Causality constraints on modifications to gravity

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On work with:

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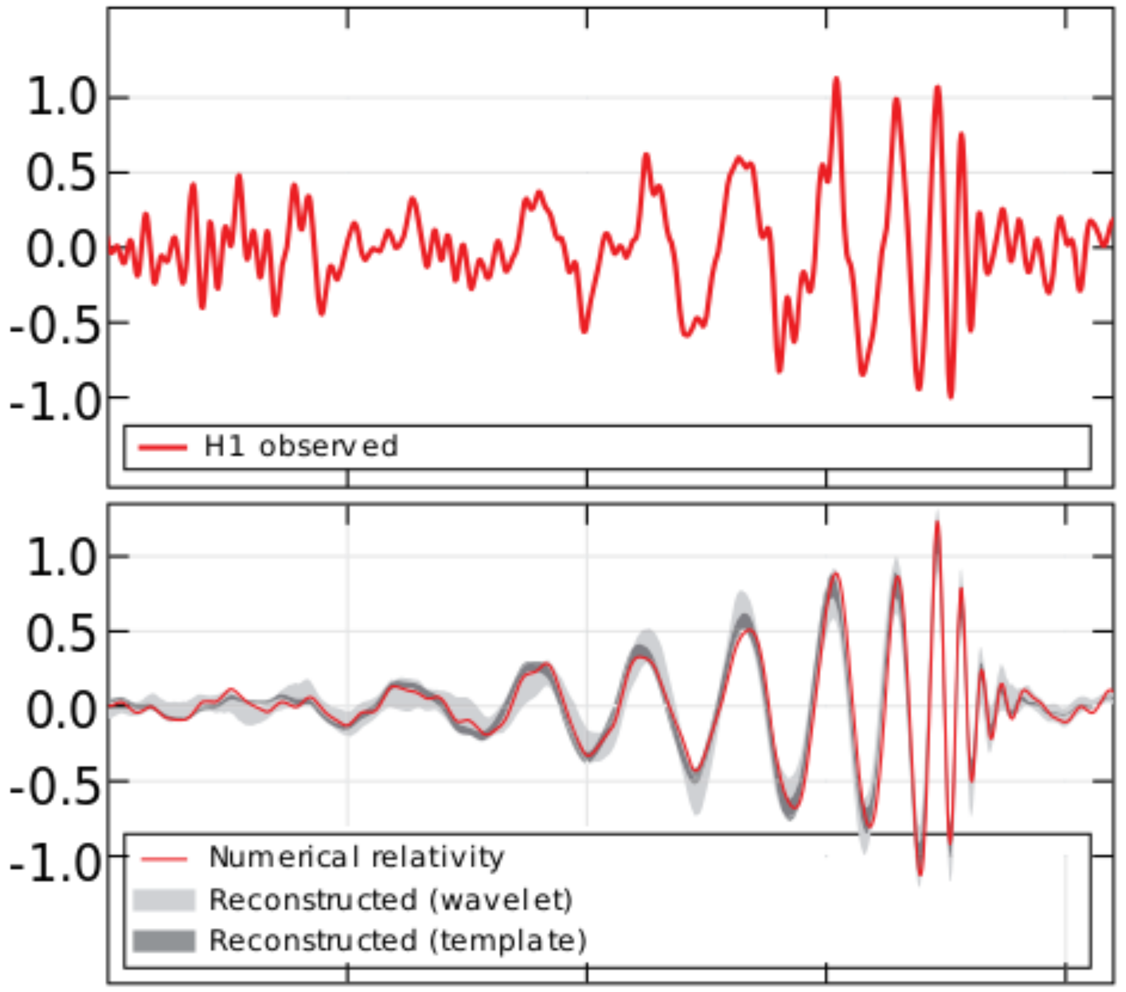
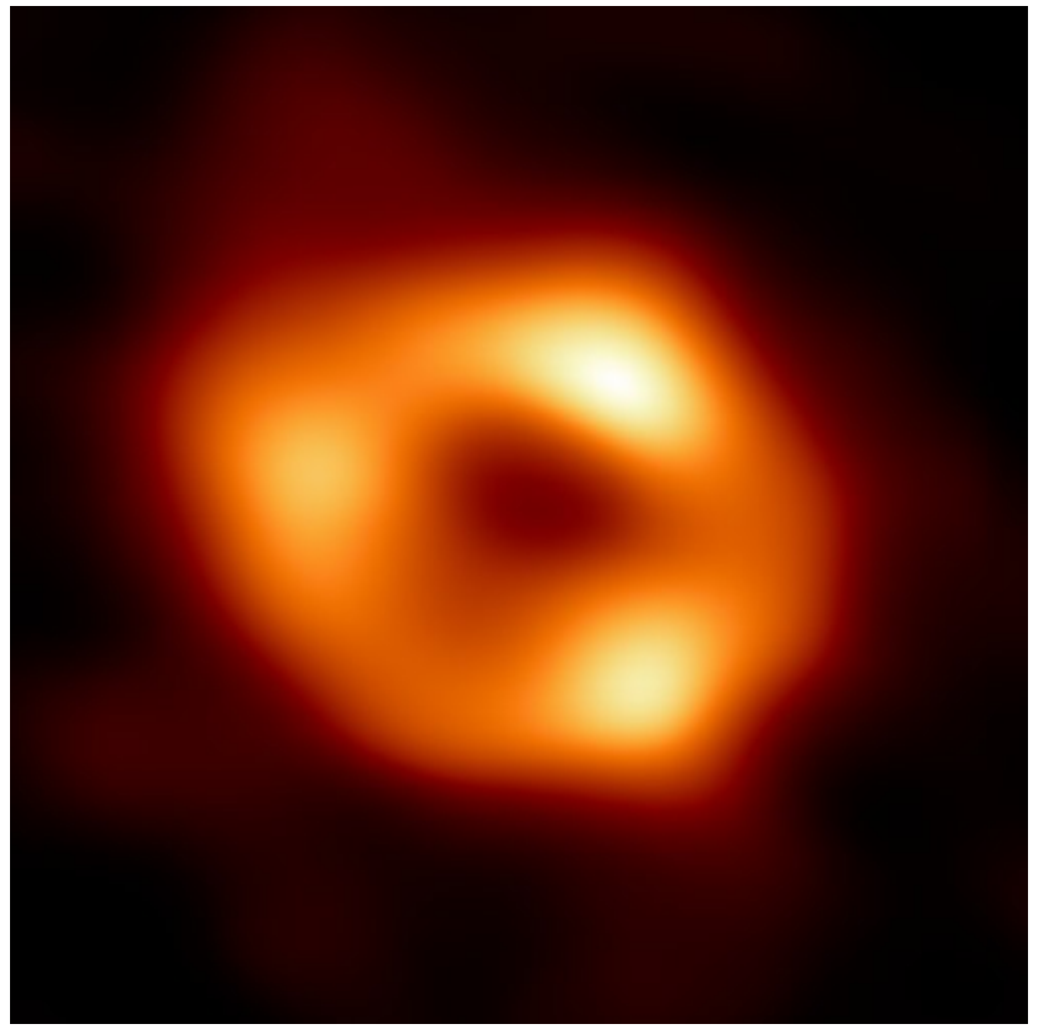
Einstein's General Relativity works well





Suppose it didn't.

Size of corrections $\Leftrightarrow$ Mass $M_{\text{higher-spin}}$ of new states ?

Treat gravity as an EFT below $M_{\text{higher-spin}}$ ($\ll M_{pl}$)

$$S = \frac{1}{16\pi G}\int R + g_3 \text{Riem}^3 + g_4 \text{Riem}^4 + \ldots + \text{matter}$$

How large can g's be ?

$\Rightarrow$ causality of graviton scattering will require:

$$|g_3| \leq \frac{\#}{M^4_{\text{higher-spin}}}, \qquad 0 < g_4 \leq \frac{\#'}{M^6_{\text{higher-spin}}}, \qquad \ldots$$

How small: [Guerrieri, Penedones& Vieira '21]

Outline

1. The question: What modifications can we bound?
   - Graviton scattering
   - causality+unitarity

2. The method
   - dispersive sum rules
   - scalar effective theories

3. Results
   - would colliders see it?

On:		
	SCH, Mazac,Rastelli& Simmons-Duffin ‘20	CFT
	SCH& van Duong ’20	Flat
	SCH, Mazac,Rastelli& Simmons-Duffin ‘21	Flat
	SCH, Mazac,Rastelli& Simmons-Duffin ’21	CFT
→	SCH, Li, Parra Martinez& Simmons-Duffin '22	Flat
	SCH, Li, Parra Martinez& Simmons-Duffin ’22	Flat
	…	

+ Arkani-Hamed, Bellazzini, Bern, Chiang, de Rham, Du, Henriksson, Huang, Huang, Kosmopou;os, McPeak, Rattazzi, Rivera, Rodina, Russo, Tolley, Vichi, Wang, Zhang, Zhiboedov, Zhou…

Low energy graviton scattering in 3+1D

$M_{\text{higher-spin}} \ll M_{\text{pl}}$: neglect loops.

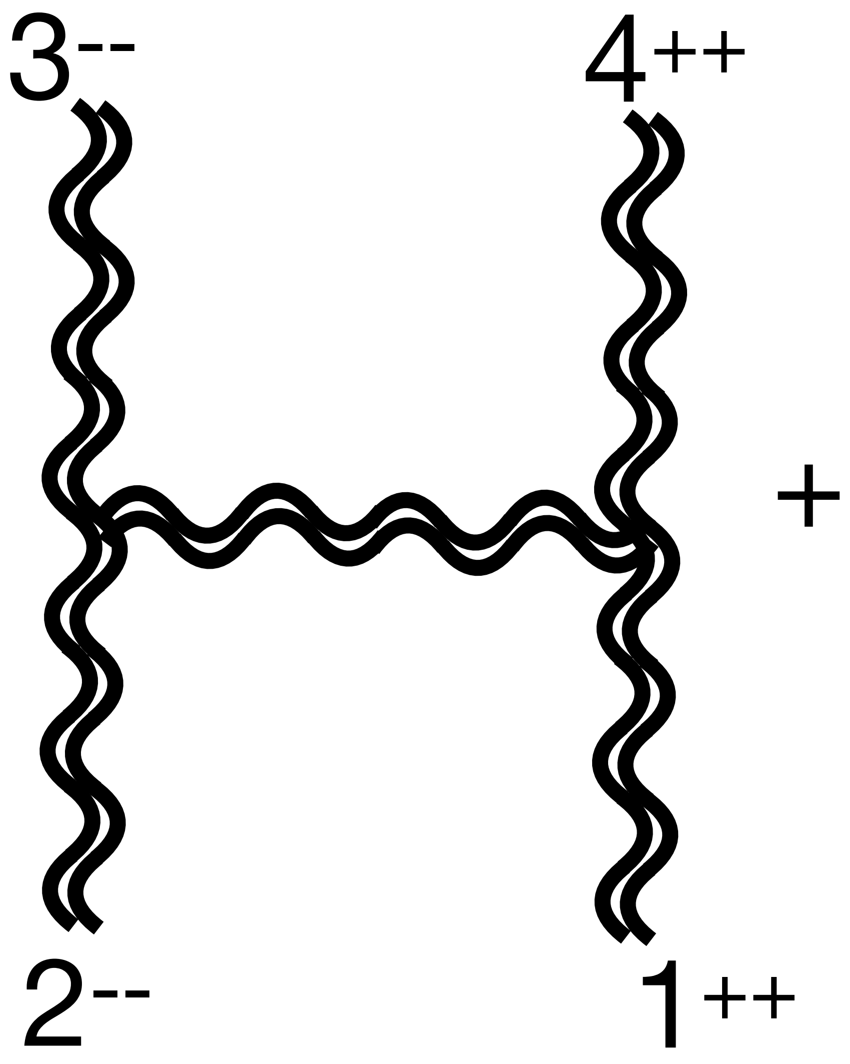
$$\mathscr{M}^{+--+} = [14]^4\langle 23\rangle^4 \times 8\pi G \left[\frac{1}{stu} + \frac{|g_3|^2 su}{4t} + \frac{|g_s|^2}{-t} + g_4 + g_5 t + \dots \right]$$

Einstein!

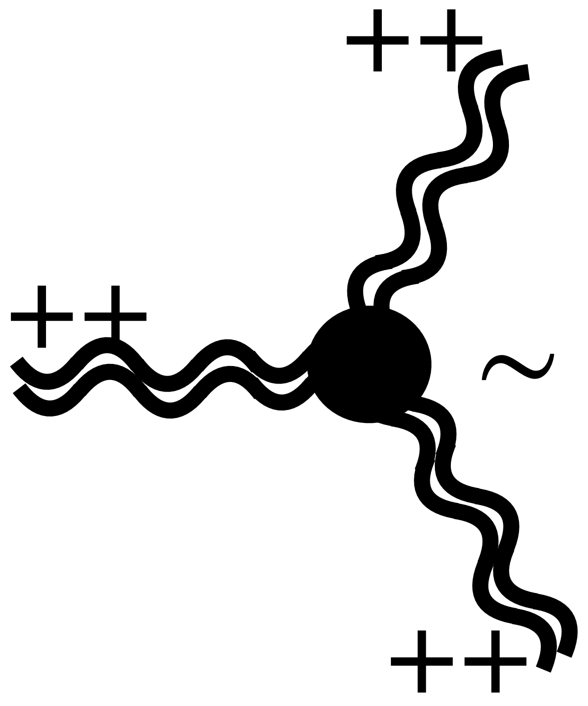
Riem^3 at vertices

ϕRiem^2

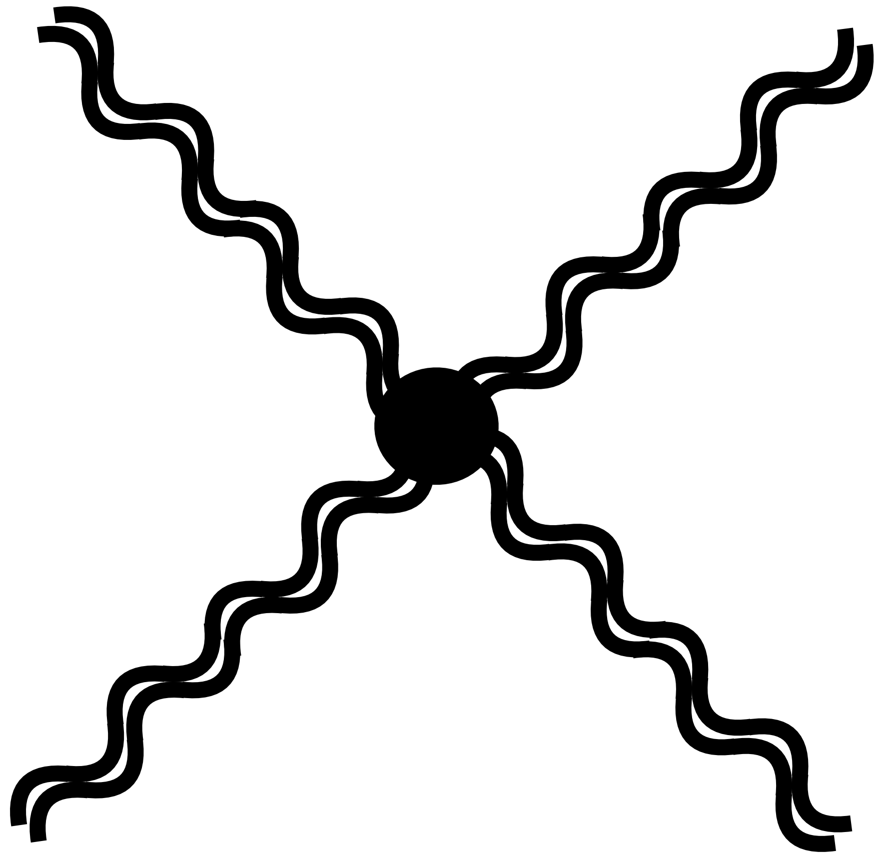
contacts: Riem^4 and derivatives



+…



$$\sim \sqrt{8\pi G}\, g_3 [12]^2[23]^2[13]^2$$



We don’t bound:

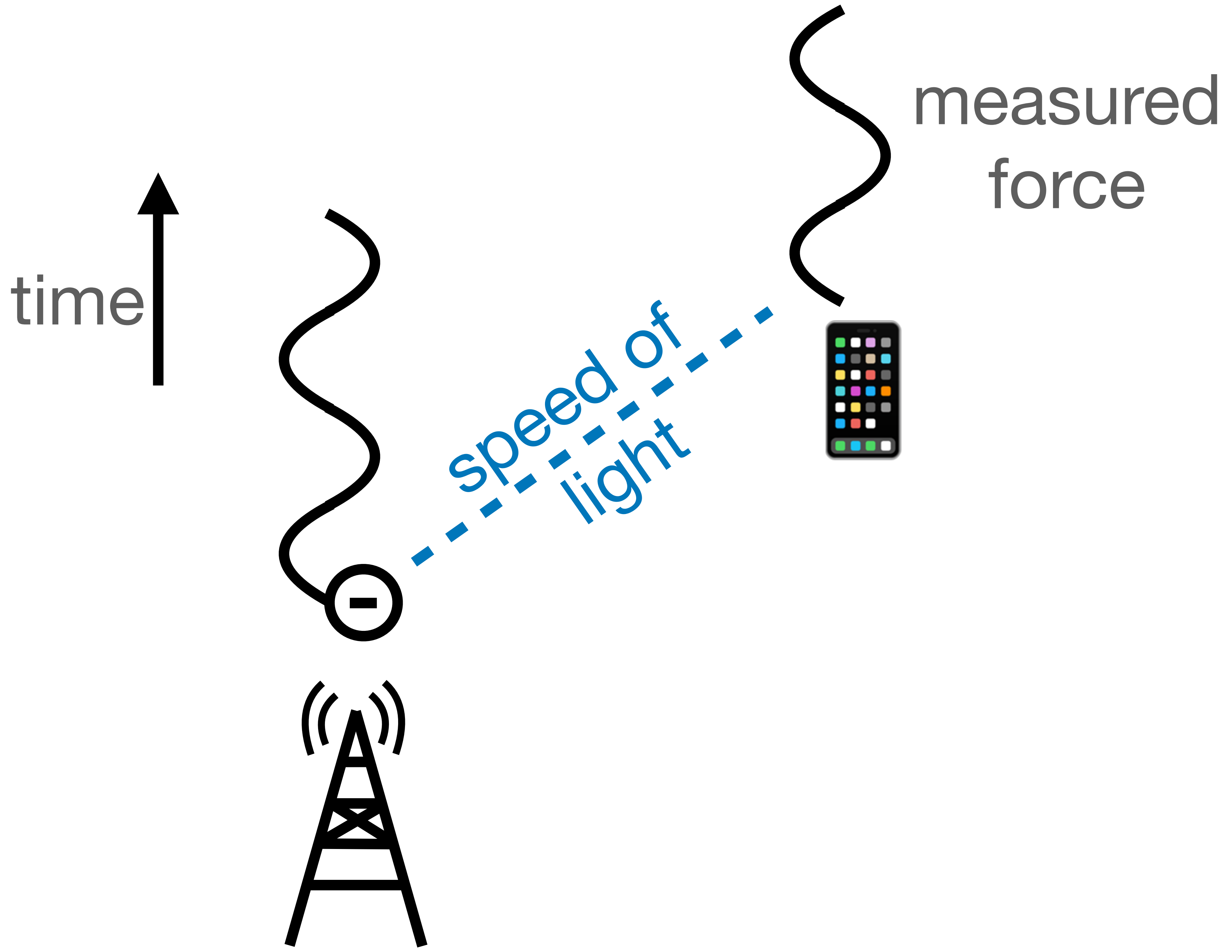
- $f(R)$ ($\simeq$ Einstein + scalar field : no imprint in graviton scattering)
- Any term with Ricci tensor/scalar: removable by field redefinition (no imprint)
- Scalar potentials (don’t grow with energy)
- ‘Fifth forces', torsion, etc: treat as extra matter fields (minimally coupled or not)

Briefly, we bound amplitudes, not Lagrangians.

Causality

"signals can't travel faster than light"

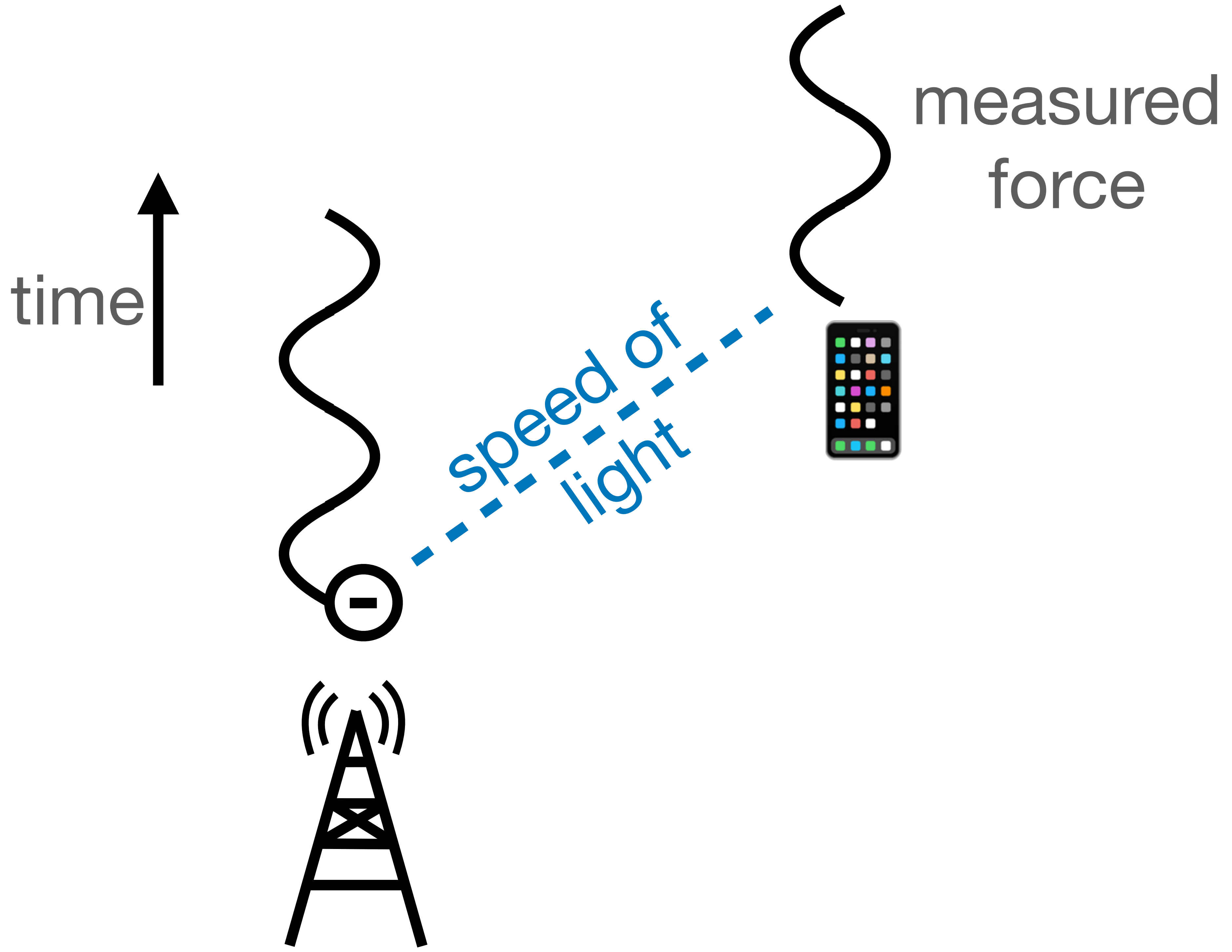
- Why waves, fields



Causality

"signals can't travel faster than light"

- Why waves, fields
- Why particles



Causality

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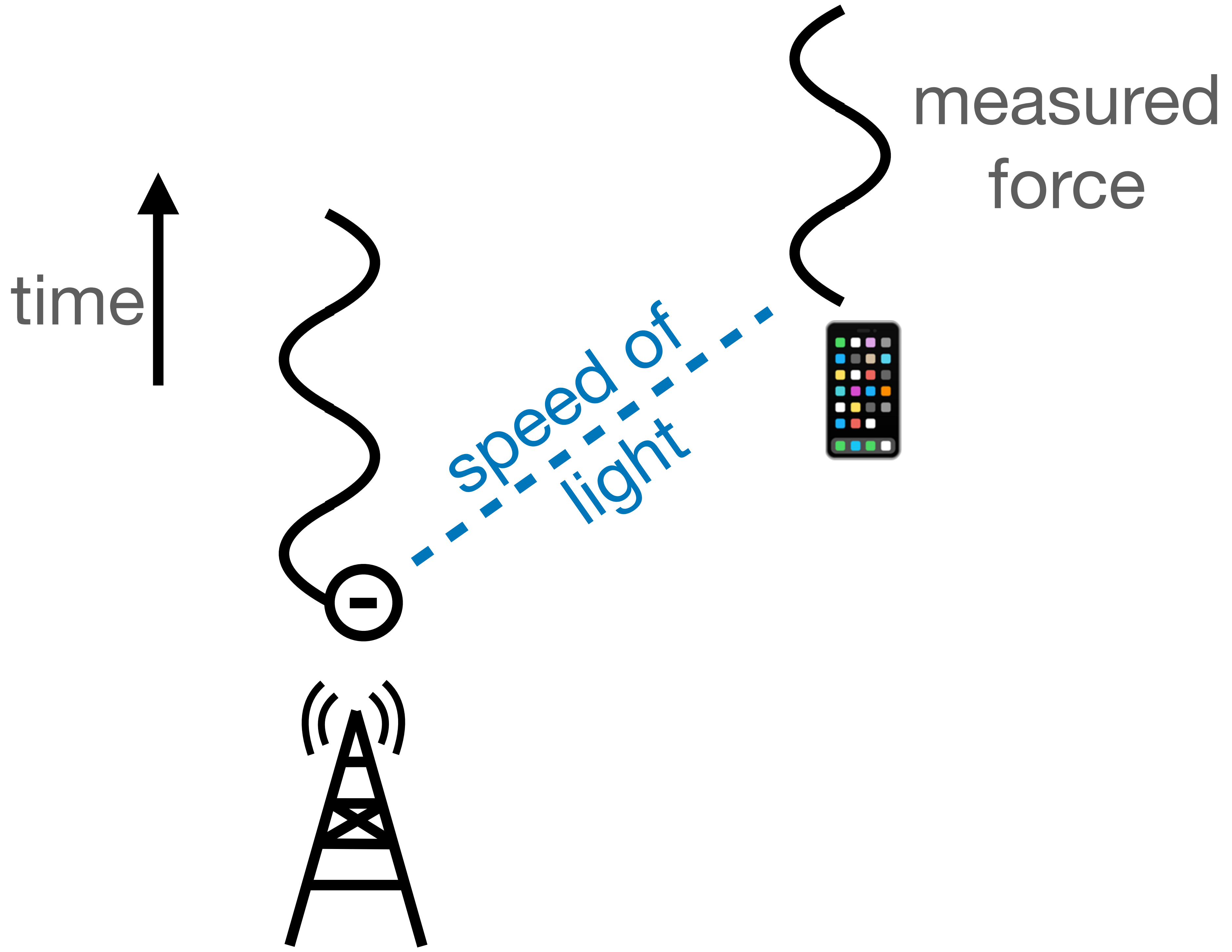
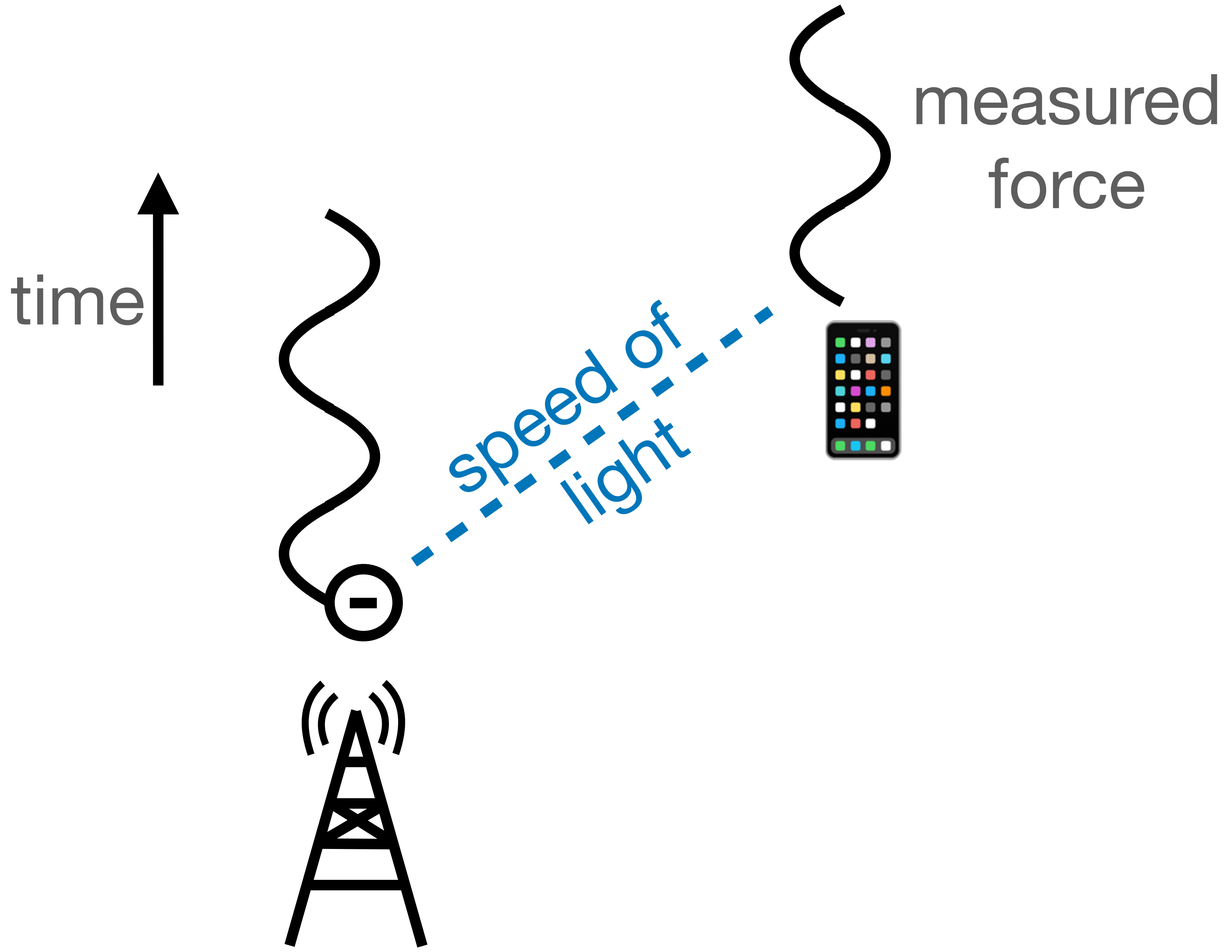
- Why antiparticles



Causality

"signals can't travel faster than light"

- Why waves, fields
- Why particles
- Why antiparticles
- ...
- Why EFTs have to work
- Why gravity is attractive
- Experimentally tested to exquisite accuracy
- ...



(doesn't imply it's exact in Nature!)

Causality vs. gravity: some known results

0) at long distances, any Lorentz-invariant S-matrix of a massless spin-2 particle must reproduce GR [Weinberg]

1) positivity of classical time delays at impact parameters $b_{\rm min} \gg 1/M_{\rm higher-spin}$ requires

$$|g_3| \lesssim \frac{1}{M^4_{\rm higher-spin}}$$

[Camanho,Edelstein,Maldacena& Zhiboedov '14]

2) positivity of forward amplitudes (imaginary parts) implies various sign constraints

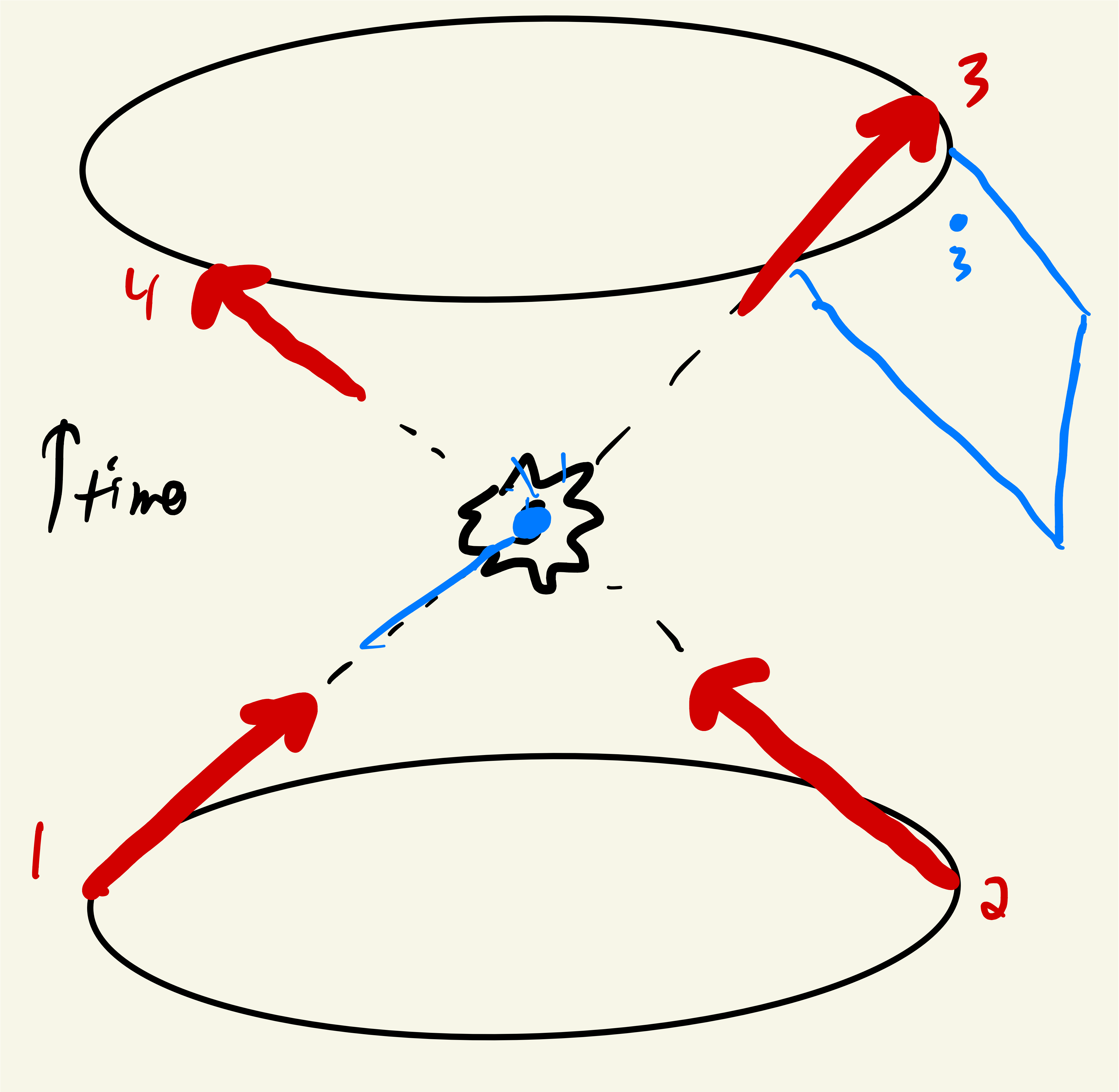
[Adams,Arkani-Hamed,Dubovsky,Nicolis&Rattazzi '06]
[Bellazzini,Cheung& Remmen '15]

$$g_4 = \int_{M^2_{\rm heavy}}^{\infty} \frac{ds}{s}\, {\rm Im}\, f(s, t=0) \geq 0$$

Method

Dispersion relations

Causality for 2->2 scattering

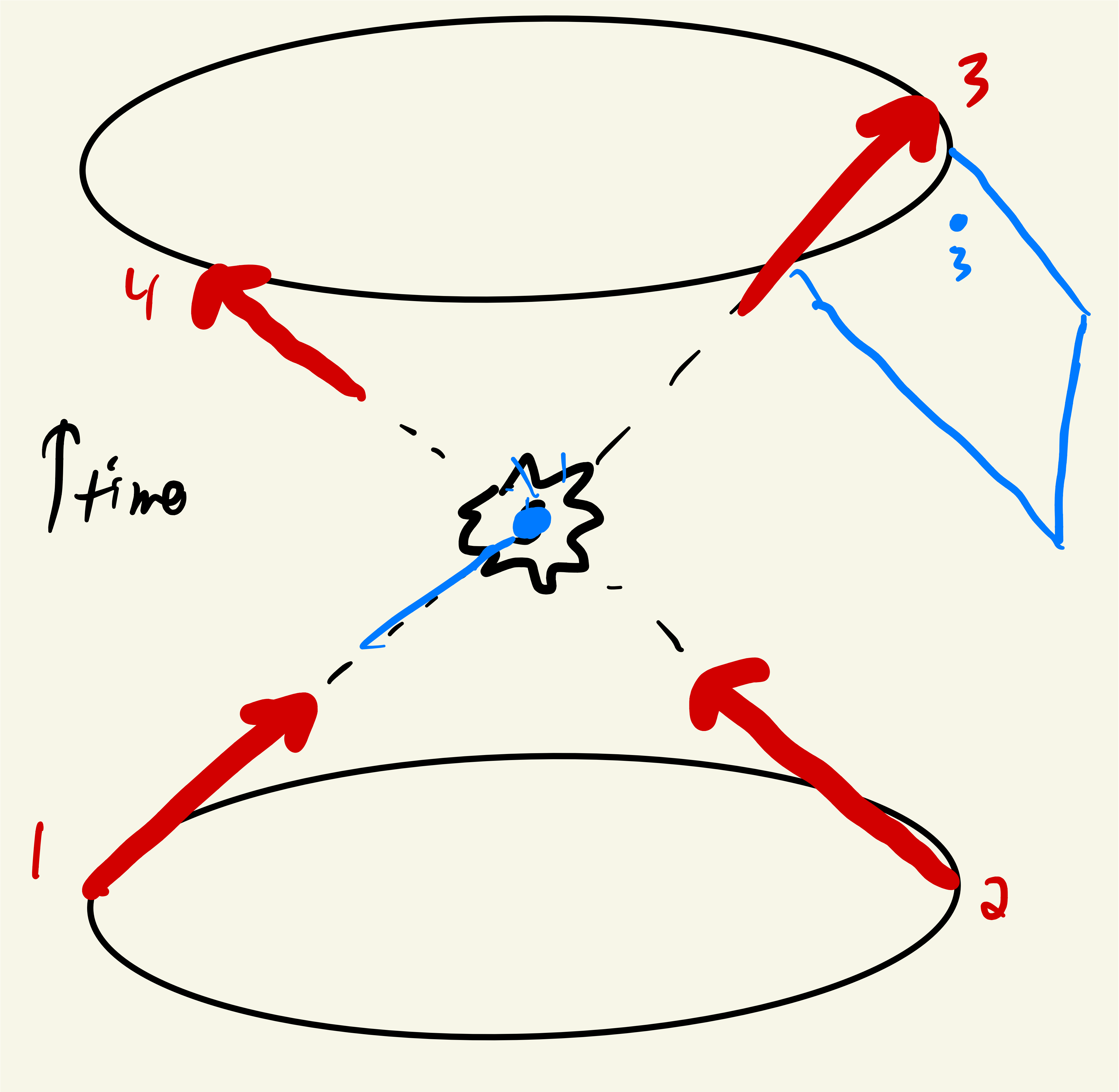


i) Fixed angle scattering can show time advances

[Giddings+Porto '09]

$\Rightarrow$ causality controls Regge limit $\begin{array}{l} s \to \infty \\ t \text{ or } b \text{ fixed} \end{array}$

Causality for 2->2 scattering



i) Fixed angle scattering can show time advances

[Giddings+Porto '09]

$\Rightarrow$ causality controls Regge limit $\begin{array}{l} s \to \infty \\ t \text{ or } b \text{ fixed} \end{array}$

ii) Strongest statement involves crossing:

particle $1 \to 3 \simeq$ antiparticle $3 \to 1$

Particles are waves. What is causality for waves?

Wavefront not superluminal $\Leftrightarrow$ Medium response is analytic in frequency

& sub-exponential in upper half-plane

[Brillouin-Sommerfeld]

Kramers-Kronig dispersion relations:

propagation

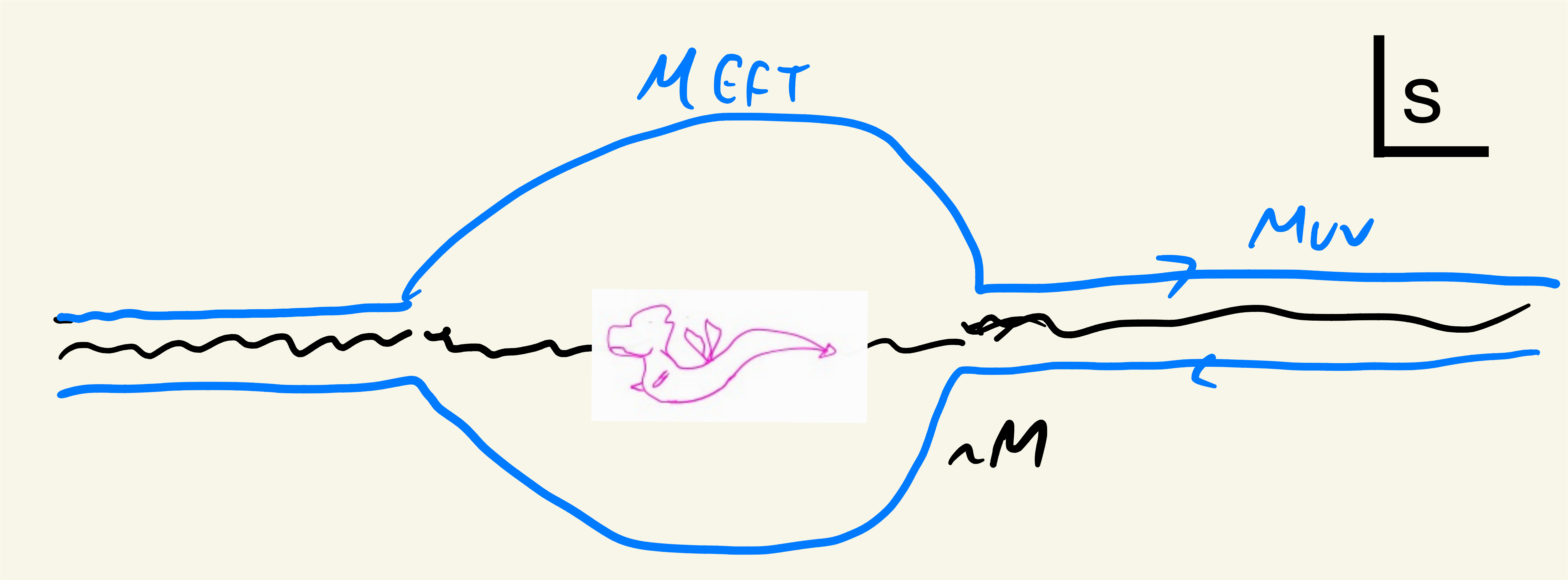
$$n(\omega) = 1 + \frac{1}{\pi} \int \frac{d\omega' \, \mathrm{Im}(n(\omega'))}{\omega' - \omega - i0}$$

absorption

For us: low-energy scattering $\Leftrightarrow$ production of heavy particles

Minimal axioms:

$M_{low}(s,t)$ has a causal+unitary (relativistic) UV completion



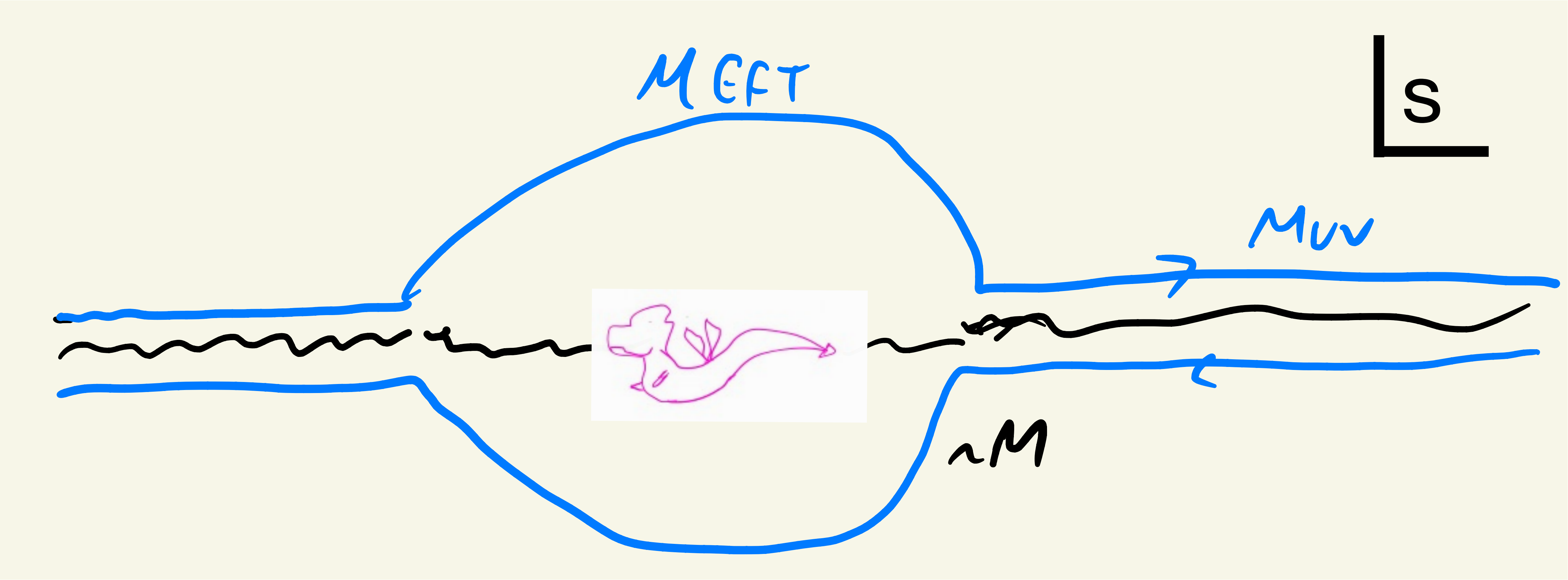
Dispersive sum rules: relate IR& UV

$$0 = \oint_{|s|=\infty} ds \frac{M_\psi(s)}{s^{k+1}} \Rightarrow \qquad \underset{\text{EFT}}{\oint} (\cdots) = \underset{\text{heavy}}{\int} \frac{ds}{s^{k+1}} \mathrm{Im} M_\psi(s) \qquad (k > 1)$$

$$\| $$

$$\sum_J |c_J|^2 P_J(1 - 2p^2/s)_\psi$$ (Im M = sum of Legendre's with positive coefficients)

(low-energy couplings) = (sums of high-energy unknowns)



Minimal axioms:

i) Analyticity of M(s,t) in $\{t \in (-M^2, 0)\} \times \left\{ \begin{array}{l} \text{real } s > M^2 \cup \text{real } u > M^2 \\ \cup \text{ upper-half-plane connecting them} \end{array} \right\}$

ii) Boundedness $|M_\psi(s)/s| \leq \text{const as } |s| \to \infty$

for smeared amplitude: $M_\psi(s) = \int_0^M \psi(p) M(s, -p^2)$

ψ: compact support $p<M$, fast decay in b

holds for AdS gravity / large-N large-gap CFTs: [SCH,Mazac,Rastelli& Simmons-Duffin '21]

Warm-up: non-gravitational real scalar

- weakly coupled EFT below M
- anything above M, just causal and unitary

$$\mathcal{L}_{\text{low}} = -\frac{1}{2}(\partial_\mu\phi)^2 - \frac{g}{3!}\phi^3 - \frac{\lambda}{4!}\phi^4 + \frac{g_2}{2}\left[(\partial_\mu\phi)^2\right]^2 + \frac{g_3}{3}(\partial_\mu\partial_\nu\phi)^2(\partial_\sigma\phi)^2 + 4g_4\left[(\partial_\mu\partial_\nu\phi)^2\right]^2 + \cdots$$

$$\updownarrow$$

$$\mathcal{M}_{\text{low}}(s,t) = -g^2\left[\frac{1}{s}+\frac{1}{t}+\frac{1}{u}\right] - \lambda + g_2(s^2+t^2+u^2) + g_3(stu) + g_4(s^2+t^2+u^2)^2 + g_5(s^2+t^2+u^2)(stu) + \ldots$$

Goal: bound higher-derivative terms

First few sum rules: (k=2, 4, …)

$$B_2: \quad 2g_2 - g_3 t + 8g_4 t^2 + \ldots \qquad = \left\langle \frac{\left(2m^2+t\right)\mathcal{P}_J\left(1+\frac{2t}{m^2}\right)}{m^2\left(m^2+t\right)^2} \right\rangle_{m \geq M}$$

$$B_4: \quad 4g_4 + \ldots \qquad = \left\langle \frac{\left(2m^2+t\right)\mathcal{P}_J\left(1+\frac{2t}{m^2}\right)}{m^4\left(m^2+t\right)^3} \right\rangle_{m \geq M}$$

(t<0)

Expand around t=0 (requires stronger axioms)

$$g_2 = \left\langle \frac{1}{m^4} \right\rangle, \qquad g_3 = \left\langle \frac{3 - \frac{4}{d-2}\mathcal{J}^2}{m^6} \right\rangle \qquad g_4 = \left\langle \frac{1}{2m^8} \right\rangle \qquad 0 = \left\langle \frac{\mathcal{J}^2(2\mathcal{J}^2 - 5d + 4)}{m^8} \right\rangle$$

clearly: $g_2 \geq 0$

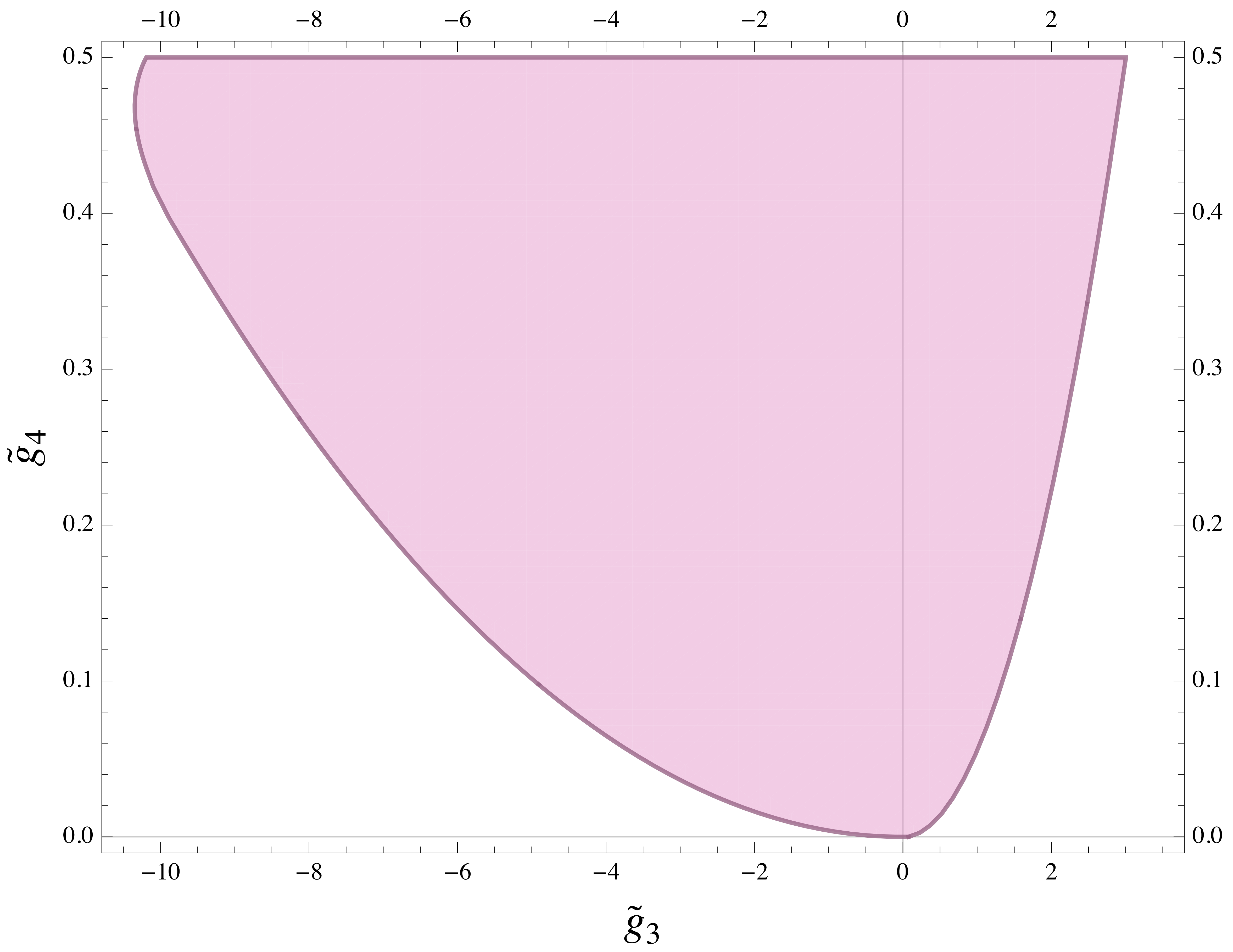
$? \leq g_3 \leq \frac{3g_2}{M^2}$

$0 \leq g_4 \leq \frac{g_2}{2M^4}$

‘null constraints’ from crossing symmetry enable 2-sided bounds

[Tolley, Wang& Zhou ’20]
[SCH& van Duong ’20]

‘dimensional analysis scaling’ is a theorem [for operators of dim≥8]



$\tilde{g}_k \equiv g_k M^{\#} / g_2$

EFT coefficient	Lower bound	Upper bound
$\tilde{g}_3$	-10.346	3
$\tilde{g}_4$	0	0.5
$\tilde{g}_5$	-4.096	2.5
$\tilde{g}_6$	0	0.25
$\tilde{g}'_6$	-12.83	3
$\tilde{g}_7$	-1.548	1.75
$\tilde{g}_8$	0	0.125
$\tilde{g}'_8$	-10.03	4
$\tilde{g}_9$	-0.524	1.125
$\tilde{g}'_9$	-13.60	3
$\tilde{g}_{10}$	0	0.0625
$\tilde{g}'_{10}$	-6.32	3.75

geometric growth like $\dfrac{1}{M^2 - s} \sim \dfrac{1}{M^2} + \dfrac{s}{M^4} + \ldots$

[Tolley, Wang& Zhou ’20]
[SCH& van Duong ’20]
[Arkani-Hamed, Huang& Huang ’20]
[Chiang, Huang, Li, Rodina& Weng ’21]
[mixed correlators numerics: Du, Zhang& Zhou ‘21]

Gravity: new results

« Spin is GREAT »

- Dispersive sum rules for gravitons:

$$M^{+--+} = \overbrace{[14]^4\langle 23\rangle^4}^{\propto t^4} \times 8\pi G \left[\frac{1}{stu} + \overbrace{\frac{|g_3|^2 su}{4t} + \frac{|g_s|^2}{-t} + g_4 + g_5 t + \ldots}^{f(s,t)} \right]$$

- Prefactor grants superconvergent sum rules:

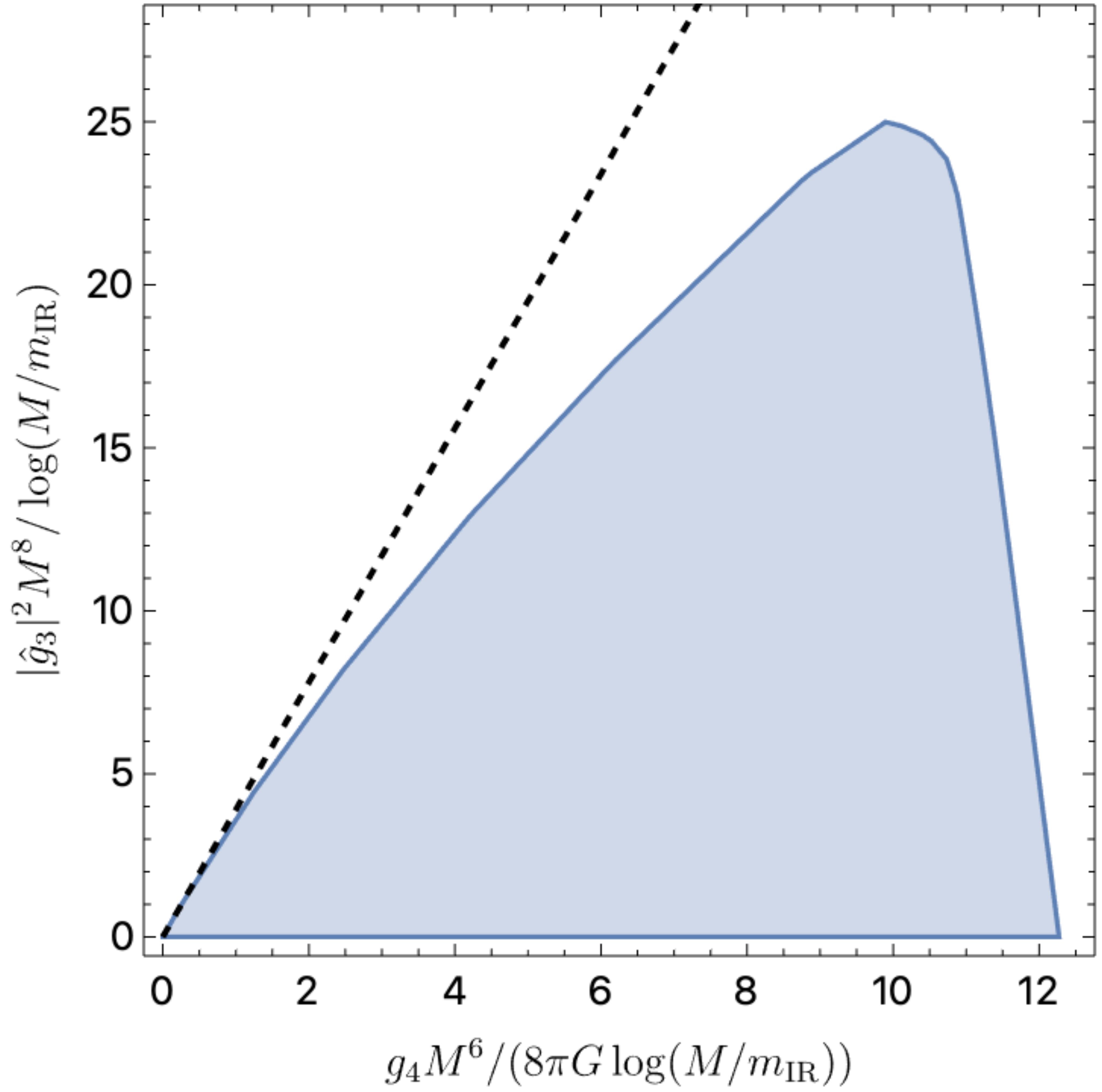
$$B_2(u) : 0 = \oint_{s=\infty} (s-t)ds[f(s,t)+f(t,s)]\,, \qquad B_3(u) : 0 = \oint_{s=\infty} ds[f(s,t)-f(t,s)]$$

- Any MAGIC combination which writes G = positive sum will dominate all else:

$$G = \sum_k \int_0^{-M^2} du \Psi_k(u) B_k(u) > 0 \qquad \Rightarrow G - \# g_3 \geq 0, \text{ etc}$$

- Require positive contributions from: -light particles of spin<=2 (SM, KK modes, etc)
  -heavy states with $M > M_{\text{higher-spin}}$ of arbitrary spin

Riem³ and Riem⁴ can't exceed GR



$$S = \frac{1}{16\pi G}\int\left(R + \frac{\tilde{g}_3\ \mathrm{Riem}^3}{M^4_{\rm higher-spin}} + \frac{\tilde{g}_4\ \mathrm{Riem}^4}{M^6_{\rm higher-spin}} + \ldots\right)$$

$\tilde{g}$'s can't exceed O(1) without violating causality at scale ~$M_{\text{higher-spin}}$

[SCH, Li, Parra Martinez, Simmons-Duffin]

A tale of 3 effective field theorists:

$$\sim h\partial^2 h + c''\frac{\partial^6 h^3}{M^5}$$

$$L \supset m_{\rm pl}^2 R + c\frac{\text{Riem}^3}{M^2}$$

"c<O(1) since couplings at cutoff should be O(1)"

$$L \supset m_{\rm pl}^2\left(R + c'\frac{\text{Riem}^3}{M^4}\right)$$

"c'<O(1): corrections can never dominate GR below the cutoff"

$$L \supset m_{\rm pl}^2 R + c'' m_{\rm pl}^3\frac{\text{Riem}^3}{M^5}$$

"c"<O(1) so gravitons stay weakly coupled below M"

When $M \ll m_{pl}$, what is the correct scaling of higher-derivative corrections with M & m_{pl} ?

A tale of 3 effective field theorists:

$$\cancel{L \supset m_{\rm pl}^2 R + {\color{red}c}\frac{\text{Riem}^3}{M^2}}$$

~~"c<O(1) since couplings at cutoff should be O(1)"~~

too restrictive
(untrue in string theory...)

$$L \supset m_{\rm pl}^2\left(R + {\color{red}c'}\frac{\text{Riem}^3}{M^4}\right)$$

"c'<O(1): corrections can never dominate GR below the cutoff"

= what we find!

$$\sim h\partial^2 h + {\color{red}c''}\frac{\partial^6 h^3}{M^5}$$

$$\cancel{L \supset m_{\rm pl}^2 R + {\color{red}c''} m_{\rm pl}^3 \frac{\text{Riem}^3}{M^5}}$$

~~"c"<O(1) so gravitons stay weakly coupled below M"~~

too permissive
(ruled out by our causality bounds!)

When $M \ll m_{pl}$, what is the correct scaling of higher-derivative corrections with M & m_{pl} ?

Our results are insensitive to the large-scale curvature of spacetime:
one only needs a flat local patch of size >> $1/M_{\text{higher-spin}}$

In AdS spacetime, localized scattering -> rigorous bounds on CFT central charges:

AdS_5/CFT_4:
$$\left|\frac{a-c}{c}\right| \leq \frac{23.0}{\Delta_{\text{gap}}^2}$$

[SCH, Mazac, Rastelli& Simmons-Duffin '21]
[SCH, Li, Parra Martinez& Simmons-Duffin '22]

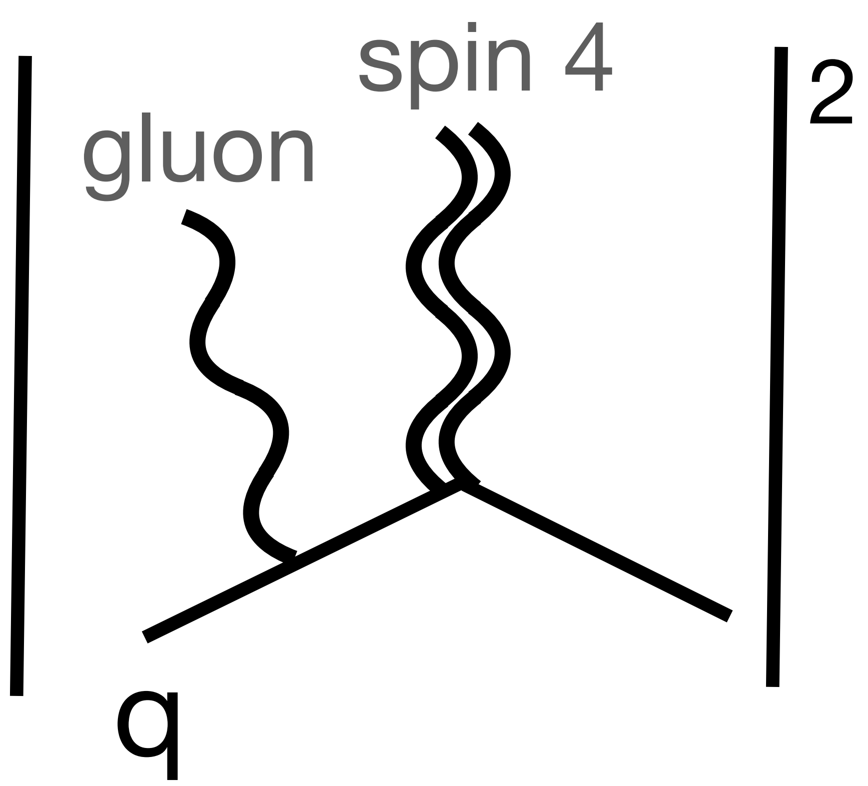
Summary

- Gravitational scattering below $M_{\text{higher-spin}}$ can't significantly differ from GR without violating causality.

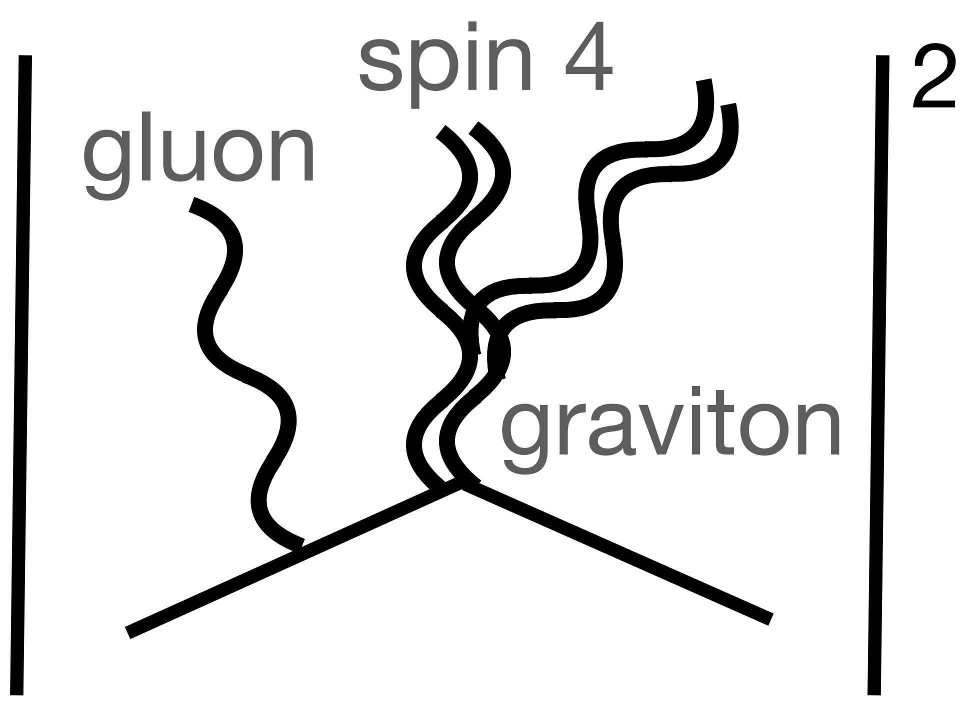
Open questions

- Interactions between [higher-spin states] and Standard Model matter?
- Expect loops only $O(N/M_{\text{pl}}^4)$. Check?
- Remove Log[IR]'s (dressing, …)?
- Higher spacetime dimensions? ✓ massive graviton(s)?
- What if M~M_{pl}: how close to classical GR can 4d quantum gravity be?
- …

What do we know about $M_{\text{higher-spin}}$?



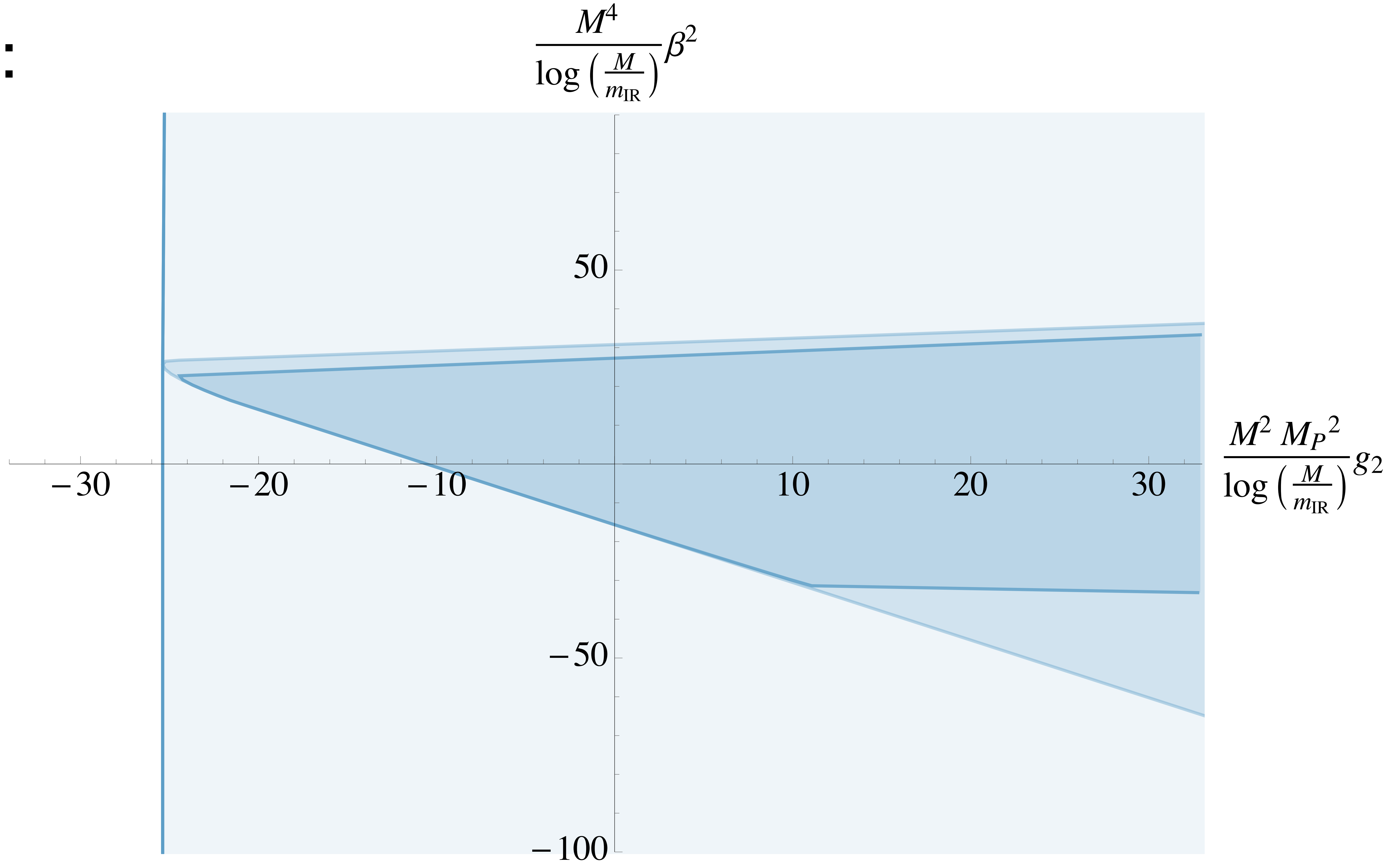
$$\sim \frac{\alpha_s E^6}{M^4 M_{\rm pl}^2}$$



$$\sim \frac{\alpha_s E^{16}}{M^{12} M_{\rm pl}^4} \quad \text{or} \sim \frac{\alpha_s E^8}{M^4 M_{\rm pl}^4}$$

- Very conservatively: hard to imagine not seeing 'missing energy' at LHC from a gravitationally-coupled spin-4 particle with M<MeV.
- Corresponds to a length scale: $M_{\rm higher-spin}^{-1} < 10^{-13}\text{m}\ldots$
- Phenomenological constraints should be analyzed carefully.

Comments on photon scattering:



F^4

$$g_2 \sim \sum_{\text{charged fields}} \frac{e_i^4}{m_i^4} + \sum_{\text{axions}} \frac{1}{f_a^2 m_a^2}$$

RF^2

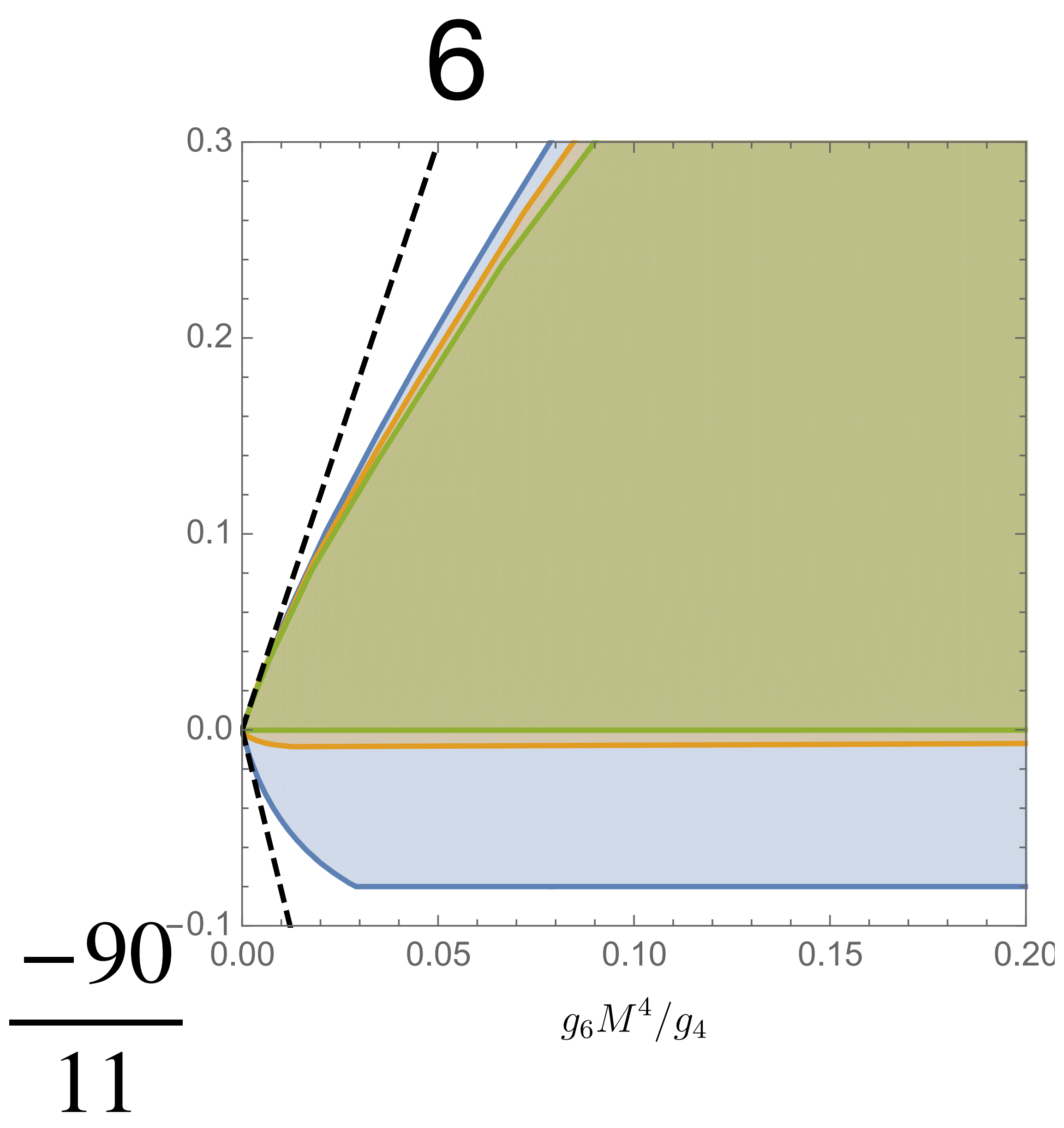
$$\beta \sim \sum_{\text{charged fields}} \frac{e_i^2}{m_i^2}$$

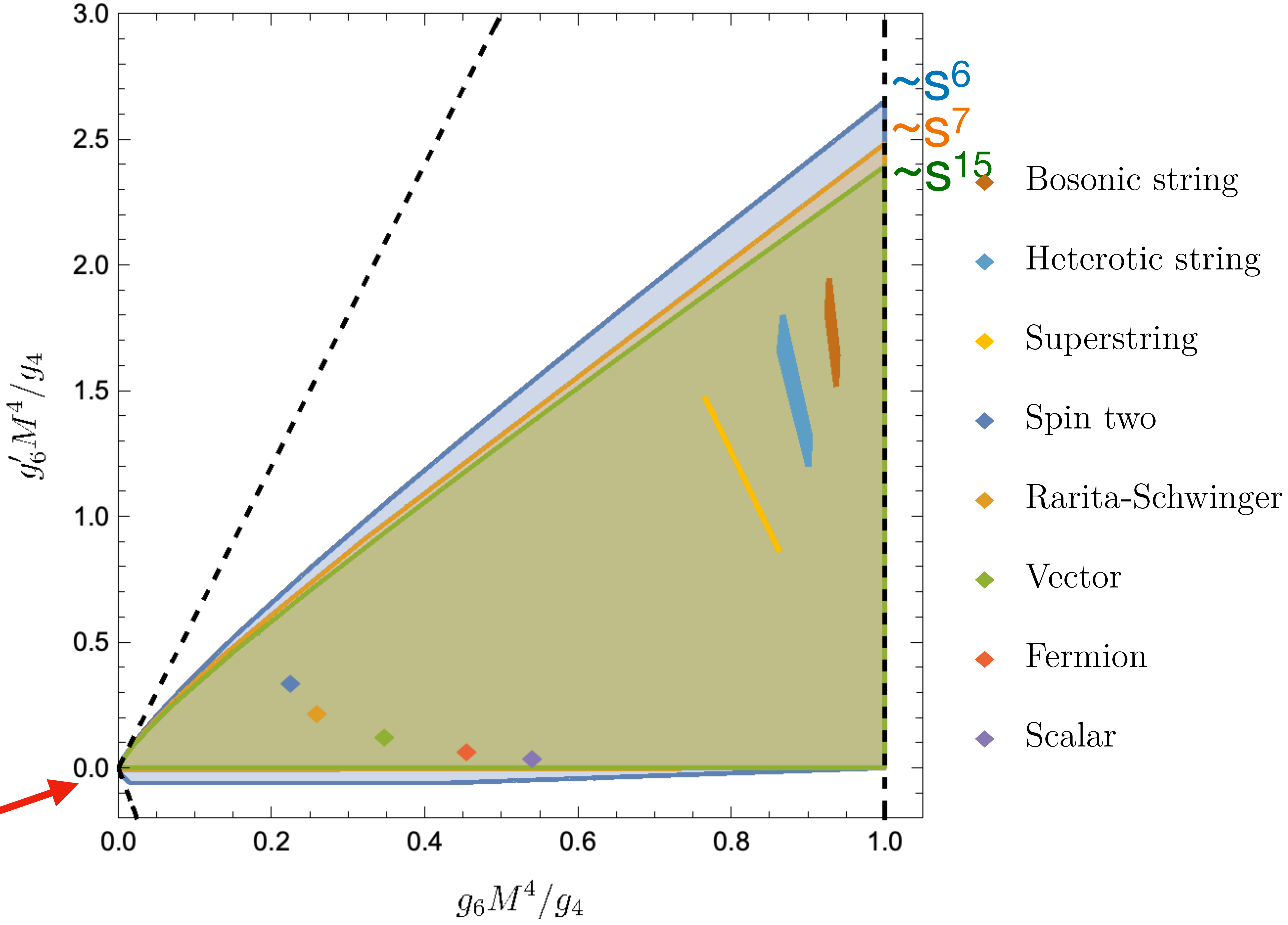
[Henriksson, McPeak, Russo, Vichi '22]

- WCG upper bound on β/g_2 not seen in dispersion relation bounds:

- true bounds at large g2 are much stronger (axions don't contribute to β)
- small negative g2 allowed: time delay from graviton swamps possible ~e^2 possible advance from matter loop.

more on contact interactions using (more) spin>=4 null constraints: (two D^4R^4)/R^4

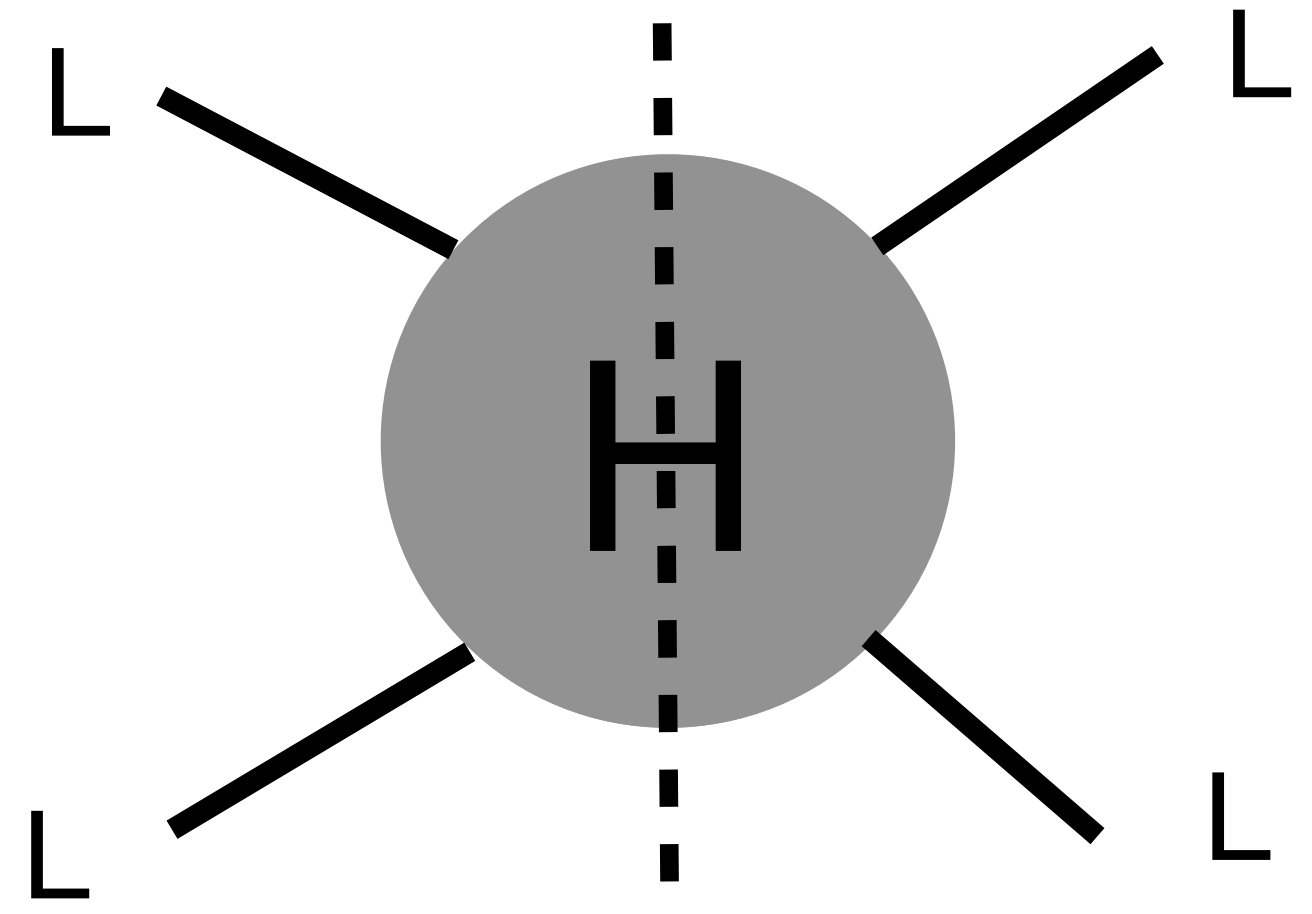




extremal slopes are only realized in region that disappears asymptotically!

[Bern, Kosmopoulous, Zhiboedov '21]
[photons: Henriksson, McPeak, Russo, Vichi'21]

null constraints
from IR crossing:



this constrains UV spectral density! (light-light-heavy couplings)

[Tolley, Wang& Zhou '20]
[SCH& van Duong '20]

$$\left\langle \frac{1}{m^4} \overbrace{\frac{J^2}{m^2}}^{\sim b^2} \right\rangle_{m \geq M} \leq \frac{\#}{m^2} \left\langle \frac{1}{m^4} \right\rangle_{m \geq M}$$

⇒ As far as sum rules are concerned,

heavy states with large spin (large b) can't couple strongly

(ie. large black holes, long strings, etc, can never dominate sum rules)