Meson spectroscopy using holographic QCD plus 't Hooft equation

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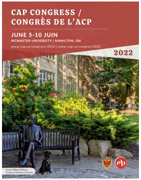

Overview

1. Light-front wavefunction

2. Longitudinal dynamics

3. Predicting Meson spectrum

Light-front coordinates

Lorentz transformation mixes the components of the space-time 4-vector. $x^\mu \equiv (x^0, x^1, x^2, x^3)$.

However, one can define combinations of the 4-vector components which are the eigenstates of the Lorentz Transformation and so get scaled under Lorentz boost:

$$x^+ = x^0 + x^3\,,\ \ x^- = x^0 - x^3\,,\ \ x^\perp = x^1,\, x^2$$

$$x^2 = x^\mu x_\mu = x^+ x^- - {x^\perp}^2$$

$x^+ \rightarrow$ Light-front time $\qquad$ $x^- \rightarrow$ Light-front distance

For energy momentum 4-vector $p^\mu \equiv (p^0, p^1, p^2, p^3)$
Light-front energy $\rightarrow p^- = p^0 - p^3$
Light-front momentum $\rightarrow p^+ = p^0 + p^3$

LFWF

$$H_{\rm QCD}^{\rm LF}|\Psi(P)\rangle = M^2|\Psi(P)\rangle \tag{1}$$

where $H_{\rm QCD}^{\rm LF} = P^+P^- - P_\perp^2$ is the LF QCD Hamiltonian and M is the hadron mass. At equal light-front time ($x^+ = 0$) and in the light-front gauge $A^+ = 0$, the hadron state $|\Psi(P)\rangle$ admits a Fock expansion, i.e.

$$|\Psi(P^+, \mathbf{P}_\perp, S_z)\rangle = \sum_{n,h_i} \int [\mathrm{d}x_i][\mathrm{d}^2\mathbf{k}_{\perp i}] \frac{1}{\sqrt{x_i}} \Psi_n(x_i, \mathbf{k}_{\perp i}, h_i)|n : x_i P^+, x_i \mathbf{P}_\perp + \mathbf{k}_{\perp i}, h_i\rangle$$

where $\Psi_n(x_i, \mathbf{k}_{\perp i}, h_i)$ is the LFWF of the Fock state with n constituents and the integration measures are given by

$$[\mathrm{d}x_i] \equiv \prod_i^n \mathrm{d}x_i \delta(1 - \sum_{j=1}^n x_j) \qquad [\mathrm{d}^2\mathbf{k}_{\perp i}] \equiv \prod_{i=1}^n \frac{\mathrm{d}^2\mathbf{k}_{\perp i}}{2(2\pi)^3} 16\pi^3 \delta^2(\sum_{j=1}^n \mathbf{k}_{\perp i}) \, .$$

$(k_i^+, k_i^-, \mathbf{k}_{\perp i})$ and h_i are the momentum and helicity of the $i^{\rm th}$ constituent and $x_i = k_i^+/P^+$.

The valence meson LFWF

For $n = 2$,

$$\mathbf{k}_{\perp 1} = -\mathbf{k}_{\perp 2} = \mathbf{k}_{\perp}$$

$$x_1 = 1 - x_2 = x$$

The position-space conjugate of $\mathbf{k}_{\perp}$, denoted by $\mathbf{b}_{\perp} = b_{\perp} e^{i\varphi}$, is the transverse separation between the quark and the antiquark.

Introduce a new light-front variable $\boldsymbol{\zeta} = \sqrt{x(1-x)}\mathbf{b}_{\perp} = \zeta e^{i\varphi}$ leads to the meson LFWF in the position-space:

$$\Psi(\zeta, x, \phi) \xlongequal{\text{factorization}} \frac{\phi(\zeta)}{\sqrt{2\pi\zeta}} e^{iL\phi} X(x)$$

$\phi(\zeta)$ and $X(x)$ are referred to as the transverse and longitudinal modes.

Holographic Schrödinger equation

Brodsky, de T´eramond (PRL, 09)
Brodsky, de T´eramond, Dosch, Erlich (Phys. Rep. 15)

In the semi-classical limit, i.e. zero quark mass and no quantum loop, based on AdS/CFT, one can show that the transverse mode of LFWF of the valence ($n = 2$ for mesons) state can be obtained from a 1-dimensional Schrödinger-like wave equation for the:

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U_\perp(\zeta)\right)\phi(\zeta) = M_\perp^2\phi(\zeta)$$

the potential is uniquely determined from the conformal symmetry breaking mechanism and correspondence with weakly coupled string modes in AdS_5 space, which results in a light-front harmonic oscillator potential in physical spacetime with confinement scale κ:

$$U_\perp(\zeta, J) = \kappa^4\zeta^2 + \kappa^2(J-1)$$

$J = L + S$ is the total meson angular momemtum.

Solutions to holographic Schrödinger equation

With the confining potential specified, one can solve the holographic Schrödinger equation to obtain the meson mass spectrum,

$$M_{\perp}^2 = 4\kappa^2 \left(n_{\perp} + L + \frac{S}{2} \right)$$

which, as expected, predicts a massless pion. The corresponding normalized eigenfunctions are given by

$$\phi_{nL}(\zeta) = \kappa^{1+L} \sqrt{\frac{2n!}{(n+L)!}} \zeta^{1/2+L} \exp\left(\frac{-\kappa^2 \zeta^2}{2} \right) L_n^L(x^2 \zeta^2) \, .$$

To completely specify the holographic meson wavefunction, we need the analytic form of the longitudinal mode $\mathcal{X}(x)$. This is obtained by matching the expressions for the pion EM or gravitational form factor in physical spacetime and in AdS space. Either matching consistently results in $X(x) = \sqrt{x(1-x)}$

Meson holographic LFWF

The meson holographic LFWFs for massless quarks can thus be written in closed form:

$$\Psi_{nL}(\zeta, x, \phi) = e^{iL\phi}\sqrt{x(1-x)}(2\pi)^{-1/2}\kappa^{1+L}\sqrt{\frac{2n!}{(n+L)!}}\zeta^L \exp\left(\frac{-\kappa^2\zeta^2}{2}\right) L_n^L(x^2\zeta^2)$$

For non-zero quark mass, Brodsky and de Teramond prescription is to shift the longitudinal mode:

$$X(x) = \sqrt{x(1-x)} \longrightarrow X_{\mathrm{BdT}}(x) = \sqrt{x(1-x)} \exp\left(-\frac{(1-x)m_q^2 + xm_{\bar{q}}^2}{2\kappa^2 x(1-x)}\right)$$

Example: pion LFWF ($m_q = m_{\bar{q}}$)

$$\Psi^\pi(x, \zeta^2) = \mathcal{N}\sqrt{x(1-x)} \exp\left[-\frac{\kappa^2\zeta^2}{2}\right] \exp\left[-\frac{m_q^2}{2\kappa^2 x(1-x)}\right]$$

Meson spectrum

The shift in meson mass when moving away from chiral limit:

$$\Delta M^2_{\mathrm{BdT}} = \int \frac{\mathrm{d}x}{x(1-x)} \times X^2_{\mathrm{BdT}}(x) \left(\frac{m_q^2}{x} + \frac{m_{\bar{q}}^2}{1-x} \right)$$

$$M^2 = M_\perp^2 + \Delta M^2_{\mathrm{BdT}}$$

For pion $M_\perp = 0$ and the above prescription leads to $M_\pi^2 = \Delta M^2_{\mathrm{BdT}} \propto m_q^2$ which is not in agreement with Gell-Mann-Oakes-Renner (GMOR) relation $M_\pi^2 \propto m_q$.
Another problem with this prescription is that it is the same for ground state and excited states mesons.

The 't Hooft Equation

G. 't Hooft, A Two-Dimensional Model for Mesons, Nucl. Phys. B 75 (1974) 461–470

In an earlier approach, 't Hooft derived a Schrödinger-like equation for the longitudinal mode, starting from the QCD Lagrangian in $(1+1)$-dim in the $N_c \gg 1$ approximation. This Lagrangian now contains two mass scales: the quark mass and the gauge coupling. The resulting 't Hooft Equation is:

$$\left(\frac{m_q^2}{x} + \frac{m_{\bar{q}}^2}{1-x}\right)\chi(x) + U_L(x)\chi(x) = M_L^2\chi(x)$$

with

$$U_L(x)\chi(x) = \frac{g^2}{\pi}\mathcal{P}\int \mathrm{d}y \frac{\chi(x)-\chi(y)}{(x-y)^2} \qquad (2)$$

The longitudinal mode $\longrightarrow X(x) = \sqrt{x(1-x)}\chi(x)$

Numerical solutions

Expand the longitudinal mode onto a Jacobi polynomial basis:

$$\chi(x) = \sum_n c_n f_n(x) \tag{3}$$

with

$$f_n(x) = N_n x^{\beta_1}(1-x)^{\beta_2} P_n^{(2\beta_2, 2\beta_1)}(2x-1), \tag{4}$$

where $P_n^{(2\beta_2, 2\beta_1)}$ are the Jacobi polynomials and

$$\begin{aligned} N_n = & \sqrt{(2n + \tilde{\beta}_1 + \tilde{\beta}_2)} \\ & \times \sqrt{\frac{n!\Gamma(n + \tilde{\beta}_1 + \tilde{\beta}_2)}{\Gamma(n + \tilde{\beta}_1 + 1)\Gamma(n + \tilde{\beta}_2)}} \end{aligned} \tag{5}$$

Solve the eigenvalue problem for eigenvalues M_L^2 and eigenvectors $\{c_n\}$.

Predicting Meson spectrum-input parameters

$$M^2(n_L, n_T, J, L) = M_T^2(n_T, J, L) + M_L^2(n_L)$$

Mesons	Light	Heavy-light	Heavy-heavy
g	0.128	0.680	0.523
$m_{u/d}$	0.046	0.046	-
m_s	0.357	0.357	-
m_c	-	1.370	1.370
m_b	-	4.640	4.640

Table: The quark masses and 't Hooft couplings in GeV. Note that we use $\kappa = 0.523$ GeV for all mesons.

Parity and charge conjugation quantum numbers for the meson to be $P = (-1)^{L+1}$ and $C = (-1)^{L+S+n_L}$ respectively.
Finding: $n_L \geq n_T + L$, i.e. in any hadron, an orbital and radial excitations in the transverse dynamics is always accompanied by an excitation in the longitudinal dynamics.

Predicting Meson spectrum: light-light

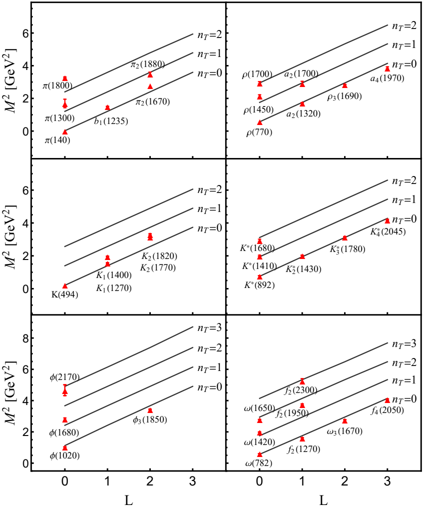

Figure: Our predictions for the Regge trajectories of light mesons. Data from the Particle Data Group.

Predicting Meson spectrum: Heavy-light

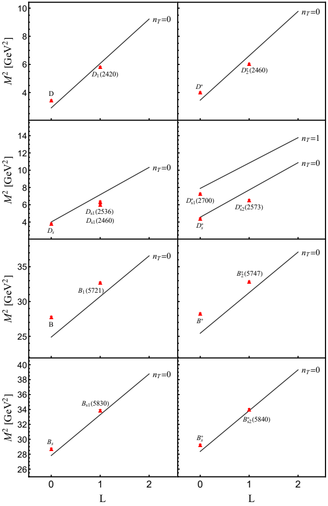

Figure: Our predictions for the Regge trajectories of heavy-light mesons. Data from the Particle Data Group.

Predicting Meson spectrum: Heavy-heavy

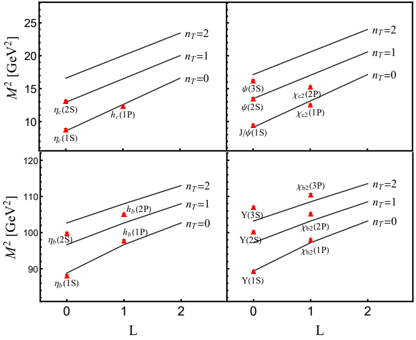

Figure: Our predictions for the Regge trajectories of heavy-heavy mesons. Data from the Particle Data Group.

Conclusion

- The meson spectrum can be very well described by using the holographic Schrödinger Equation in conjunction with the 't Hooft Equation.
- This is achieve with keeping the transverse confinement scale universal across the full spectrum.
- For heavy-heavy mesons, The transverse coincides with the longitudinal confinement scale ('t Hooft coupling), as expected from the restoration of manifest 3-dimensional rotational symmetry in the nonrelativistic limit.