On the Global Extraction of GPDs from DVCS

Nucleon Structure from JLAB and EIC

Kyle Shiells June 9, 2022

Outline

- Introduction: probing nucleon structure
  - Deep inelastic scattering
  - Exclusive processes
  - Spin sum rules of the nucleon
- Process: deeply virtual compton scattering
  - Comparison to other exclusive processes
  - Historical development of the theory
  - Compton form factors and azimuthal dependence
  - Harmonic Analysis

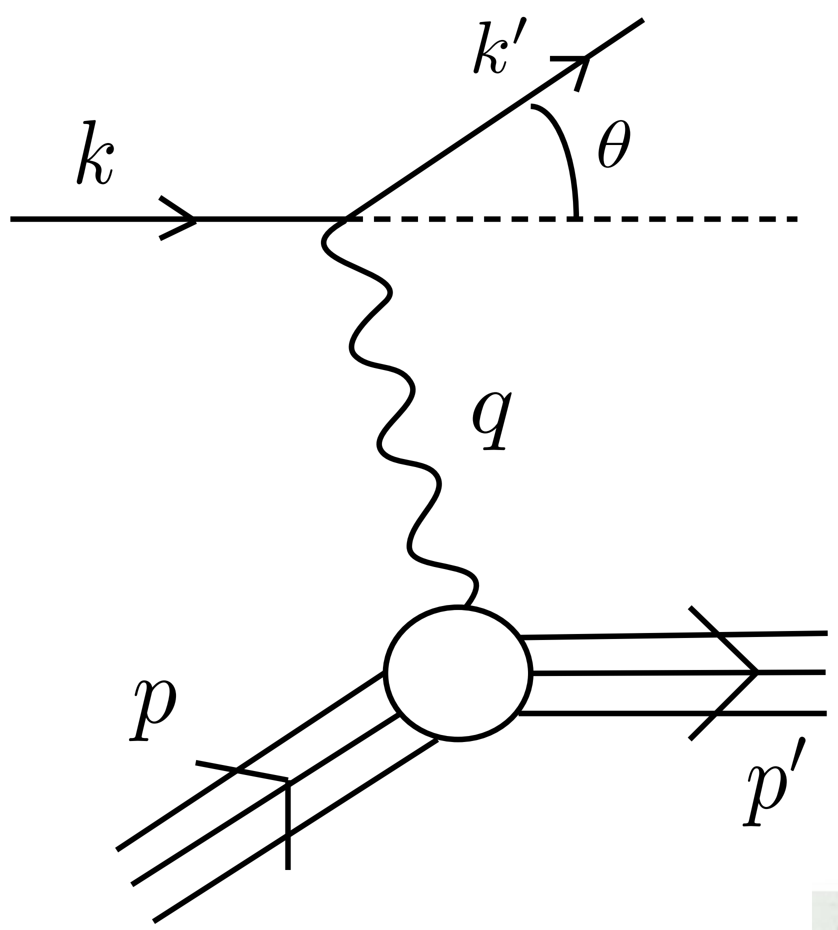
Prerequisites for Global Analysis/phenomenology

- Global analysis: using DVCS and lattice data
  - CFF extraction
  - GPD models
  - Universal moment parameterization
  - Significance of lattice observables
  - Overview of DVCS data and its role
- Conclusion & outlook

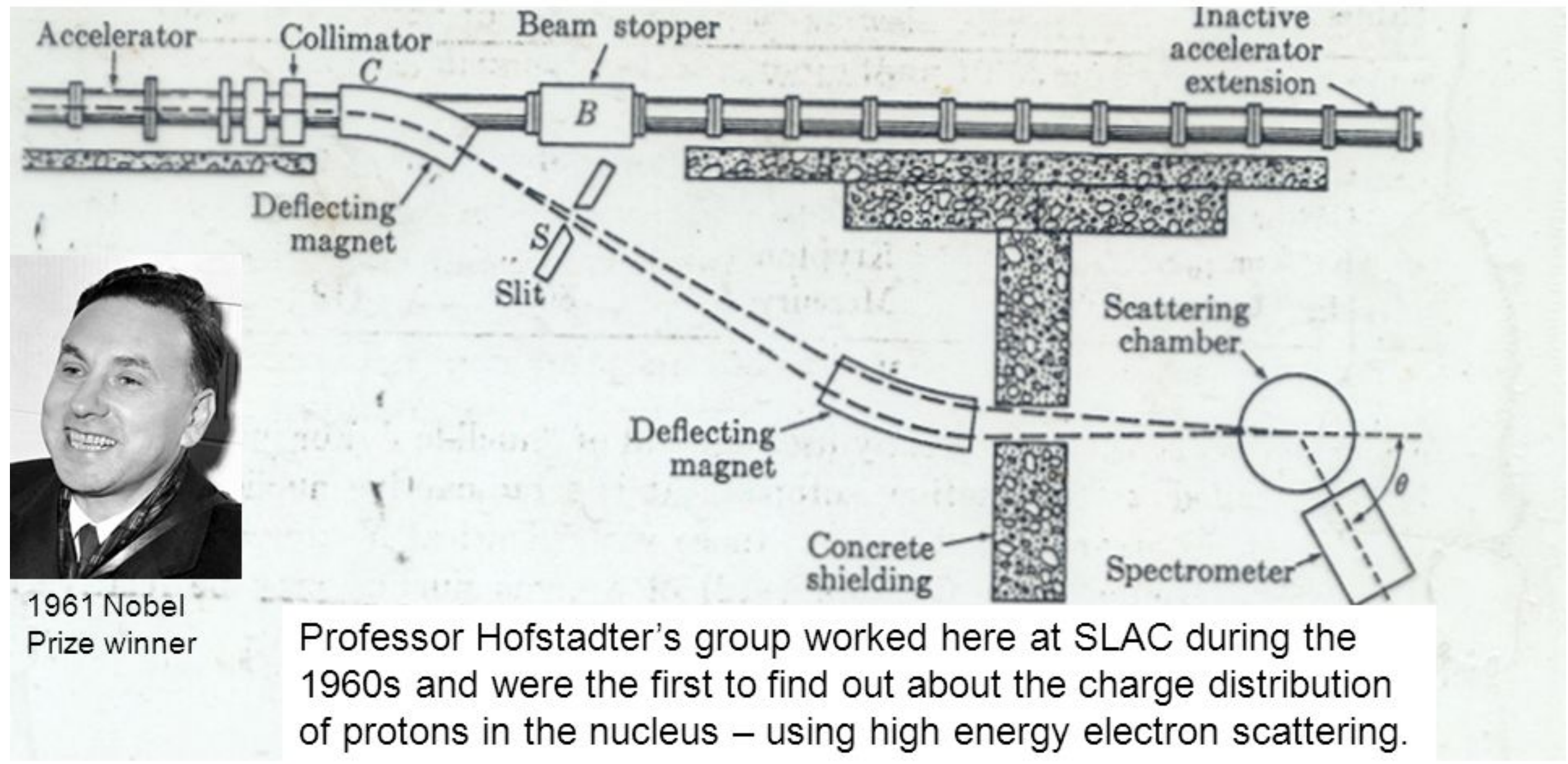
Global Analysis/ phenomenology

Introduction

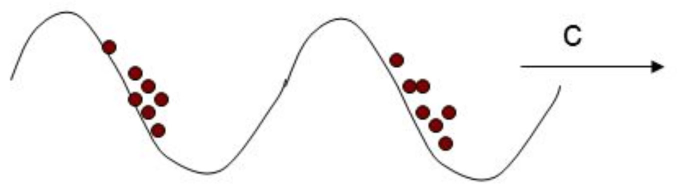
Electron diffraction on Proton



Electron scattering at Stanford 1954 - 57



Professor Hofstadter's group worked here at SLAC during the 1960s and were the first to find out about the charge distribution of protons in the nucleus – using high energy electron scattering.



A linear accelerator LINAC was used to accelerate the electrons

- High energy electrons can probe the short distances in a proton
- Cross section measured by detectors sees an "intensity" pattern

$$\frac{d\sigma}{d\Omega} \sim |F(q)|^2$$

Form factor

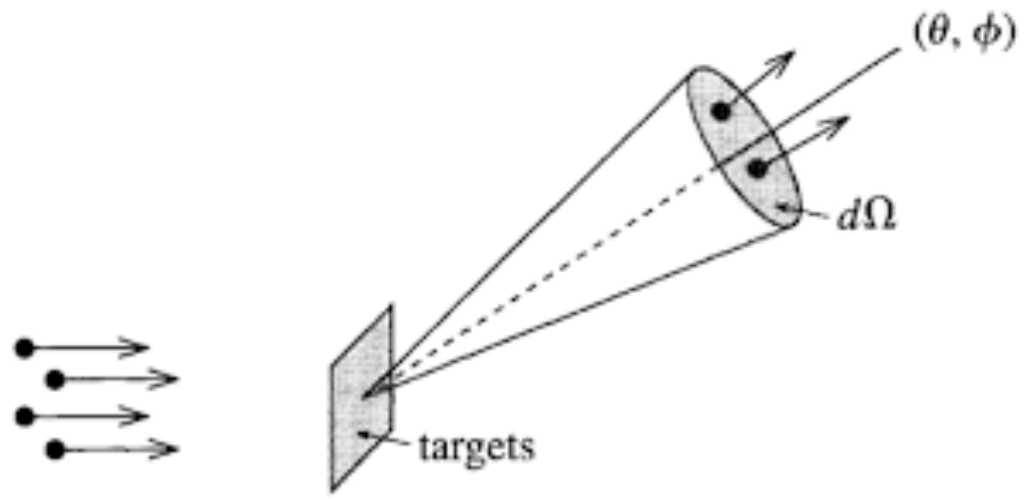
- Nucleon charge distribution can be found by Fourier transform

$$\rho_e(r) = \int F(q) e^{-iq\cdot r} d^3q$$

Further Imaging the Proton

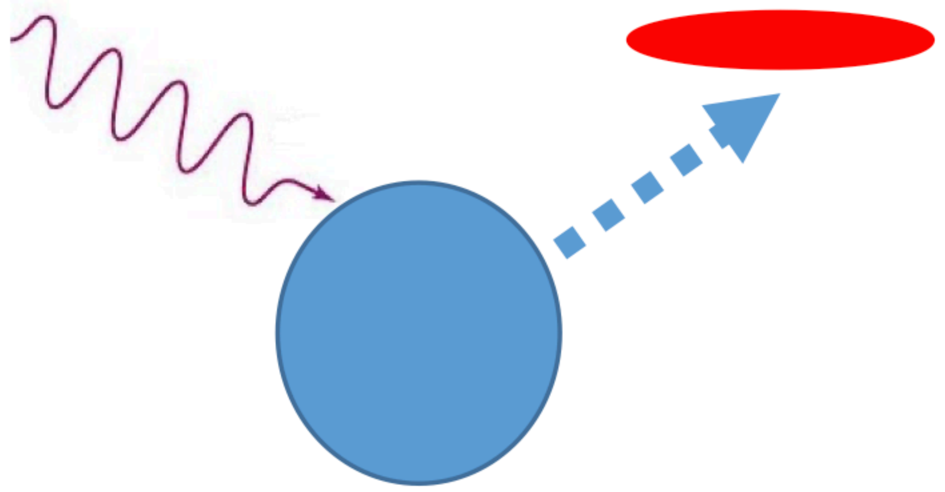
Additional challenges:

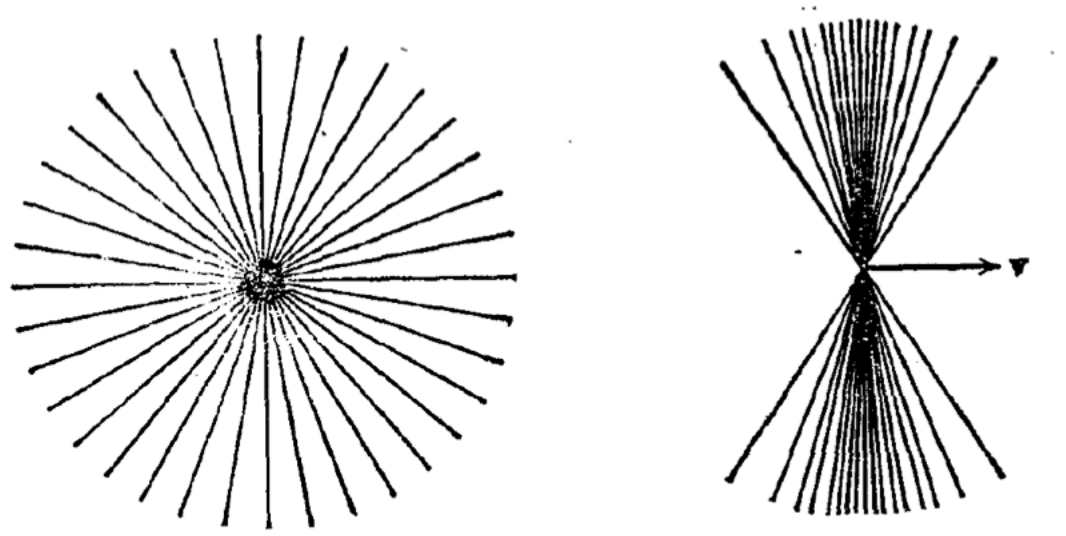
- High energy electrons have energy on the order of proton's mass



- Coherence: smaller cross sections ⇒ lower imaging efficiency

- Recoil: proton recoil makes diffraction pattern harder to relate to spacial distributions

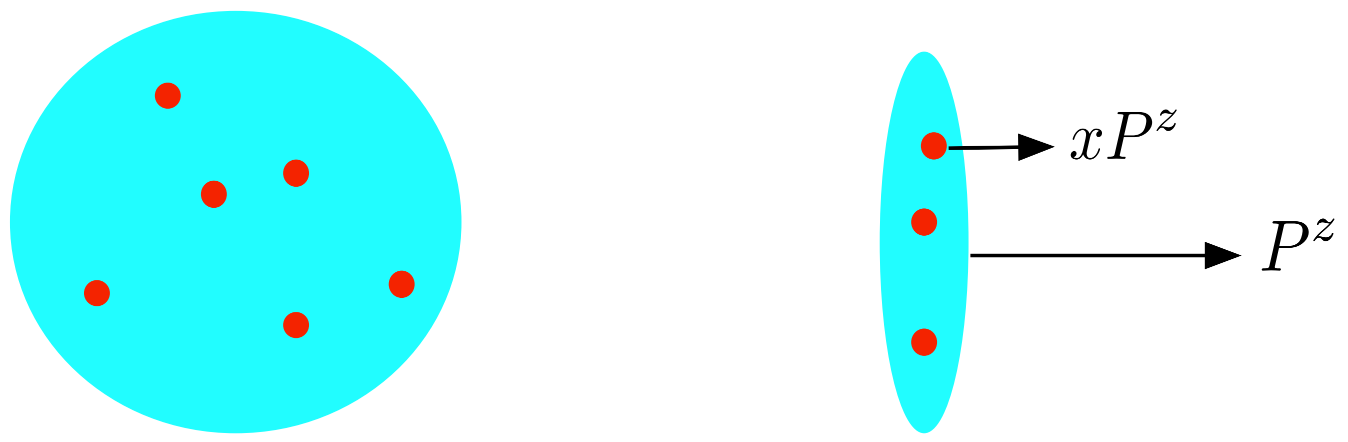




- Due to time dilation the “stuff” inside the proton doesn’t have much time to interact with itself

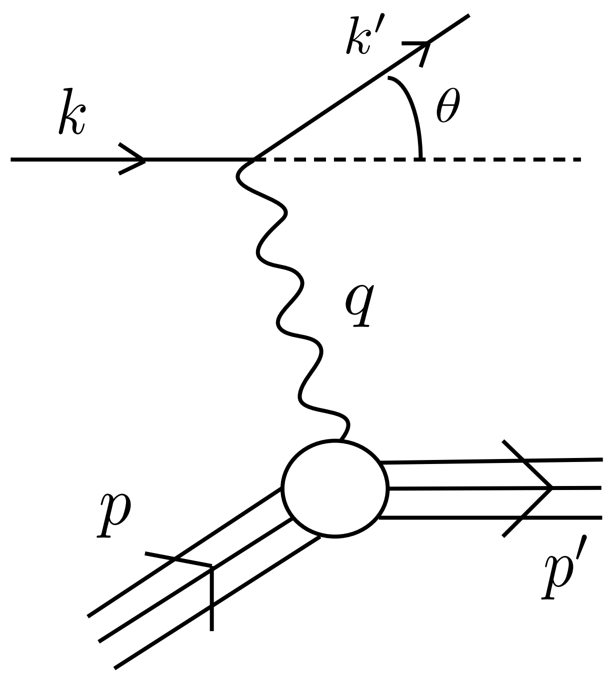
Infinite Momentum Frame:

$M_{eff} = \gamma M \to \infty$



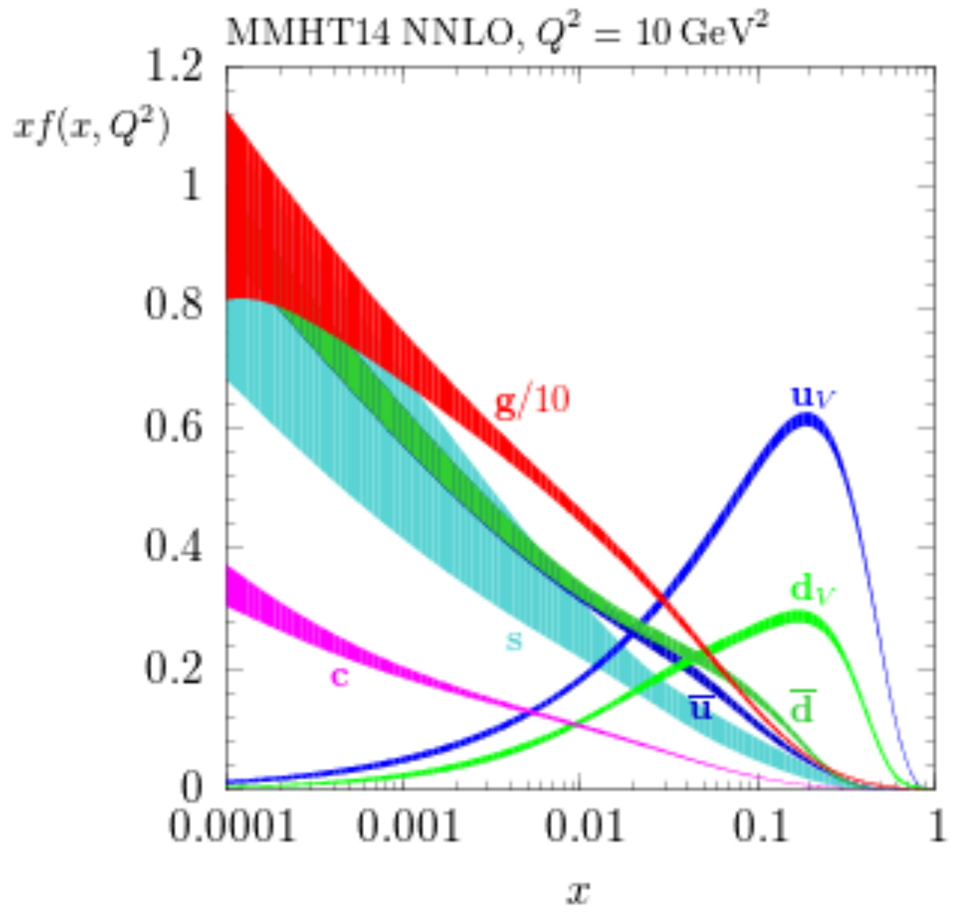
$0 \leq x \leq 1$ Momentum fraction

- Feynman: the densities probed in form factors are related to partons
- Realized in deep inelastic scattering (DIS) at high Q^2



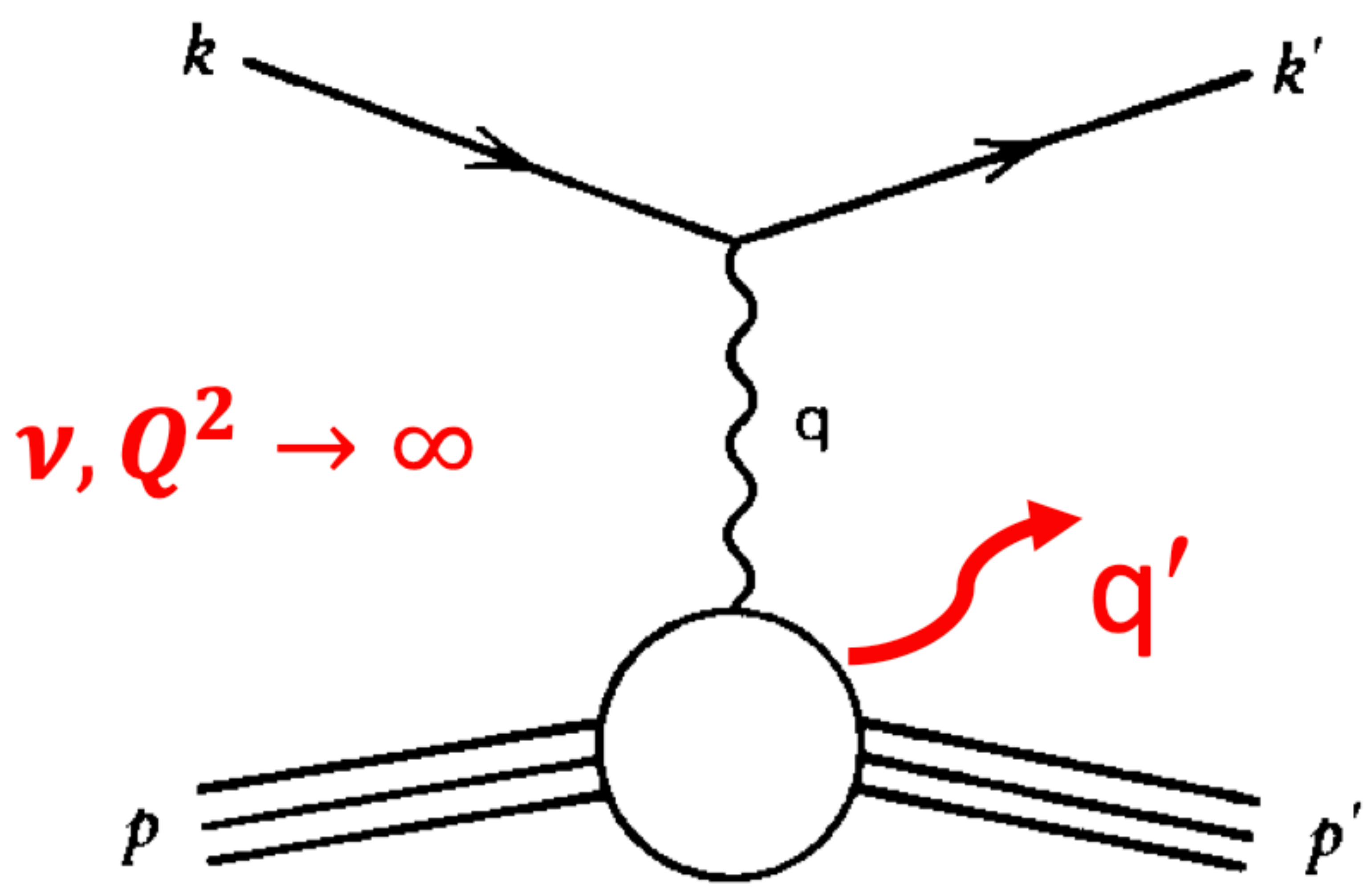
$f = f(x, Q^2)$

Parton distribution function (PDF)



These PDFs here only tell you about the longitudinal momentum of the partons

Deeply Virtual Exclusive Processes



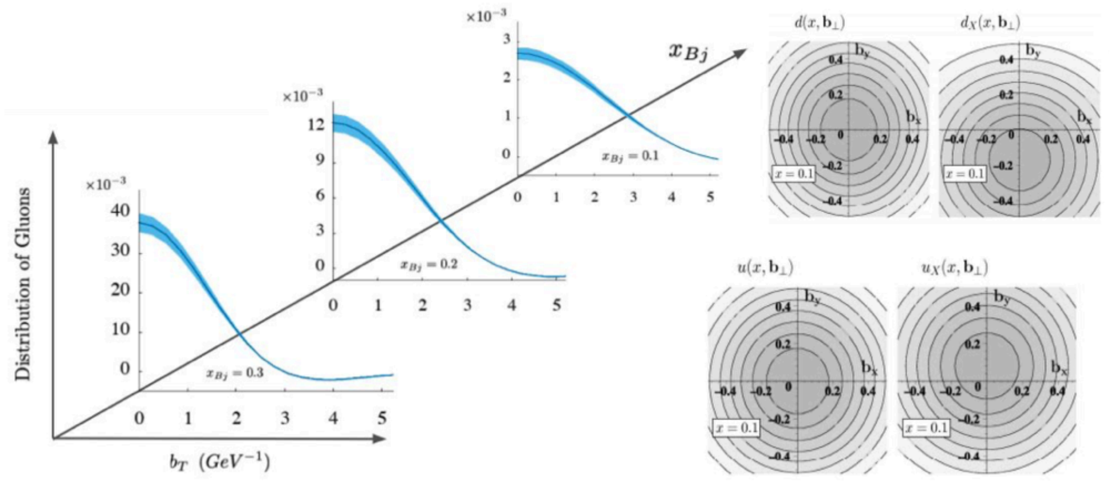
- Now includes the emission of a photon, or a meson
- Includes 2 more kinematic variables

  $f(x, Q^2) \to F(x, \xi, t, Q^2)$ Generalized Parton distribution (GPD)

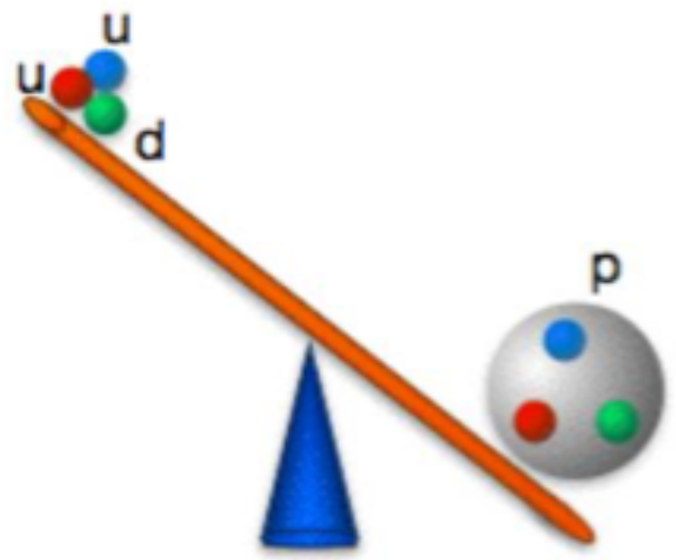
- More challenging than Hofstadter's (Inclusive) process: more difficult to detect all final state particles and much smaller cross sections

- What are GPDs good for?

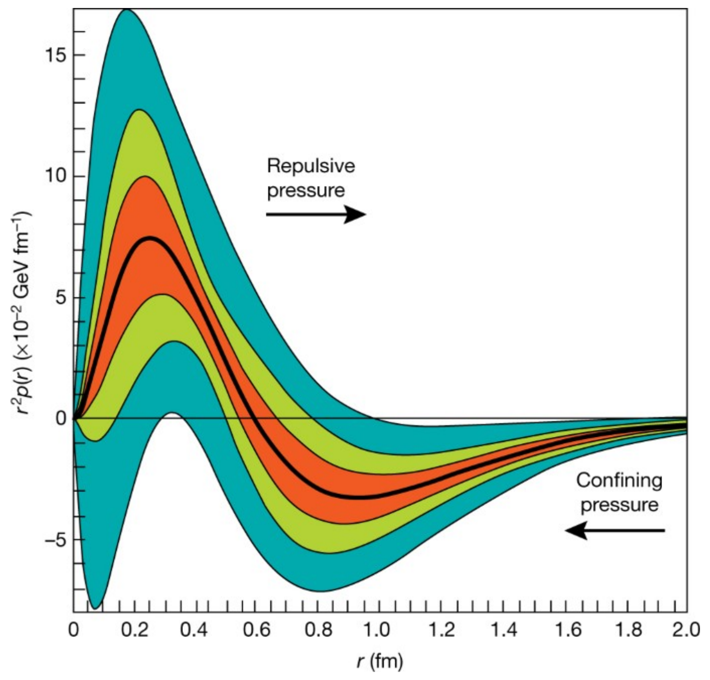
- 2D Imaging of partons in the proton



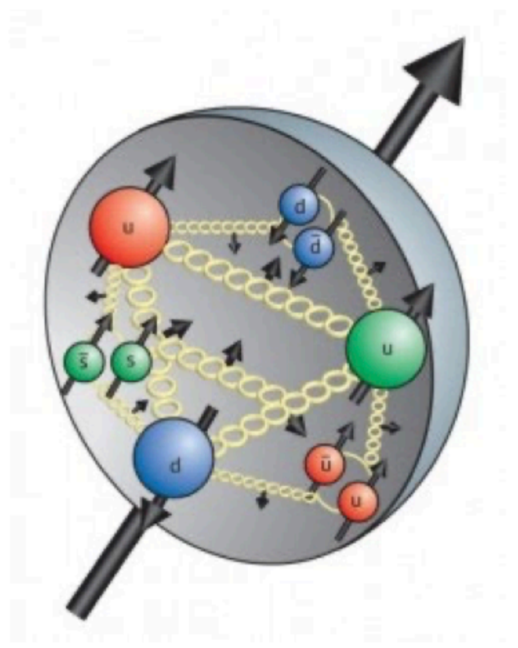
- Origin of the proton's mass



- Pressure distribution inside proton



- Proton spin structure



And more!

Proton Spin Sum Rules

- To solve the problem, we need AM sum rules from QCD

$$\mathcal{L}_{\rm QCD} = -\frac{1}{4}F^{\mu\nu}_a F_{\mu\nu a} + \sum_f \bar{\psi}_f (i\not{D} - m_f)\psi_f$$

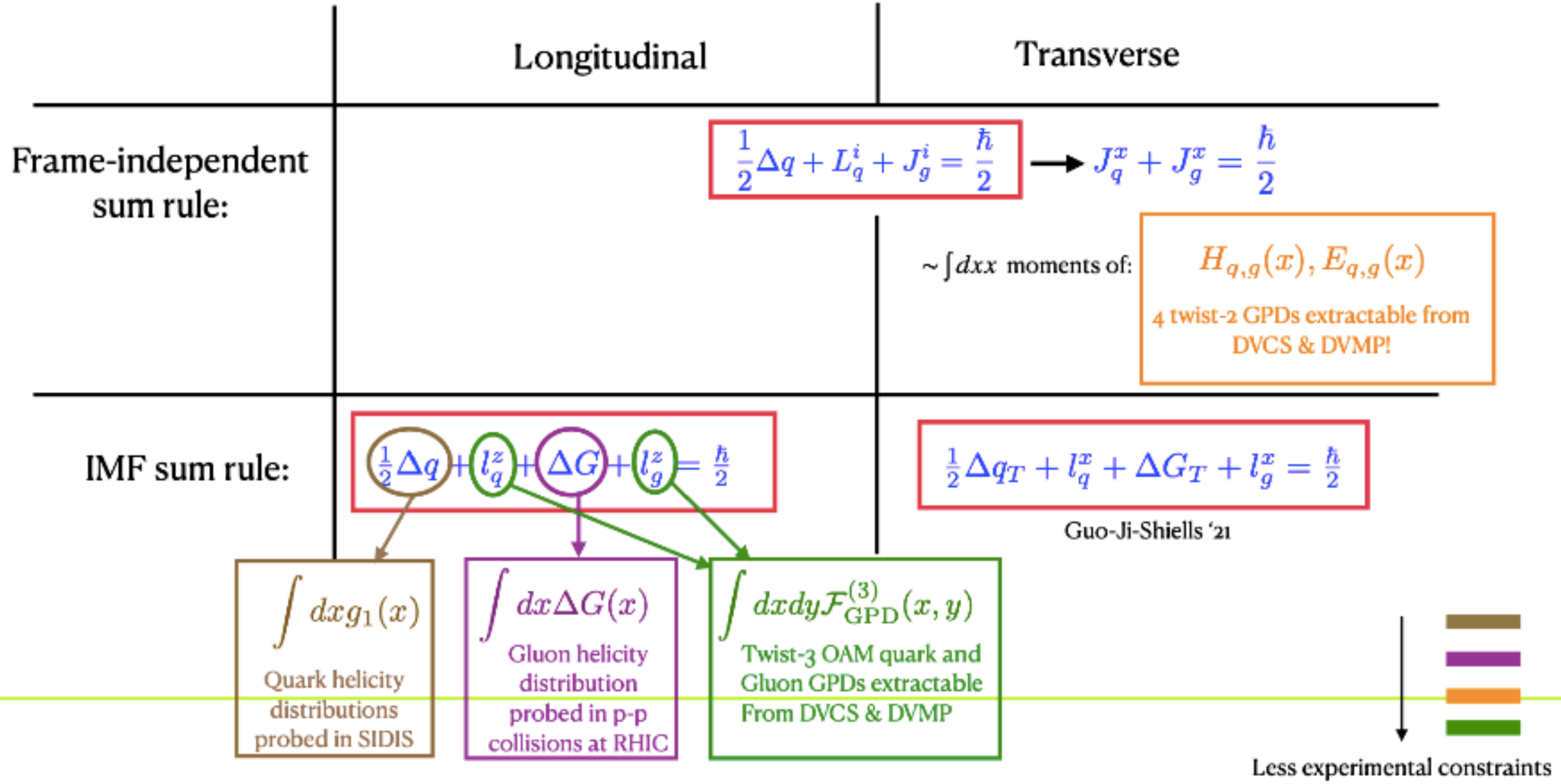
$$T^{\mu\nu} = \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)}\partial^\nu \phi - g^{\mu\nu}\mathcal{L}$$

$$J^i = \frac{\epsilon_{0ikl}}{2}\int d^3x (x^k T^{0l} - x^l T^{0k})$$

$$\langle P, S | J^i | P, S \rangle = \frac{\hbar}{2}$$

- Important factors for deriving sum rules:
  - Nucleon polarization
  - Choosing a frame or be frame-independent
  - Gauge-invariance
  - One needs to isolate INTRINSIC AM from CM contributions

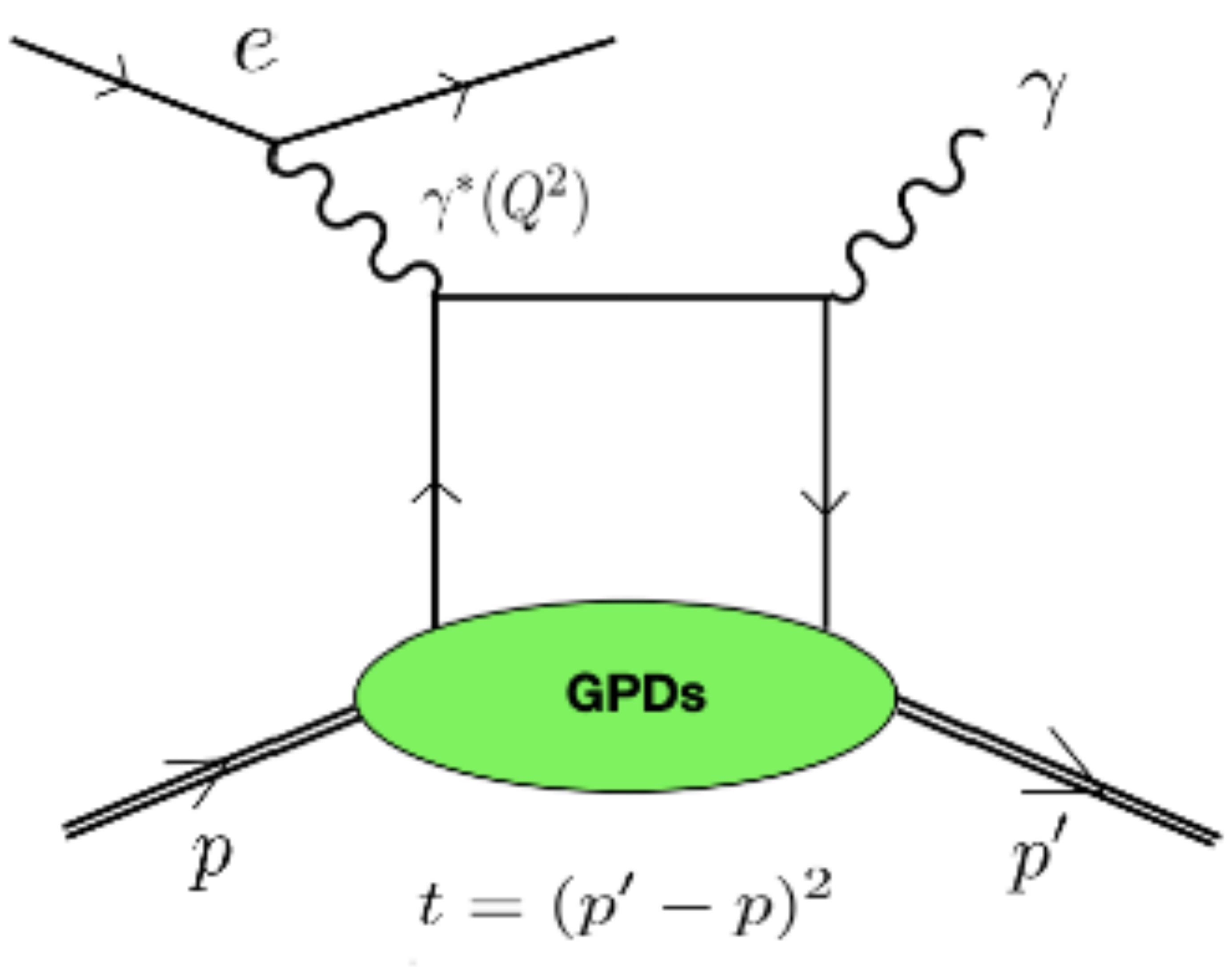
Spin Sum Rule Roadmap



Deeply Virtual Compton Scattering

Choosing an Exclusive Process

- Choosing a scattering process is like a parasite choosing a host
- Candidates: DVCS (γ production), DVMP, DDVCS, TCS, ...
- DVCS is "clean" and has a growing ensemble of data (HERMES, JLAB, COMPASS, ZEUS) and dominated by quark GPDs → H_q, E_q
- DVMP example: J/ψ production (Gluon-X, EIC) = gateway to gluon GPDs → H_g, E_g



Focus on DVCS for extraction twist-2 quark GPDs

Timeline of DVCS Cross Section Calculations

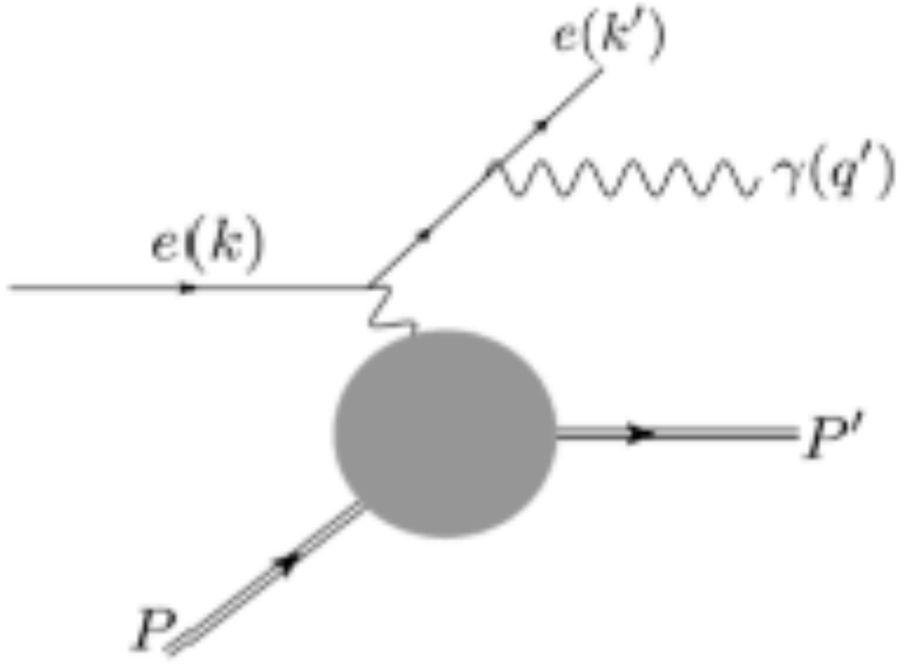
X. Ji, PRD 55 (1997) 7114 **(Ji)**	• First attempt, twist-2
Belitsky, Mueller, Kirchner, Nuc Phys B 629 (2002) 323	• Full twist-2 + WW twist-3, certain light cone choice made, kinematical approximations made, all polarization channels covered
Belitsky, Mueller, Kirchner, Phys Rev D 82 (2010) 074010 **(BMK)**	• Kinematic improvements made to 2001 work, but doesn't cover all polarization channels
Braun, Manashov, Muller, Pirnay, PRD89, (2019) 074022 **(BMMP)**	• Extension of BMK's work, incorporating higher order target and mass corrections
B. Kriesten et al., Phys Rev D 101 (2020) 054021 **(UVa)**	• Genuine twist-3 CFFs used, physics connections to other processes made, all polarizations covered
Y. Guo, X. Ji, K. Shiells, JHEP 12 (2021) 103 **(GSJ)**	• Full twist-2 + WW twist-3, optimal light cone choice found, no kinematical approximations used, all polarization channels covered
Y. Guo, X. Ji, K. Shiells, B. Kriesten (2022) 2202.11114	• Extension of 2021 work with genuine twist-3 CFFs

GSJ Formalism (Guo, Shiells, Ji)

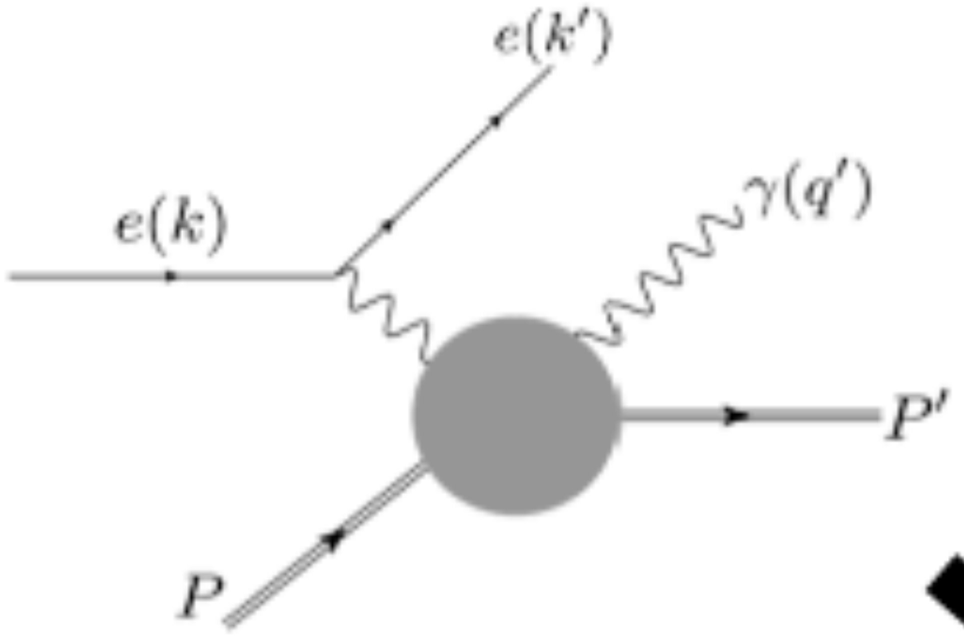
- Considers the 5-fold differential DVCS Cross section $\frac{\mathrm{d}^5\sigma}{\mathrm{d}x_B \mathrm{d}Q^2 \mathrm{d}|t| \mathrm{d}\phi \mathrm{d}\phi_S} = \frac{\alpha_{\mathrm{EM}}^3 x_B y^2}{16\pi^2 Q^4 \sqrt{1+\gamma^2}} |\mathcal{T}|^2$

- Comes from 2 amplitudes:

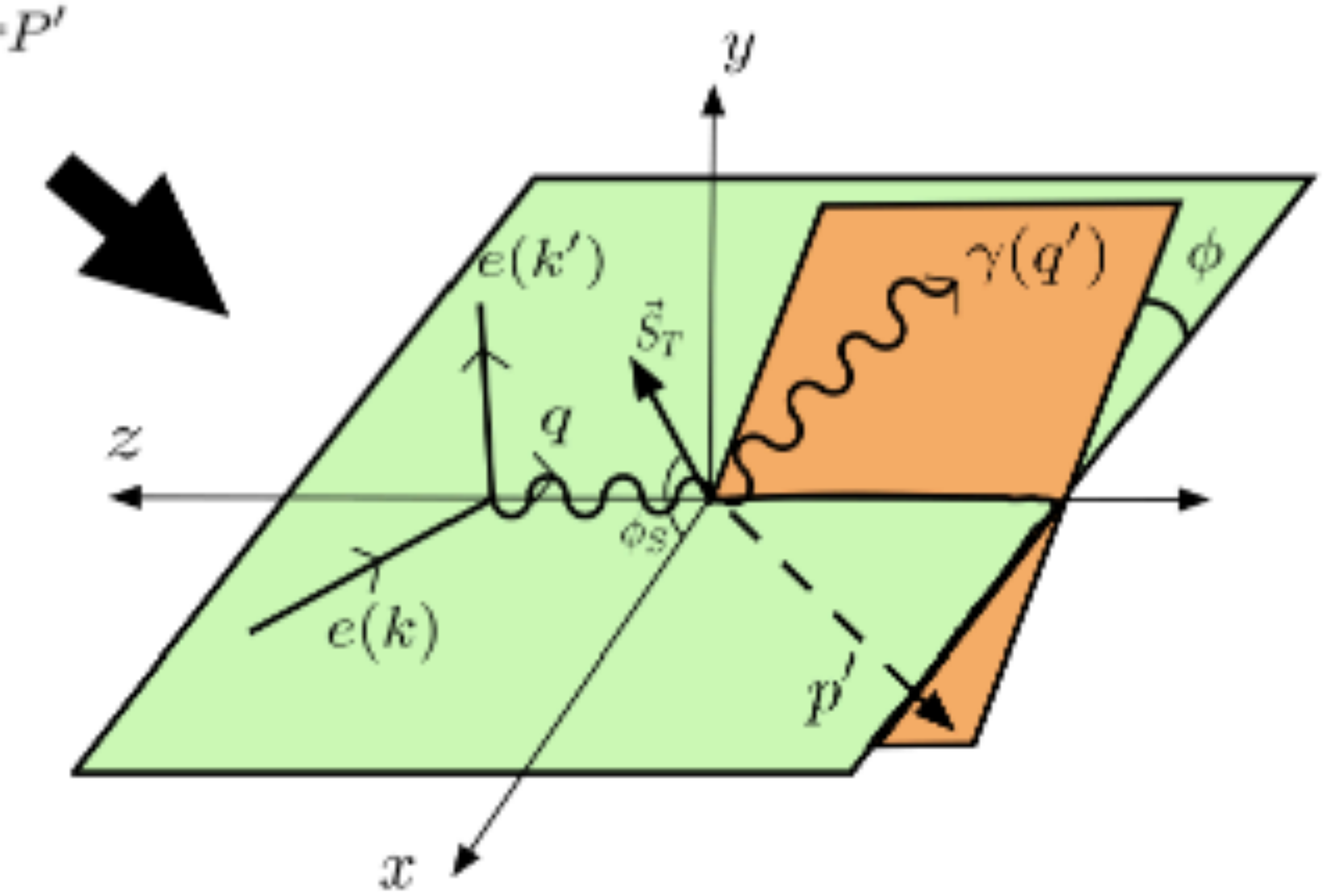
$$\mathcal{T} = \mathcal{T}_{\mathrm{BH}} + \mathcal{T}_{\mathrm{DVCS}}$$



Bethe-Heitler



DVCS



- Bethe-Heitler is QED-calculable
- Twist expands the Compton amplitude with respect to a general light-cone direction, expressing in terms of universally-defined twist-2 quark-quark GPDs
- Allow for a polarized beam and target

- Cannot measure final photon polarization ⇒ sum over its polarizations

- The total cross section has 3 parts:

Bethe-Heitler (QED)

Compton (~ CFF^2)

Interference (CFF)

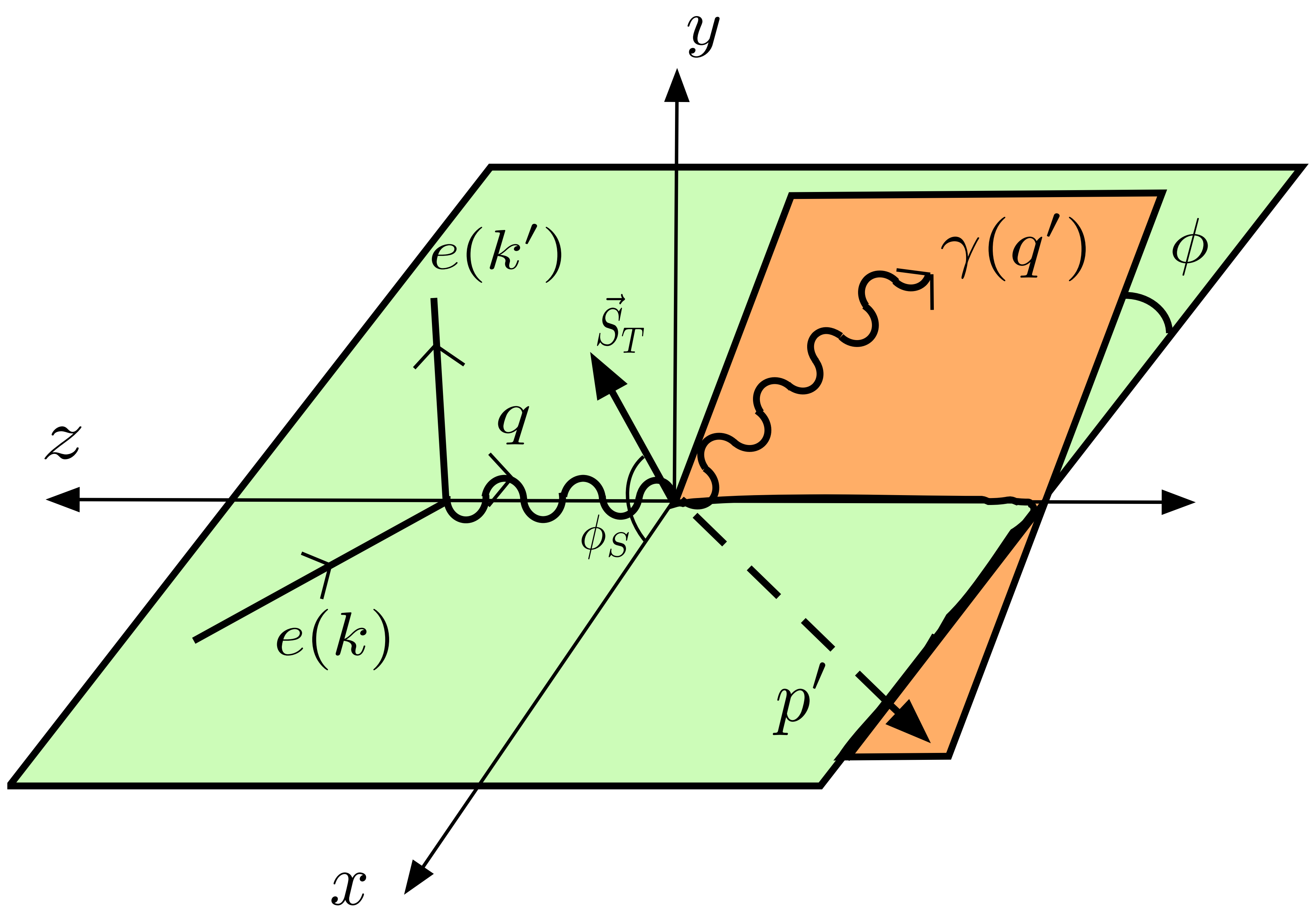
$$\sigma_{\text{Tot}}(y, x_B, t, Q, \phi, \phi_S) \equiv \frac{\mathrm{d}^5\sigma}{\mathrm{d}x_B \mathrm{d}Q^2 \mathrm{d}|t| \mathrm{d}\phi \mathrm{d}\phi_S} = \frac{\alpha_{\text{EM}}^3 x_B y^2}{16\pi^2 Q^4 \sqrt{1+\gamma^2}} \left(|\mathcal{T}_{\text{DVCS}}|^2 + |\mathcal{T}_{\text{BH}}|^2 + \mathcal{I} \right)$$

- We can always express the Compton & Interference σ's into a product of a (scalar coefficient) × (CFF expression)

Example: UU cross section

$$\sigma_{\text{DVCS}}^{UU} = \frac{\Gamma}{Q^4} 4h^U \mathcal{D}_1^{\text{DVCS}}(\mathcal{F}_i^2)$$

$$\sigma_{\mathcal{I}}^{UU} = -\frac{e_l \Gamma}{Q^2 t} \left[A^{I,U} \mathcal{A}_{\text{Re}}^U(\mathcal{F}_i) + B^{I,U} \mathcal{B}_{\text{Re}}^U(\mathcal{F}_i) + C^{I,U} \mathcal{C}_{\text{Re}}^U(\mathcal{F}_i) \right]$$



- Twist-2 dynamics $\quad \mathcal{F}_i \sim \text{Re,Im} \ \{\mathcal{H}, \mathcal{E}, \widetilde{\mathcal{H}}, \widetilde{\mathcal{E}}\}$

= 8 unknowns

- We want data at as many polarization channels as possible:
  UU, LU, UL, LL, 2 × (UT, LT)

= 8 channels

Using Harmonics

- All scalar coefficients can be expressed in terms of harmonic series

e.g.

$$\sigma^{UU}_{\mathrm{DVCS}} = \frac{\Gamma}{Q^4} 4h^U \mathcal{D}_1^{\mathrm{DVCS}}(\mathcal{F}_i^2)$$

$$h^U = \sum_{n=0}^{3} h_n^U \cos(n\phi)$$

$$\sigma^{UU}_{\mathcal{I}} = -\frac{e_l \Gamma}{Q^2 t}\left[A^{I,U} \mathcal{A}^U_{\mathrm{Re}}(\mathcal{F}_i) + B^{I,U} \mathcal{B}^U_{\mathrm{Re}}(\mathcal{F}_i) + C^{I,U} \mathcal{C}^U_{\mathrm{Re}}(\mathcal{F}_i)\right]$$

$$A^{I,U} = \frac{Q^4}{\mathcal{P}_1(\phi)\mathcal{P}_2(\phi)} \sum_{n=0}^{3} a_n^{I,U} \cos(n\phi)$$

...

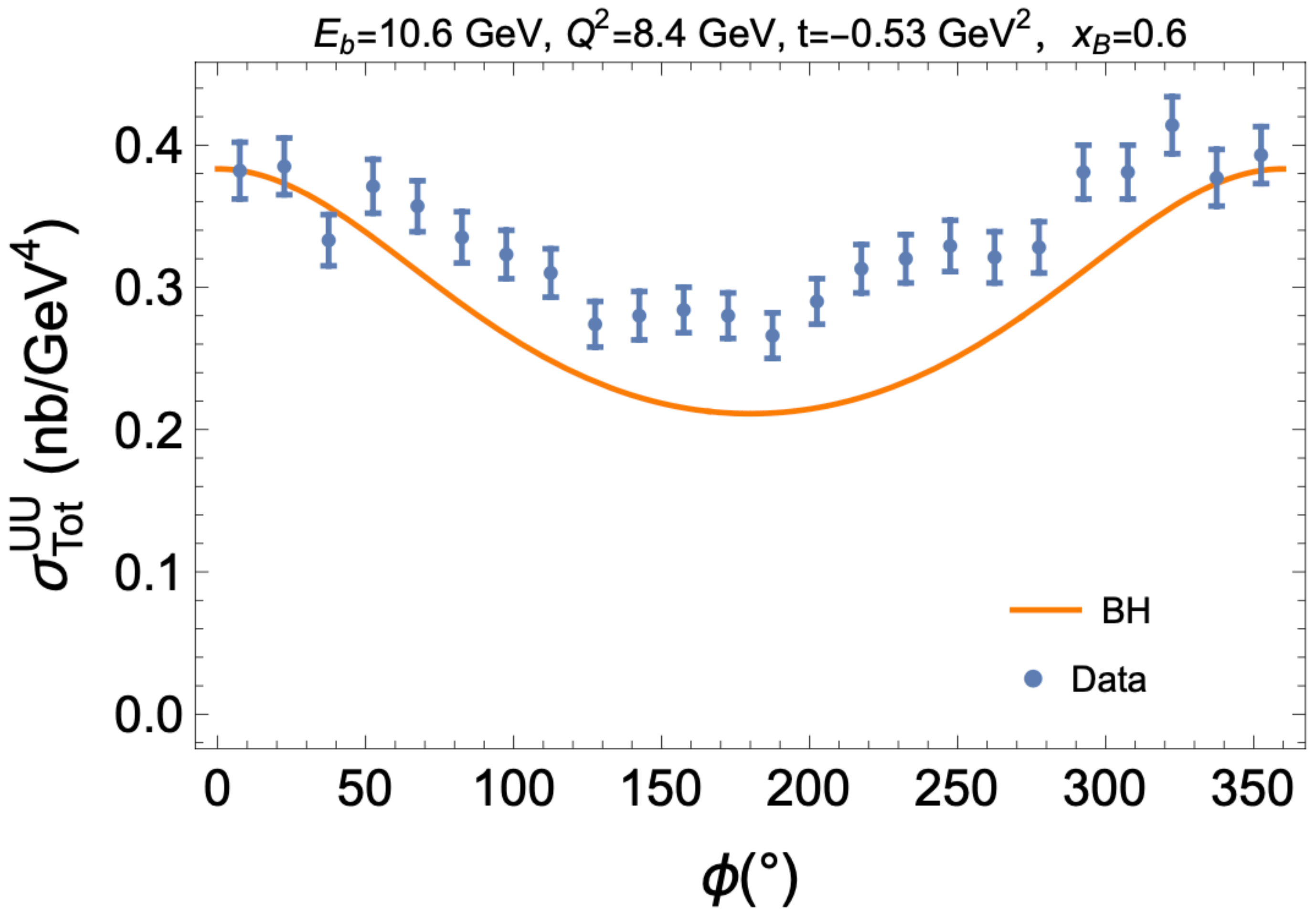
- In general **leading twist dominates the lower-order harmonic coefficients**, while the higher-order harmonics involve higher twist contributions and are kinematically suppressed

- General idea: we can **fit harmonic coefficients to the data**, acquiring equations which constrains the CFFs — this works for both cross sections and asymmetries

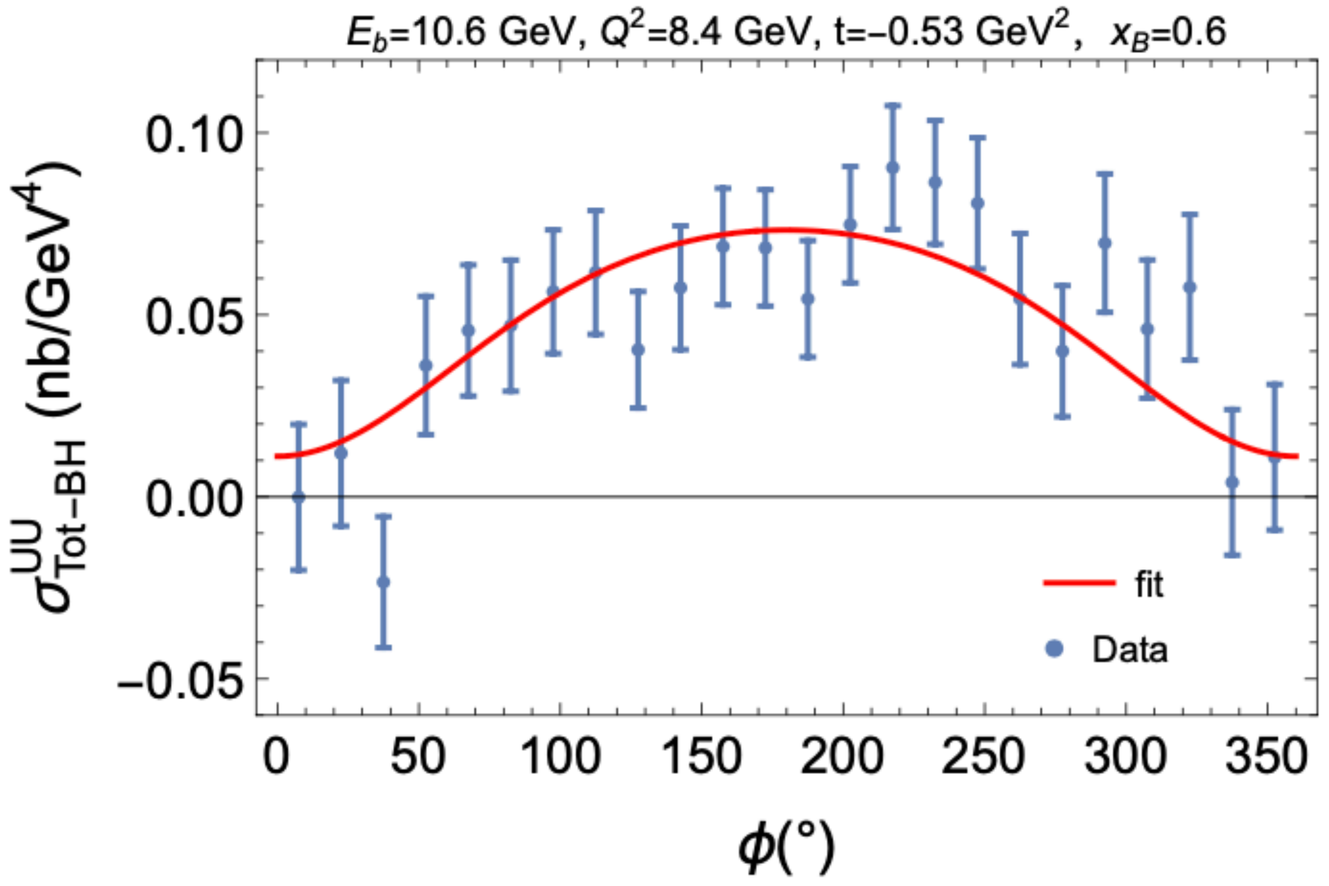
Example of Harmonic Fitting

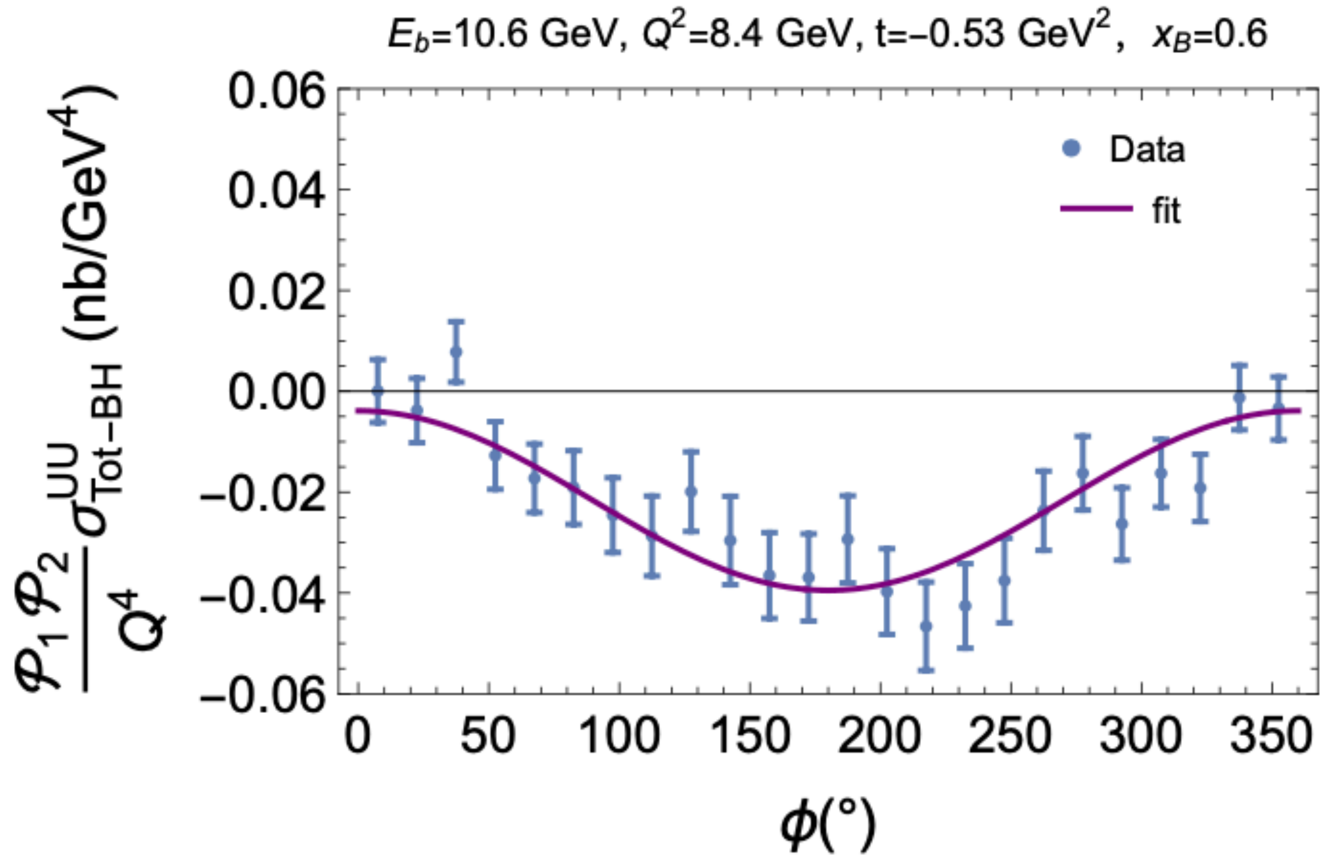
Real DVCS data: (Hall A)

2201.03714



Subtract BH contribution





Fit harmonic coefficients

$$\frac{\mathcal{P}_1\mathcal{P}_2}{Q^4}\sigma^{UU}_{\text{Tot-BH}} = \widetilde{c}_0 + \widetilde{c}_1 \cos\phi$$

Equate to predicted coefficients

$$\frac{\mathcal{P}_1\mathcal{P}_2}{Q^4}\sigma^{UU}_{\text{Tot-BH}} \approx \left[\frac{4\Gamma}{Q^4}(BH \otimes h^U)_0 \mathcal{D}_1^{\text{DVCS}} + \frac{\Gamma}{Q^2 t}\left(a_0^{I,U}\mathcal{A}^U_{\text{Re}} + c_0^{I,U}\mathcal{C}^U_{\text{Re}}\right)\right]\cos(0\phi)$$
$$+ \left[\frac{4\Gamma}{Q^4}(BH \otimes h^U)_1 \mathcal{D}_1^{\text{DVCS}} + \frac{\Gamma}{Q^2 t}\left(a_1^{I,U}\mathcal{A}^U_{\text{Re}} + c_1^{I,U}\mathcal{C}^U_{\text{Re}}\right)\right]\cos(1\phi)$$

Global Extraction of GPDs

Data Analysis: Extraction of twist-2 CFFs

- 3 general approaches:

1. Local Extraction in (x_B, t, Q^2) →

   M. Boer, M. Guidal, JPG Nucl & Part 42 (2015) 034023
   K. Kumericki, D. Muller, M. Murray, Phys of Part & Nucl 45 (2014) 723
   B. Kriesten, S. Liuti, (2020) 2011.04484

2. Global Extraction with ML (no biased model)

   M. Cuic et al., PRL 125 (2020) 232005
   H. Moutarde et al., EPJC 79 (2019) 614
   Grigsby et al. 2012.04801 (2021)

3. Global Extraction with a parametrized model

   ↓

   I'LL FOCUS ON THIS ONE

2112.15144

PREPARED FOR SUBMISSION TO JHEP — CNF-UMD-2021

On Extraction of Twist-Two Compton Form Factors from DVCS Observables Through Harmonic Analysis

Kyle Shiells[a], Yuxun Guo[b] and Xiangdong Ji[a,b]
[a] *Center for Nuclear Femtography, 1201 New York Ave., NW, Washington DC, 20005, USA*
[b] *University of Maryland, College Park, MD 20742 USA*

$$\sigma_{\rm DVCS} = \sigma_{\rm DVCS}\left(x_B, t, Q^2, E_b, \phi, \mathcal{F}_i(x_B, t, Q^2)\right)$$

A few remarks...

- This LOCAL extraction of CFFs is the most **model-independent** first step towards extracting GPDs from DVCS data
- However, as we have **8 unknown parameters**, it's very difficult to get enough data at the same kinematical point for a stable extraction
- Even if we do find the CFFs at several (x_B, t, Q^2) points, we are still left with the difficult **inversion problem** to get GPDs
- This motivates us to directly **model (parameterize) GPDs**, and fit them GLOBALLY to DVCS data

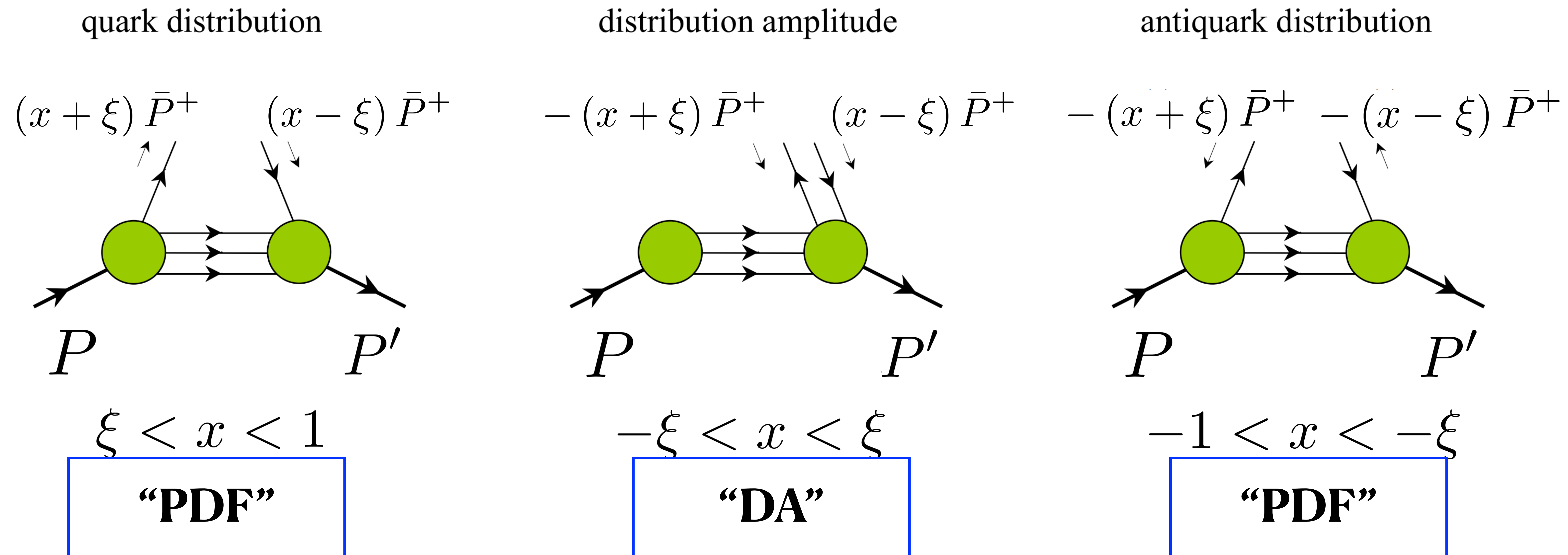
GPD Basics

- Generalized parton distributions are the generalization of parton distributions to include an elastic recoil of nucleon.

$$F_q(x,\xi,t) \sim \int \frac{\mathrm{d}\lambda}{2\pi} e^{i\lambda x} \left\langle P' \middle| \bar{\psi}\left(-\frac{\lambda n}{2}\right) \gamma^+ \psi\left(\frac{\lambda n}{2}\right) \middle| P \right\rangle$$

$\Delta \equiv P' - P \neq 0$ ⟹ $t \equiv \Delta^2 \neq 0$ $\quad \xi \equiv -\Delta \cdot n/(2\bar{P}\cdot n) \neq 0$

- This non-zero momentum transfer will change the physical pictures

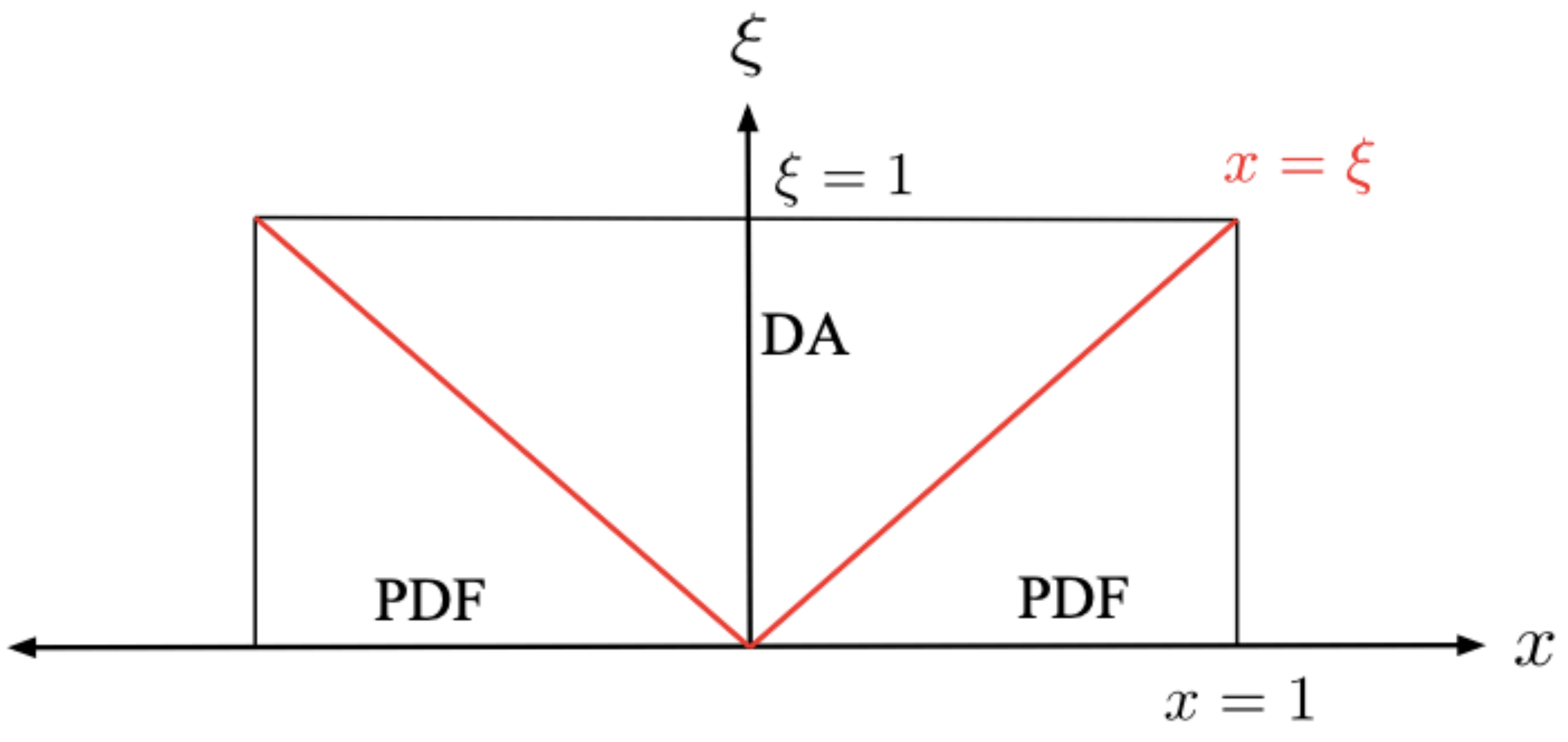


GPD Models

- Double Distributions
  - Integral representation, originally intended as a toy model
  - Since then they've been used extensively
  - Do not work well with scale evolution
  - Cannot fit all the data well

- Dynamical models
  - Assumes a physical sub-process for the GPDs
  - Can be difficult to reconcile with the PDF and DA interpretations
  - May or may not scale evolve well

- Conformal Moment Expansion
  - Opposite to dynamical models, they are very mathematical in construction
  - However they're very general and meet required GPD properties
  - Scale evolve very nicely



Conformal moment expansion

Therefore, we expansion GPD in terms of its conformal moments

$$F(x,\xi,t)=\sum_n (-1)^n p_n\left(\frac{x}{\xi}\right)\xi^{-n-1}\mathcal{F}_n(\xi,t)$$

$$|\psi\rangle=\sum_n |n\rangle\langle n|\,\psi\rangle$$

need to model this function now

where the so-called conformal wave functions are

$$p_n(|x|<1)=\frac{2^n\Gamma\left(\frac{5}{2}+n\right)}{\Gamma\left(\frac{3}{2}\right)\Gamma(3+n)}\left(1-x^2\right)C_n^{\frac{3}{2}}(-x)$$

problem: $p_n(x)$ only defined for $|x|\leq 1$ solution: analytically continue to $|x|>1$ with the help of the Schlafli integral

$$p_j(x,\xi)=-\frac{\Gamma(5/2+j)}{\Gamma(1/2)\Gamma(2+j)}\frac{1}{2i\pi}\oint_{-1}^{1}du\frac{\left(u^2-1\right)^{j+1}}{(x+u\xi)^{j+1}}$$

$C_n^{\frac{3}{2}}(x)$ are Gegenbauer polynomials, and they renormalize multiplicatively under LO evolution kernel

$$\int_{-1}^{1}\frac{\mathrm{d}x'}{|\xi|}\left[V\left(\frac{x}{\xi},\frac{x'}{\xi}\right)\right]_+ C_j^{\frac{3}{2}}\left(\frac{x}{\xi}\right)=\gamma_j C_j^{\frac{3}{2}}\left(\frac{x'}{\xi}\right)$$

Modeling the small - ξ dependence

We can start to put in the xi-dependence with the polynomiality condition in mind.

$$\mathcal{F}_j(\xi, t) = \sum_{l=0}^{2l \leq (j+1)} \xi^{2l} \bar{\mathcal{F}}_{jl}(t) \quad \xrightarrow{\text{small } \xi} \quad \mathcal{F}_j(\xi, t) = \sum_{k=0,\text{even}}^{k_{\text{cut}}} \xi^k \mathcal{F}_{j,k}(t)$$

- ❑ To start with we can take the (semi-)forward limit, which must obey a certain Regge behaviour

$$\lim_{\xi \to 0} \mathcal{F}_j(\xi, t) = \bar{\mathcal{F}}_{j0}(t)$$

- ❑ Then one could add more t-dependent functions for the higher order ξ terms

$$\lim_{\xi \to 0} \mathcal{F}_j(\xi, t) = \bar{\mathcal{F}}_{j0}(t) + \xi^2 \bar{\mathcal{F}}_{j2}(t) + \cdots$$

GUMP: GPDs from Universal Moment Parameterization

- The motivation is the Mellin moments modeling

PDF $q(x) = N x^{-\alpha}(1-x)^{\beta}$ $\xrightarrow{q_J = \int_0^1 \frac{\mathrm{d}x}{x} x^J q(x)\,,}$ $q_J = N_0 B(J-\alpha, 1+\beta)$

- We have the following model of conformal moments

$$B(x,y) = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)}$$

$$\mathcal{F}_j(\xi=0,t) = N_0 \frac{B(1+j-\alpha, 1+\beta)}{B(1-\alpha, 1+\beta)} \frac{1+j-\alpha}{1+j-\alpha(t)} \beta(t)$$

(KM Model)

such that $$\lim_{t,\xi\to 0} \mathcal{F}_j(\xi,t) = \frac{B(1+j-\alpha, 1+\beta)}{B(1-\alpha, 1+\beta)}$$

$$\alpha(t) = \alpha + \alpha' t$$

Regge trajectory

- Simplest model includes 4 parameters/flavour/order of ξ

$$\mathcal{F} \sim \boxed{N, \alpha, \beta, \alpha'}$$

Putting it all together

$$F(x,\xi,t) = \sum_n (-1)^n p_n\left(\frac{x}{\xi}\right)\xi^{-n-1}\mathcal{F}_n(\xi,t)$$

Conformal wave functions spell out x/ξ dependence

$$p_j(|x|\le\xi,\xi) = \frac{2^{j+1}\Gamma(5/2+j)\xi^{-j-1}}{\Gamma(1/2)\Gamma(1+j)}(1+x/\xi)_2F_1\left(-1-j,j+2,2\,\middle|\,\frac{\xi+x}{2\xi}\right),$$

and

$$p_j(x>\xi,\xi) = \frac{\sin(\pi[j+1])}{\pi}x^{-j-1}{}_2F_1\left(\begin{matrix}(j+1)/2,(j+2)/2\\ 5/2+j\end{matrix}\middle|\,\frac{\xi^2}{x^2}\right).$$

Remaining ξ, t dependence expanded into polynomial in ξ and phenomenological t-dep FF

$$\mathcal{F}_j(\xi,t) = \sum_{k=0,\text{even}}^{k_{\text{cut}}}\xi^k\mathcal{F}_{j,k}(t)$$

$$\mathcal{F}_{j,k}(t) = N_k B(j+1-\alpha_k,1+\beta_k)\frac{j+1-k-\alpha_k}{j+1-k-\alpha_k+\alpha'_k t}$$

Mellin -Barnes Integral Representation

$$F(x,\xi,t) = \frac{1}{2i}\int_{c-i\infty}^{c+i\infty}dj\,\frac{p_j(x,\xi)}{\sin(\pi[j+1])}\mathcal{F}_j(\xi,t)$$

Becomes inverse Mellin moment in forward limit!

$$f(x) = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty}x^{-s}f_s\mathrm{d}s$$

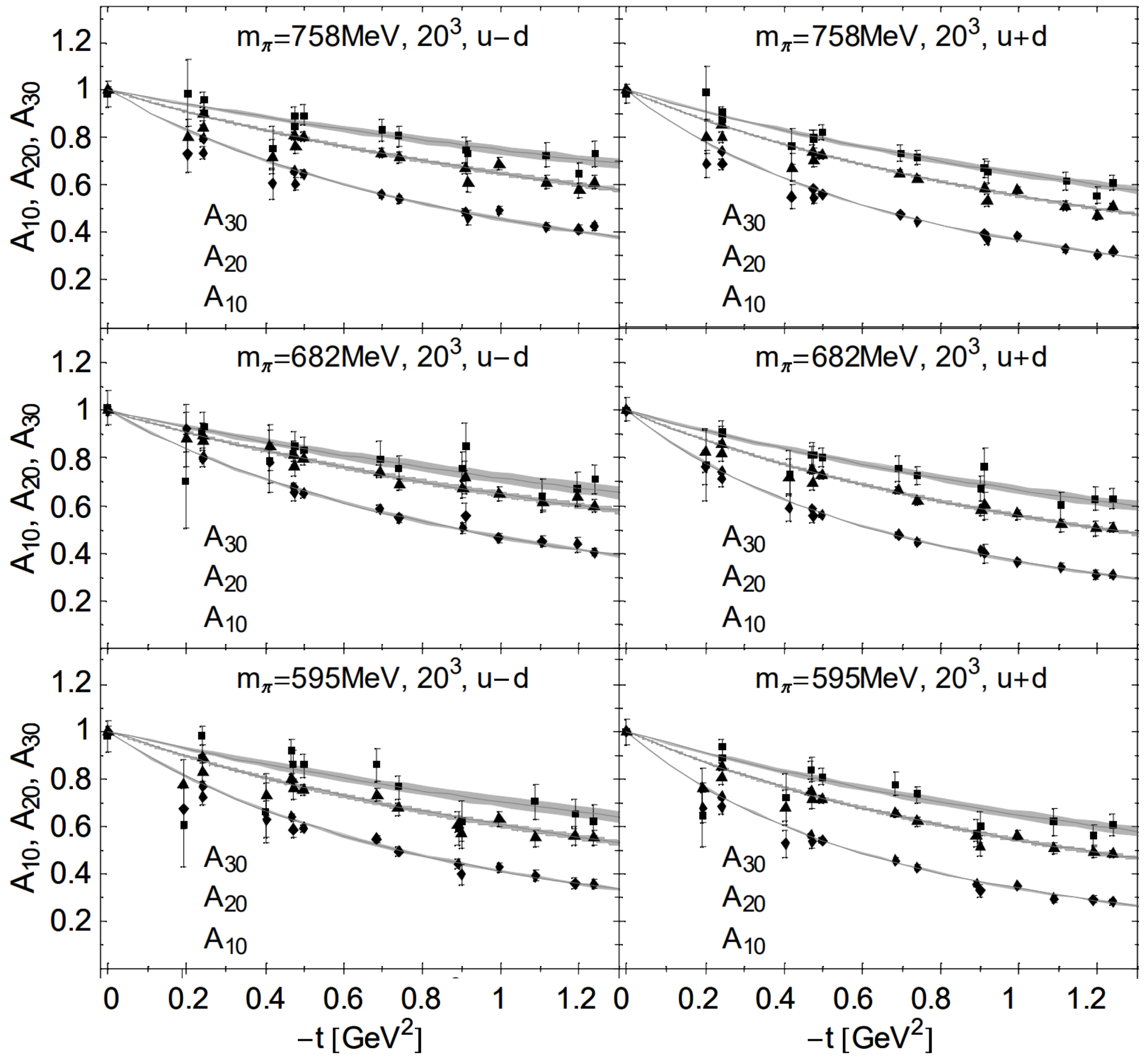
Lattice Data

- There are four twist-2 GPDs for quarks, and their moments can be written as

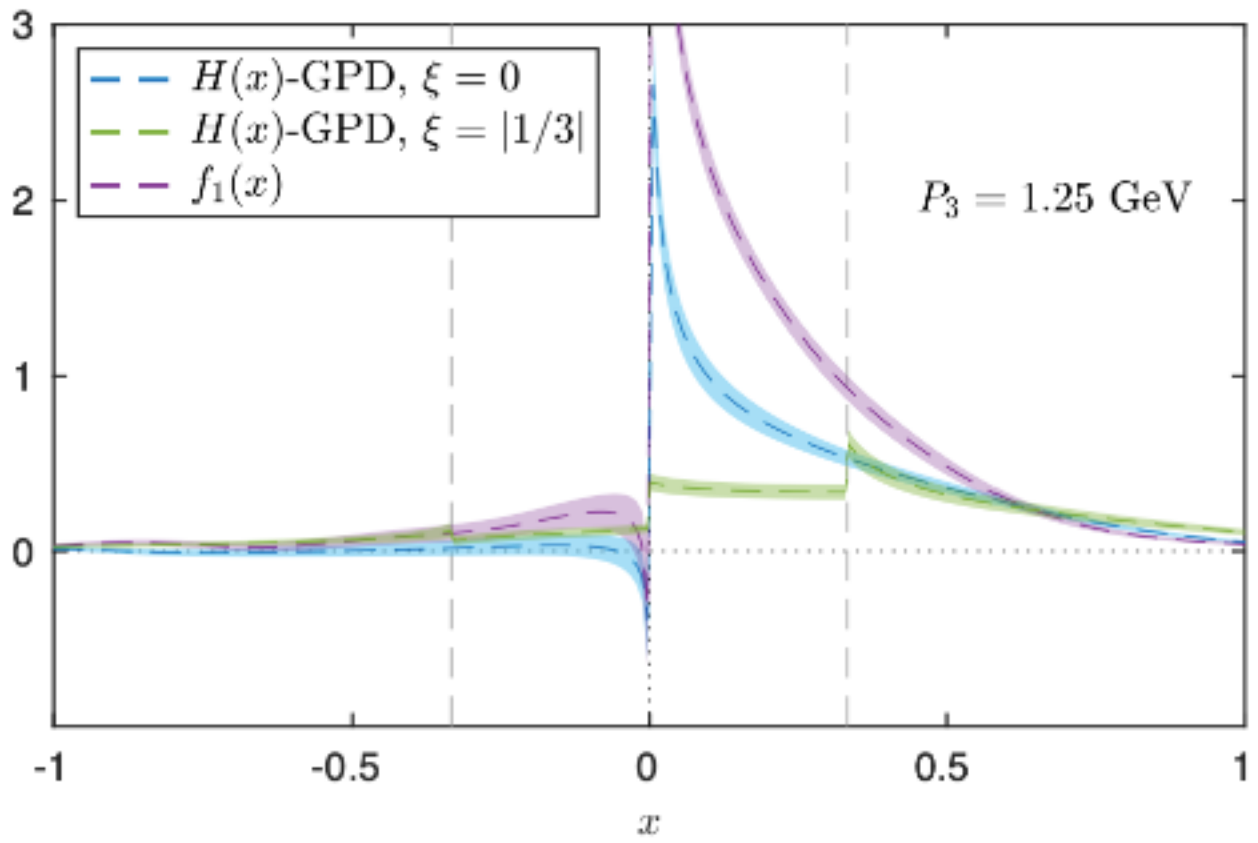
$$\int_{-1}^{+1} dx x^{n-1} H(x,\xi,t) = \sum_{i=0,\text{even}}^{n-1} (-2\xi)^i A_{ni}(t) + (-2\xi)^n C_{n0}(Q^2)\Big|_{n \text{ even}} ,$$

$$\int_{-1}^{+1} dx x^{n-1} E(x,\xi,t) = \sum_{i=0,\text{even}}^{n-1} (-2\xi)^i B_{ni}(t) - (-2\xi)^n C_{n0}(Q^2)\Big|_{n \text{ even}} ,$$

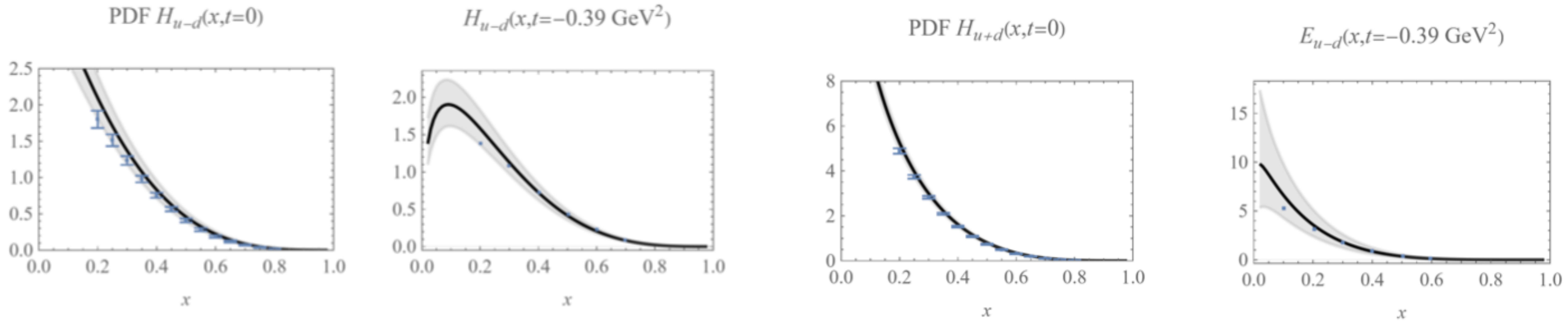
are related to the generalized form factors (GFFs).



- Thanks to large momentum effective theory (LaMET), GPDs can also be explicitly computed over *x* at fixed (*t*, *ξ*)

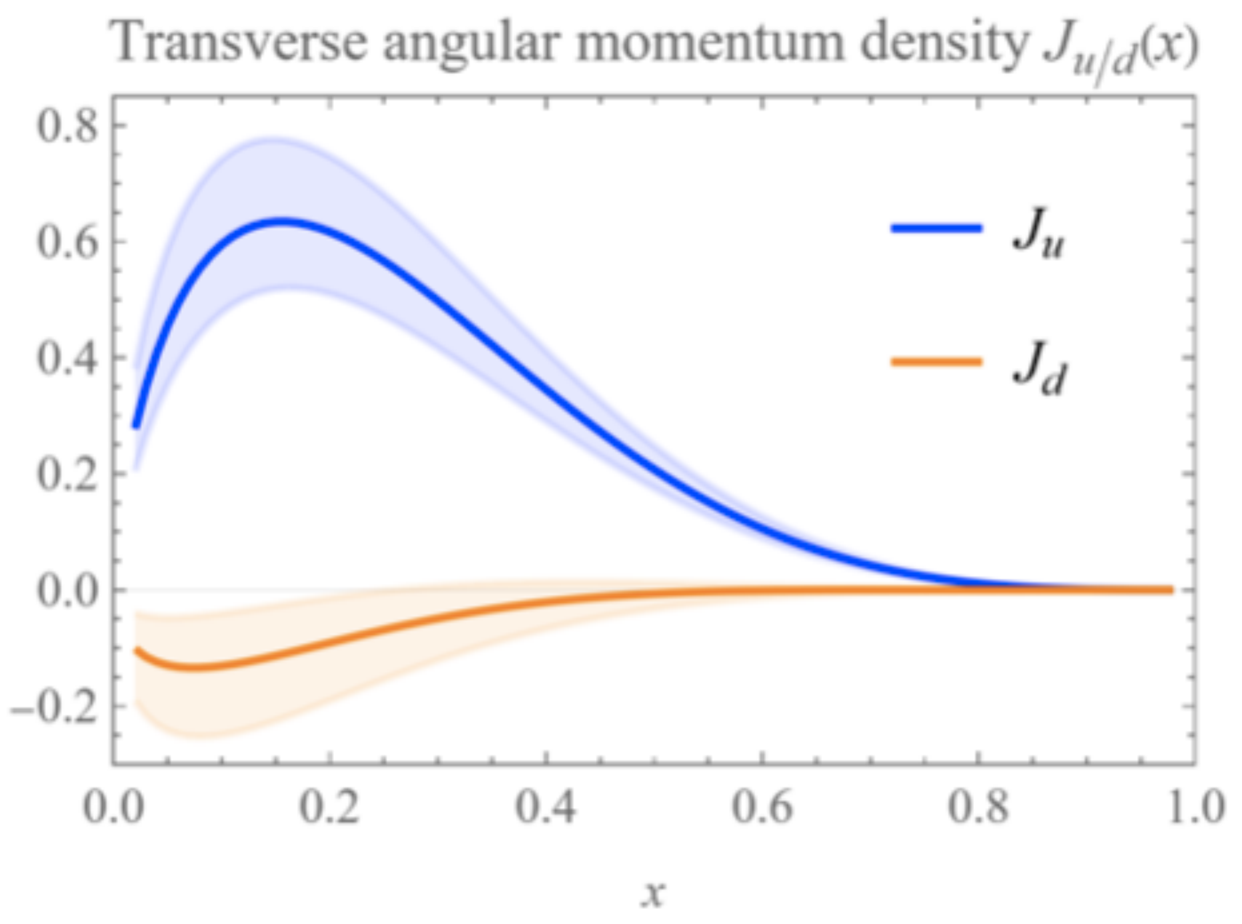


GPD Fit to Lattice Constraints

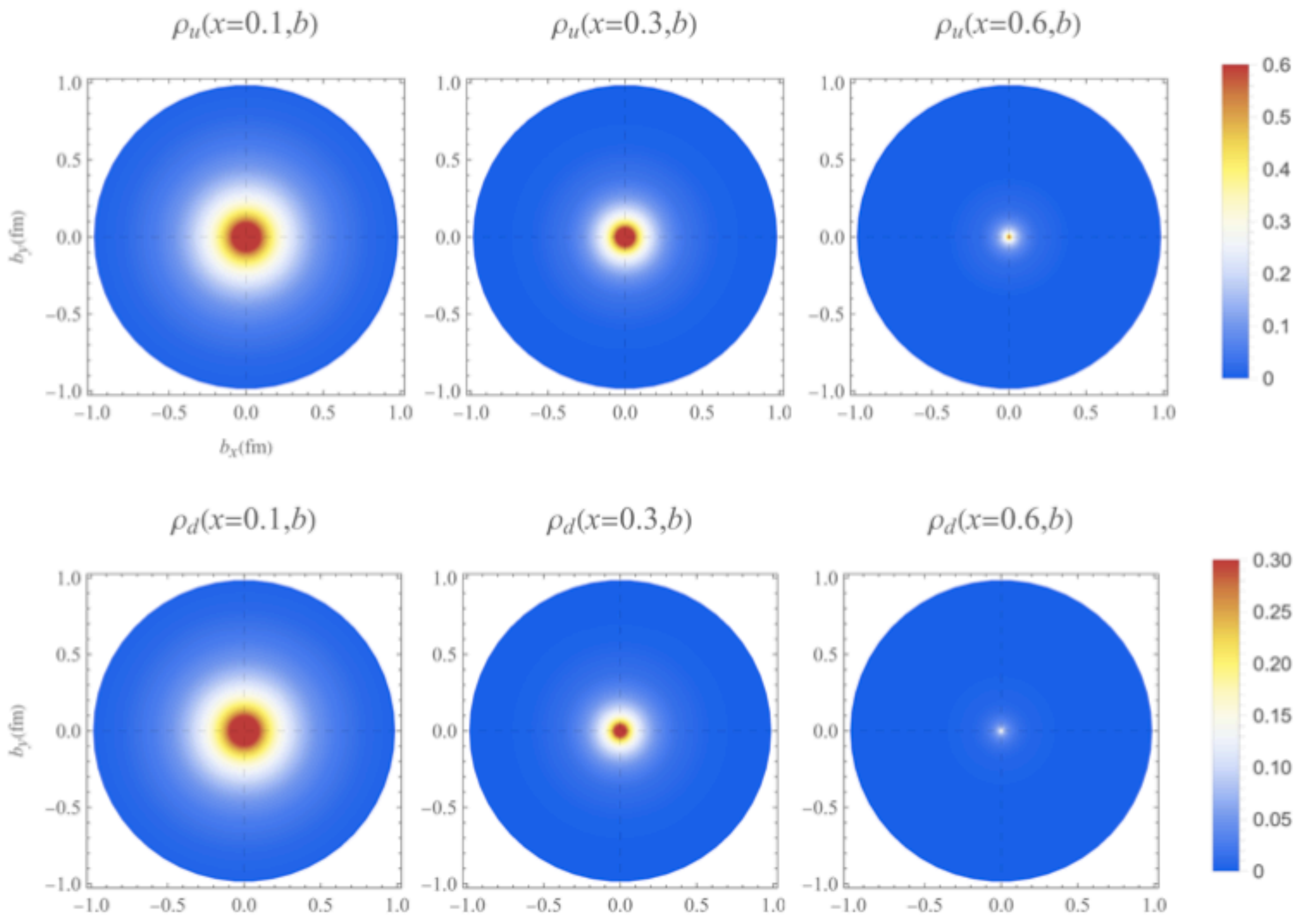


, . . .

$$J_q^{x(2)}(x) = \frac{x}{2}\Big(H_q(x) + E_q(x)\Big)$$



$$\rho_q(x,\boldsymbol{b}) = \int \frac{d^2\boldsymbol{\Delta}}{(2\pi)^2} e^{-i\boldsymbol{\Delta}\cdot\boldsymbol{b}} H_q(x,-\boldsymbol{\Delta}^2) = \mathscr{H}_q(x,\boldsymbol{b})$$



Separation of small and large x

$$f^+(x) = f(x) + \bar{f}(x) = f_{\text{val}}(x) + 2f_{\text{sea}}(x)$$

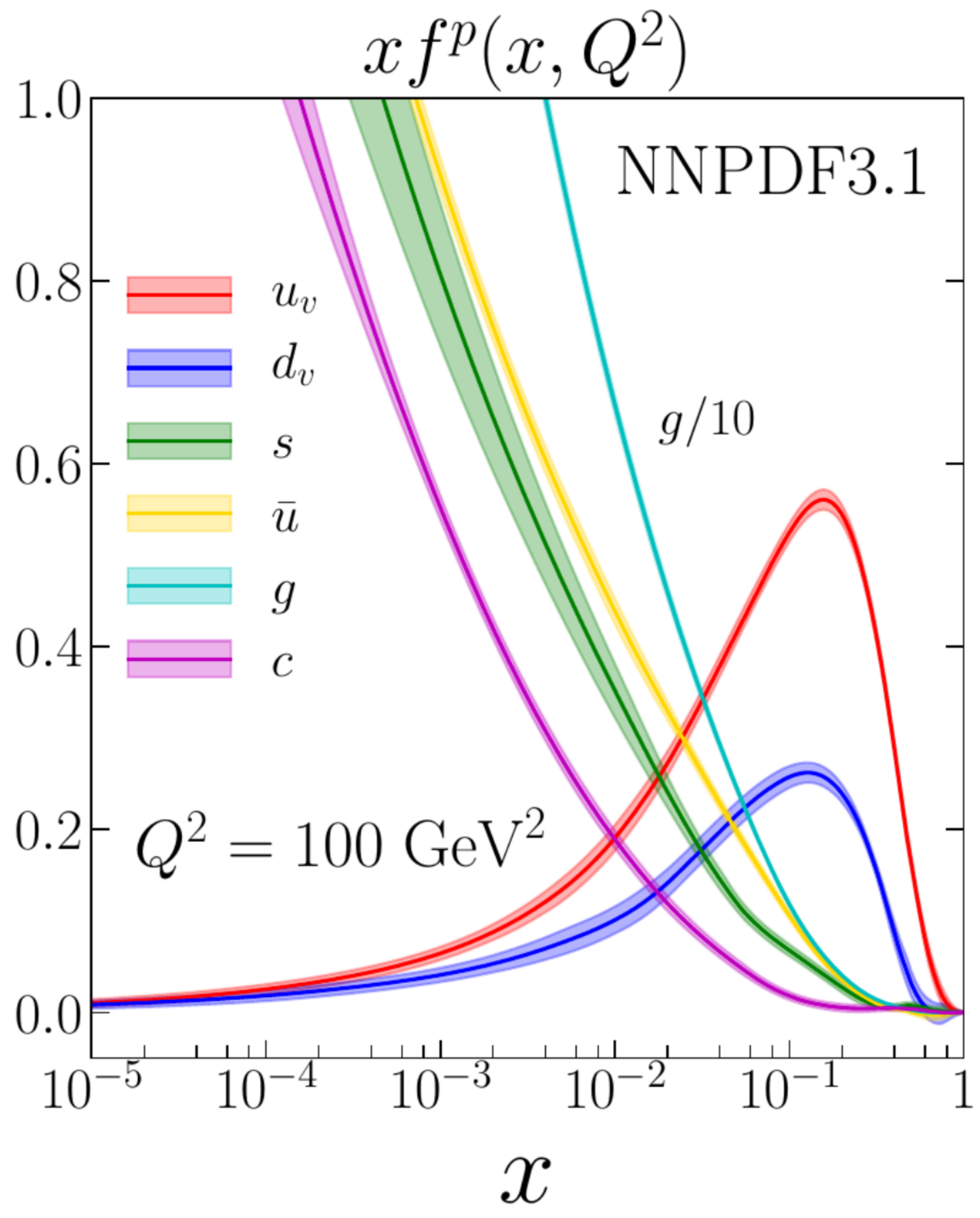
Form factors calculated on lattice only have the positive moments

$$f_n(t) = \int \mathrm{d}x x^n f(x,t)$$

which are dominated by valence contributions

$\langle x \rangle_{u_v} = 0.325$ $\langle x \rangle_{\bar{u}} = 0.028$

$$f_n(t) \approx \int \mathrm{d}x x^n f_{\text{val}}(x,t)$$



On the other hand, CFFs are associated with the inverse moments

$$\mathcal{F}_{CFF}(\xi,t) = \int \mathrm{d}x \left(\frac{1}{x-\xi+i\epsilon} + \frac{1}{x+\xi+i\epsilon} \right) F(x,\xi,t)$$

The moments calculations on lattice are mostly constraining the valence distributions.

Compton Form Factors in GUMP

Observables: $\sigma_{\mathrm{DVCS}} = \sigma_{\mathrm{DVCS}}\left(x_B, t, Q^2, E_b, \phi, \mathcal{F}_i(x_B, t, Q^2)\right)$

$$\mathcal{F}_i = \mathcal{H}, \mathcal{E}, \cdots$$

- WE can simply express CFFs directly in moment space because the *x*-integration can be done explicitly!

$$\mathcal{H}_{CFF}(\xi,t) = -Q_q^2 \int_{-1}^{1} \mathrm{d}x \left(\frac{1}{x-\xi+i0} + \frac{1}{x+\xi-i0}\right) H(x,\xi,t)$$

$$F(x,\xi,t) = \frac{1}{2i}\int_{c-i\infty}^{c+i\infty} dj \frac{p_j(x,\xi)}{\sin(\pi[j+1])}\mathcal{F}_j(\xi,t)$$

$$\left.\right\} \mathcal{H}_{CFF}(\xi,t) = \frac{1}{2i}\int_{c-i\infty}^{c+i\infty} dj\xi^{-j-1}\left[i + \tan\left(\frac{\pi j}{2}\right)\right] \mathbb{C}_j \mathcal{H}_j(\xi,t)$$

$$\mathbb{C}_j \overset{\mathrm{LO}}{=} \frac{2^{j+1}\Gamma(j+5/2)}{\Gamma(3/2)\Gamma(j+3)} \qquad \mathcal{H}_j = \mathcal{H}_j(\xi, t; N_{q,k}, \alpha_{q,k}, \beta_{q,k}, \alpha'_{q,k})$$

- Consequently, our CFFs will carry our GUMP parameters, which can be fit to real DVCS cross section data

Measurements

JLab 12 GeV

Electron Ion Collider (EIC)

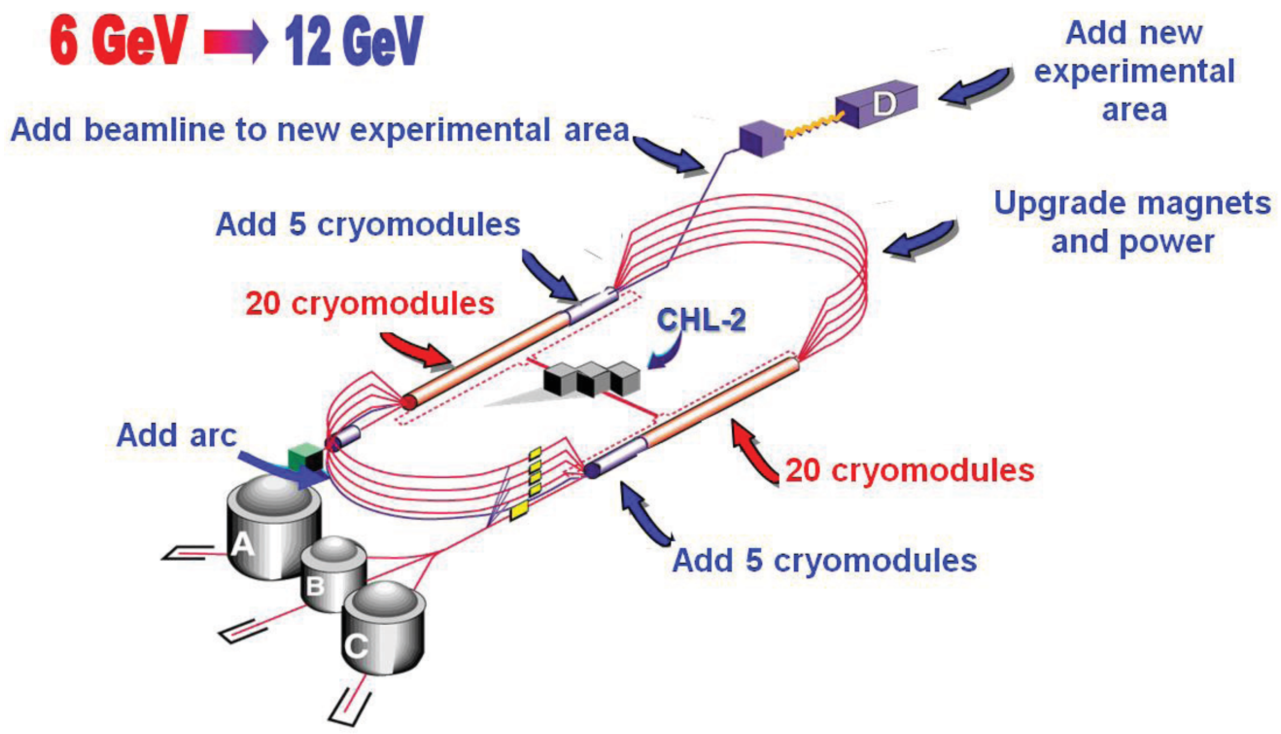
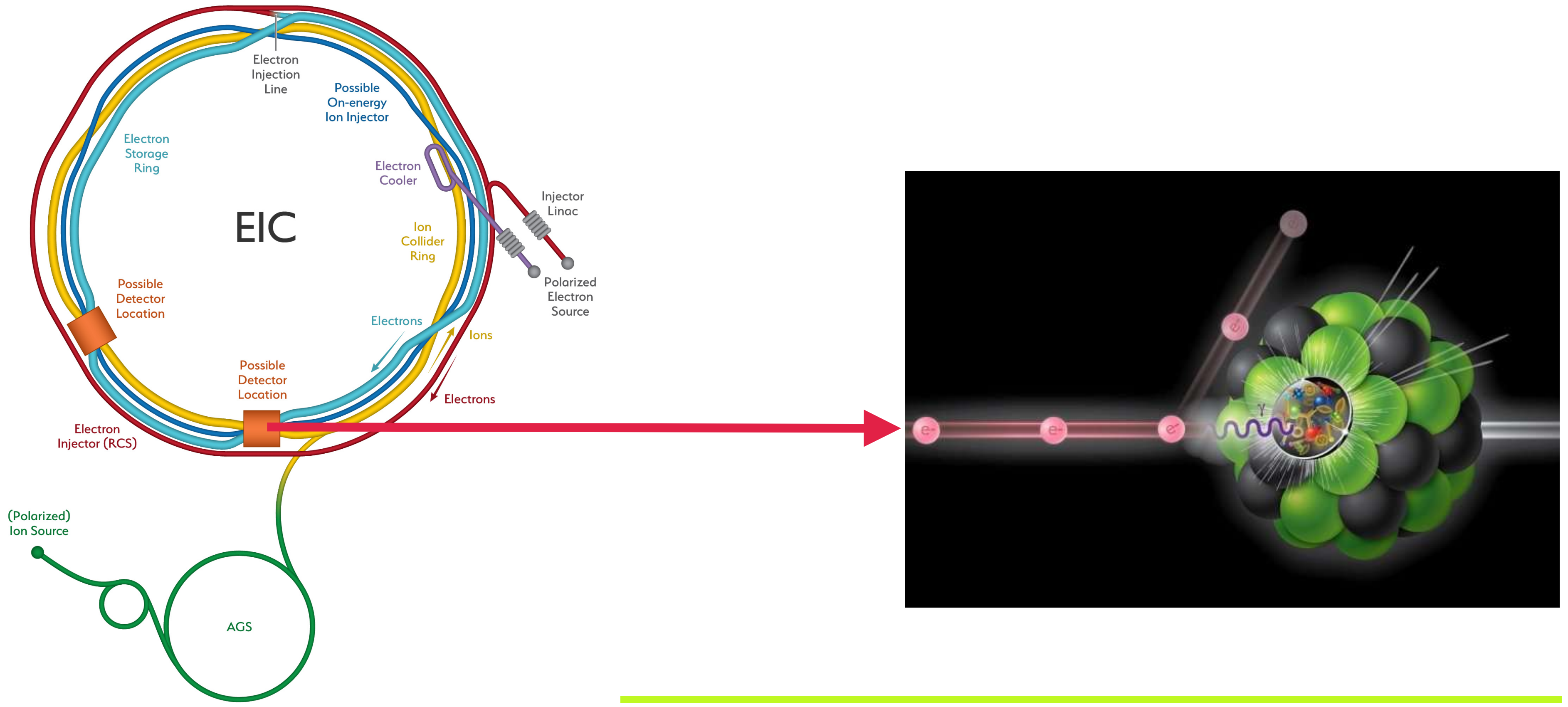


Figure 1: Changes to CEBAF.



Global DVCS Data

No.	Collab.	Year	Observable		Kinematic dependence	No. of points used / all
1	HERMES	2001	A^{+}_{LU}		ϕ	10 / 10
2		2006	$A^{\cos i\phi}_{C}$	$i=1$	t	4 / 4
3		2008	$A^{\cos i\phi}_{C}$	$i=0,1$	$x_{\rm Bj}$	18 / 24
			$A^{\sin(\phi-\phi_S)\cos i\phi}_{UT,\rm DVCS}$	$i=0$		
			$A^{\sin(\phi-\phi_S)\cos i\phi}_{UT,\rm I}$	$i=0,1$		
			$A^{\cos(\phi-\phi_S)\sin i\phi}_{UT,\rm I}$	$i=1$		
4		2009	$A^{\sin i\phi}_{LU,\rm I}$	$i=1,2$	$x_{\rm Bj}$	35 / 42
			$A^{\sin i\phi}_{LU,\rm DVCS}$	$i=1$		
			$A^{\cos i\phi}_{C}$	$i=0,1,2,3$		
5		2010	$A^{+,\sin i\phi}_{UL}$	$i=1,2,3$	$x_{\rm Bj}$	18 / 24
			$A^{+,\cos i\phi}_{LL}$	$i=0,1,2$		
6		2011	$A^{\cos(\phi-\phi_S)\cos i\phi}_{LT,\rm DVCS}$	$i=0,1$	$x_{\rm Bj}$	24 / 32
			$A^{\sin(\phi-\phi_S)\sin i\phi}_{LT,\rm DVCS}$	$i=1$		
			$A^{\cos(\phi-\phi_S)\cos i\phi}_{LT,\rm I}$	$i=0,1,2$		
			$A^{\sin(\phi-\phi_S)\sin i\phi}_{LT,\rm I}$	$i=1,2$		
7		2012	$A^{\sin i\phi}_{LU,\rm I}$	$i=1,2$	$x_{\rm Bj}$	35 / 42
			$A^{\sin i\phi}_{LU,\rm DVCS}$	$i=1$		
			$A^{\cos i\phi}_{C}$	$i=0,1,2,3$		
8	CLAS	2001	$A^{-,\sin i\phi}_{LU}$	$i=1,2$	—	0 / 2
9		2006	$A^{-,\sin i\phi}_{UL}$	$i=1,2$	—	2 / 2
10		2008	A^{-}_{LU}		ϕ	283 / 737
11		2009	A^{-}_{LU}		ϕ	22 / 33
12		2015	A^{-}_{LU}, A^{-}_{UL}, A^{-}_{LL}		ϕ	311 / 497
13		2015	$d^4\sigma^{-}_{UU}$		ϕ	1333 / 1933
14	Hall A	2015	$\Delta d^4\sigma^{-}_{LU}$		ϕ	228 / 228
15		2017	$\Delta d^4\sigma^{-}_{LU}$		ϕ	276 / 358
16	COMPASS	2018	$d^3\sigma^{\pm}_{UU}$		t	2 / 4
17	ZEUS	2009	$d^3\sigma^{+}_{UU}$		t	4 / 4
18	H1	2005	$d^3\sigma^{+}_{UU}$		t	7 / 8
19		2009	$d^3\sigma^{\pm}_{UU}$		t	12 / 12
					SUM:	2624 / 3996

- HERMES contains many observables, but somewhat poor statistics
- JLAB has high luminosity and much better statistics, but so far lacks the breadth of DVCS observables
- Much more data is expected over next 10 years with JLAB 12 GeV (& 24 GeV?) as well as the highly anticipated EIC

Conclusions

1. **Nucleon Spin:**

$$J_q + J_g = \frac{1}{2} \qquad \frac{1}{2}\Delta q + \Delta G + l_q^z + l_g^z = \frac{1}{2}$$

- 1st order: reduce uncertainty of Spin PDFs $g_1(x)$ and $\Delta G(x)$

- 2nd order: measure twist-2 GPDs H & E for quark and gluon

- 3rd order: measure the twist-3 GPDs of AM

- JLab and EIC will be essential in solving the nucleon spin structure

High CM energy, high luminosity (polarized) electron/positron beams scattering off polarized nucleons needed in DIS, SIDIS, DVCS, DVMP, etc.

2. Global Analysis of GPDs:

- One needs a sufficient number of unique (non degenerate) observables to find a stable extraction of either CFFs

- This is achieved by fitting harmonic dependence of DVCS cross section for multiple beam energies and for multiple polarization observables (UU, LU, UL, LL, UT, LT)

- One can model GPDs using universal moment parameterization, which also allows one to also naturally incorporate lattice data

$$\chi^2_{\text{fit}}(N_j, \alpha_j, \beta_j, \alpha'_j) = \sum_i \frac{(O_i - E_i)^2}{\delta_i^2}$$

Lattice GFFs   Lattice GPDs   PDFs   DVCS cross sections   ......