First results from the Muon g-2 Experiment at Fermilab

Chris Polly, Fermi National Accelerator Laboratory

The results heard round the world!

- Worldwide press coverage
  - Over 3000 media outlets covered the story
  - Total estimated media reach of those outlets **> 6 billion people!** (Pop. Earth 7.7 billion)


"All the News That's Fit to Print"

The New York Times

Late Edition

VOL. CLXX .... No. 59,022 NEW YORK, THURSDAY, APRIL 8, 2021 $3.00

Biden Tax Plan Aims to Curtail

Contagious Variant Is Fueling Surge in Infections Across the U.S.

PAIR OF SETBACKS

A Particle's Tiny Wobble Could Upend the Known Laws of Physics

By DENNIS OVERBYE

A ring at the Fermi National Accelerator Laboratory in Illinois is used to study the wobble of muons.

Adventurers Fleeing Pandemic Strain the West's Rescue Teams

By ALI WATKINS

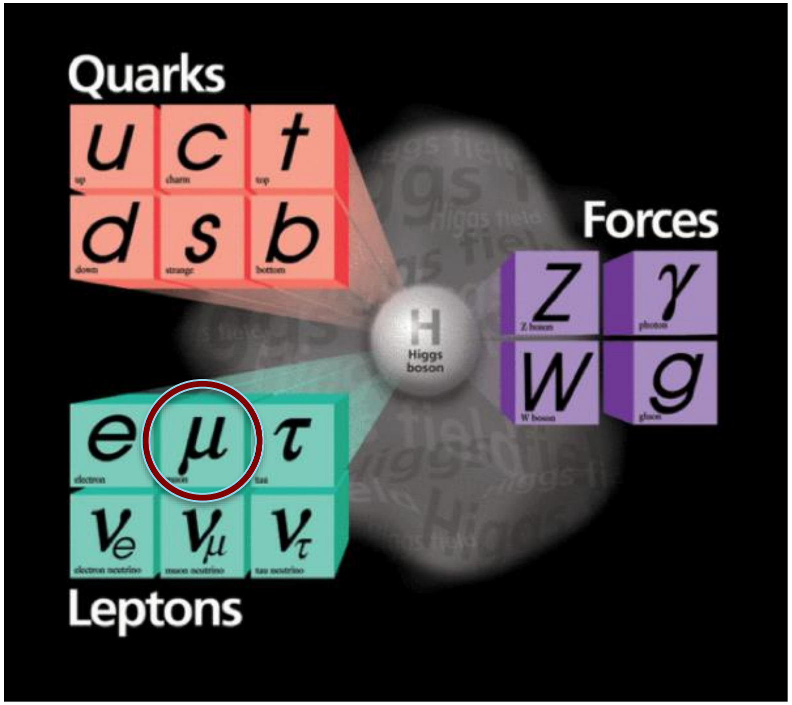
A trail in the Wind River Range in western Wyoming.

Slide from first FNAL colloquium on g-2

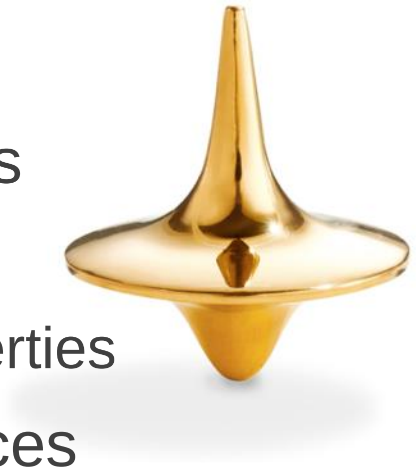
Muon Campus today

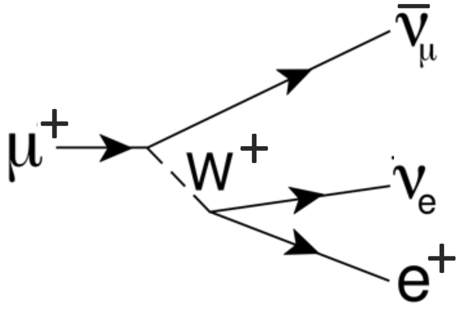
Muons in the Standard Model

Fundamental building blocks of the Standard Model



- Similar to electrons
  - Same charge
  - Same spin properties
- Important differences
  - 200x more massive
  - Unstable, live ~2 millionths of a second before they decay



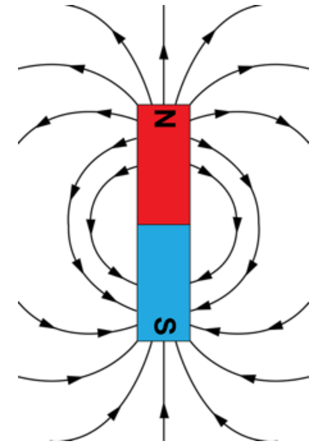


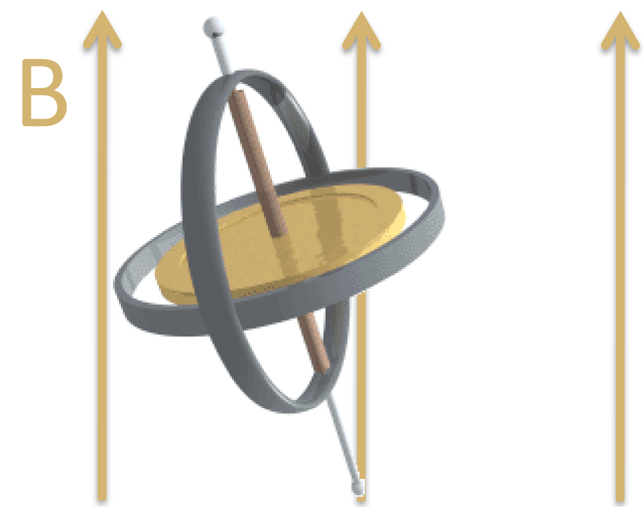
Muon g-2 measures the muon's magnetic moment

- Because of their spin & charge, muon's act like little bar magnets and have a magnetic moment, **μ**
- Like a bar magnet, they feel a torque when placed in a magnetic field

$$\vec{\tau} = \vec{\mu} \times \vec{B}, \quad U = -\vec{\mu} \cdot \vec{B}$$

- That torque causes the muon spin to precess around the magnetic field at a rate that increases or decreases depending on the strength of μ & B





$$\vec{\mu} = g \frac{e}{2m} \vec{S}$$

The g-factor

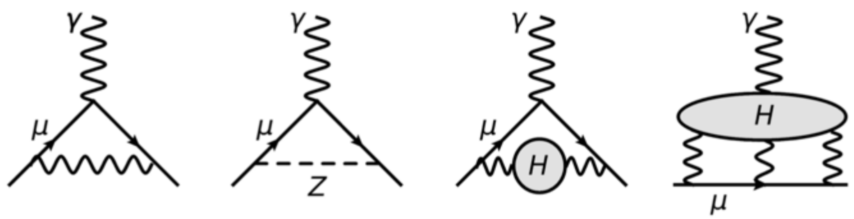
- The strength of the magnetic moment can be written in terms of fundamental constants and an overall coefficient called the g-factor

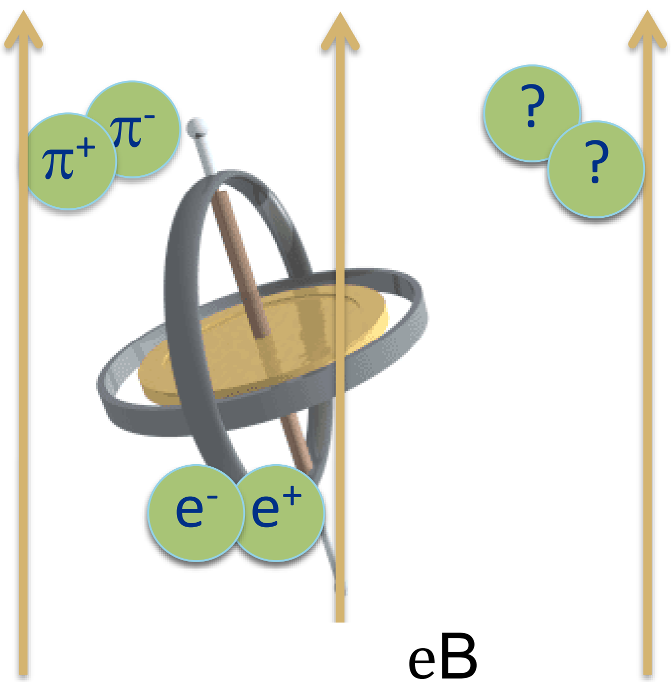
$$\vec{\mu} = g\frac{e}{2m}\vec{S}$$

- g = 1
  - This was the classical expectation around 1900
- g = 2
  - Folding in relativistic quantum mechanics, the expectation was shown to be 2 by Thomas and predicted by Dirac's wave equation
- As you can guess from the experiment name, Muon g-2, there is more to the story...

The anomalous magnetic moment, a_μ

- Particles are never truly alone, constantly surrounded by an entourage of other particles blinking in and out of existence
- What particles? All of them!





- The anomalous magnetic moment, a_μ, is the interesting part

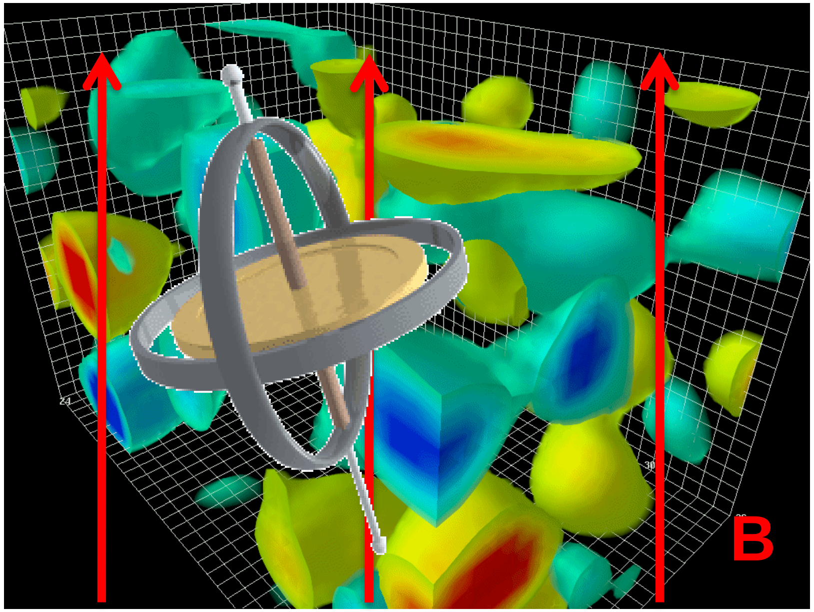
$$a_\mu = \frac{g-2}{2}$$

$$\omega_s = g\frac{eB}{2mc}$$

- m^2 scaling → 40,000x sensitivity compared to the e-

New physics search

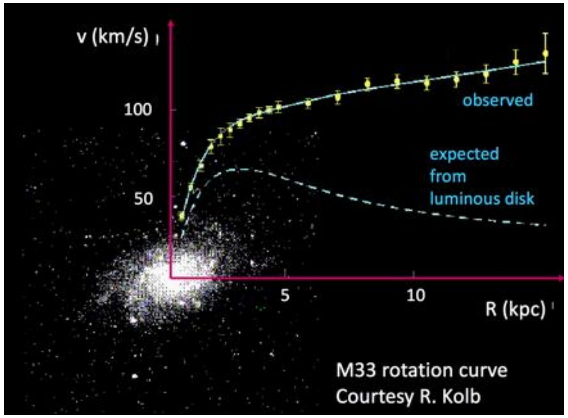
Image Credits: Derek Leinweber



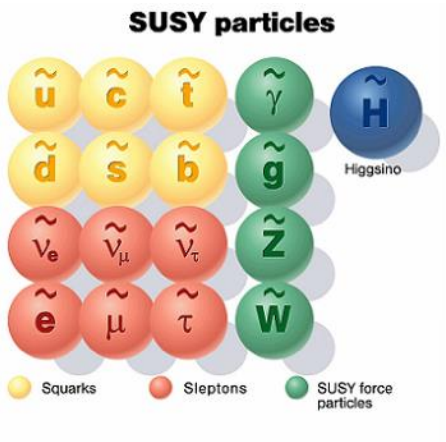
- Measuring the precession tells us the muon magnetic moment
- The high precision allows us to ‘see’ if new particles or forces are contributing to the anomaly!

$$a_\mu = \frac{g-2}{2}$$

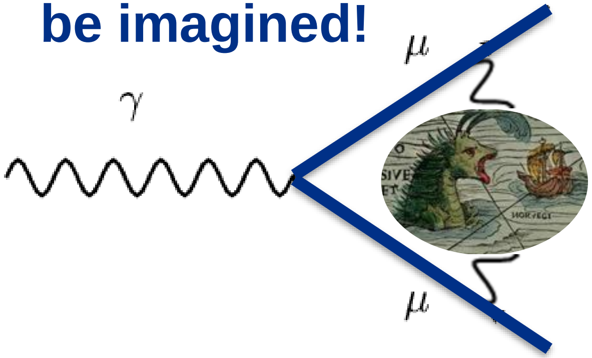
Dark matter!



SUSY!



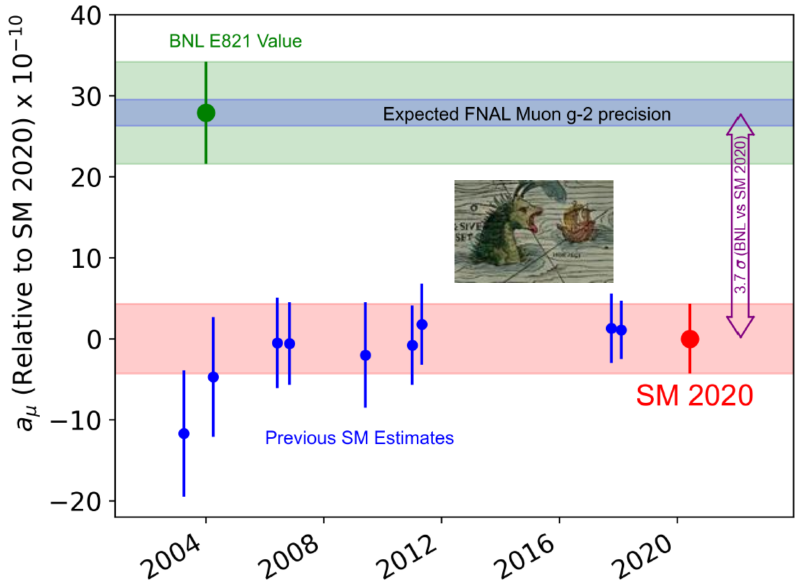
Monsters yet to be imagined!

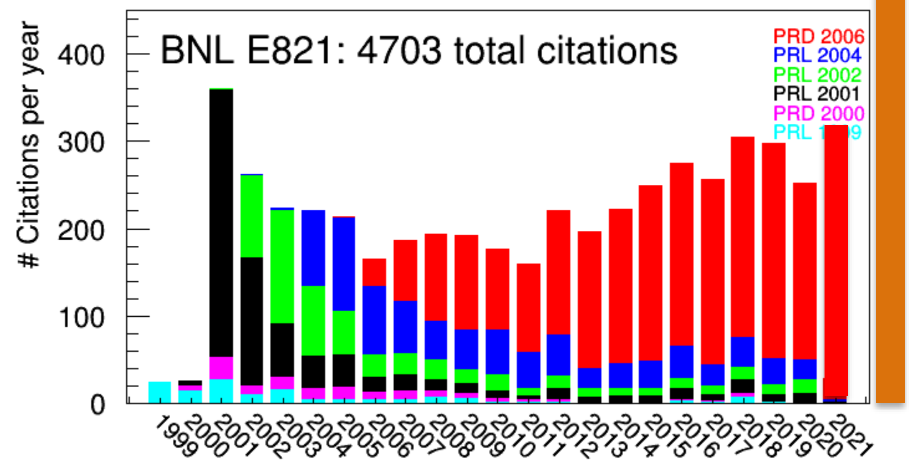


A hint of new physics from BNL

- a_μ last measured 20 years ago at Brookhaven National Lab (BNL) where an interesting 2.7σ hint of new physics was discovered
  - Over time it grew to 3.7σ with improvements in theory

$$a_\mu = \frac{g-2}{2}$$



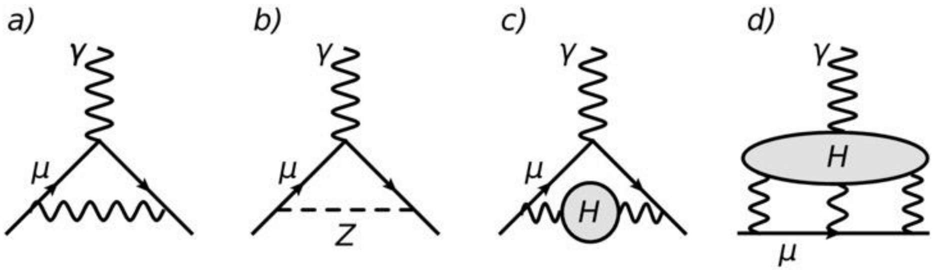


698 FNAL PRL

- The difference has intrigued physicists for years
  - Difference is ~270 x 10^{-11} in a_μ

Fermilab

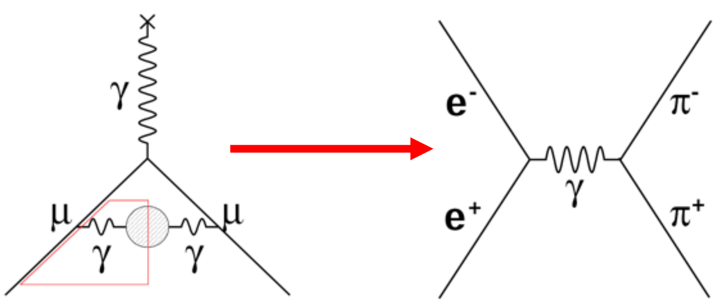
Theoretical status of a_μ



Source	Value ($a_\mu \times 10^{-11}$)	Error
a) QED	116 584 718.9	0.1
b) EW	154	1
c) HVP	6845	40
d) HLBL	92	18

Muon g-2 Theory Initiative arXiv:2006.04822

- QED and EW are extremely well known
- Hadronic terms are more difficult due to non-perturbative nature of QCD
  - HVP can be determined from $e^+e^- \rightarrow$ hadrons data
  - Lattice calculations starting to reach required precision



$$a_\mu^{had,1} \propto \int_{2m_\pi}^{\infty} ds \frac{K(s)}{s} R(s)$$

$$R(s) = \frac{\sigma(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \text{muons})}$$

Bringing g-2 to Fermilab

- Goal: Bring the container used to hold the muons from BNL and couple it to Fermilab's powerful accelerator beam

- Reduce the overall error by a factor of 4 to 140 ppb
  - 20x the muons → 100 ppb stat error from (4.5x better)
  - systematics at the same 100 ppb level (3x better)

Brookhaven Muon Storage Ring

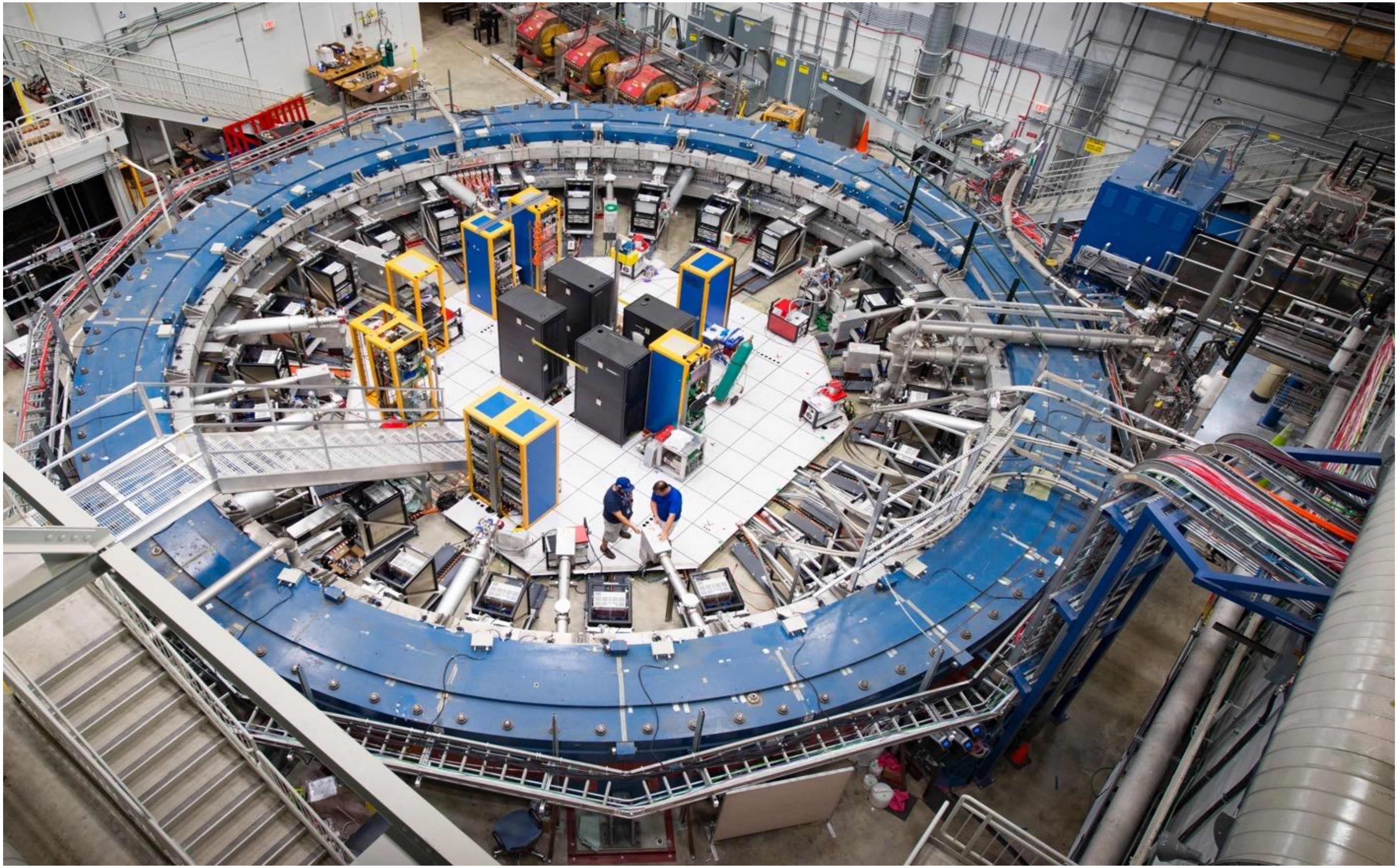
Parts of the 50' diameter storage ring could not come apart!!

Storage ring transported by land/sea in 2013

Including 30 miles of Chicago suburbs!

Amazing photo ops

All put back together at Fermilab!



Why use a storage ring?

- The rate that the muon spin rotates, ω_s, with respect to the cyclotron frequency, ω_c, is given by

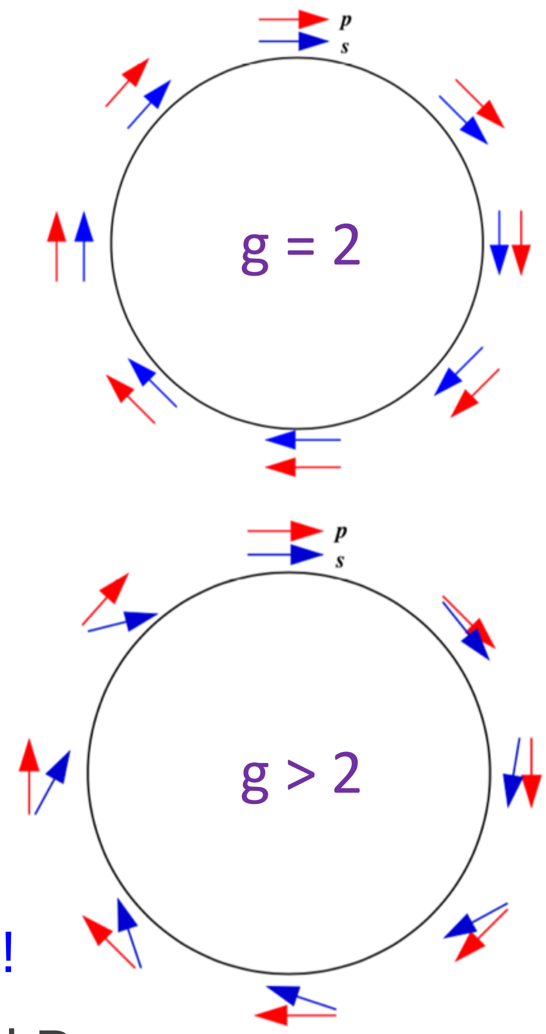
$$\vec{\omega}_a = \vec{\omega}_s - \vec{\omega}_c = -\left(\frac{g_\mu - 2}{2}\right)\frac{q\vec{B}}{m} = -a_\mu \frac{q\vec{B}}{m}$$

- If g = 2 exactly, the spin and momentum vectors remain locked together → $\omega_a = 0$
  - But g = 2.0023... & (g-2)/2 = 0.0023...
- ω_a is directly proportional to a_μ

  → 800x more sensitive than expt at rest!

- To extract a_μ, we need to determine ω_a and B

$$a_\mu = \left(\frac{e}{m}\right)^{-1}\frac{\omega_a}{B}$$



Not quite as simple as $a_\mu = \left(\frac{e}{m}\right)^{-1} \frac{\omega_a}{B}$

- Full BMT equation → spin precession modified by E-fields and non-perpendicular motion relative to B

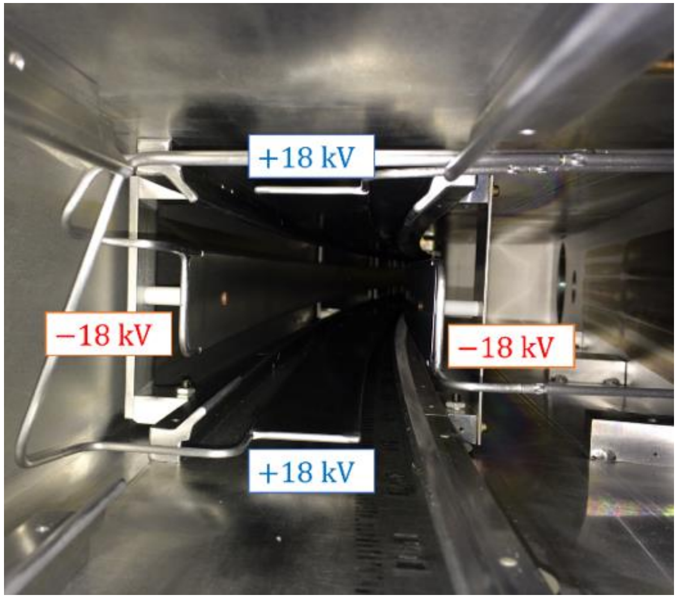
Electric field correction

$$\vec{\omega}_a \equiv \vec{\omega}_s - \vec{\omega}_c = -\frac{q}{m_\mu}\left[a_\mu \vec{B} - a_\mu \left(\frac{\gamma}{\gamma+1}\right)(\vec{\beta}\cdot\vec{B})\vec{\beta} - \left(a_\mu - \frac{1}{\gamma^2-1}\right)\frac{\vec{\beta}\times\vec{E}}{c}\right]$$

Pitch correction

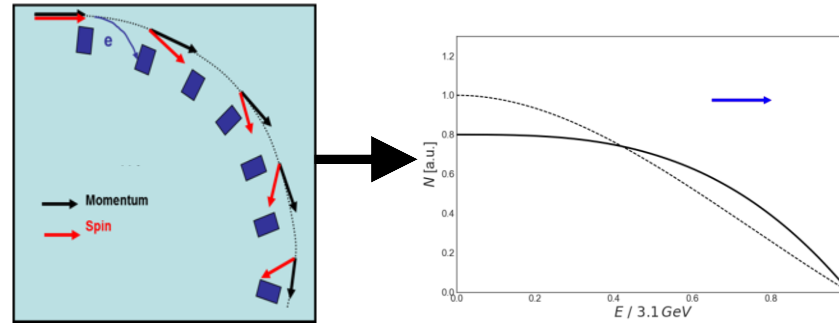
0, for γ = 29.3, v = 99.94c

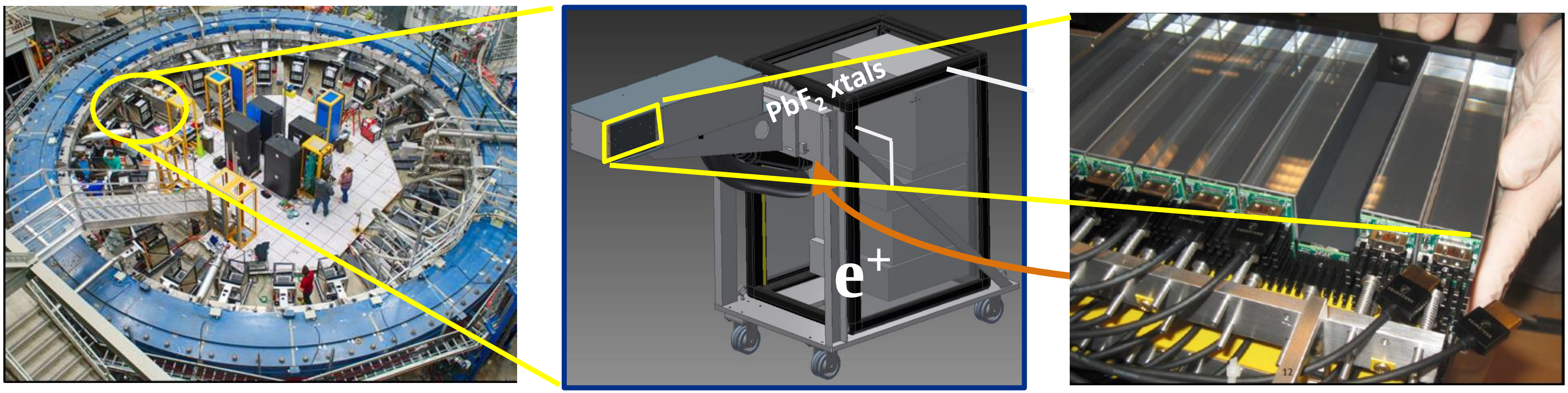
- Experiment requires a quadrupole E-field to keep muon vertically confined → horizontal and vertical harmonic coherent betatron oscillation (CBO)
- Choosing to run at the 'magic momentum' minimizes impact of E-field



How do we measure a_μ?

- Parity violation in muon decay → high energy decay positrons are preferentially emitted in the muon spin direction
- Measure the energy spectrum with detectors around the inside of the ring

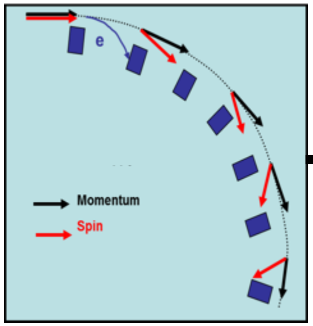


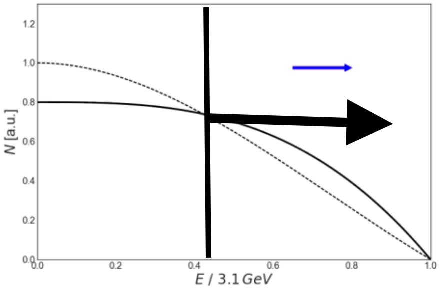


- Major upgrades from BNL:
  - 6x9 array to PbF2 crystals allows us to spatially separate pileup with better Cerenkov timing relative to the PbSciFi monolithic BNL calorimeters
  - 800 MHz waveform digitizers sample at twice the rate
  - Modern computing bandwidth allows us to keep data down to 0 threshold (1 GeV at BNL)
  - Sophisticated laser systems allows us to monitors gain changes at 1 part in 10^4

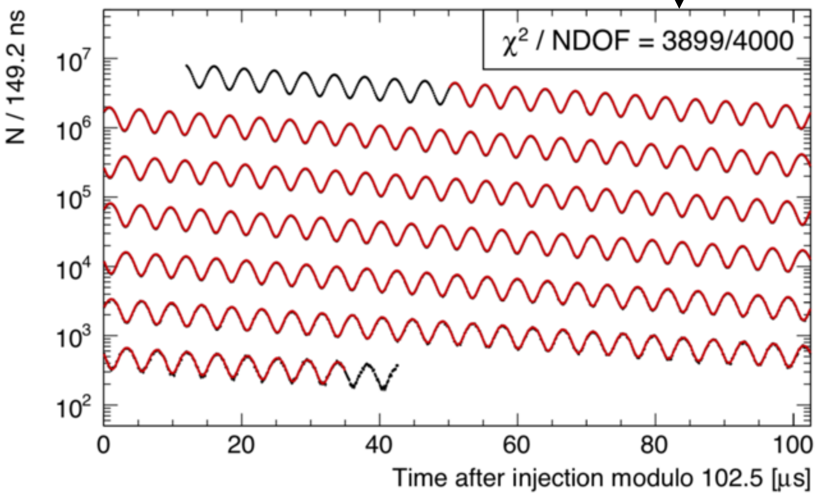
Fermilab

Generating the 'wiggle plot'





"g-2 wiggle plot"



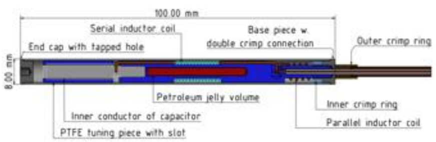
$$f(t) \simeq N_0 e^{-\lambda t}[1 + A\cos(\omega_a t + \phi)]$$

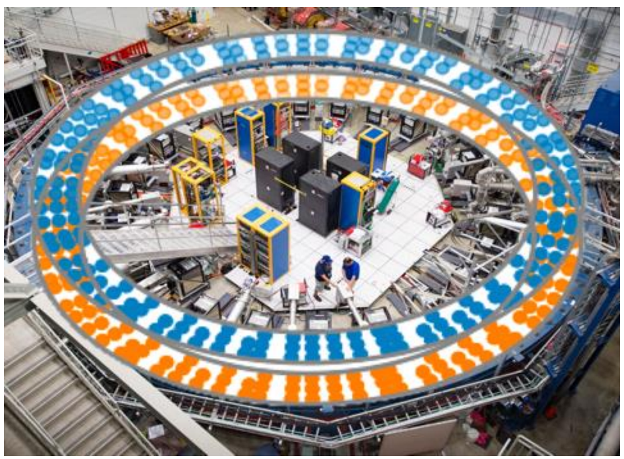
We also need B at < 100 ppb to determine a_μ

$$\omega_a = \omega_s - \omega_c = a_\mu \frac{eB}{mc}$$

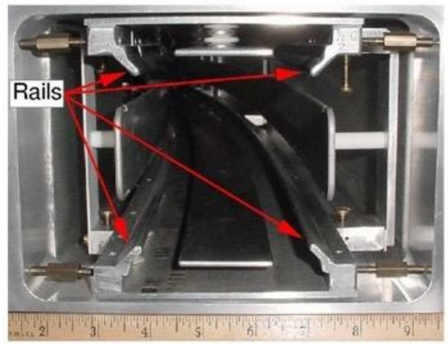
- Use NMR to find B-field in terms of proton precession frequency ω_p (comagnetometer)

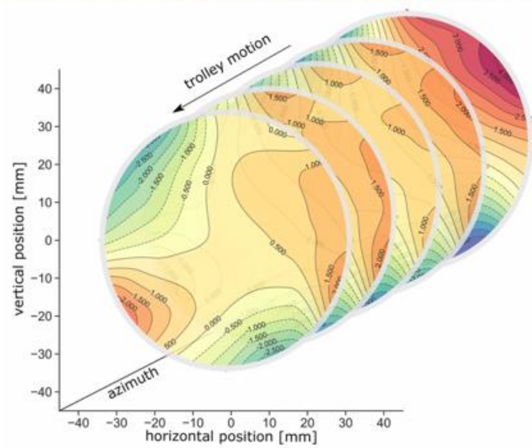
378 fixed probes monitor 24/7



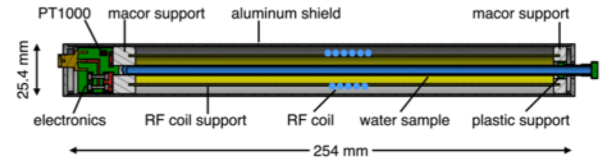


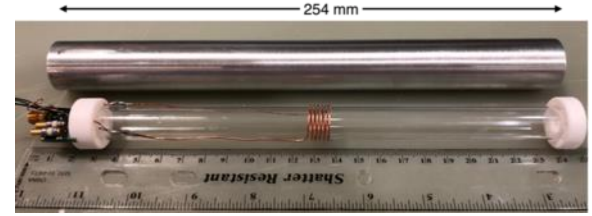
NMR trolley maps field every 3 days





Trolley cross-calibrated to absolute probes



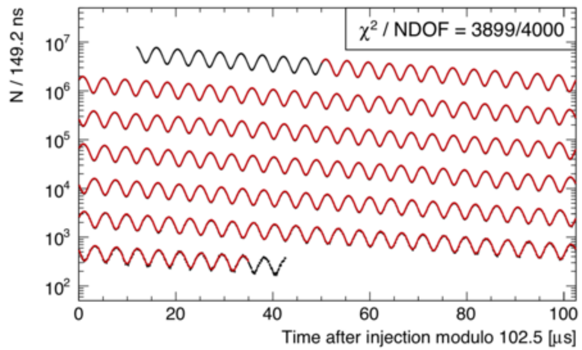


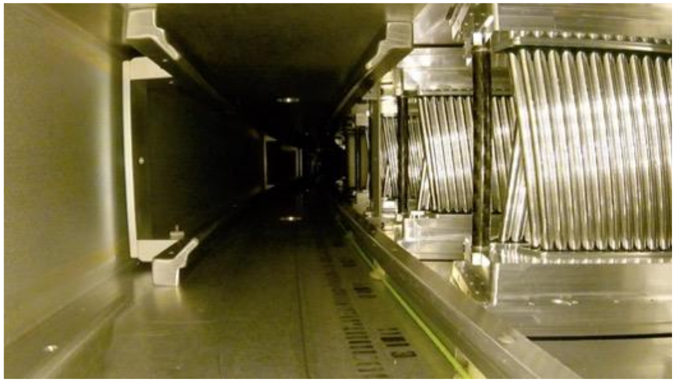
Absolute probes all cross-calibrated at ANL test magnet

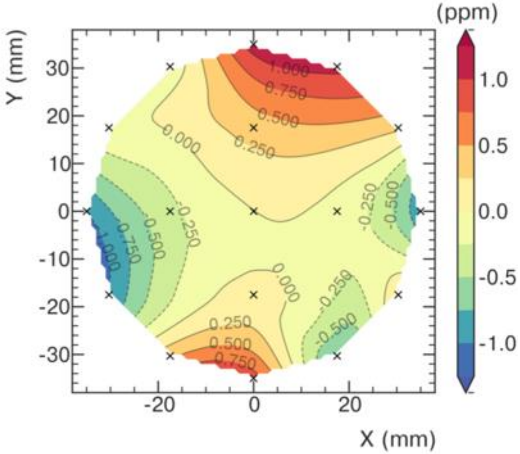
- Every field system upgraded, shimmed field 3x better, hall temp to +/- 1C

The analysis ‘big’ picture

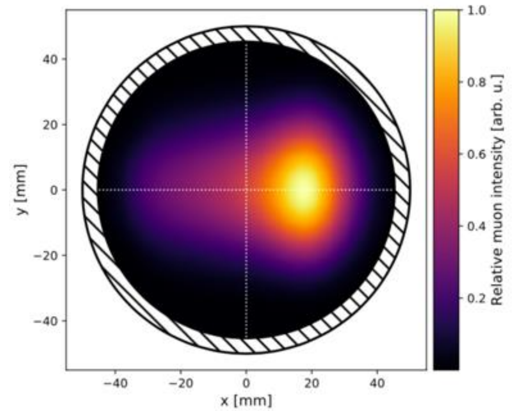
$$\frac{\omega_a}{\omega_p \otimes \rho(r)} \Rightarrow$$







$\otimes$



*All plots actual Run 1 data

- *In vacuo* straw trackers tell us the spatial distribution and many other muon beam properties (CBO, p-dist)
  - Also a major addition compared with BNL

Fermilab

Systematic error progress - ω_a

Run 1 stat error 434 ppb

	BNL actual [ppb]	FNAL TDR [ppb]	FNAL Run 1 [ppb]
Gain + residuals	120	20	19
Pileup	80	40	37
Lost muons	90	20	5
CBO	70	30	40
E-field/pitch	50	30	55
Phase acceptance	N/A	N/A	75
Total	180	70	108

*Run 1 ω_a systematics are simple averages over 4 data sets, correlations approximate, BNL ←→ FNAL mapping not perfect but close enough

- Run 1 only 6% total statistics, many systematics errors scale down with stats
- CBO driven by increased amplitude due to poor kick in Run 1&2, reduced x2 with kicker upgrades by Run 3
- E-field/pitch driven by impact of time/momentum correlations of the muon bunch at injection, will improve with better simulation and measurements
- Phase acceptance primarily due to failed quad resistors that led to beam instability, fixed in Run 2 and beyond
- On track to beat 70 ppb goal from Run 2 and beyond!

Systematic error progress - B

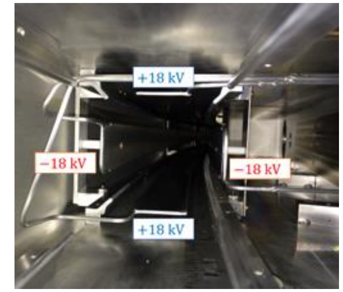
Run 1 stat error 434 ppb

	BNL actual [ppb]	FNAL TDR [ppb]	FNAL Run 1 [ppb]
Trolley calibration	90	30	32
Trolley B measurements	50	30	25
Fixed probe tracking	70	30	23
Muon weighting	30	10	20
Absolute calibration	50	35	19
Configuration	Under other	Under other	23
Kicker transients	Under other	Under other	37
Quad transients	Under other	Under other	92
Other	100	50	negligible
Total	170	70	114

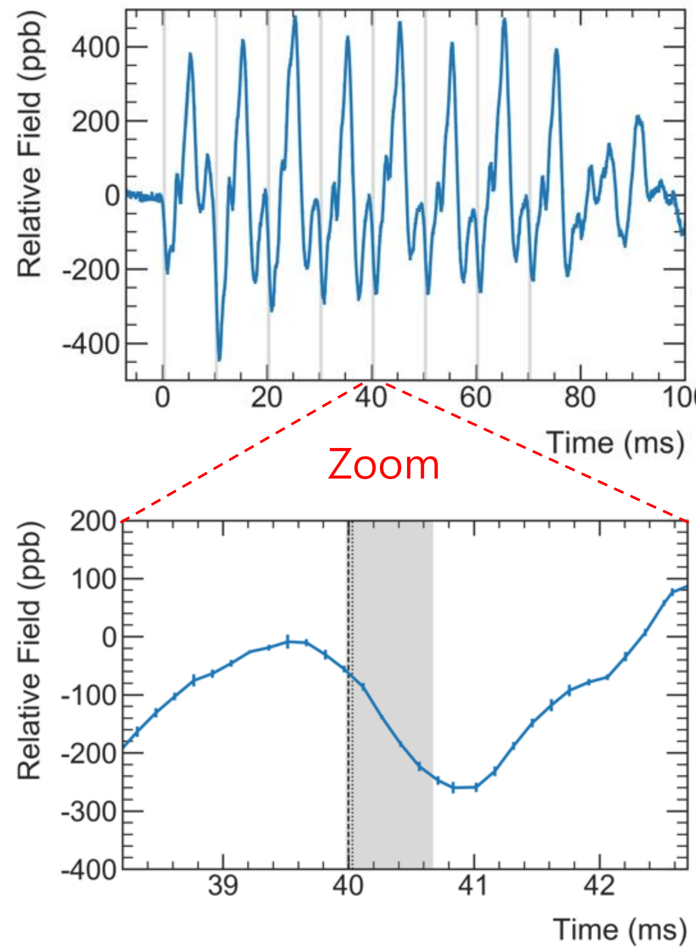
*BNL ←→ FNAL mapping not perfect but close enough

- Trolley calibration improves with more calibrations, trolley NMR sample temperature dependence better determined
- Muon weighting will improve due to better centered beam (kicker upgrade)
- Kicker/quad transients reduced to < 30 ppb with better mapping for Run 2 and beyond
- On track to beat 70 ppb goal from Run 2 and beyond!

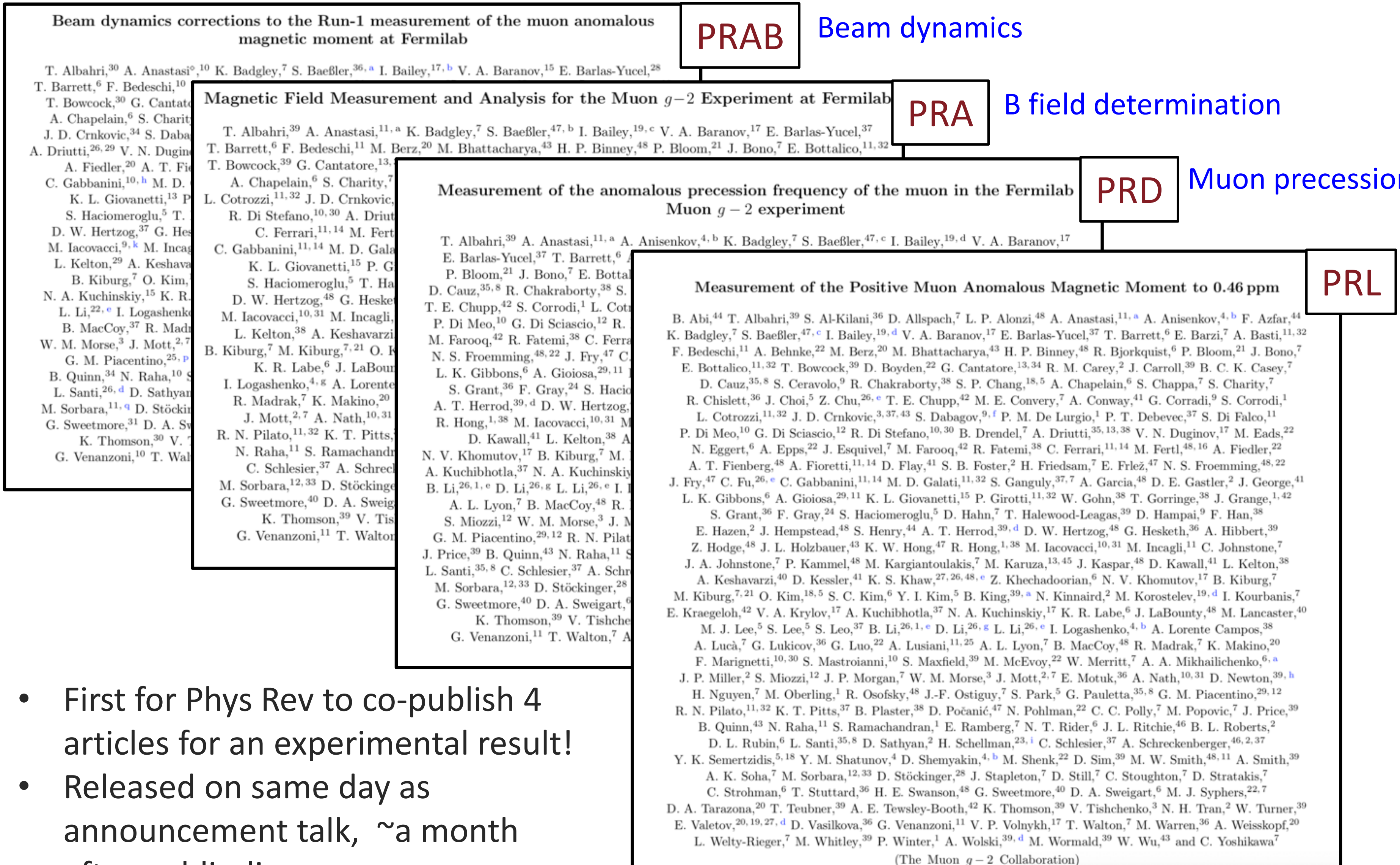
B_q – Quad transients



- Recall, E- field keeps muons vertically confined
- Quads pulsed → induces mech. vibrations → oscillating conductor perturbs B field
  - Deliver 8 muon bunches with 10 ms spacing → 3x closer to 100 Hz natural resonance than BNL
- Built special NMR probes to map the effect
  - Long process to make measurements
- Overall correction is 17 ppb
  - Only matters in window when muons are present, averaged over 8 bunches, averaged over 43% of ring with quad coverage
- 92 ppb Run 1 uncertainty is dominated by not having a complete map for Run 1
  - Analysis of more complete map is nearly done
  - Expect uncertainty to be reduced x3 for Run 2 and beyond



Four articles on arXiv and published in Phys Rev

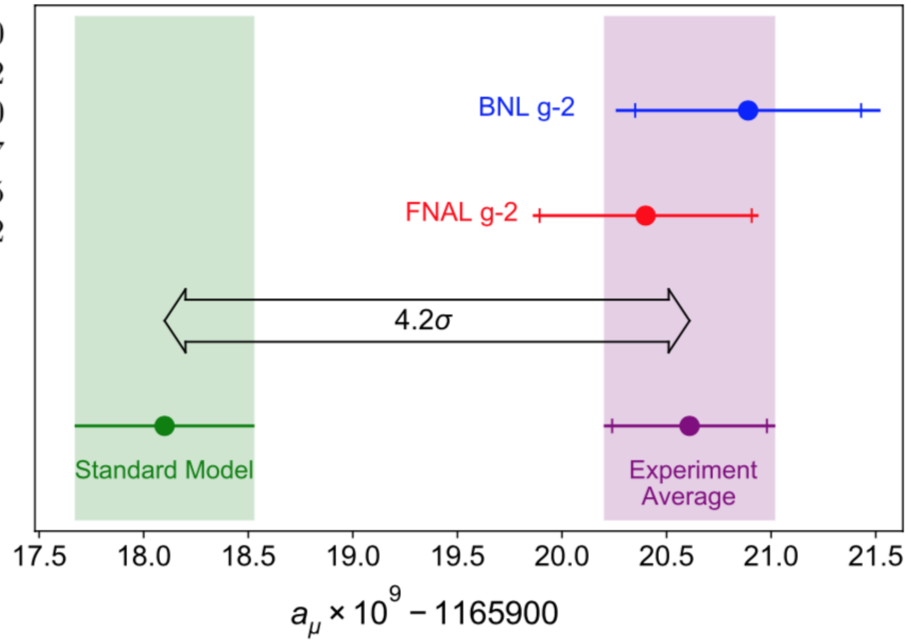


- First for Phys Rev to co-publish 4 articles for an experimental result!
- Released on same day as announcement talk, ~a month after unblinding

Final results from Run 1

Quantity	Correction Terms (ppb)	Uncertainty (ppb)
ω_a^m (statistical)	–	434
ω_a^m (systematic)	–	56
C_e	489	53
C_p	180	13
C_{ml}	-11	5
C_{pa}	-158	75
$f_{\text{calib}}\langle\omega_p(x,y,\phi)\times M(x,y,\phi)\rangle$	–	56
B_k	-27	37
B_q	-17	92
$\mu_p'(34.7^\circ)/\mu_e$	–	10
m_μ/m_e	–	22
$g_e/2$	–	0
Total systematic	–	157
Total fundamental factors	–	25
Totals	544	462

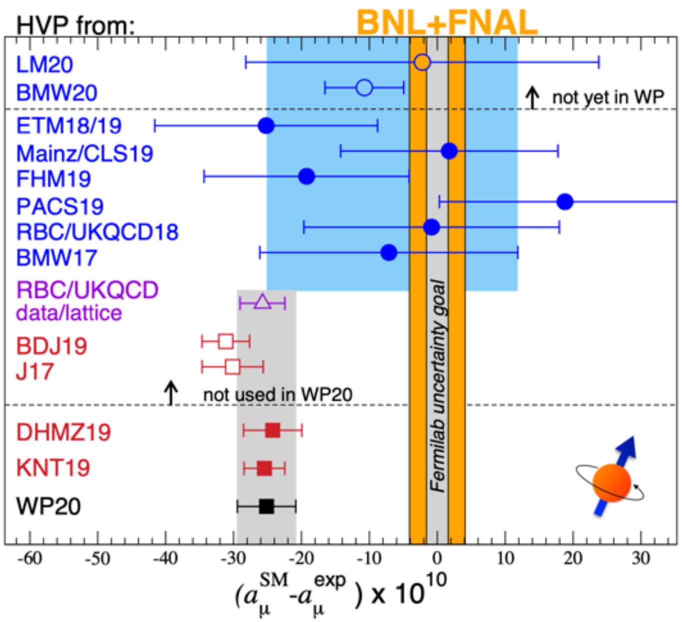
- 462 ppb overall error
  - 434 ppb statistical
  - 157 ppb systematic
  - 25 ppb CODATA inputs
- Good agreement with BNL
- Combined tension with SM increases to 4.2σ



Conclusions

- We have determined a_μ to an unprecedented 460 ppb precision!

$$a_\mu(\text{FNAL}) = 116\,592\,040(54) \times 10^{-11}$$



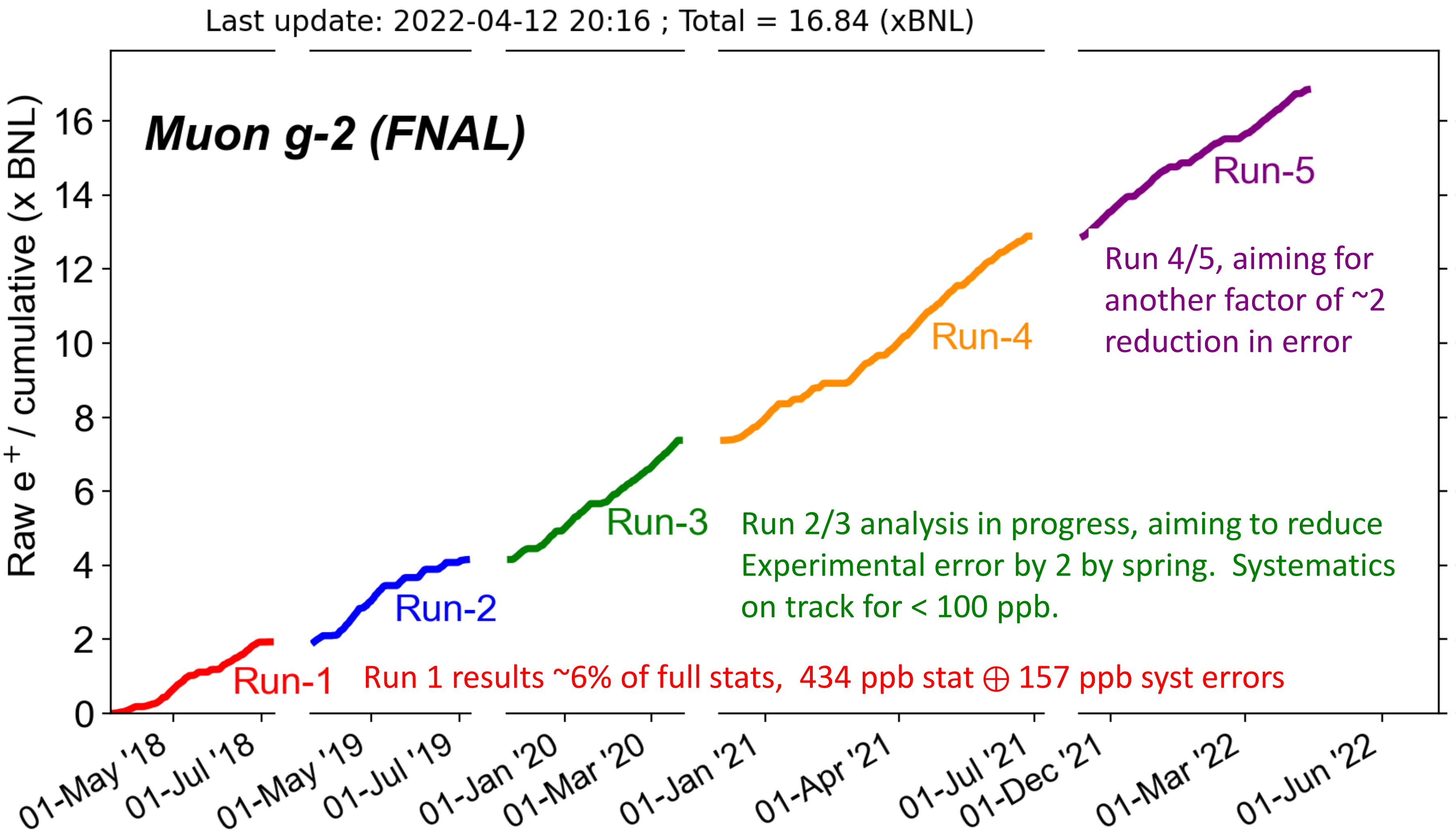
- The Run 1 result
  - 6% of ultimate data sample
  - 15% smaller error than BNL
  - 3.3σ tension with e+e- SM

- After 20 years, we confirm the BNL experimental results!

- Combining BNL/FNAL and comparing to e+e- based theory recommended by the Theory Initiative → 4.2σ tension with the SM
  - Lattice QCD (blue band) are becoming competitive, particularly BMW20, and indicate quark contributions might be larger (stay tuned)

Outlook

Much more data to come!



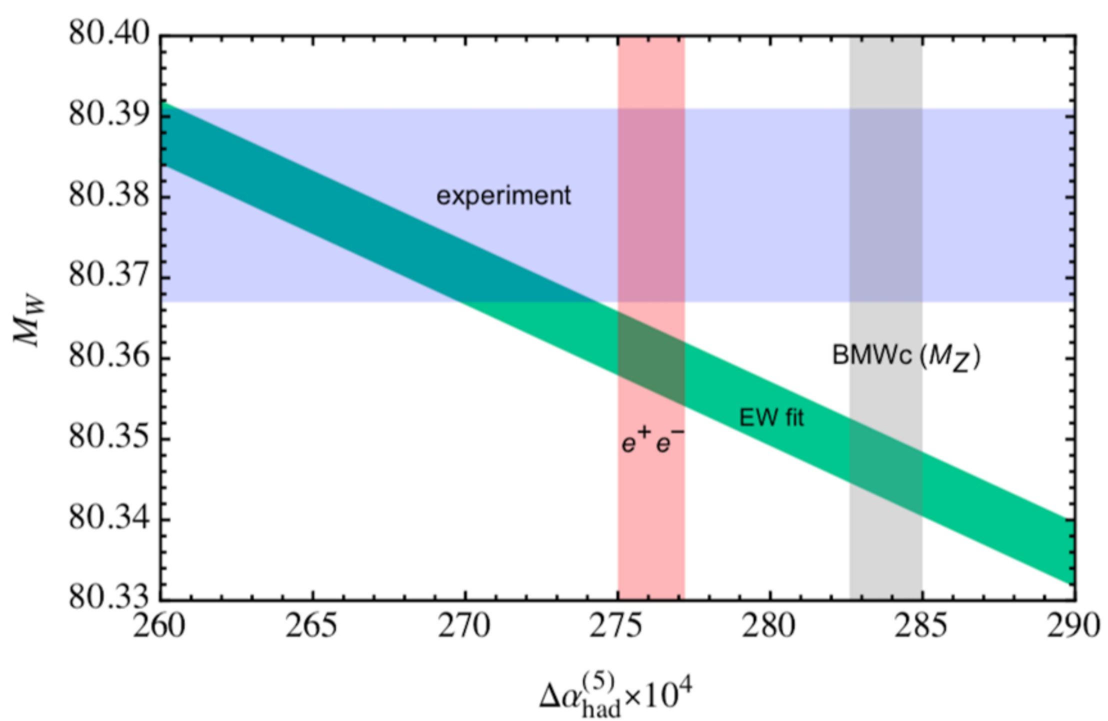
- Switching to μ- next year and aiming for 4x BNL μ- stats in Run 6

(The Muon $g-2$ Collaboration)
[1]*Argonne National Laboratory, Lemont, IL, USA*
[2]*Boston University, Boston, MA, USA*
[3]*Brookhaven National Laboratory, Upton, NY, USA*
[4]*Budker Institute of Nuclear Physics, Novosibirsk, Russia*
[5]*Center for Axion and Precision Physics (CAPP) / Institute for Basic Science (IBS), Daejeon, Republic of Korea*
[6]*Cornell University, Ithaca, NY, USA*
[7]*Fermi National Accelerator Laboratory, Batavia, IL, USA*
[8]*INFN Gruppo Collegato di Udine, Sezione di Trieste, Udine, Italy*
[9]*INFN, Laboratori Nazionali di Frascati, Frascati, Italy*
[10]*INFN, Sezione di Napoli, Napoli, Italy*
[11]*INFN, Sezione di Pisa, Pisa, Italy*
[12]*INFN, Sezione di Roma Tor Vergata, Roma, Italy*
[13]*INFN, Sezione di Trieste, Trieste, Italy*
[14]*Istituto Nazionale di Ottica - Consiglio Nazionale delle Ricerche, Pisa, Italy*
[15]*Department of Physics and Astronomy, James Madison University, Harrisonburg, VA, USA*
[16]*Institute of Physics and Cluster of Excellence PRISMA+, Johannes Gutenberg University Mainz, Mainz, Germany*
[17]*Joint Institute for Nuclear Research, Dubna, Russia*
[18]*Department of Physics, Korea Advanced Institute of Science and Technology (KAIST), Daejeon, Republic of*
[19]*Lancaster University, Lancaster, United Kingdom*
[20]*Michigan State University, East Lansing, MI, USA*
[21]*North Central College, Naperville, IL, USA*
[22]*Northern Illinois University, DeKalb, IL, USA*
[23]*Northwestern University, Evanston, IL, USA*
[24]*Regis University, Denver, CO, USA*
[25]*Scuola Normale Superiore, Pisa, Italy*
[26]*School of Physics and Astronomy, Shanghai Jiao Tong University, Shanghai, China*
[27]*Tsung-Dao Lee Institute, Shanghai Jiao Tong University, Shanghai, China*
[28]*Institut fr Kern - und Teilchenphysik, Technische Universität Dresden, Dresden, Germany*
[29]*Università del Molise, Campobasso, Italy*
[30]*Università di Cassino e del Lazio Meridionale, Cassino, Italy*
[31]*Università di Napoli, Napoli, Italy*
[32]*Università di Pisa, Pisa, Italy*
[33]*Università di Roma Tor Vergata, Rome, Italy*
[34]*Università di Trieste, Trieste, Italy*
[35]*Università di Udine, Udine, Italy*
[36]*Department of Physics and Astronomy, University College London, London, United Kingdom*
[37]*University of Illinois at Urbana-Champaign, Urbana, IL, USA*
[38]*University of Kentucky, Lexington, KY, USA*
[39]*University of Liverpool, Liverpool, United Kingdom*
[40]*Department of Physics and Astronomy, University of Manchester, Manchester, United Kingdom*
[41]*Department of Physics, University of Massachusetts, Amherst, MA, USA*
[42]*University of Michigan, Ann Arbor, MI, USA*
[43]*University of Mississippi, University, MS, USA*
[44]*University of Oxford, Oxford, United Kingdom*
[45]*University of Rijeka, Rijeka, Croatia*
[46]*Department of Physics, University of Texas at Austin, Austin, TX, USA*
[47]*University of Virginia, Charlottesville, VA, USA*
[48]*University of Washington, Seattle, WA, USA*

Thank you!

Interpretation: Implication for Precision EW fits?

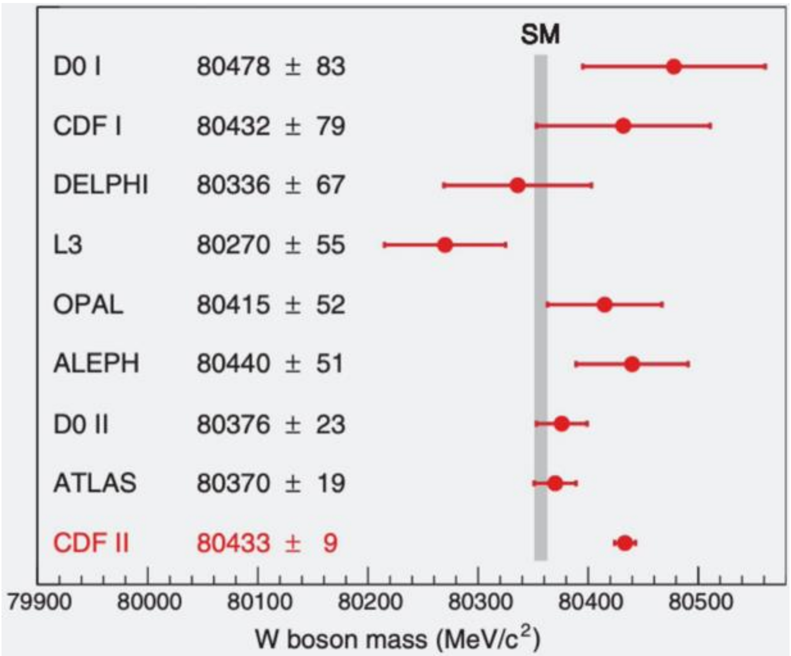
- Increasing the cross-section for e+e- → hadrons also increases the strength of the fine structure constant
- Creates additional tension in the precision EW fits



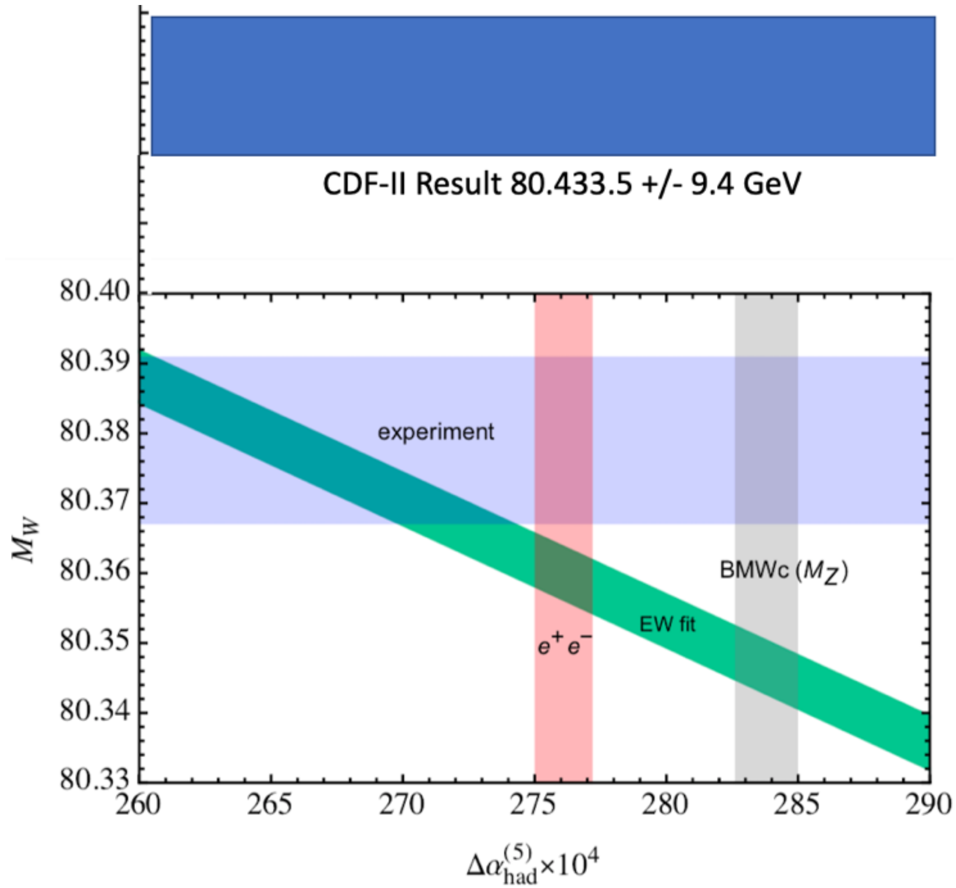
A. Crivellin, et al. (link)

Fermilab

Interpretation: Relationship to M_W

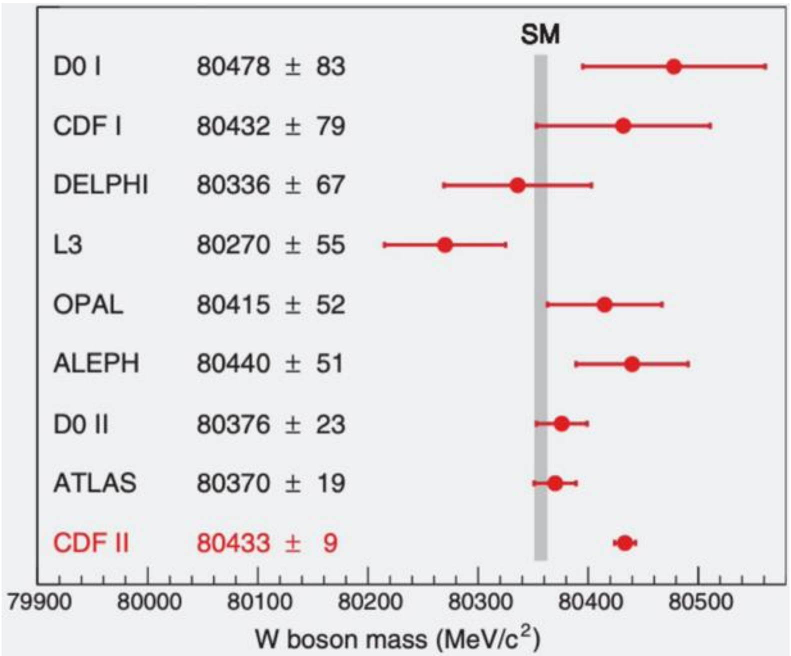


- CDF-II collaboration just published a result finding a much larger M_W
- Tension is much larger

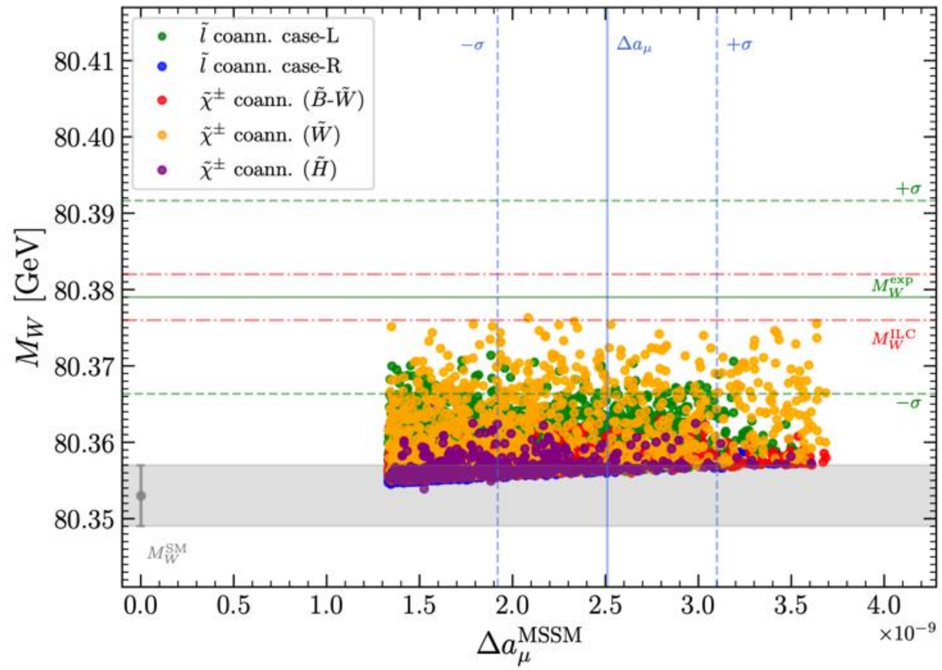


Fermilab

Interpretation: Relationship to M_W

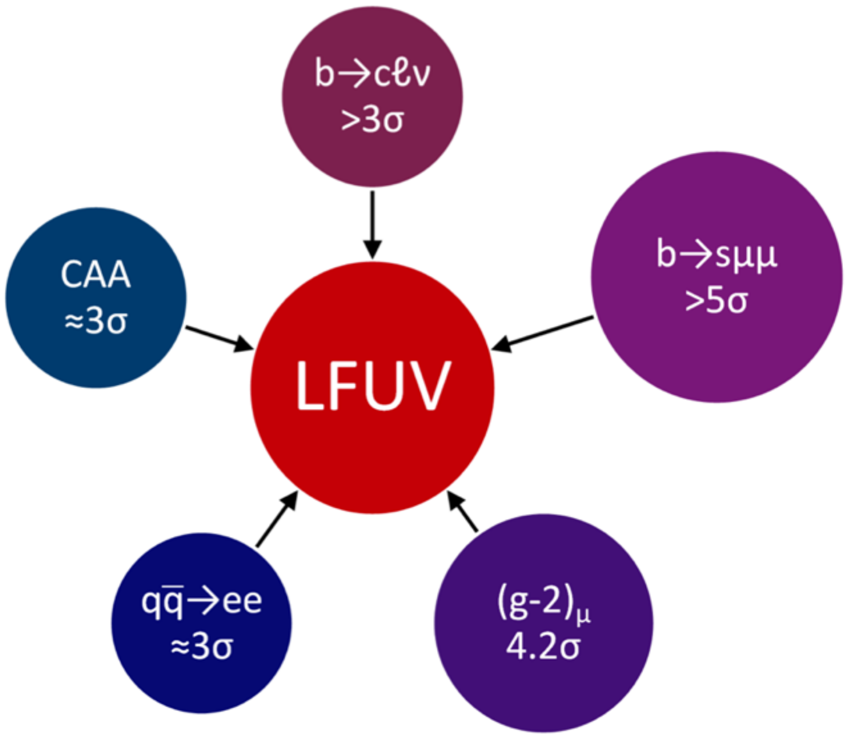


E. Bagnaschi, et al. (arXiv:2203.15710)



- Still need to understand why the result is so much higher than others
- General feature that supersymmetric models predict larger than SM values for M_W and a_μ

Interpretation: Are we seeing evidence that lepton universality is violated?



Mounting Evidence for the Violation of Lepton Flavor Universality
https://arxiv.org/pdf/2111.12739.pdf
(A. Crivellin, M. Hoferichter)

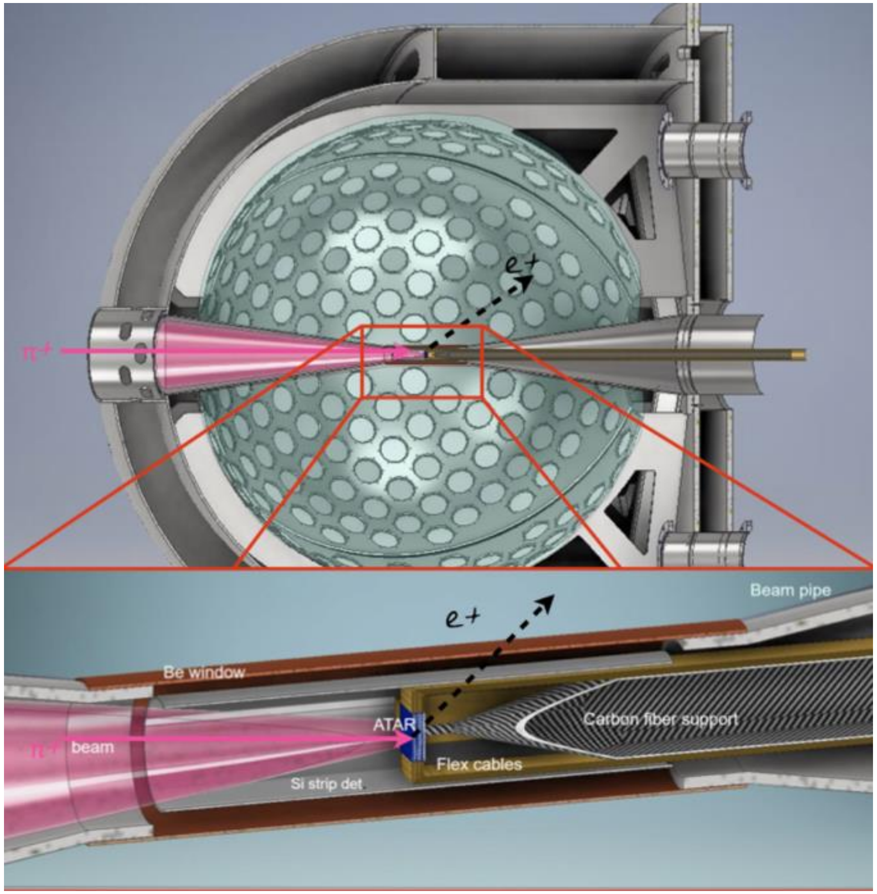
- Quite a few measurements that include leptons in the final state are starting to show tension with SM predictions
- Many of these have avenues for continued improvement
- New efforts to test lepton universality being proposed

PIONEER Experiment

- Primary goal is to improve $R_{e/\mu}$, the charged pion branching ratio to electrons vs muons, by an order of magnitude
  - $R_{e/\mu}$ thy uncertainty ~15x smaller than current exp (PIENU)
- Secondary goal to study pion beta decay

$$\pi^+ \rightarrow \pi^0 e^+ \nu(\gamma)$$

  and improve V_{ud} by an order of magnitude for theoretically clean CKM unitarity test
- Recently rate a high priority by the PSI PAC



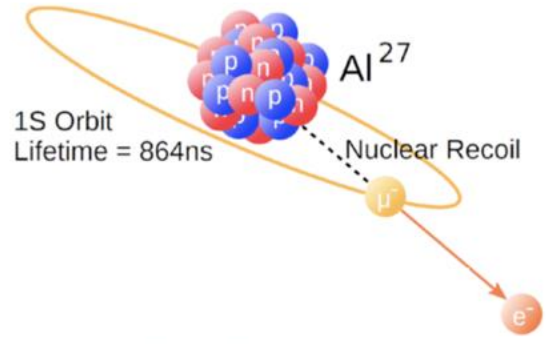
PIONEER PSI Proposal (arXiv:2203.01981)
PIONEER Snowmass (arXiv:2203.05505)

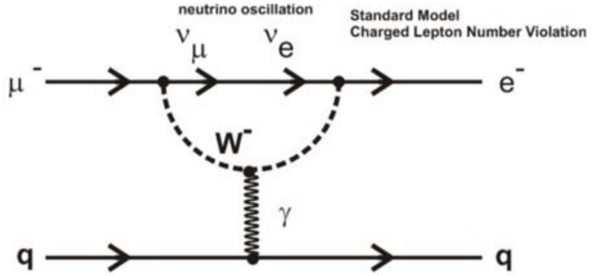
Mu2e Experiment

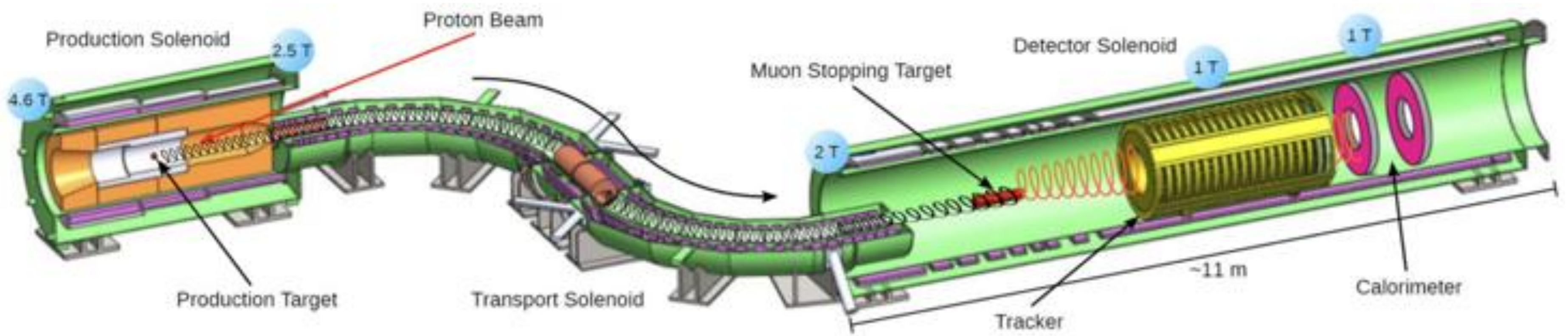
- Mu2e is searching for muons spontaneously converting to electrons in the field of a nucleus

$$R_{\mu e} = \frac{\mu^- + A(Z,N) \to e^- + A(Z,N)}{\mu^- + A(Z,N) \to \nu_\mu + A(Z-1,N)}$$

- Goal of the experiment is to reach a sensitivity to branching ratios of 3×10^{-17}
  - 4 order of magnitude improvement over last experiment (SINDRUM II)
- Any signal is unambiguously due to new physics
  - SM contribution enters at below < 10^{-50} level





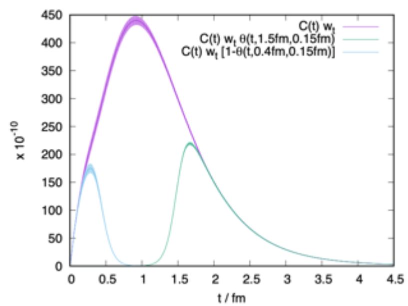


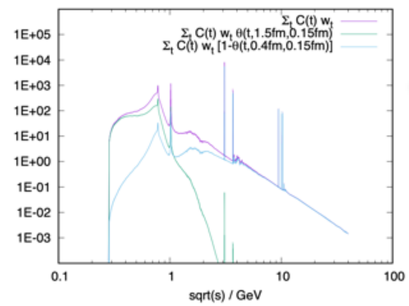
Hybrid approach data/lattice

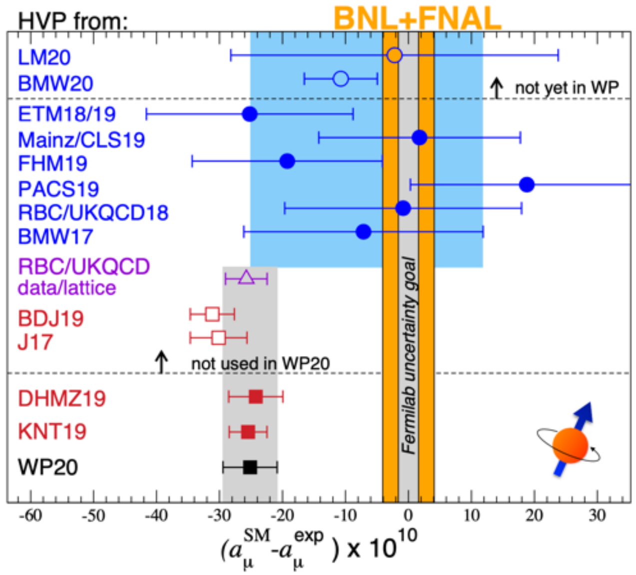
- Can take the e+e- → hadrons data and convert it from Minkowski into Euclidean space to directly compare to the lattice calculation

RBC-UKQCD PRL 121, 022003 (2018)

$$a_\mu^{\mathrm{HVP\ LO}} = \sum_{t=0}^{+\infty} w(t)C(t) \qquad w(t) = w^{\mathrm{SD}}(t) + w^{\mathrm{W}}(t) + w^{\mathrm{LD}}(t)$$







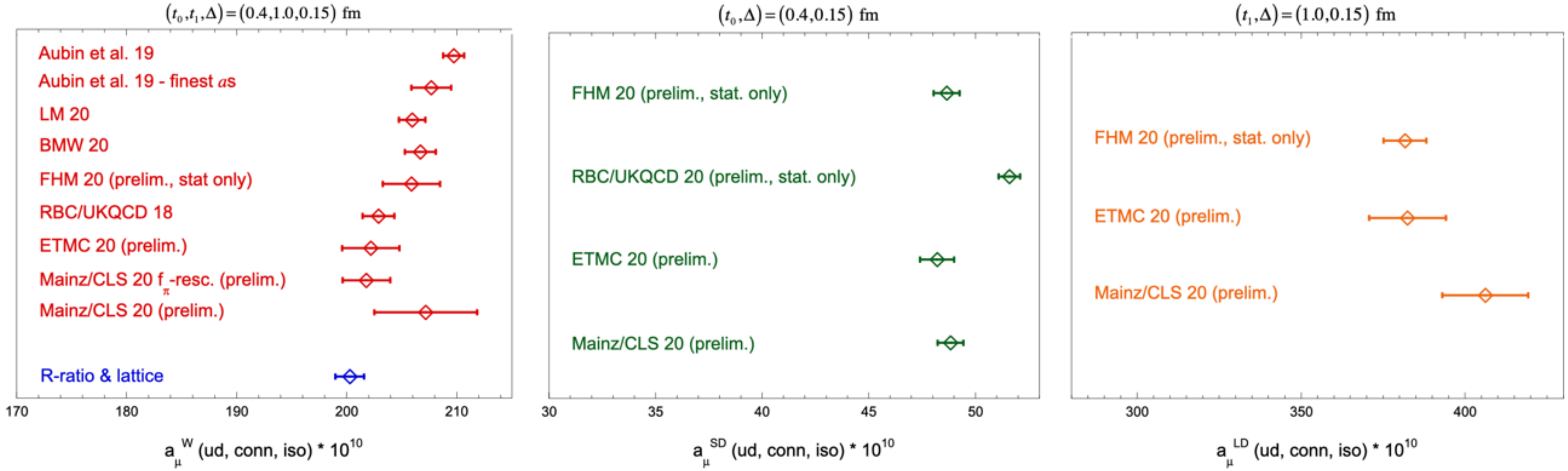
- Very short and long distances are where the lattice calculation errors grow, and systematics become more dominant
- Intermediate distances are where most of the e+e- → 2π data end up and is where there is tension in the data-driven approach
- Suggests a best-of-both-worlds approach combining e+e- data with lattice results wherever the errors are minimized

Lattice HVP outlook

$$a_\mu = a_\mu^{\mathrm{SD}} + a_\mu^{\mathrm{W}} + a_\mu^{\mathrm{LD}}$$

"Window" quantities

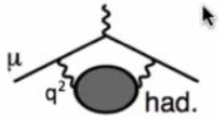
(Plots from Davide Giusti)



- Comparing lattice calculation (and R-ratio) in the three Euclidean distance windows well help us find the tension
- Many other lattice groups aiming for similar precision to BMWc in the next year or two

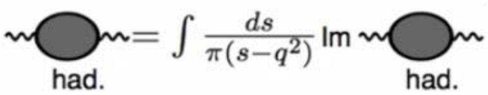
Dispersion integral

a_μ^{HVP} : Basics principles of dispersive method for HVP



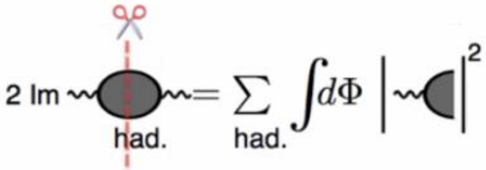
Original loop with hadronic VP

➡ loop integral over q^2 of virtual photon(s)



Causality ➡ analyticity ➡ dispersion integral:

obtain HVP from its imaginary part only



Unitarity ➡ Optical Theorem:

imaginary part (`cut diagram') =

sum over |cut diagram|², i.e.

sum over all hadronic cross sections

$$a_\mu^{\mathrm{had,LO}} = \frac{m_\mu^2}{12\pi^3}\int_{s_{\mathrm{th}}}^{\infty} ds\ \frac{1}{s}\hat{K}(s)\sigma_{\mathrm{had}}(s)$$

- Weight function $\hat{K}(s)/s = \mathcal{O}(1)/s$

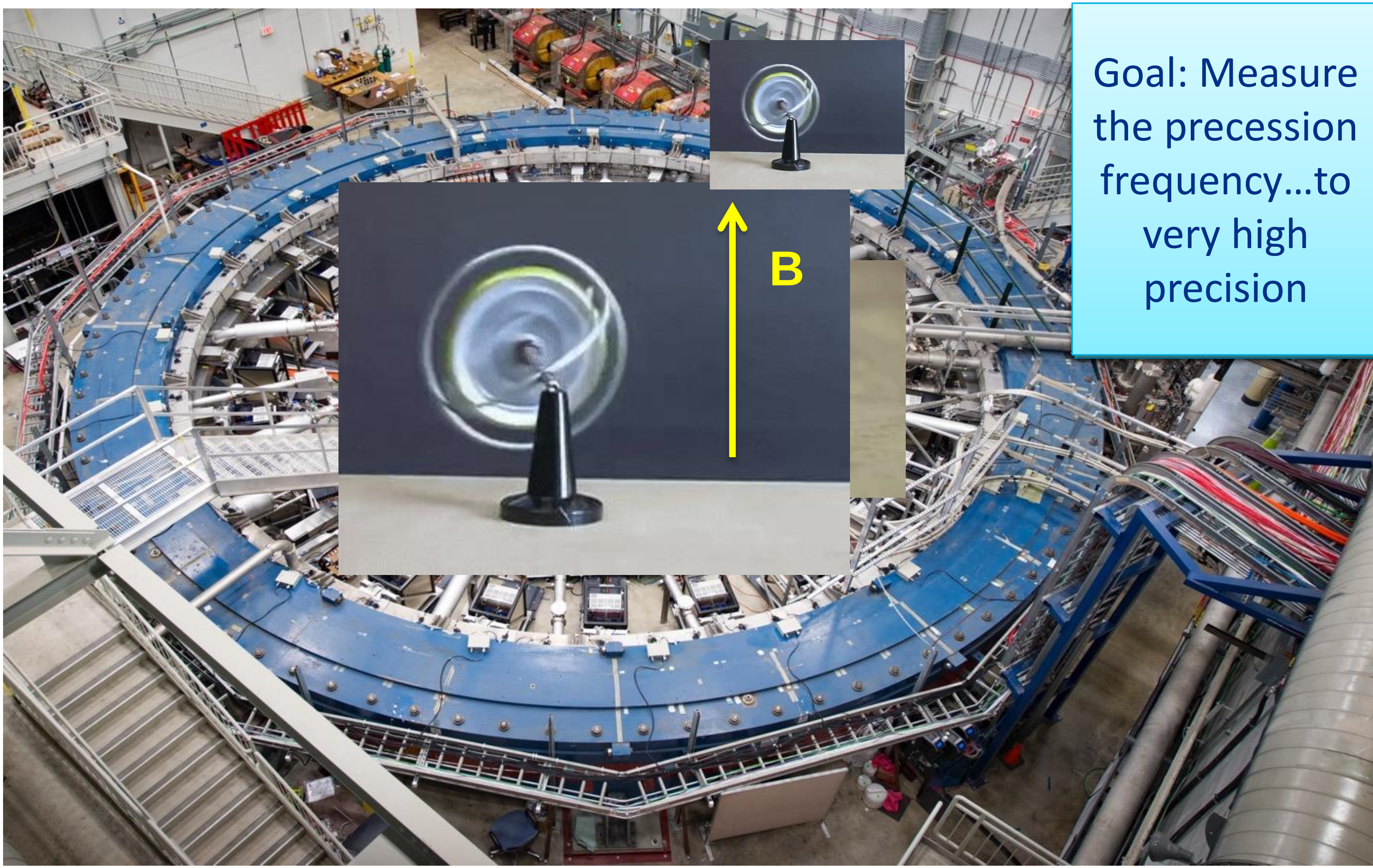
$\Longrightarrow$ Lower energies more important

$\Longrightarrow$ $\pi^+\pi^-$ channel: 73% of total $a_\mu^{\mathrm{had,LO}}$

- Hadronic cross sections from > 100 data sets for $e^+e^- \rightarrow$ hadrons in > 35 hadronic final states
- Uncertainty of a_μ^{HVP} prediction from statistical & systematic uncertainties of data
- Perturbative QCD only at large energies √s, no modelling of $\sigma_{had}(s)$, maximally data-driven

Fermilab

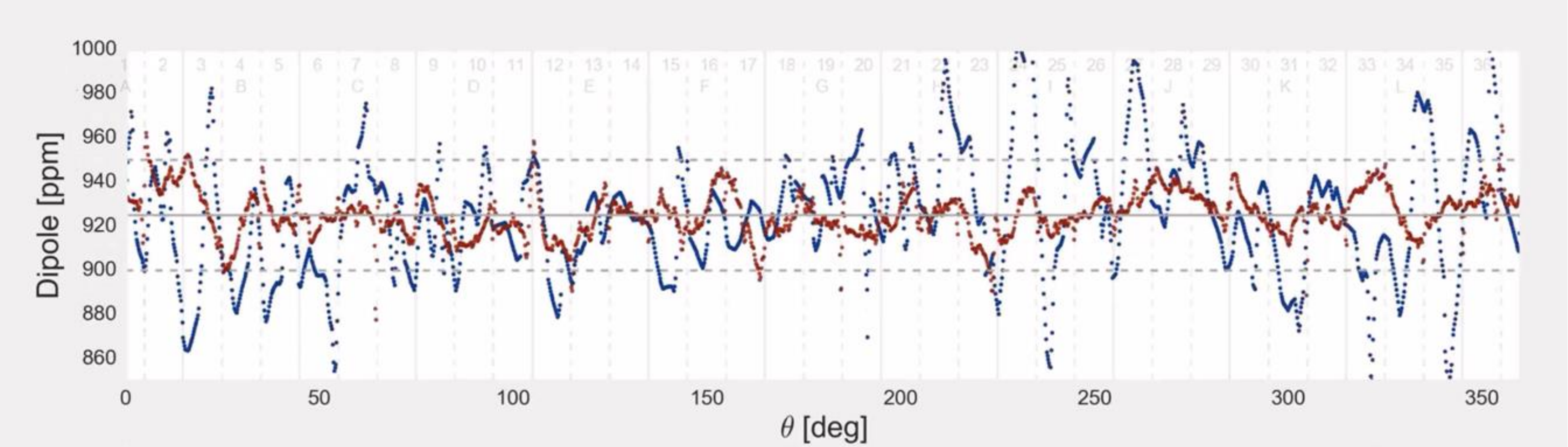
All put back together at Fermilab!



Goal: Measure the precession frequency…to very high precision

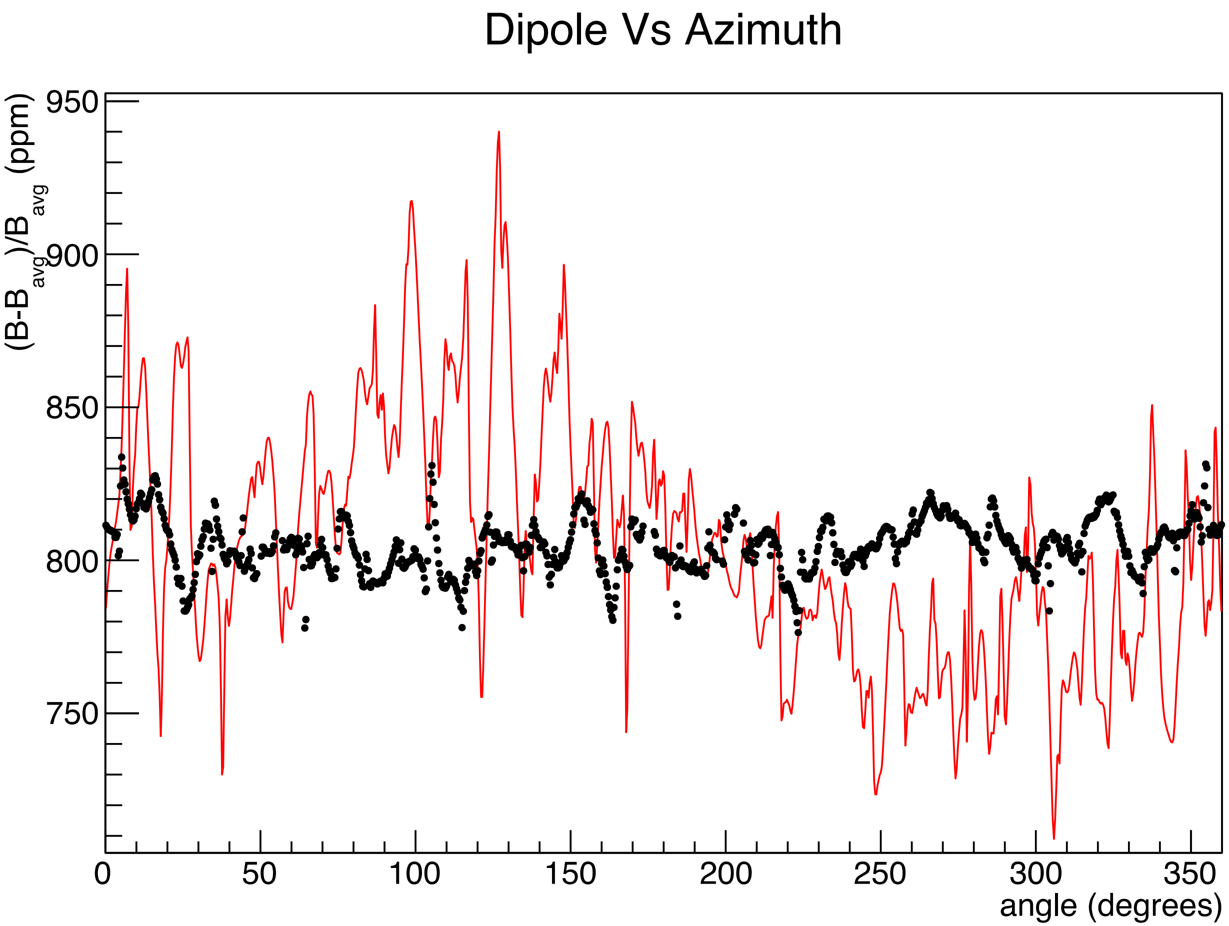
Not just the same BNL exp with more stats...

- Spent over a year shimming the field to be 3x more uniform



- Environmental improvements
  - 3' thick reinforced concrete floor for added stability (3 slabs at BNL)
  - Hall temperature control to < +/- 1C to prevent magnet gap from opening and closing (5C variations at BNL)
- Beam at BNL had a 50% hadronic contamination that momentarily blinded detectors, no hadrons at FNAL –> better gain detector gain stability

Comparison with BNL field



BNL Typical Scan

- 39 ppm RMS (dipole)
- 230 ppm peak-to-peak

FNAL Rough Shimming

- 10 ppm RMS (dipole)
- 75 ppm peak-to-peak

Not just the same BNL exp with more stats...

- Upgrades to 24 calorimeters arrayed around the ring
  - PbWO4 crystals vs PbSciFi → Cerenkov vs scintillation → better temporal separation of pileup
  - Segmented into 6x9 arrays vs monolithic block → spatial separation of pileup
  - Vertically taller → less beam falls of top and bottom → less sensitivity to CBO
- Waveform digitizers that sample twice as fast at 800 MHz → better pileup separation
- DAQ capable of recording all events down to ~10 MeV compared to BNL where most events < 800 MeV were tossed → smaller pileup errors
- Two straw tracker stations installed inside vacuum chambers → unprecedented view of beam dynamics → improved knowledge of all systematics arising from beam motion/detector acceptance
- Laser system that can monitor gain at a part in 10-5 → smaller gain error
- New kickers center the stored muon beam better → smaller electric field correction

We rely on others for e/m and absolute H_2O calib

Us | Them

$$a_\mu = \frac{\omega_a}{\tilde{\omega}'_p(T_r)} \frac{\mu'_p(T_r)}{\mu_e(H)} \frac{\mu_e(H)}{\mu_e} \frac{m_\mu}{m_e} \frac{g_e}{2}$$

ω_a : the muon spin precession frequency

$\tilde{\omega}'_p(T_r)$: precession of protons in water sample mapping the field and weighted by the muon distribution

Goal: 140 ppb =
100 ppb (stat) ⨁ 100 ppb (syst)

$\tilde{\omega}'_p(T)$ Proton Larmor precession frequency in a spherical water sample. Temperature dependence known to < 1ppb/°C. Metrologia **13**, 179 (1977), Metrologia **51**, 54 (2014), Metrologia **20**, 81 (1984)

$\frac{\mu_e(H)}{\mu'_p(T)}$ Measured to 10.5 ppb accuracy at T = 34.7°C
Metrologia **13**, 179 (1977)

$\frac{\mu_e}{\mu_e(H)}$ Bound-state QED (exact)
Rev. Mod. Phys. **88** 035009 (2016)

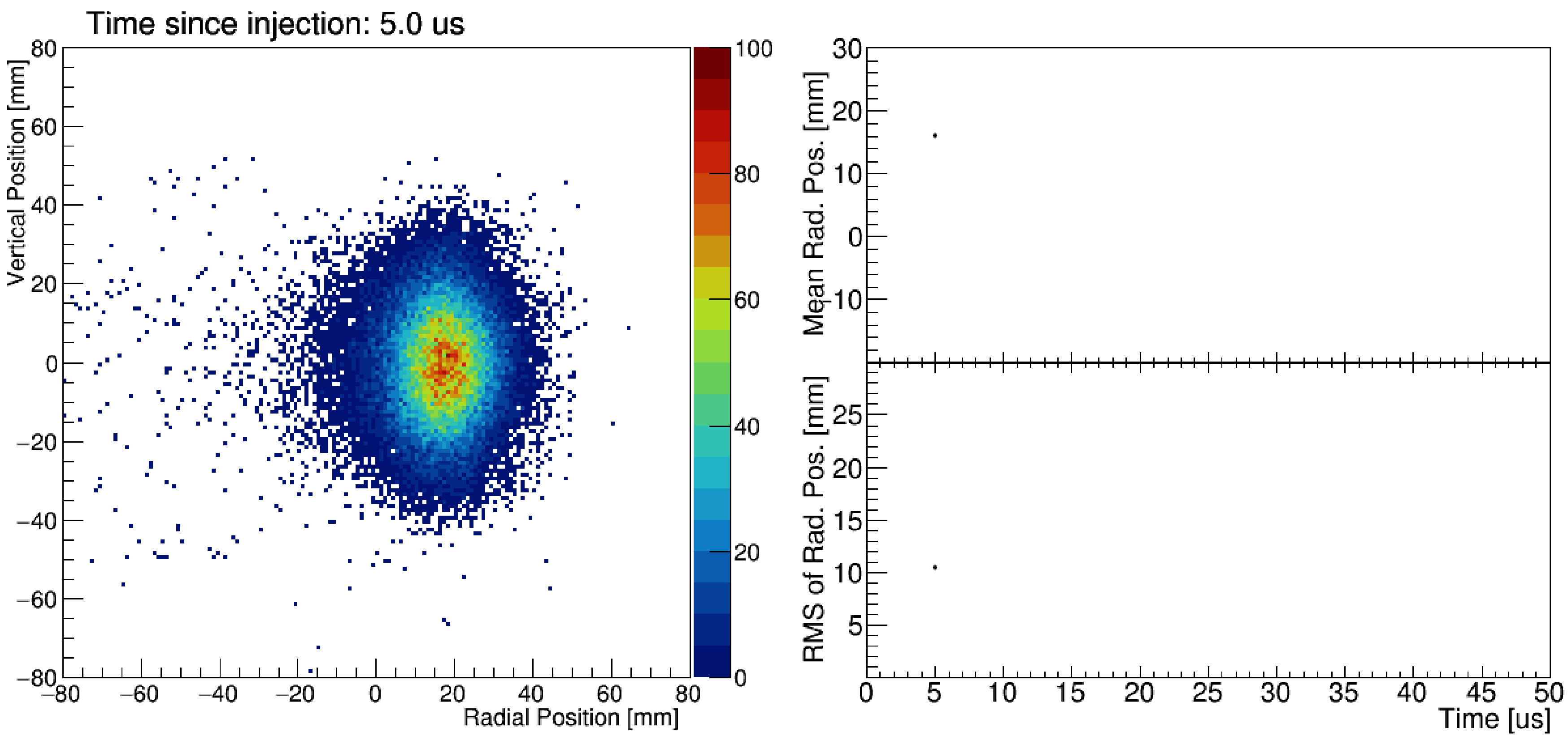
$\frac{m_\mu}{m_e}$ Known to 22 ppb from muonium hyperfine splitting
Phys. Rev. Lett. **82**, 711 (1999)

$\frac{g_e}{2}$ Measured to 0.28 ppt
Phys. Rev. A **83**, 052122 (2011)

All < 22 ppb

Fermilab

Imaging CBO with the trackers



- The *in vacuo* straw trackers give us a much better understanding of beam-related systematic than BNL.

Fermilab

But wait, there's more...

E-field & pitch corrections

Muon loss & phase acceptance corrections

$$\frac{\omega_a}{\tilde{\omega}_p} = \left(\frac{f_{\text{clock}}\, \omega_a (1 + C_e + C_p + C_{ml} + C_{pa})}{(1 + B_{QT} + B_{Eddy})\, f_{\text{field}}\, \omega_p \otimes \rho(\mathbf{r})} \right)$$

Field transients

Field calibration

- Every one of these terms has been studied in extraordinary detail. How much?

Systematics (numerator)

Source	Uncertainty
Frequency Standard	1 ppt
Frequency Synthesizers	0.1 ppb
Digitization Frequency	2 ppb
Total Systematic	2 ppb

Data Set	Run-1a	Run-1b	Run-1c	Run-1d
C_{pa}	-184	-165	-117	-164
Stat. uncertainty	23	20	15	14
Tracker & CBO	73	43	41	44
Phase maps	52	49	35	46
Beam dynamics	27	30	22	45
Total uncertainty	96	74	60	80

$R(\omega_a)$ with detailed systematics categories [ppb]				
Total systematic uncertainty	65.2	70.5	54.0	48.8
Time randomization	14.8	11.7	9.2	6.9
Time correction	3.9	1.2	1.1	1.0
Gain	12.4	9.4	8.9	4.8
Pileup	39.1	41.7	35.2	30.9
Pileup artificial dead time	3.0	3.0	3.0	3.0
Muon loss	2.2	1.9	5.2	2.4
CBO	42.0	49.5	31.5	35.2
Ad-hoc correction	21.1	21.1	22.1	10.3

*Run 1 ω_a data analyzed in four subsets

	1a	1b	1c	1d
C_p (ppb)	**176**	**199**	**191**	**166**
Statistical uncertainty	<0.1	<0.1	<0.1	<0.1
Tracker alignment/reco.	11.0	12.3	12.0	10.7
Tracker res. & acc. removal	3.3	3.9	3.7	3.0
Azimuthal avg. & calo. acc.	1.0	1.3	2.2	1.1
Amplitude fit	1.2	0.4	1.0	2.9
Quad alignment/voltage	4.4	4.4	4.4	4.4
Systematic uncertainty	**12.4**	**13.7**	**13.6**	**12.3**

Data Set	Run-1a	Run-1b	Run-1c	Run-1d
C_{ml}	-14	-3	-7	-17
Phase-momentum	2	0	1	3
Form of $l(t)$	2	0	1	1
f_{loss} function	2	1	2	2
Linear sum ($\sigma_{C_{ml}}$)	6	2	4	6

	1a	1b	1c	1d
C_e (ppb)	**471**	**464**	**534**	**475**
Statistical uncertainty	0.4	0.5	0.4	0.2
Fourier method	8.4	13.4	14.4	3.9
Momentum-time correlation	52	52	52	52
Quad alignment/voltage	6.4	6.4	6.4	6.4
Field index	1.7	1.5	1.7	4.0
Systematic uncertainty	**53**	**54**	**54**	**53**

Systematics (denominator)

run-1 (substructure)	77.4 ppb
azimuthal shape*	7.6 ppb
skin depth	12.6 ppb
frequency extraction (0.4/1ms)	4.6 ppb
Q3L: fit, position	1.5 ppb
repeatability	13.3 ppb
drift	10.2 ppb
radial dependency	4.4 ppb
2^{nd} 8-pulses	14.0 ppb
total −15.0 ppb	81.7 ppb

Source	Uncertainty (ppb)
Temperature	**15 – 28**
Configuration	**22**
Trolley	**25**
Fixed Probe Production	**<1**
Fixed Probe Baseline	**8**
Tracking Drift	**22 – 43**
Total	**43 – 62**

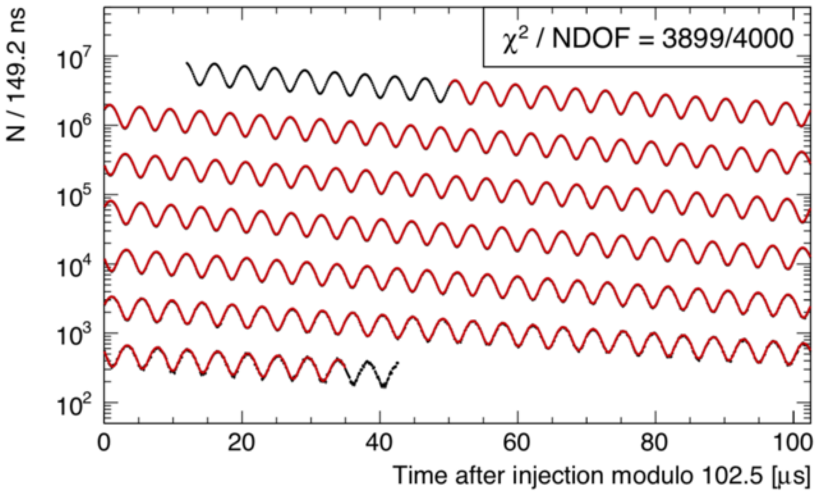
PROBE	Calibration Coefficients Value (Hz)	Stat (Hz)	Syst (Hz)
1	90.81	0.38	2.02
2	84.21	0.65	1.18
3	95.02	0.53	2.19
4	86.03	0.25	1.28
5	92.96	0.51	1.10
6	106.24	0.46	1.35
7	116.64	0.96	1.61
8	76.39	0.60	1.21
9	83.52	0.23	1.64
10	24.06	1.39	1.26
11	177.55	0.22	1.99
12	110.85	0.44	1.73
13	122.89	2.08	1.93
14	77.11	0.53	1.88
15	74.82	1.06	1.59
16	20.35	0.44	2.94
17	172.12	1.23	1.96
AVG		**0.70**	**1.70**

Quantity	Symbol	Value	Unit
Diamagnetic Shielding T dep	$(1/\sigma)d\sigma/dT$	-10.36(30)	ppb/℃
Bulk Susceptibility	δ_b	-1504.6 ± 4.9	ppb
Material Perturbation	δ_s	15.2 ± 13.3	ppb
Paramagnetic Impurities	δ_p	0 ± 2	ppb
Radiation Damping	δ_{RD}	0 ± 3	ppb
Proton Dipolar Fields	δ_d	0 ± 2.3	ppb

Run-1 Estimate:
B_k = -27.4 ± 37 ppb

	correction [ppb]				**uncertainty [ppb]**			
Dataset	**1a**	**1b**	**1c**	**1d**	**1a**	**1b**	**1c**	**1d**
1. Tracker and calo effects	-	-	-	-	9.2	13.3	15.6	19.7
2. COD effects	1.6	1.5	1.7	1.4	5.2	4.7	5.2	4.9
3. In-fill time effects	-1.9	-2.3	-1.2	-4.1	-	-	-	-
Total	**-0.3**	**-0.8**	**0.5**	**-2.7**	**10.6**	**14.1**	**16.5**	**20.3**

C_{PA} – Phase acceptance error

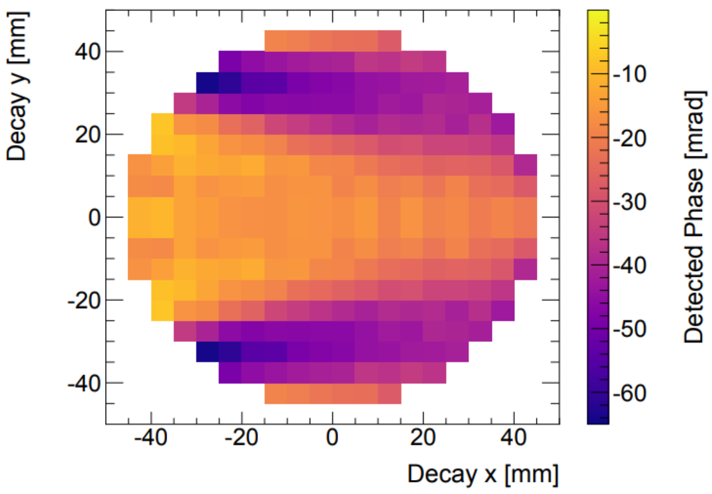


$$f(t) \simeq N_0 e^{-\lambda t}[1 + A\cos(\omega_a t + \phi)]$$

But what if the phase of the muon population changes in time $\phi(t)$?

$$cos(\omega_a t + \phi(t)) = cos(\omega_a t + \phi_0 + \phi' t + ...)$$
$$= cos((\omega_a + \phi')t + \phi_0 + ...)$$

The extracted ω_a is shifted by ϕ' 😬

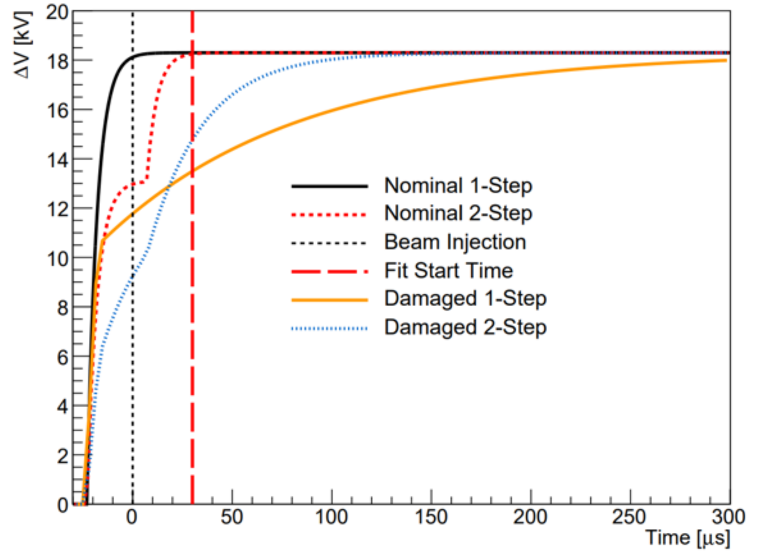


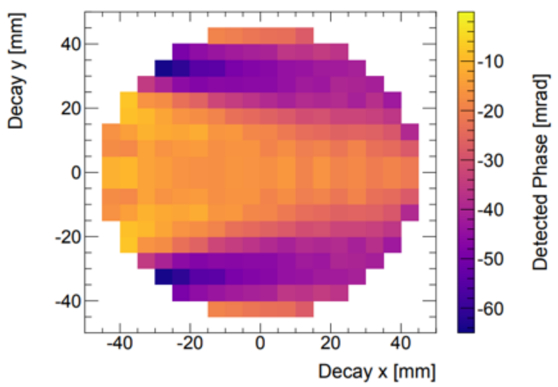
- The decay positrons we detect come from muons that have a particular phase
- That phase depends on muon decay position (x,y) and energy E
- Not a big issue if the muon distribution remains stable in the gap

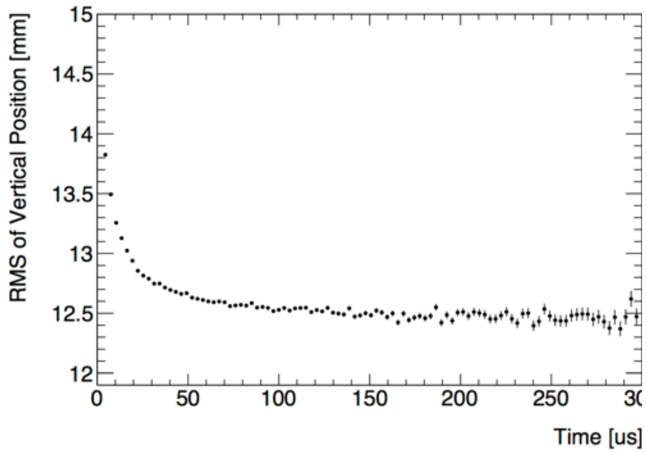
Fermilab

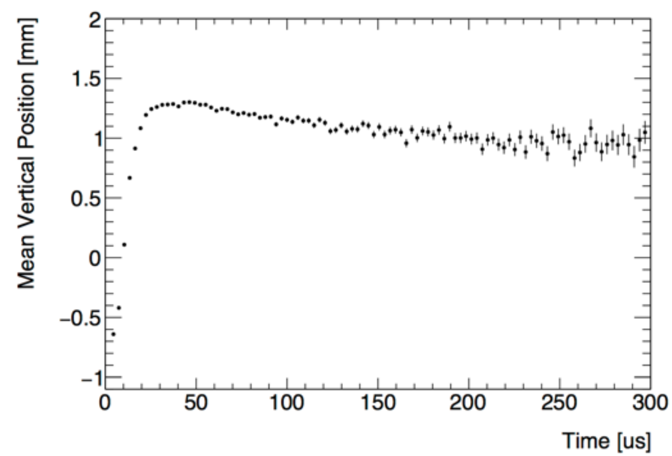
C_{PA} – Phase acceptance error

- Equipment failure led beam instability
- HV resistors died → changing E-field → beam vertical mean and width changed
- -158 ppb correction with a 75 ppb uncertainty in Run 1
- Fixed by Run 2 removing the majority of this correction and uncertainty









With better B_q maps, resistors fixed, and a few other improvements… on track for the 100 ppb systematic goal for Run 2 and beyond