Quantum resources in the future of quantum information

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We want to quantify the resourcefulness.

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It becomes more valuable if there isn't much of it.

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Common mathematical framework given by category theory [Coecke et al.].

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Main question

Can $\mathtt{A}$ be converted into $\mathtt{B}$ with free operations?

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Nielsen's theorem [Nielsen]

$|\psi\rangle_{AB} \overset{\text{LOCC}}{\mapsto} |\phi\rangle_{AB}$ if and only if $\text{tr}_B |\psi\rangle\langle\psi| \overset{\text{RU}}{\mapsto} \text{tr}_B |\phi\rangle\langle\phi|$.

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3. Export the machinery of resource theories beyond quantum information (category theory) [CMS et al.].

References

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