Gravitational time dilation, free fall, and matter waves

Outline:

- Ocean waves vs. de Broglie waves:
  - gravitational time dilation explains the free fall
  - motion along geodesics follows; not postulated

Gravitational time dilation, free fall, and matter waves

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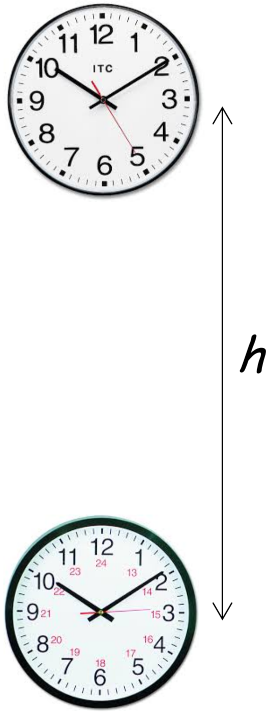
Gravitational time dilation



$$\Delta t = \frac{gh}{c^2} t$$

Example: a weekend trip from Seattle to Mount Rainier, t = 40 hours, elevation *h* = 1340 metres,

$$\Delta t = \frac{gh}{c^2} t = \frac{9.8 \cdot 1340}{9 \cdot 10^{16}} 40 \cdot 3600 \text{ s} = 21 \text{ ns}.$$

For h = 1 cm, $\Delta t/t = 10^{-18}$

This tiny difference of time flow makes things fall!

How does this tiny difference cause falling?

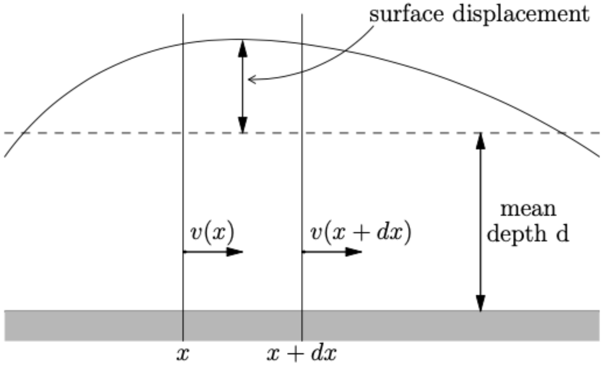
Matter waves evolve faster at a higher elevation.

Intuitive example: ocean waves; why do they always approach the beach?

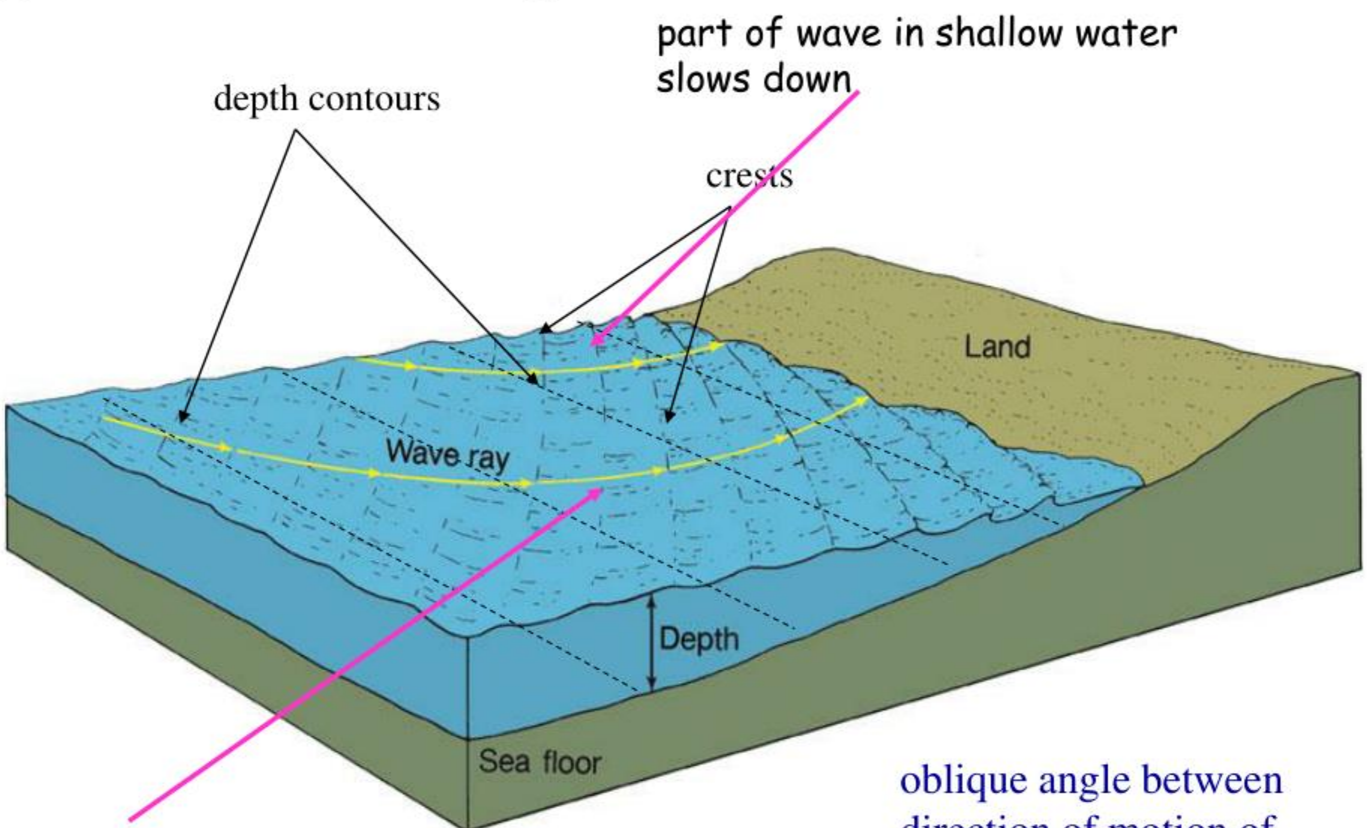
Reason: waves are slower in shallower water,

$$gd\frac{\partial^2\zeta}{\partial x^2} = \frac{\partial^2\zeta}{\partial t^2}$$

$$u = \sqrt{gd}$$



Wave Refraction – slowing and bending of waves as they approach shore at an angle



oblique angle between direction of motion of waves and depth contours

What does this mean for matter waves?

Consider a free particle traveling vertically: a plane wave

$$\psi = \exp \frac{ipz - i\sqrt{m^2c^4 + p^2c^2}\,t}{\hbar}$$
$$\simeq \exp \frac{1}{\hbar}\left(ipz - imc^2t - i\frac{p^2}{2m}t\right)$$

we usually neglect this term: an overall phase, independent of particle's motion;
but near the earth, time *t* has a slight z-dependence

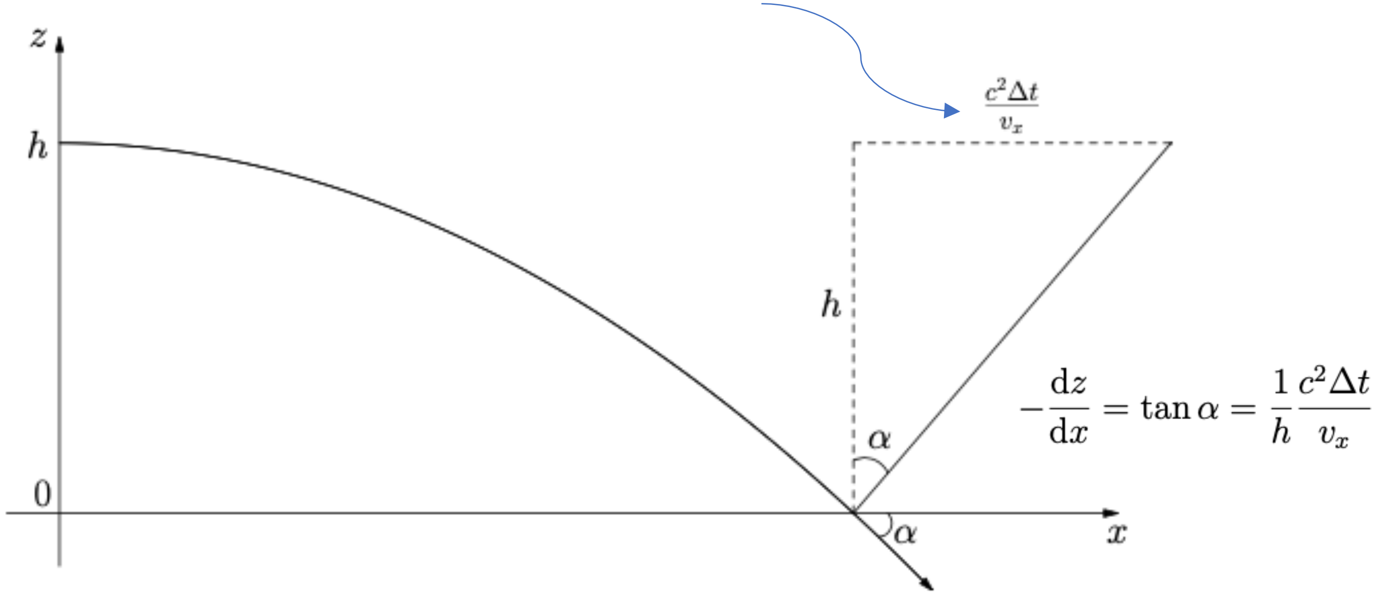
$$t \to t\left(1 + \frac{gz}{c^2}\right)$$

$$\psi \simeq \exp \frac{1}{\hbar}\left(i(p - mgt)z - imc^2t - i\frac{p^2}{2m}t\right)$$

this is just the free-fall evolution of momentum

Another point of view: projectile motion

$$\psi \simeq \exp\frac{1}{\hbar}\left(imv_x x - imc^2t - i\frac{mv_x^2}{2}t\right)$$
$$\simeq \exp\frac{1}{\hbar}\left[imv_x\left(x - \frac{c^2t}{v_x}\right)\right]$$



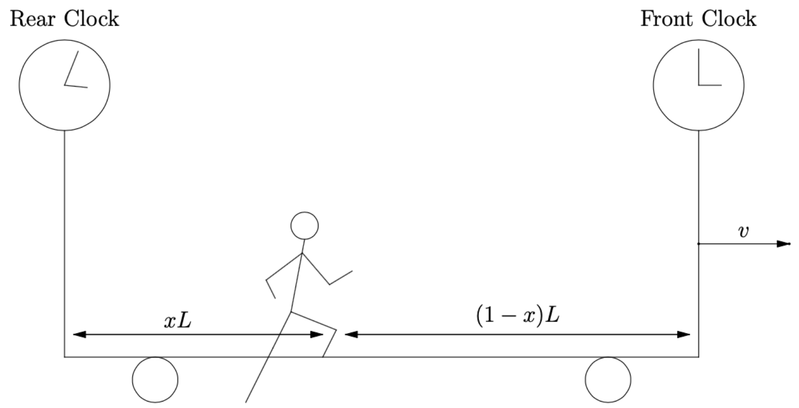
But $\mathrm{d}x = v_x \mathrm{d}t$ and $\Delta t = \frac{gh}{c^2}t$

$$-\frac{\mathrm{d}z}{\mathrm{d}t} = \frac{c^2\Delta t}{h} = gt$$

Application: twin paradox (local version)

Rear clock is ahead by

$$\Delta t = \frac{Lv}{c^2}$$



duration of the spurt is $t = v/g$ (here "g" is the average acceleration)

$$\Delta t_R = g\,xL\,t/c^2$$

$$\Delta t_F = g(1-x)Lt/c^2$$

$$\Delta t = \Delta t_F + \Delta t_R = \frac{gLt}{c^2} = \frac{Lv}{c^2}$$

When the observer settles on the train, both clocks show the same time.

Summary

Free fall explained by the evolution rate of matter waves at various altitudes.

Analogy: ocean waves near beaches.

Take-home message: non-relativistic matter follows geodesics because of the phase evolution of its wave function.

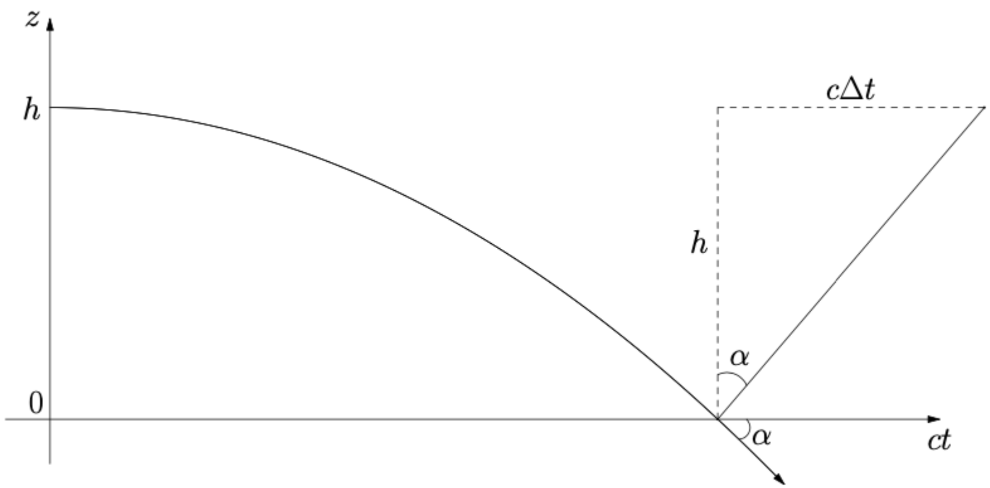
The role of the mc^2t term in the phase: discussed previously, Sagnac effect:

Relativistic aspects of nonrelativistic quantum mechanics

Dennis Dieks[a] and Gerard Nienhuis[b]

Am. J. Phys. **58** (7), July 1990

Phase of matter waves in space-time



Note: this plot shows a one-dimensional motion (only vertical), not a two-dimensional projectile motion.

Velocity from the slope: $$\frac{v}{c} = -\tan\alpha$$

$$\frac{v}{c} = -\frac{c\Delta t}{h}$$ *h* cancels with $\Delta t = \frac{gh}{c^2}t$

We reproduce the familiar result,

$$\boxed{v = -gt}$$