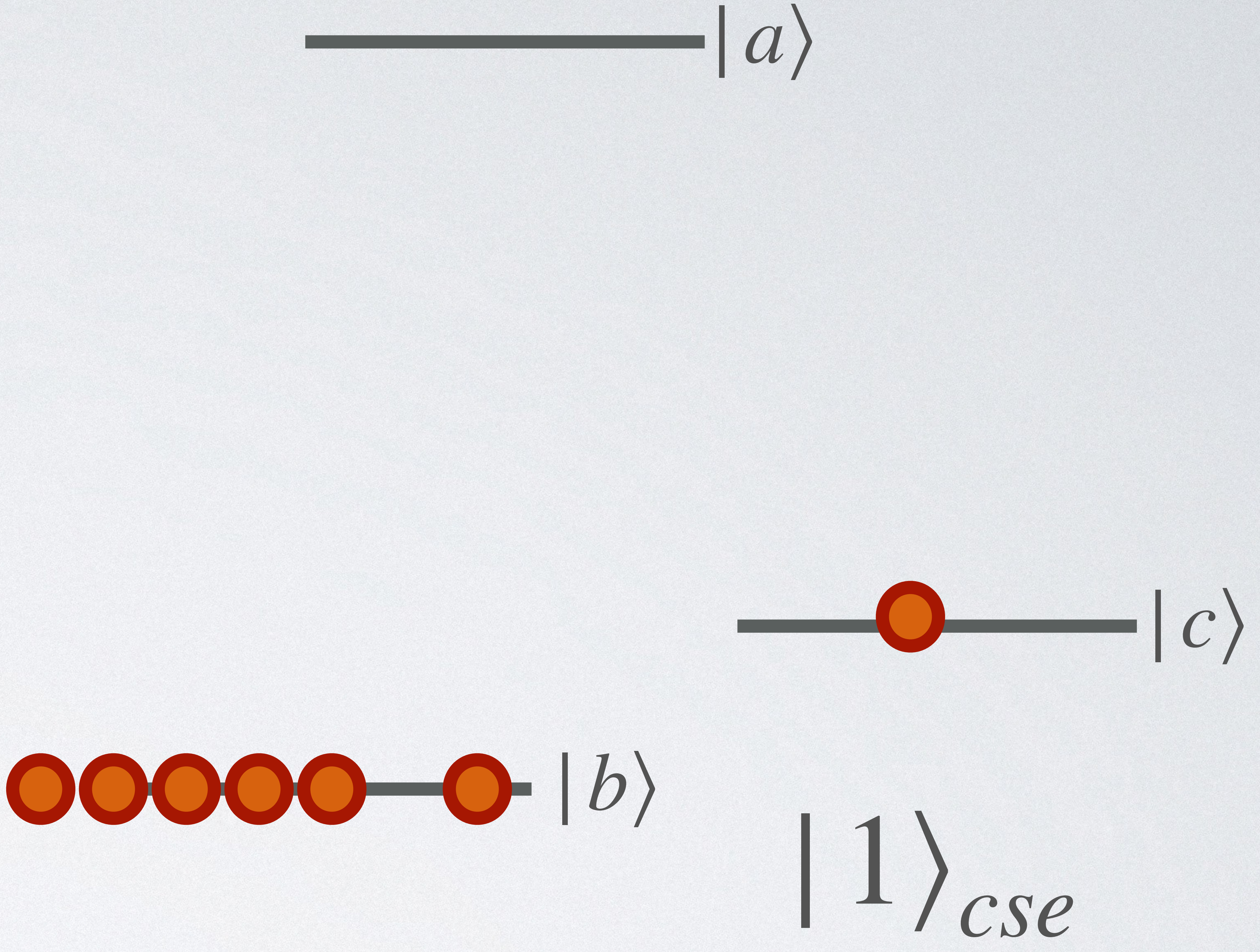

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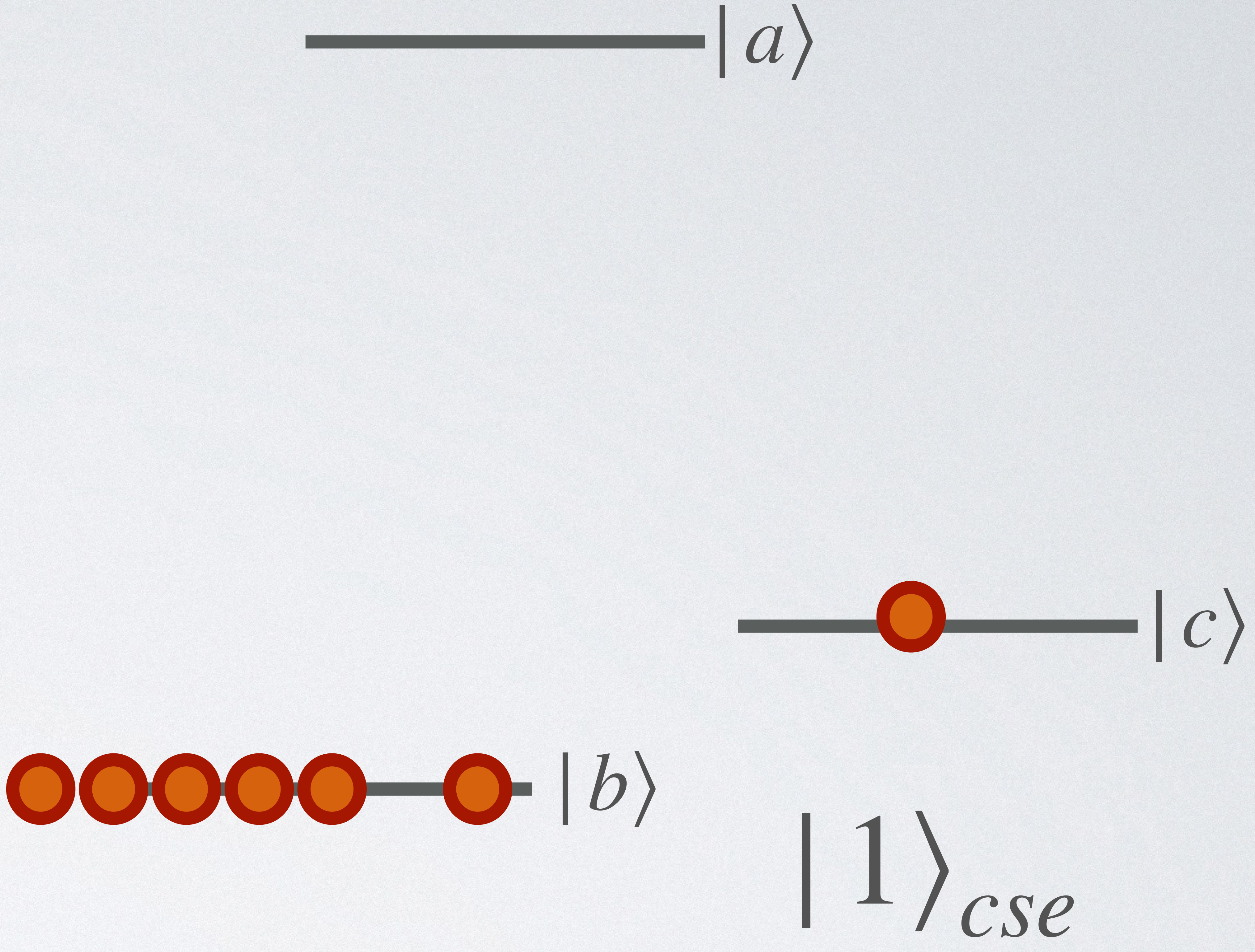
  - Same as optical 4WM case!



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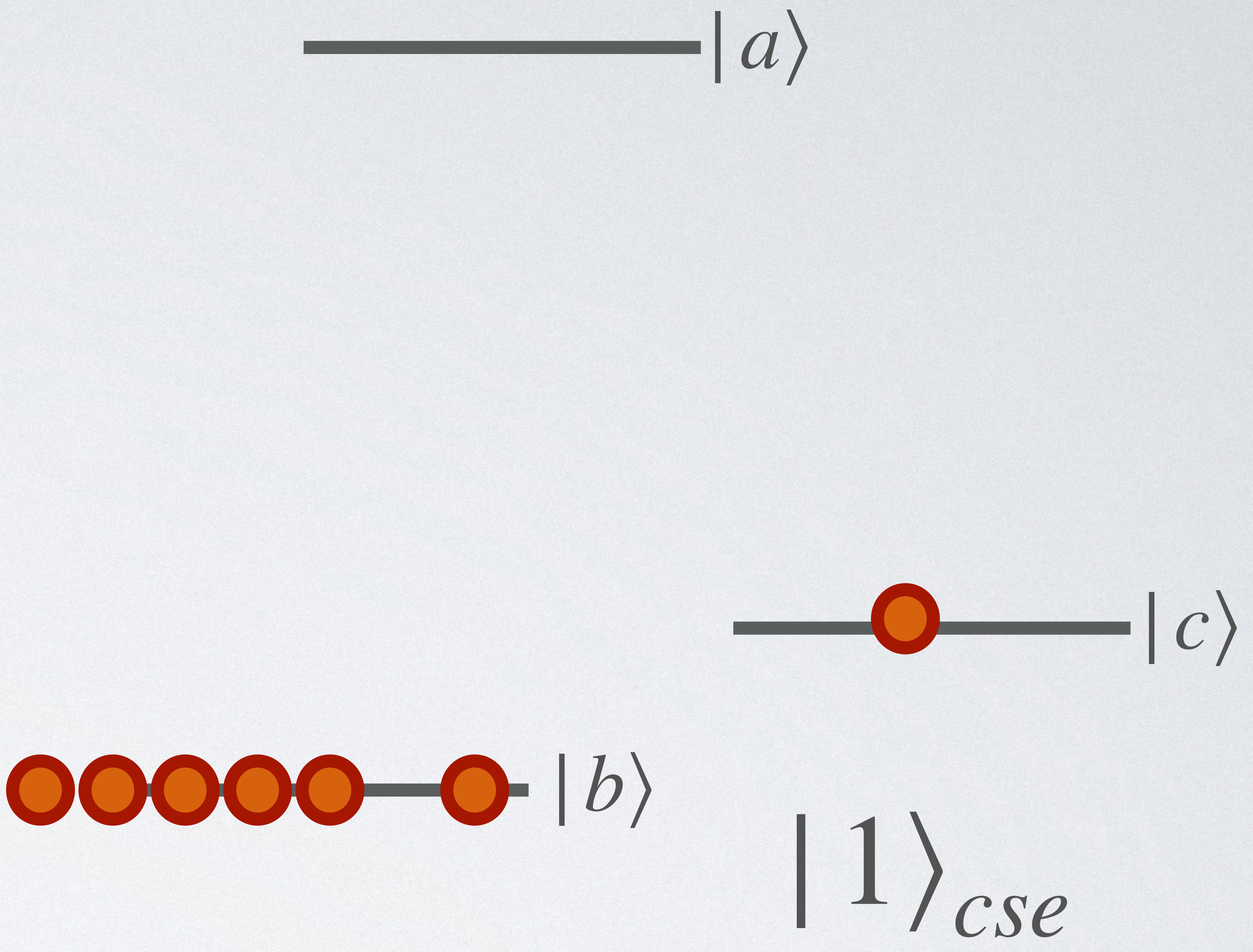

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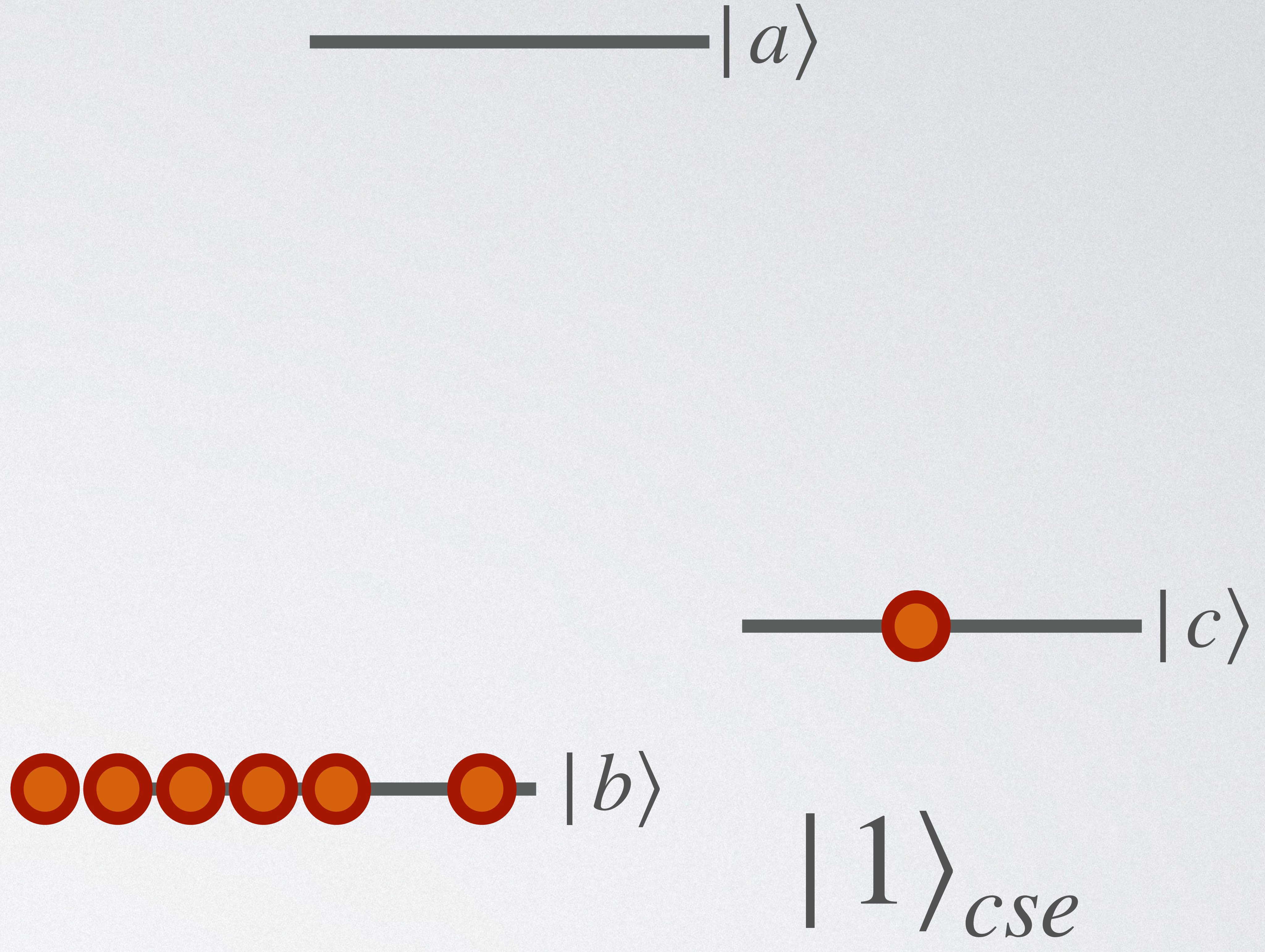

COLLECTIVE SPIN EXCITATIONS

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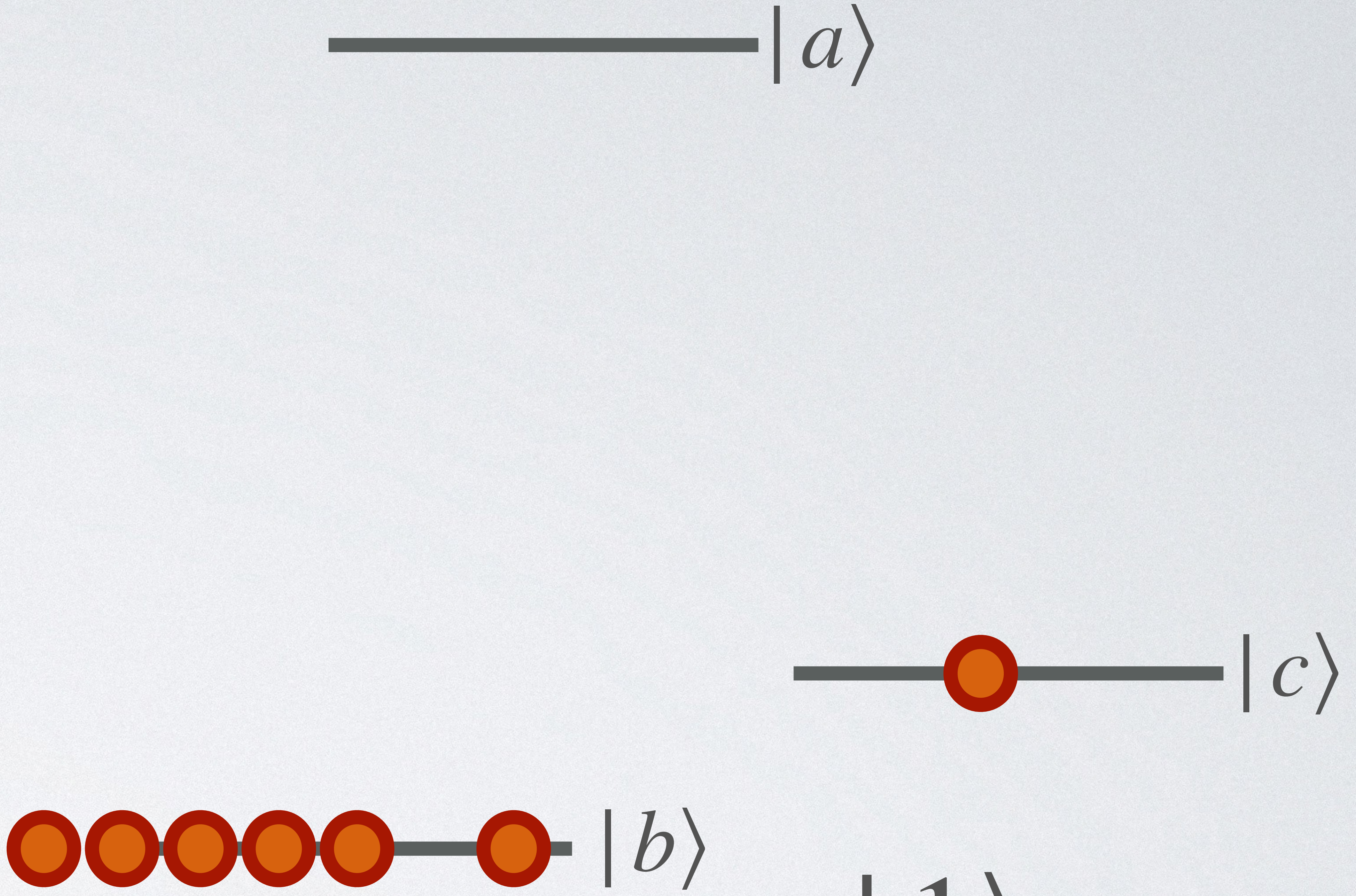

$$|1\rangle_{cse}$$

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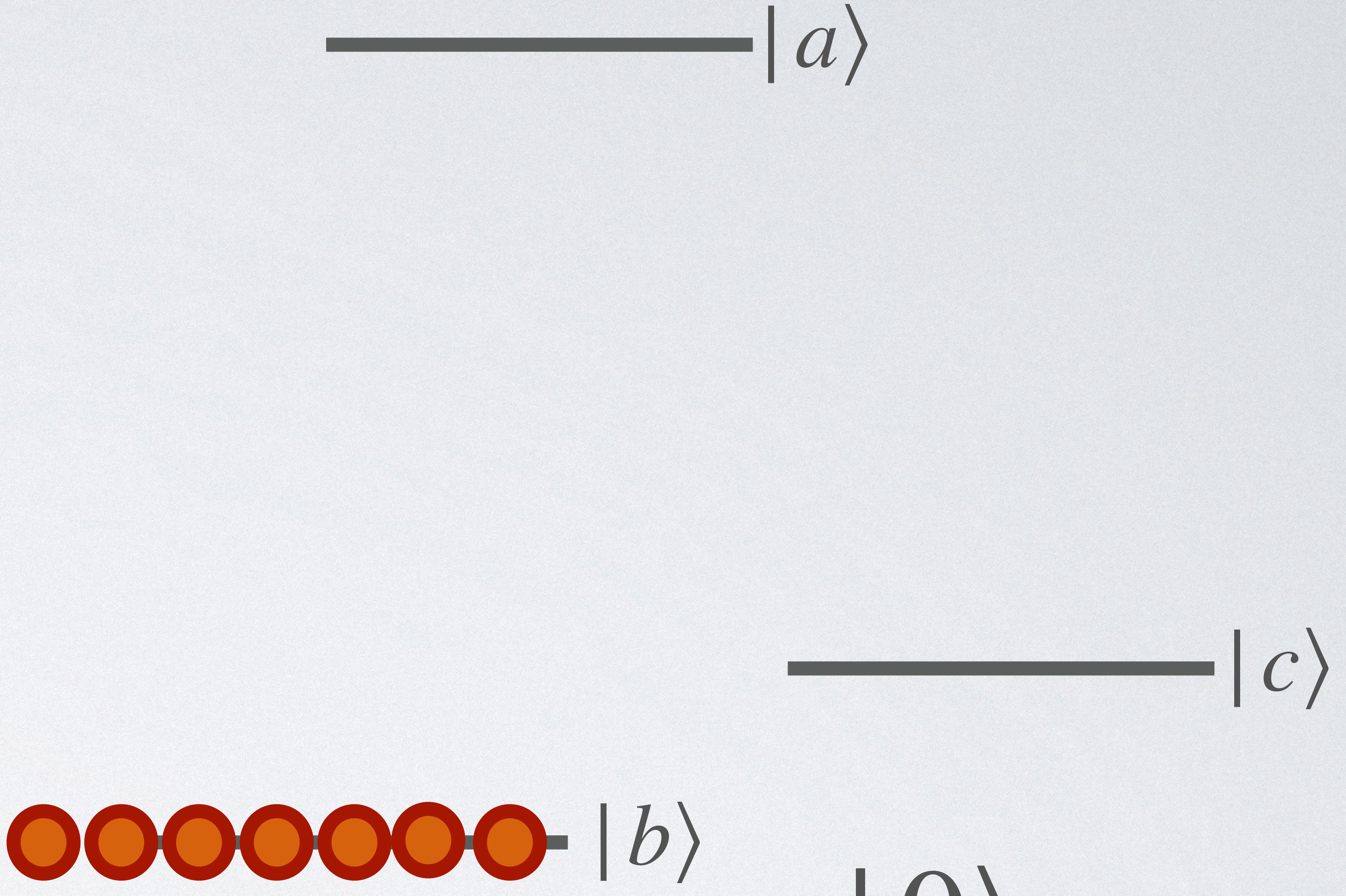
COLLECTIVE SPIN EXCITATIONS

DLCZ protocol



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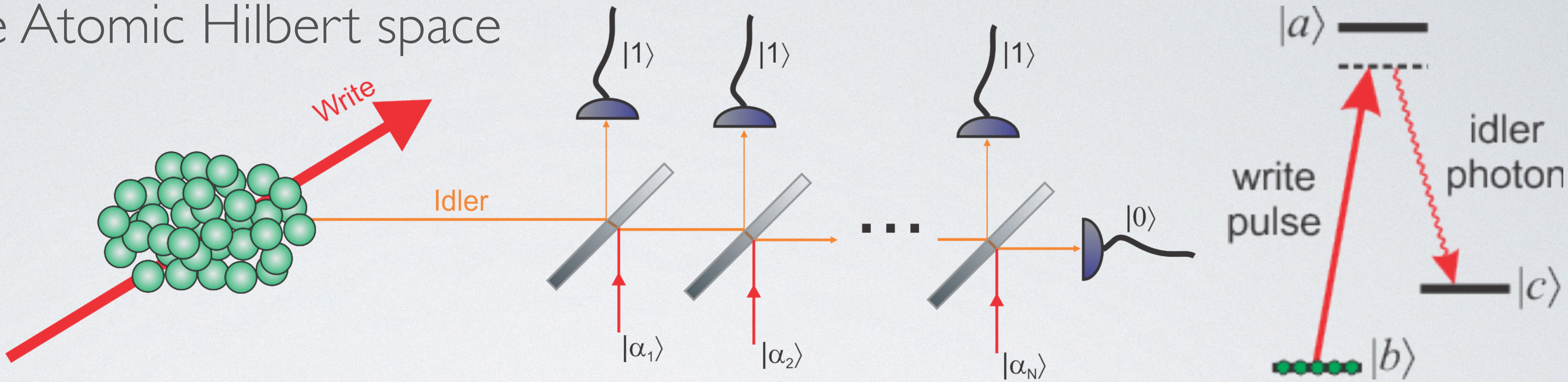
$|0\rangle_{cse}$

COLLECTIVE SPIN EXCITATIONS

Engineering the Atomic Hilbert space

WRITE

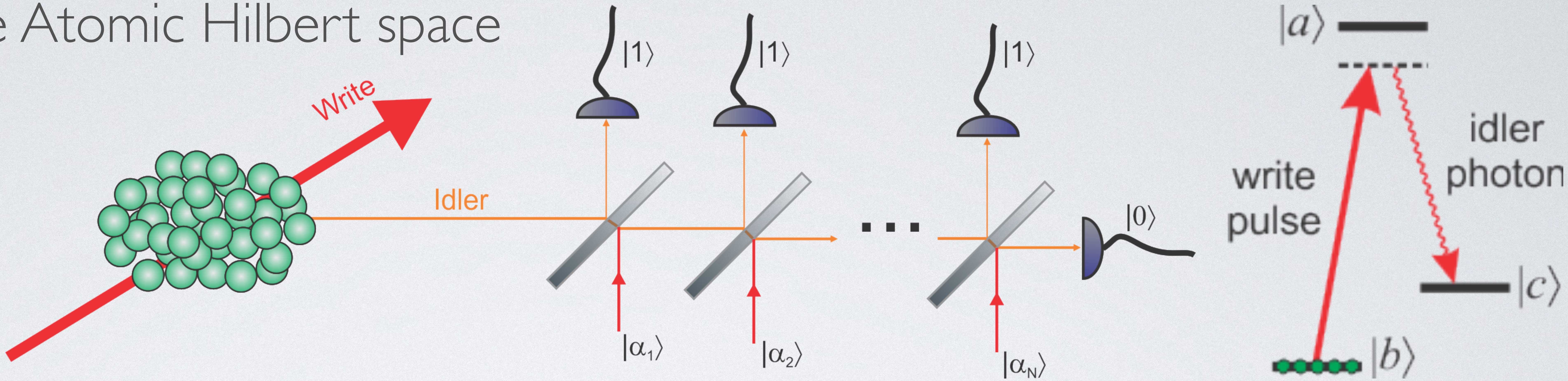


- Perform partial photon subtraction of stokes (idler) channel: **CSE engineering**

COLLECTIVE SPIN EXCITATIONS

Engineering the Atomic Hilbert space

WRITE



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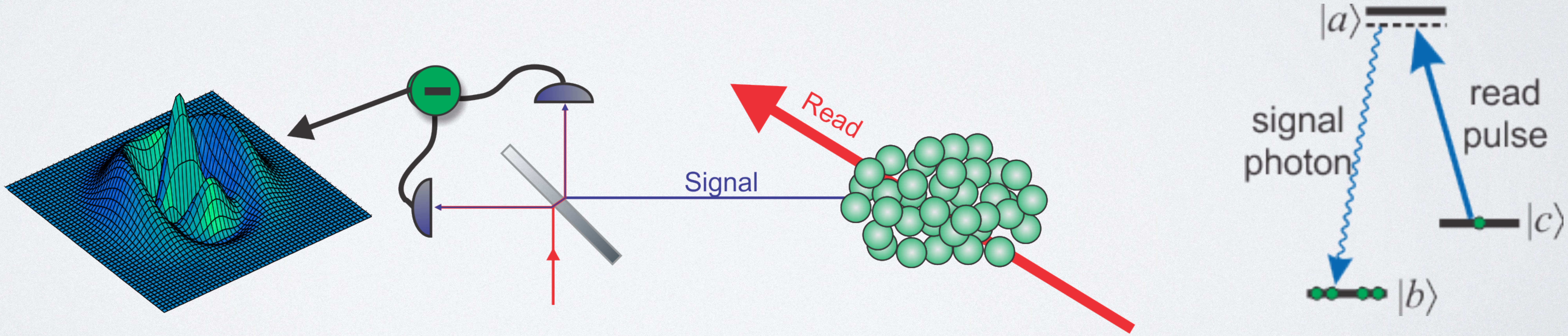
READ



- Read out CSE state into optical Hilbert space