Predicting pion and kaon EM form factors with spin-improved holographic light-front wavefunctions.

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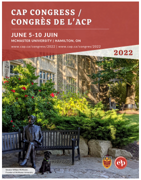

Overview

1. Light-front wavefunction
2. Dynamical spin wavefunction
3. Predictions for radii, decay constants and form factors

LFWF

$$H^{\mathrm{LF}}_{\mathrm{QCD}}|\Psi(P)\rangle = M^2|\Psi(P)\rangle \tag{1}$$

where $H^{\mathrm{LF}}_{\mathrm{QCD}} = P^+P^- - P_\perp^2$ is the LF QCD Hamiltonian and M is the hadron mass. At equal light-front time ($x^+ = 0$) and in the light-front gauge $A^+ = 0$, the hadron state $|\Psi(P)\rangle$ admits a Fock expansion, i.e.

$$|\Psi(P^+, \mathbf{P}_\perp, S_z)\rangle = \sum_{n,h_i} \int [\mathrm{d}x_i][\mathrm{d}^2\mathbf{k}_{\perp i}] \frac{1}{\sqrt{x_i}} \Psi_n(x_i, \mathbf{k}_{\perp i}, h_i)|n : x_i P^+, x_i\mathbf{P}_\perp + \mathbf{k}_{\perp i}, h_i\rangle$$

where $\Psi_n(x_i, \mathbf{k}_{\perp i}, h_i)$ is the LFWF of the Fock state with n constituents and the integration measures are given by

$$[\mathrm{d}x_i] \equiv \prod_i^n \mathrm{d}x_i \delta(1 - \sum_{j=1}^n x_j) \qquad [\mathrm{d}^2\mathbf{k}_{\perp i}] \equiv \prod_{i=1}^n \frac{\mathrm{d}^2\mathbf{k}_{\perp i}}{2(2\pi)^3} 16\pi^3 \delta^2(\sum_{j=1}^n \mathbf{k}_{\perp i}) \,.$$

$(k_i^+, k_i^-, \mathbf{k}_{\perp i})$ and h_i are the momentum and helicity of the i^{th} constituent and $x_i = k_i^+/P^+$.

The valence meson LFWF

For $n = 2$,

$$\mathbf{k}_{\perp 1} = -\mathbf{k}_{\perp 2} = \mathbf{k}_{\perp}$$

$$x_1 = 1 - x_2 = x$$

The position-space conjugate of $\mathbf{k}_{\perp}$, denoted by $\mathbf{b}_{\perp} = b_{\perp} e^{i\varphi}$, is the transverse separation between the quark and the antiquark.

Introduce a new light-front variable $\boldsymbol{\zeta} = \sqrt{x(1-x)}\mathbf{b}_{\perp} = \zeta e^{i\varphi}$ leads to the meson LFWF in the position-space:

$$\Psi(\zeta, x, \phi) \xlongequal{\text{factorization}} \frac{\phi(\zeta)}{\sqrt{2\pi\zeta}} e^{iL\phi} X(x)$$

$\phi(\zeta)$ and $X(x)$ are referred to as the transverse and longitudinal modes.

Holographic Schrödinger equation

Brodsky, de Teramond (PRL, 09)
Brodsky, de Teramond, Dosch, Erlich (Phys. Rep. 15)
In the semi-classical limit, i.e. zero quark mass and no quantum loop, based on AdS/CFT, one can show that the transverse mode of LFWF of the valence ($n = 2$ for mesons) state can be obtained from a 1-dimensional Schrödinger-like wave equation for the:

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U_\perp(\zeta)\right)\phi(\zeta) = M_\perp^2\phi(\zeta)$$

the potential is uniquely determined from the conformal symmetry breaking mechanism and correspondence with weakly coupled string modes in AdS_5 space, which results in a light-front harmonic oscillator potential in physical spacetime with confinement scale κ:

$$U_\perp(\zeta, J) = \kappa^4\zeta^2 + \kappa^2(J-1)$$

$J = L + S$ is the total meson angular momemtum.

Solutions to holographic Schrödinger equation

With the confining potential specified, one can solve the holographic Schrödinger equation to obtain the meson mass spectrum,

$$M^2 = 4\kappa^2 \left(n + L + \frac{S}{2} \right)$$

which, as expected, predicts a massless pion. The corresponding normalized eigenfunctions are given by

$$\phi_{nL}(\zeta) = \kappa^{1+L} \sqrt{\frac{2n!}{(n+L)!}} \zeta^{1/2+L} \exp\left(\frac{-\kappa^2 \zeta^2}{2} \right) L_n^L(x^2 \zeta^2) \ .$$

To completely specify the holographic meson wavefunction, we need the analytic form of the longitudinal mode $X(x)$. This is obtained by matching the expressions for the pion EM or gravitational form factor in physical spacetime and in AdS space. Either matching consistently results in $X(x) = \sqrt{x(1-x)}$

Meson holographic LFWF

The meson holographic LFWFs for massless quarks can thus be written in closed form:

$$\Psi_{nL}(\zeta, x, \phi) = e^{iL\phi}\sqrt{x(1-x)}(2\pi)^{-1/2}\kappa^{1+L}\sqrt{\frac{2n!}{(n+L)!}}\zeta^L \exp\left(\frac{-\kappa^2\zeta^2}{2}\right) L_n^L(x^2\zeta^2)$$

For non-zero quark mass, Brodsky and de Teramond prescription is to shift the longitudinal mode:

$$X(x) = \sqrt{x(1-x)} \longrightarrow X_{\mathrm{BdT}}(x) = \sqrt{x(1-x)} \exp\left(-\frac{(1-x)m_q^2 + x m_{\bar{q}}^2}{2\kappa^2 x(1-x)}\right)$$

Example: pion LFWF ($m_q = m_{\bar{q}}$)

$$\Psi^\pi(x, \zeta^2) = \mathcal{N}\sqrt{x(1-x)} \exp\left[-\frac{\kappa^2\zeta^2}{2}\right] \exp\left[-\frac{m_q^2}{2\kappa^2 x(1-x)}\right]$$

Spin wavefunction

$$\Psi(x,\mathbf{k}) \to \Psi_{h\bar{h}}(x,\mathbf{k}) = S_{h\bar{h}}(x,\mathbf{k})\Psi(x,\mathbf{k})$$

where $S_{h\bar{h}}(x,\mathbf{k})$ corresponds to the helicity wavefunction for a point-like meson-$q\bar{q}$ coupling and, in most general form, can be written as

$$S^{\pi}_{h\bar{h}}(x,\mathbf{k}) = \frac{\bar{v}_{\bar{h}}((1-x)P^+,-\mathbf{k})}{\sqrt{1-x}}\left[(A\not{P} + BM_\pi)\gamma^5\right]\frac{u_h(xP^+,\mathbf{k})}{\sqrt{x}} \,.$$

A and B are constants

Dynamical spin effects

Using the light-front spinors we obtain

$$S^{\pi}_{h\bar{h}}(x,\mathbf{k}) = \left\{ AM_{\pi}^2 + B\left(\frac{m_f M_{\pi}}{x(1-x)}\right)\right\} h\delta_{-h\bar{h}} + B\left(\frac{M_{\pi} k e^{i(2h)\theta_k}}{x(1-x)}\right)\delta_{h\bar{h}}$$

with $\mathbf{k} = ke^{i\theta_k}$

If we take $B=0$, the helicity wavefunction becomes momentum-independent:

$$S^{\pi}_{h\bar{h}}(x,\mathbf{k}) \to S^{\pi}_{h\bar{h}} = \frac{1}{\sqrt{2}} h\delta_{-h\bar{h}}$$

normalized such that $\sum_{h\bar{h}} |S^{\pi}_{h,\bar{h}}|^2 = 1$

We shall refer to this case as the non-dynamical (momentum-independent) helicity wavefunction.

Results: Pion and kaon radii

$$\sqrt{\langle r^2 \rangle} = \left[\frac{3}{2} \int \mathrm{d}x \mathrm{d}^2\mathbf{b} [b(1-x)]^2 |\Psi(x, \mathbf{b})|^2 \right]^{1/2}$$

P	B	$\sqrt{\langle r_P^2 \rangle}_{\rm Th.}$ [fm]	$\sqrt{\langle r_P^2 \rangle}_{\rm Exp.}$ [fm]
$\pi^\pm$	0	0.543 ± 0.022	
	1	0.673 ± 0.034	0.672 ± 0.008
	$\gg 1$	0.684 ± 0.035	
$K^\pm$	0	0.615 ± 0.038	
	1	0.778 ± 0.065	0.560 ± 0.031
	$\gg 1$	0.815 ± 0.070	

Table: Our predictions for the radii of $\pi^\pm$ and $K^\pm$, compared to the measured values from PDG. The theory uncertainties are due to the uncertainties in the constituent quark masses and in the AdS/QCD scale:
$[m_l, m_s] = ([330, 500] \pm 30)$ MeV and $\kappa = (523 \pm 24)$ MeV.

Predictions for pion and kaon decay constants

P	B	$f_P^{\mathrm{Th.}}$ [MeV]	$f_P^{\mathrm{Exp.}}$ [MeV]
$\pi^{\pm}$	0	162 ± 8	$130 \pm 0.04 \pm 0.2$
	1	138 ± 5	
	$\gg 1$	135 ± 6	
$K^{\pm}$	0	156 ± 8	156 ± 0.5
	1	142 ± 7	
	$\gg 1$	135 ± 6	

Table: Our predictions for the decay constants of the charged pion and kaon, compared to the measured values from PDG. The theory uncertainties result from the uncertainties in the constituent quark masses and the AdS/QCD scale: $[m_l, m_s] = ([330, 500] \pm 30)$ MeV and $\kappa = (523 \pm 24)$ MeV.

EM form factor-predictions

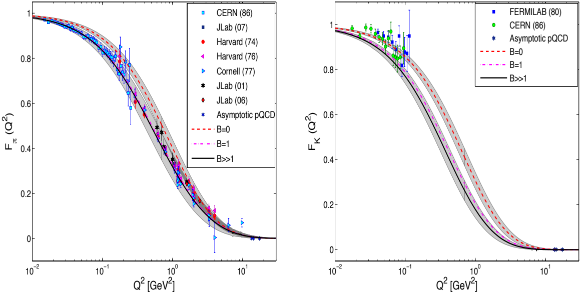

Figure: Our predictions for the pion (left) and kaon (right) EM form factors. Dashed red curves: $B = 0$. Orange dot-dashed curves: $B = 1$. Solid black curves: $B \gg 1$. The grey bands for the $B = 0$ and $B \gg 1$ curves indicate the theory uncertainty resulting from the uncertainties in the constituent quark masses and the AdS/QCD scale. $[m_l, m_s] = ([330, 500] \pm 30)$ MeV and $\kappa = (523 \pm 24)$ MeV.

Conclusion

- Dynamical spin effects are maximal in the pion holographic light-front wavefunction.
- The smaller kaon data set, on the other hand, prefer no dynamical spin effects.

Universal fundamental AdS/QCD scale κ

AdS/QCD scale κ can be chosen to fit the experimentally measured Regge slopes

- $\kappa = 590$ MeV for pseudoscalar mesons and $\kappa = 540$ MeV for vector mesons.
- A fit to the HERA data on diffractive ρ electroproduction, with $m_{u/d} = 140$ MeV, gives $\kappa = 560$ MeV.
- $\kappa = 550$ MeV (with $m_{u/d}[m_s] = 46[140]$ MeV) leads to a good simultaneous description of the HERA data on diffractive ρ and ϕ electroproduction.

Pion's LFWF

In earlier applications of LFH with massless quarks, much lower values of κ were required to fit the pion data:

- $\kappa = 375$ MeV in order to fit the pion EM form factor data Brodsky, de Teramond, PRD 77, 056007 (2008)
- All these previous studies seem to indicate that a special treatment is required at least for the pion either by using a distinct AdS/QCD scale κ or/and relaxing the normalization condition on the holographic wavefunction, i.e. invoking higher Fock states contributions.
- Pion observables are predicted using the holographic wavefunction with the helicity dependence is always assumed to decouple from the dynamics, i.e. the helicity wavefunction is taken to be momentum-independent.

Spin improved wavefunction

A two-dimensional Fourier transform of our spin-improved wavefunction to impact space gives

$$\Psi^{\pi}_{h\bar{h}}(x,\mathbf{b}) = \{(Ax(1-x)M_{\pi}^2 + Bm_f M_{\pi})h\delta_{-h\bar{h}} - BM_{\pi} ih\partial_b \delta_{h\bar{h}}\}\frac{\Psi^{\pi}(x,\zeta^2)}{x(1-x)}$$

which can be compared to the original holographic wavefunction,

$$\Psi^{\pi[\mathrm{o}]}_{h\bar{h}}(x,\mathbf{b}) = \frac{1}{\sqrt{2}} h\delta_{-h\bar{h}}\Psi^{\pi}(x,\zeta^2)$$

EM form factor

Pion EM form factor defined as

$$\langle \pi^+ : P' | J^\mu_{\rm em}(0) | \pi^+ : P \rangle = 2(P+P')^\mu F_\pi(Q^2)$$

$P' = P + q$, $Q^2 = -q^2$ and the EM current $J^\mu_{\rm em}(z) = \sum_f e_f \bar{\Psi}(z)\gamma^\mu \Psi(z)$ with $f = \bar{d}, u$ and $e_{\bar{d},u} = 1/3, 2/3$.
The EM form factor can be expressed in terms of the pion LFWF using the Drell-Yan-West formula:

$$F_\pi(Q^2) = 2\pi \int \mathrm{d}x \mathrm{d}b \; b \; J_0[(1-x)bQ] \; |\Psi^\pi(x, \mathbf{b})|^2$$

Note that the above equation implies that $F_\pi(0) = 1$ and that the slope of the EM form factor at $Q^2 = 0$ is related to the mean radius of the pion via

$$\langle r_\pi^2 \rangle = -\frac{6}{F_\pi(0)} \frac{\mathrm{d}F_\pi}{\mathrm{d}Q^2}\bigg|_{Q^2=0}$$

Decay constant

$$\langle 0|\bar{\Psi}_d \gamma^\mu \gamma_5 \Psi_u|\pi^+\rangle = f_\pi P^\mu$$

Taking $\mu = +$ and expanding the left-hand-side we obtain

$$\langle 0|\bar{\Psi}_d \gamma^+ \gamma^5 \Psi_u|\pi^+\rangle = \sqrt{4\pi N_c} \sum_{h,\bar{h}} \int \frac{\mathrm{d}^2\mathbf{k}}{16\pi^3} \mathrm{d}x \Psi^\pi_{h,\bar{h}}(x,\mathbf{k}) \left\{ \frac{\bar{v}_{\bar{h}}}{\sqrt{1-x}} (\gamma^+\gamma^5) \frac{u_h}{\sqrt{x}} \right\}$$

The light-front matrix element in curly brackets can readily be evaluated:

$$\left\{ \frac{\bar{v}_{\bar{h}}}{\sqrt{1-x}} (\gamma^+\gamma^5) \frac{u_h}{\sqrt{x}} \right\} = 2P^+(2h)\delta_{-h\bar{h}}$$

$$\Rightarrow f_\pi = 2\sqrt{\frac{N_c}{\pi}} \int \mathrm{d}x \{A((x(1-x)M_\pi^2) + B m_f M_\pi\} \frac{\Psi^\pi(x,\zeta)}{x(1-x)}\bigg|_{\zeta=0} .$$