HIGHER-ORDER LEPTONIC CORRECTIONS IN COVARIANT APPROACH

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MOTIVATION

- Standard Model (SM) is the most precise theory and can make predictions that match experiments to one part in ten billion, yet it is incomplete and cannot explain the mystery of dark matter, hierarchy problem, matter anti-matter asymmetry etc.
- For finding answers → Physics beyond the Standard Model
- We are doing precision physics → achieve by calculating the higher order corrections → our results can help many experimental programs searching for beyond the SM physics at the precision frontier.

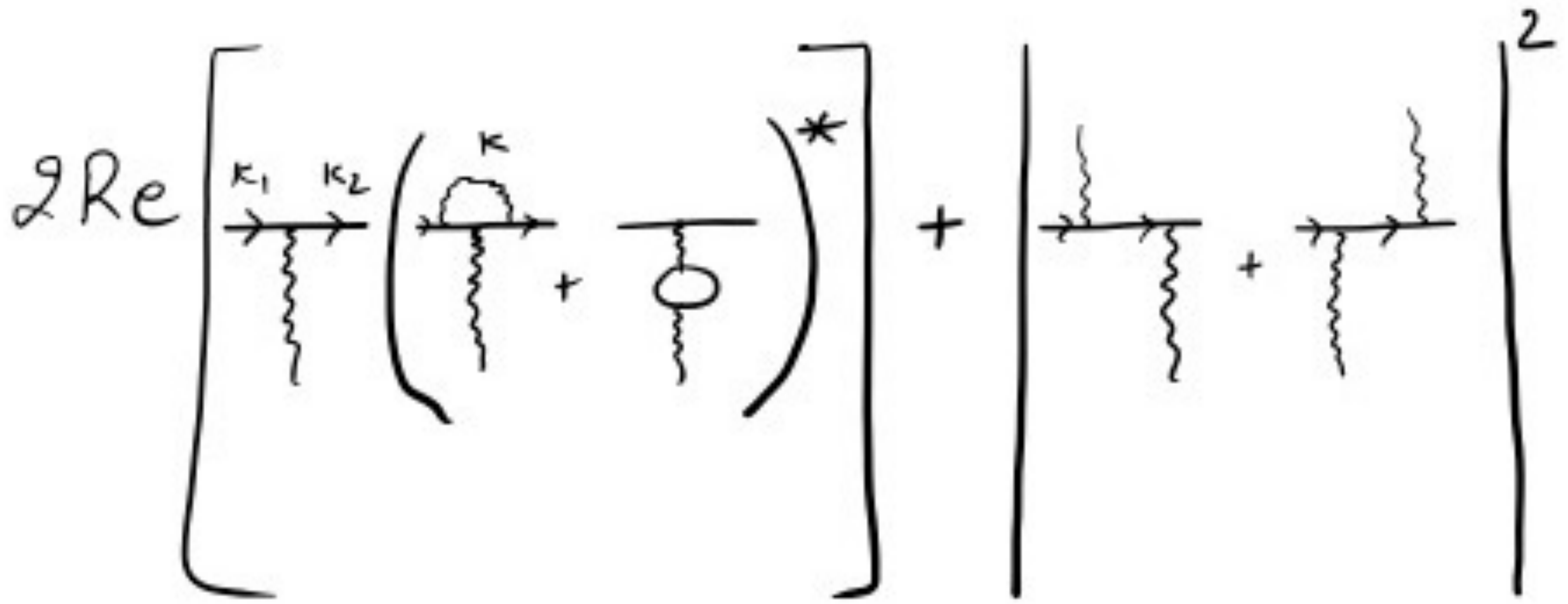
- We are studying the SM precision by calculating both polarized and unpolarized asymmetry by including all SM particles. This includes calculating the electroweak differential scattering cross sections ($e^-\mu^-$, e^-p, μ^-p) using NLO (α^3) and quadratic level (NNLO α^4) Covariant/ leptonic tensor approach.

- The more higher orders we include, the more precise results could be obtained. Any discrepancy between the results of our theoretical calculations and experimentally measured values may enable us to search for the physics beyond the Standard Model.

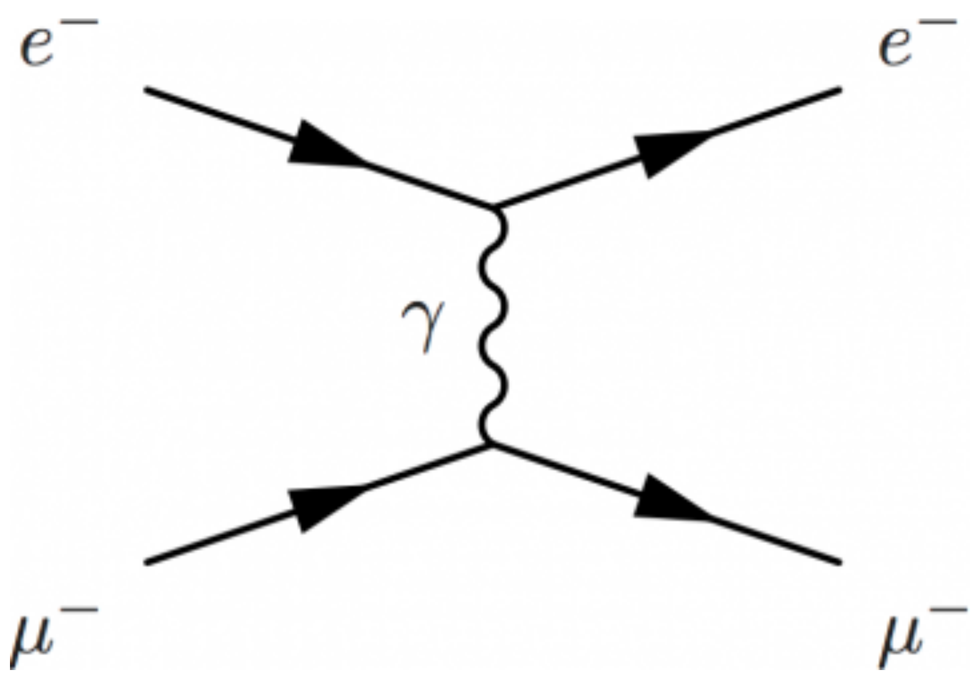
QED LEPTONIC TENSOR AND INTRODUCTION TO COVARIANT APPROACH

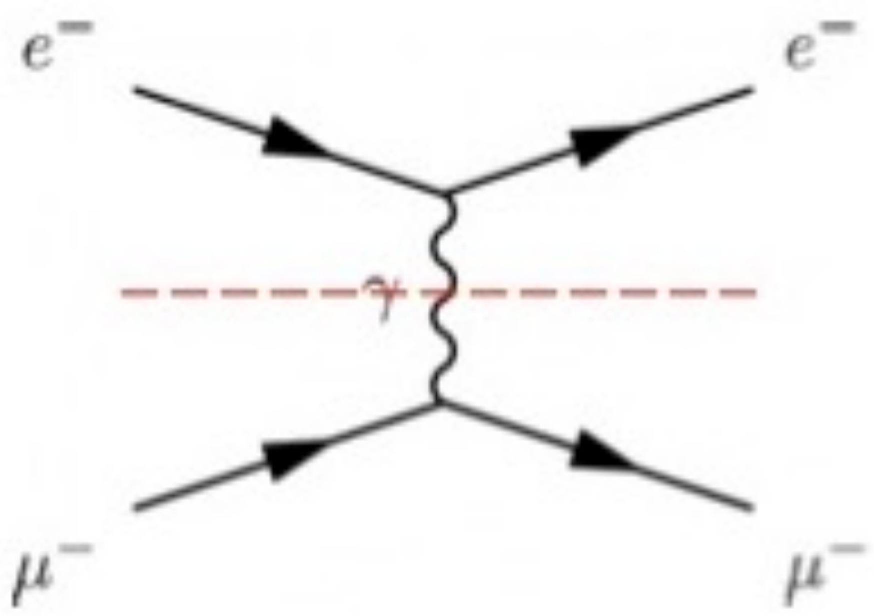
- First introduced by Bardin and Shumeiko in 1976 (Nuclear Physics **B127**) to extract the infrared divergence from the lowest-order bremsstrahlung cross section.

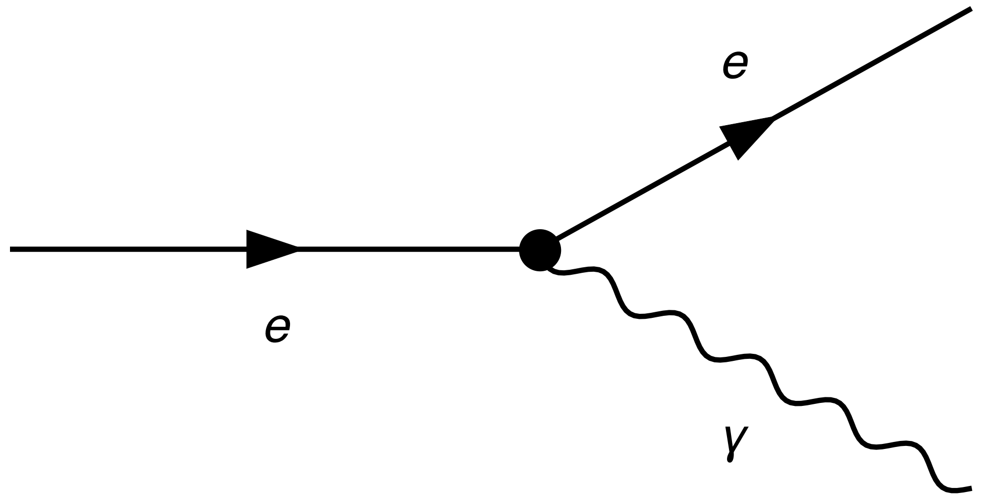
- Recently used by Afanasev et al. (Phys. Rev. D **66**) to calculate QED radiative corrections in processes of exclusive Pion electroproduction.

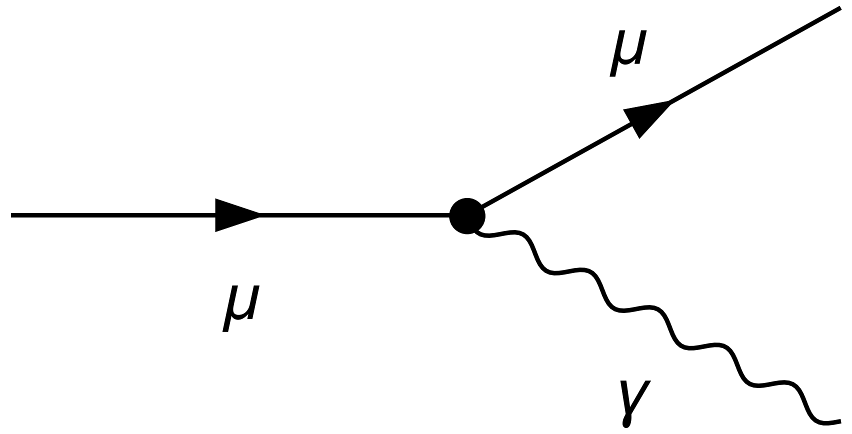


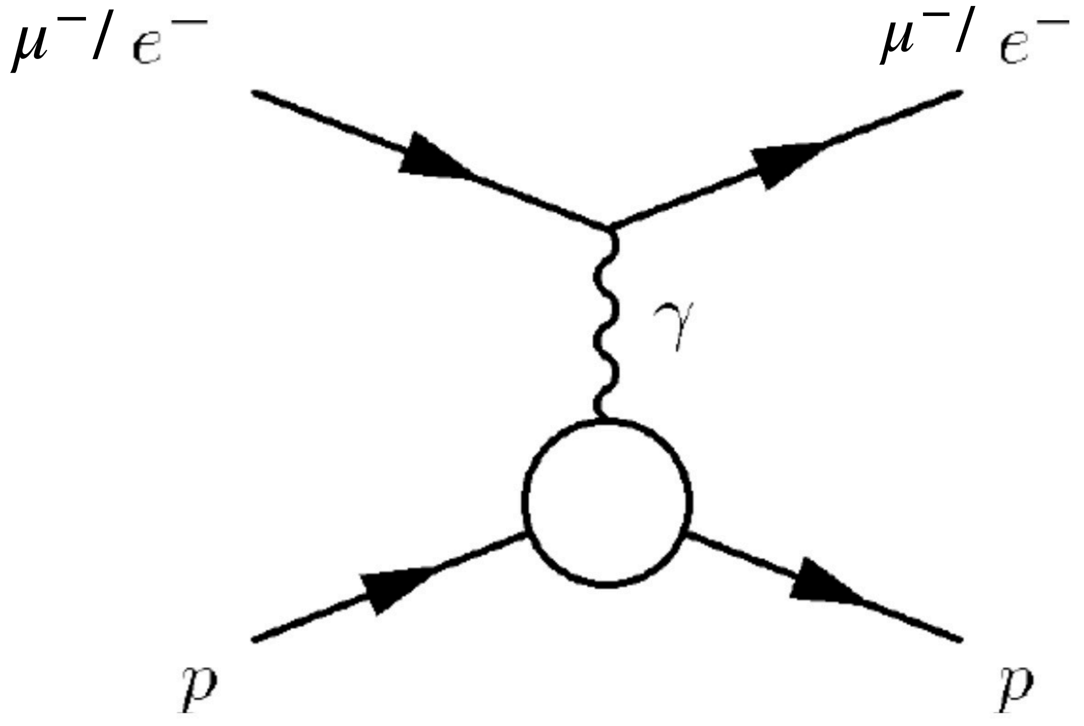
WHAT IS A COVARIANT APPROACH?

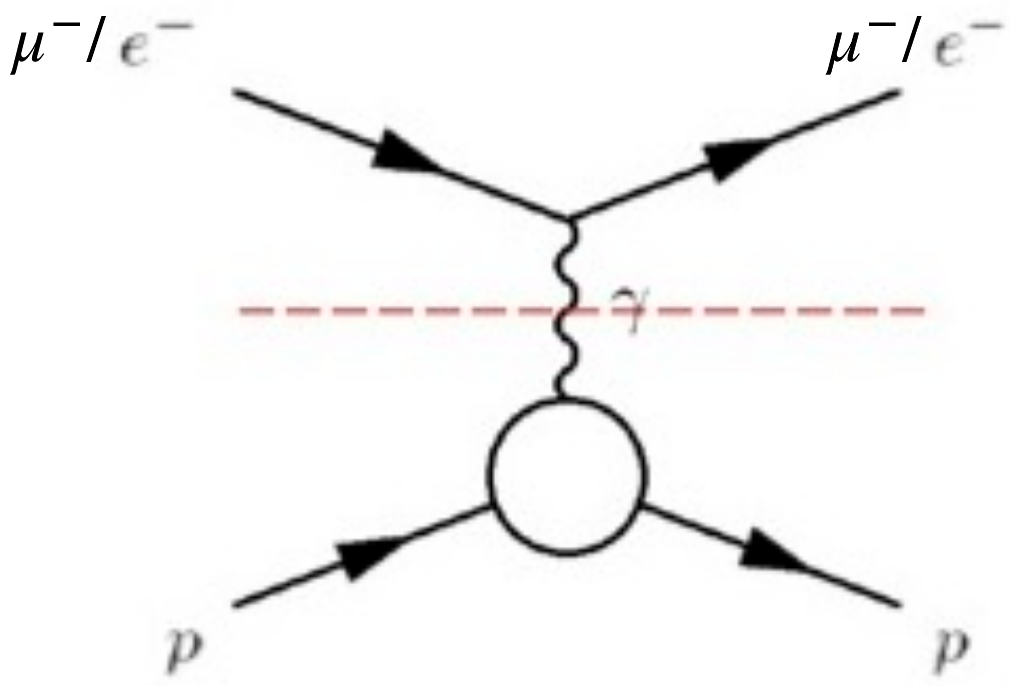


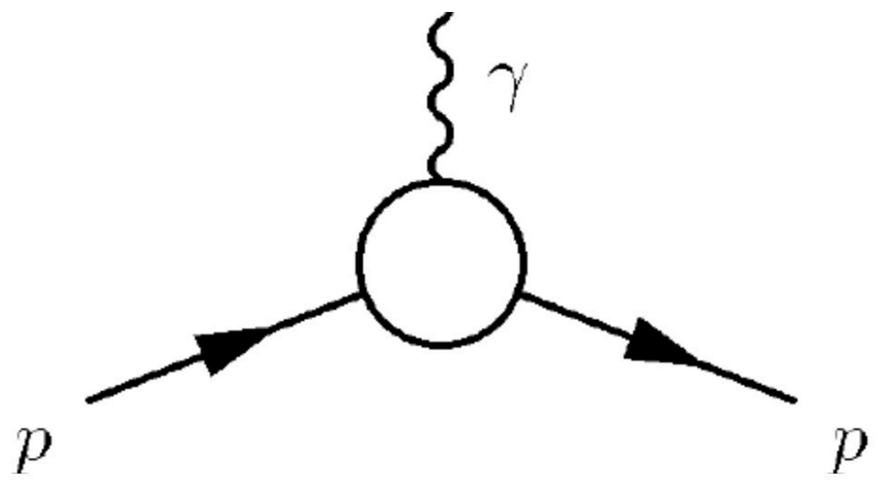












HADRONIC TENSOR AND TENSOR STRUCTURE FUNCTIONS

- The differential cross section of general lepton-lepton/hadron scattering can be obtained by:

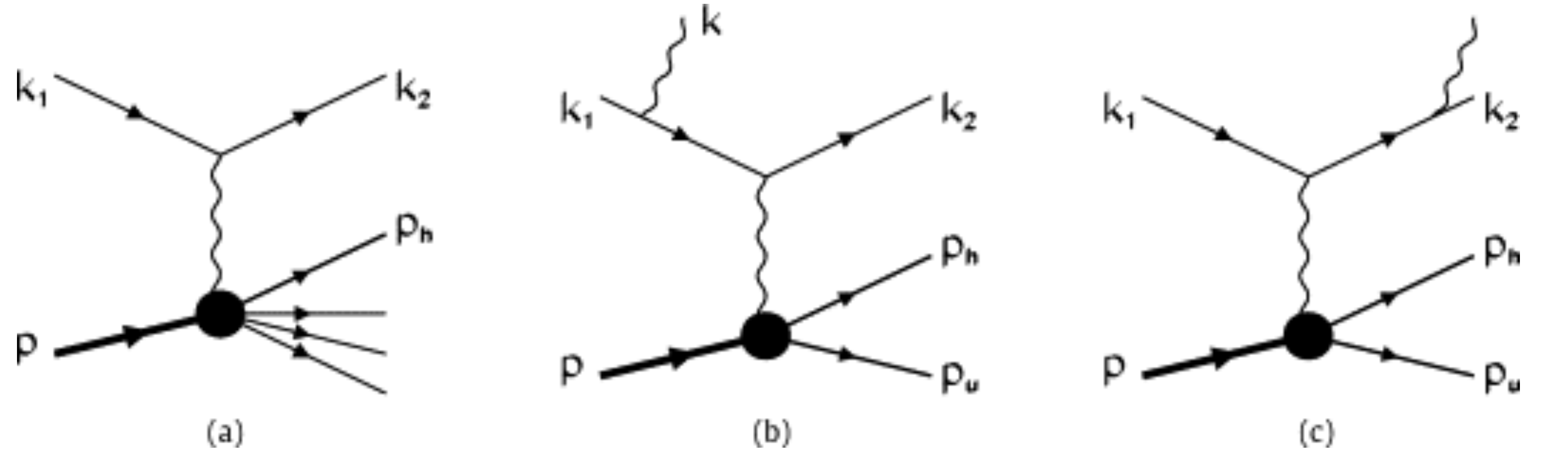
$$d\sigma \sim L^{\mu\nu}L_{\mu\nu} \text{ or } d\sigma \sim L^{\mu\nu}W_{\mu\nu}$$

- where $W_{\mu\nu}$ is the arbitrary hadronic tensor and can be obtained using general covariant form

$$W_{\mu\nu} = -\tilde{g}_{\mu\nu}H_1 + \tilde{p}_\mu\tilde{p}_\nu H_2 + \tilde{p}_{\mu h}\tilde{p}_{\nu h}H_3 + (\tilde{p}_\mu\tilde{p}_{\nu h} + \tilde{p}_{\mu h}\tilde{p}_\nu)H_4 + (\tilde{p}_{\mu h}\tilde{p}_\nu - \tilde{p}_\mu\tilde{p}_{\nu h})H_5$$

Where H_1, H_2, H_3, H_4 and H_5 are hadronic structure functions and can be extracted from experimental data.

$$\tilde{a}_\mu = a_\mu - \frac{aq}{q^2}q_\mu \quad \text{(gauge invariance)}$$



- For QED (e^-p, μ^-p) elastic scattering case:

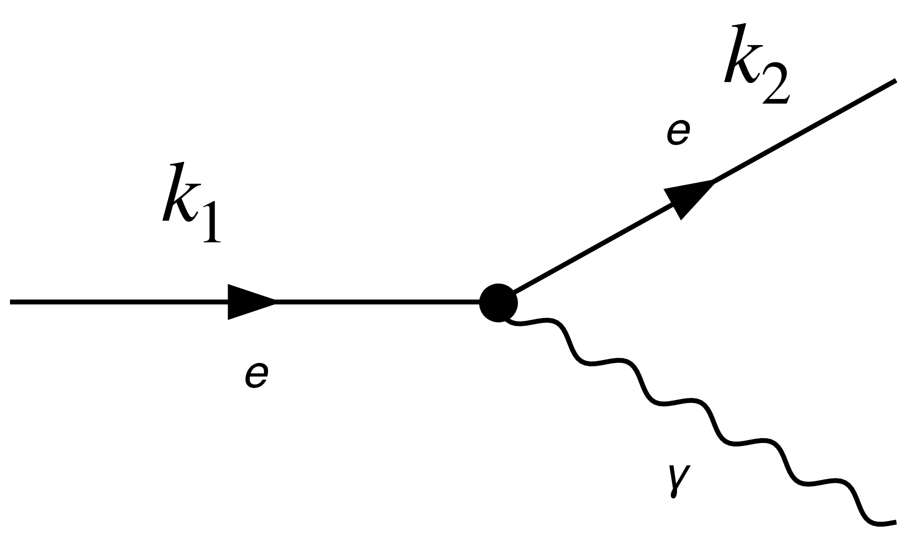
$$W^{\mu\nu} = (F_1 + F_2)^2(p_1^\mu p_2^\nu + p_1^\nu p_2^\mu - g^{\mu\nu}(p_1 . p_2 - m^2)) + (p_1 + p_2)^\mu (p_1 + p_2)^\nu \left(\frac{p_1 . p_2 + m^2}{4m^2} F_2^2 - F_2(F_1 + F_2) \right)$$

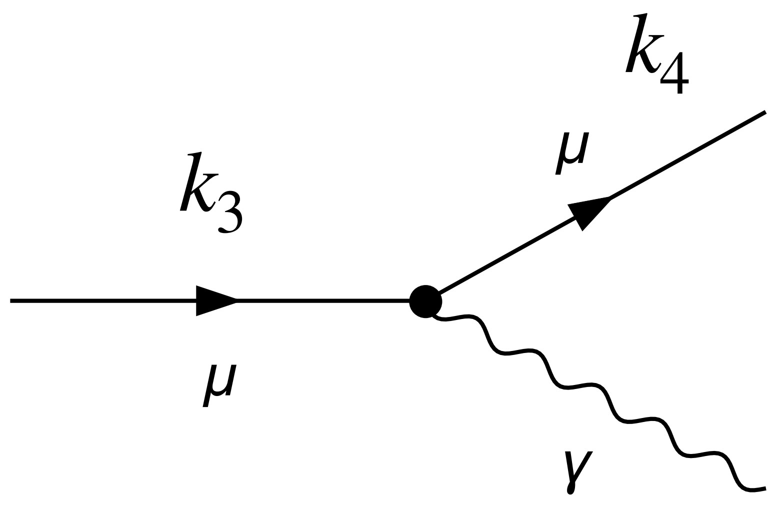
where $F_1(t)$ and $F_2(t)$ are Dirac and Pauli form factors depending on momentum transfer t, p_1 and p_2 are incoming and outgoing protons and m is the mass of proton.

where: $$F_1(t) = \frac{\tau G_M + G_E}{1 + \tau}, \quad F_2(t) = \frac{G_M - G_E}{1 + \tau},$$

$$\tau = -\frac{t}{4m^2} \text{ and } G_E = \frac{1}{\left(1 - \frac{t}{0.71}\right)^2}, \quad G_M = \frac{2.79}{\left(1 - \frac{t}{0.71}\right)^2}$$

TREE-LEVEL LEPTONIC TENSOR (α-ORDER)

$|M|^2 \propto \dfrac{1}{t^2}$ $\Bigg|$ 

$\Bigg|^2_{\mu\nu} \cdot \Bigg|$ 

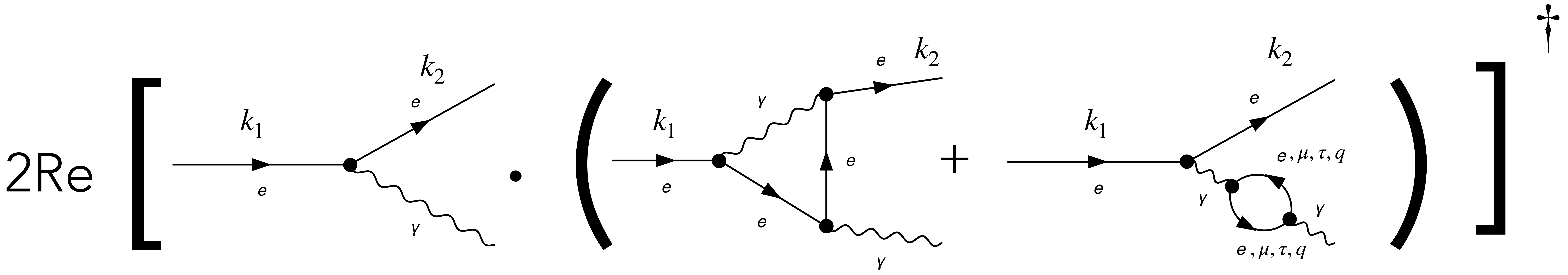
$\Bigg|^2_{\mu\nu}$

- For tree-level upper part of the diagram (say $e^-\mu^-$ scattering), one can calculate leptonic tensor which is:

$$L^0_{\mu\nu} = \frac{2\pi\alpha(tg_{\mu\nu} + 2(k_{2\mu}k_{1\nu} + k_{1\mu}k_{2\nu}))}{t}$$

Where is $t = -Q^2$ (momentum transfer) and k_1, k_2 are incoming and outgoing e^- momenta.

NEXT TO THE LEADING ORDER (NLO) QED LEPTONIC TENSOR (α^2-ORDER)

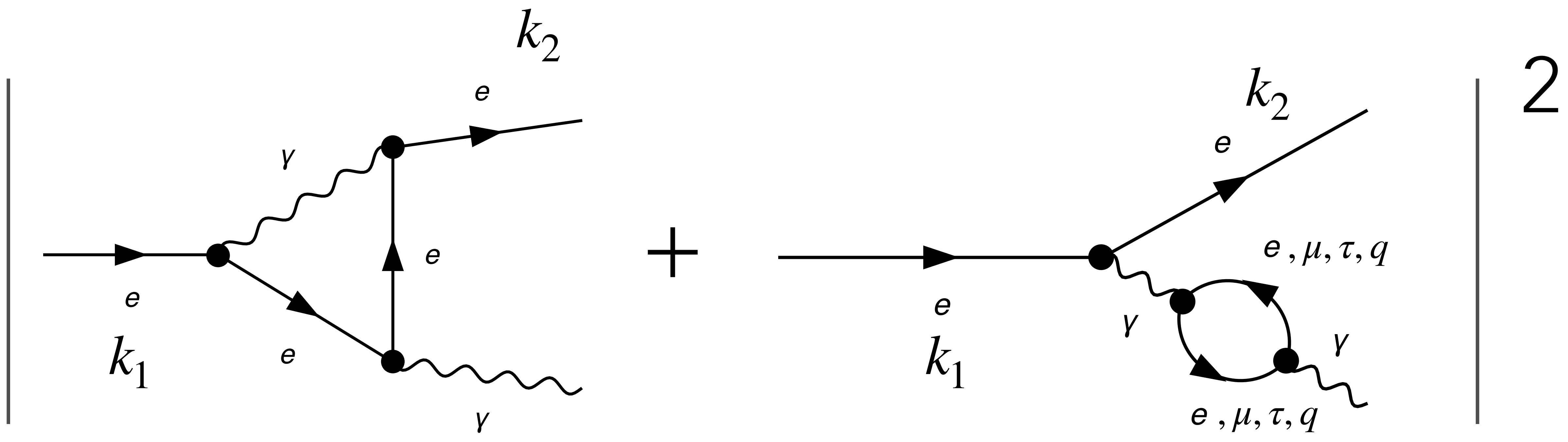


- The NLO leptonic tensor can be obtained by multiplying tree-level upper diagram with the sum of one-loop level SE and triangular diagrams.

$$L_{\mu\nu}^{NLO} = (l_1)g_{\mu\nu} + (l_2)k_{1\nu}k_{2\mu} + (l_3)k_{1\mu}k_{2\nu} + (l_4)k_{1\mu}k_{1\nu} + (l_5)k_{2\mu}k_{2\nu}$$

Where l_1, l_2, l_3, l_4 and l_5 are leptonic structure functions which depend on the momentum transfer t and written in terms of Passarino-Veltman integral functions. We used LoopTools Mathematica package to calculate them.

NEW RESULTS: QUADRATIC LEPTONIC TENSOR (α^3-ORDER)

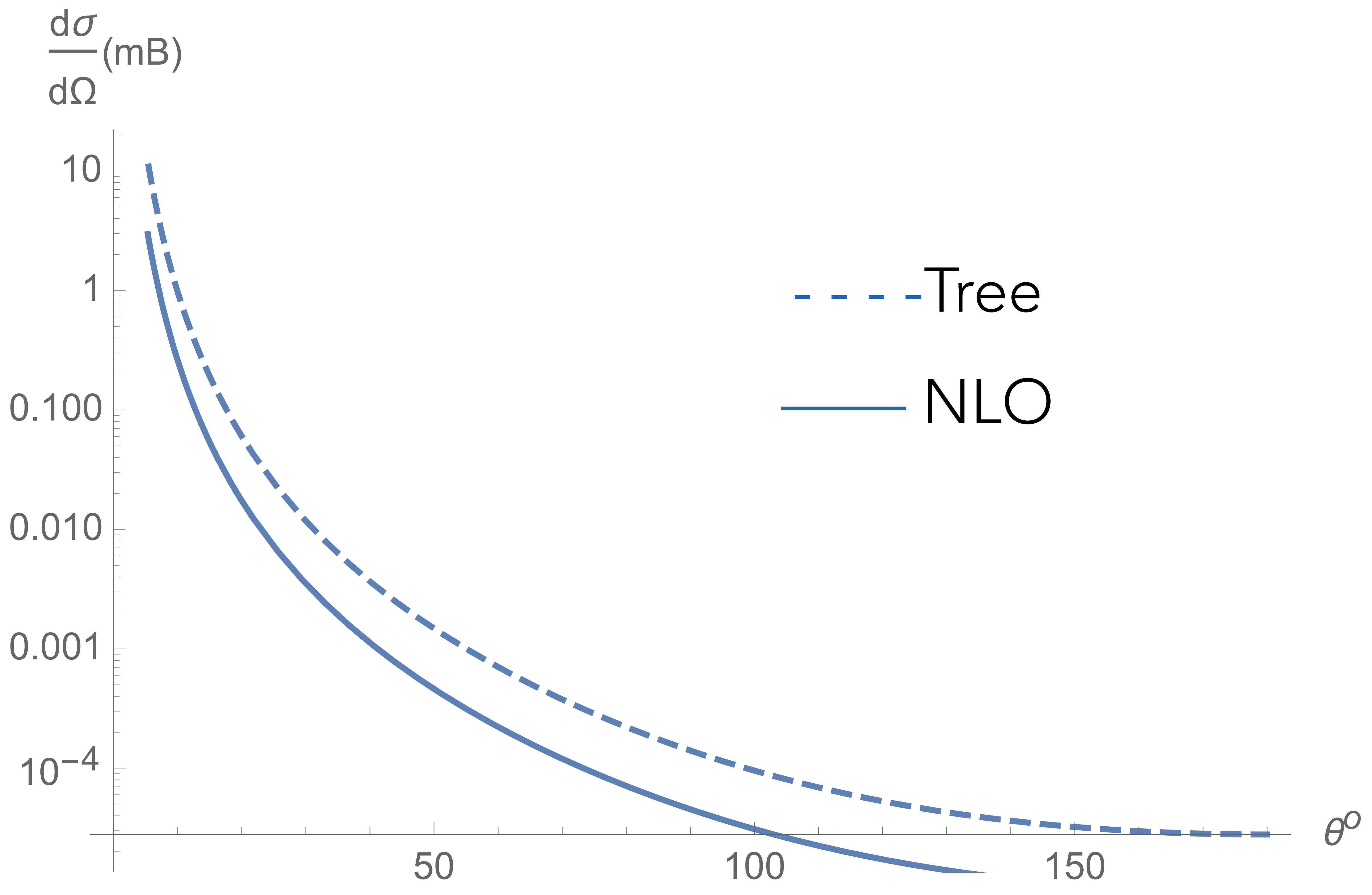


- The quadratic leptonic tensor can be obtained by squaring the sum of one-loop level SE and triangular diagrams. Tensor form is the same as that of NLO and is given by:

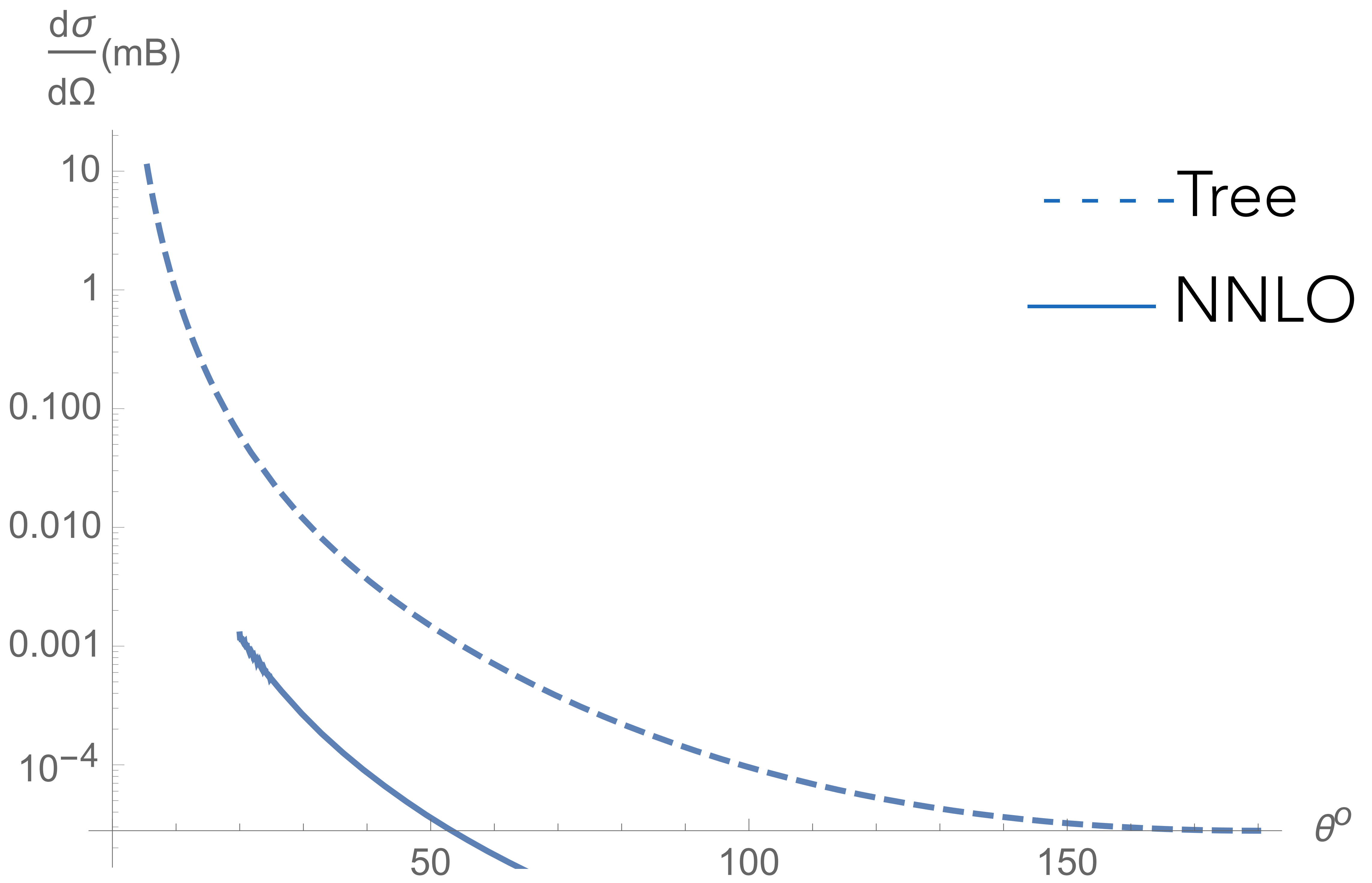
$$L_{\mu\nu}^{Quadratic} = (n_1)g_{\mu\nu} + (n_2)k_{1\nu}k_{2\mu} + (n_3)k_{1\mu}k_{2\nu} + (n_4)k_{1\mu}k_{1\nu} + (n_5)k_{2\mu}k_{2\nu}$$

Where n_1, n_2, n_3, n_4 and n_5 are quadratic leptonic structure functions of the order of α^3. Mass of electron is not neglected in our calculations.

Graphs for Tree level, NLO and Quadratic level Differential ($e^-\mu^-$) Scattering Cross Sections versus Scattering angle θ (CMS)

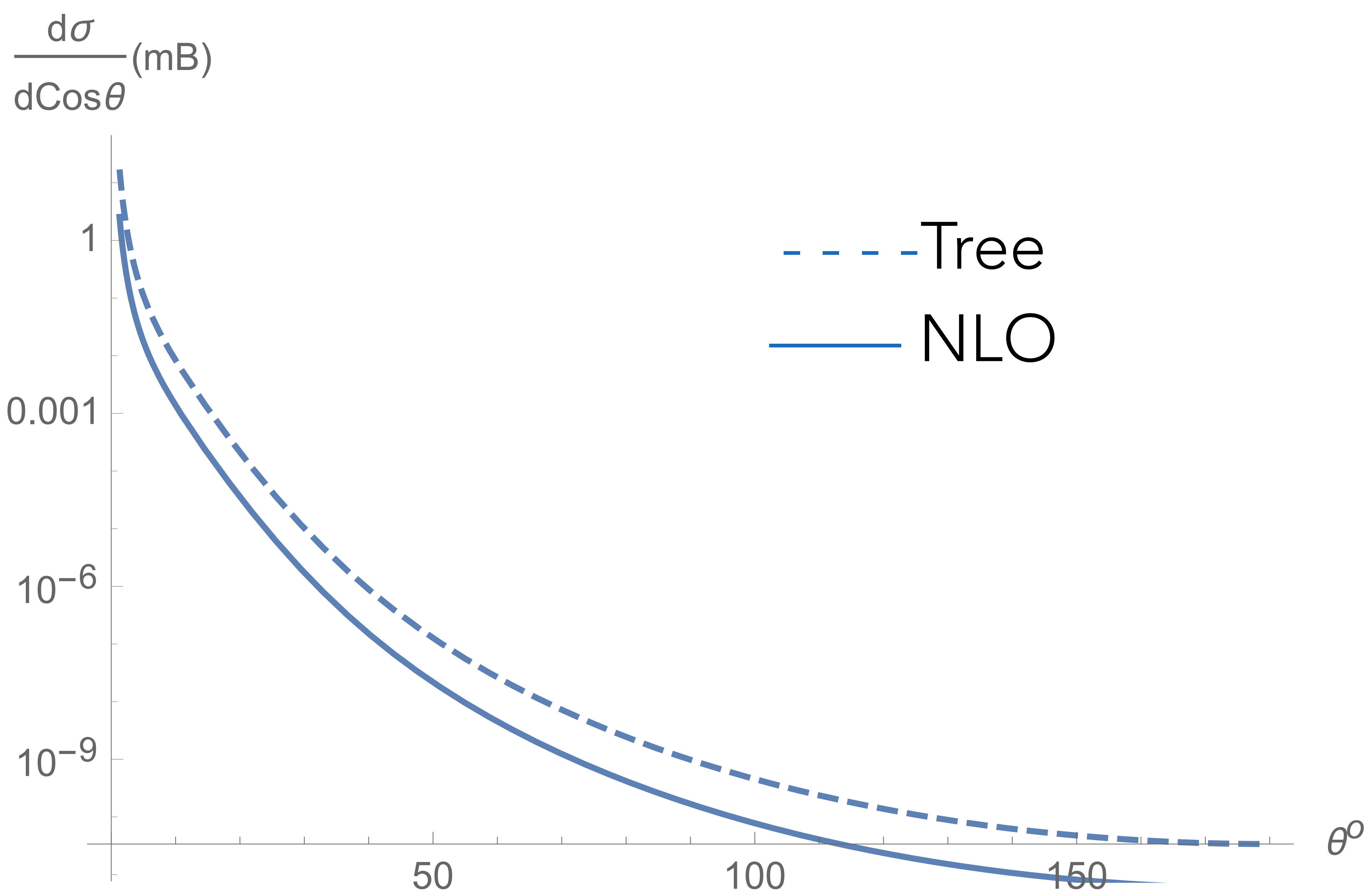


($e^-\mu^-$) Tree level and NLO level $\frac{d\sigma}{d\Omega}$

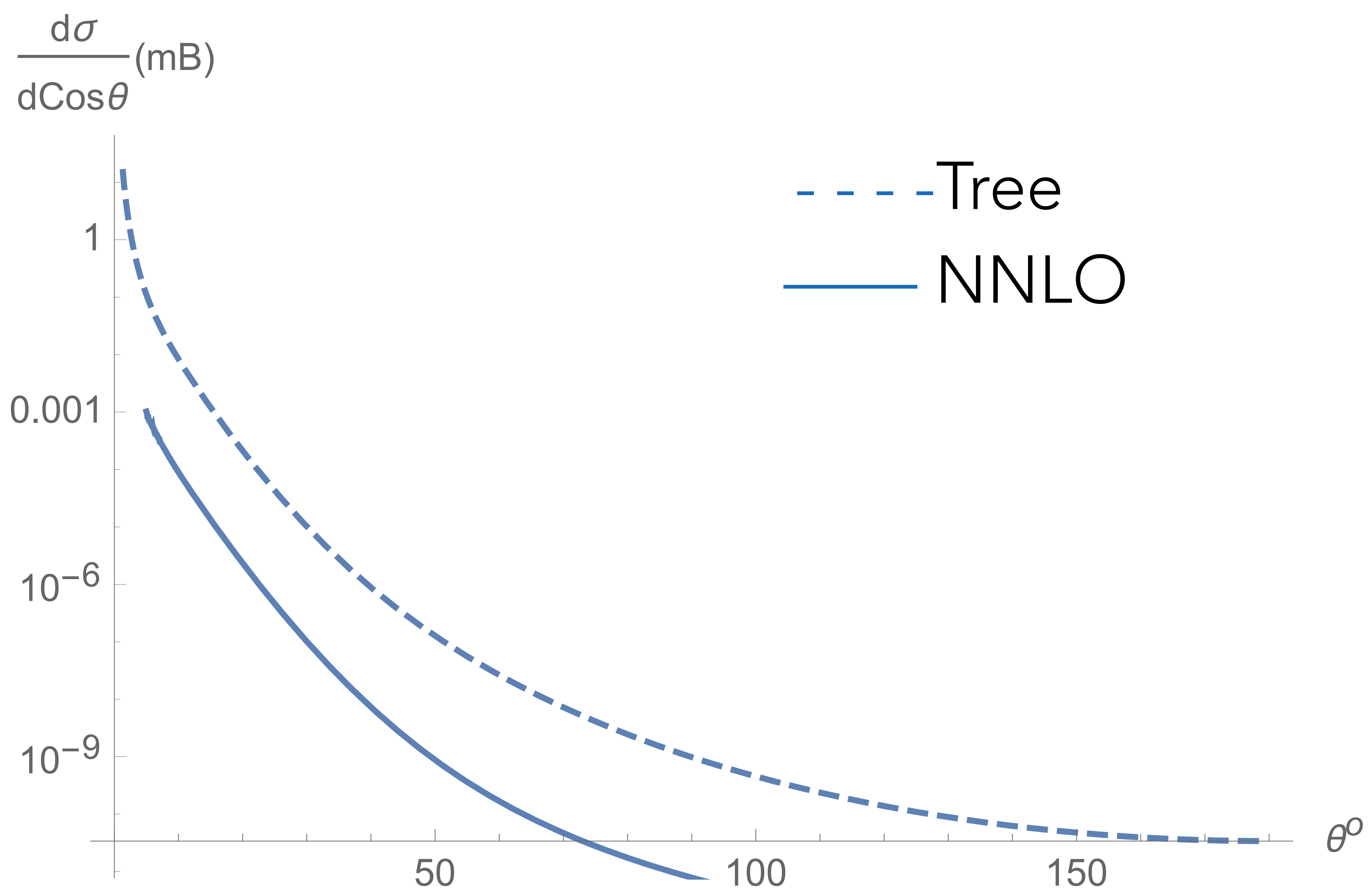


($e^-\mu^-$) Tree level and NNLO $\frac{d\sigma}{d\Omega}$

Graphs for Tree level, NLO and Quadratic level Differential $(e^{-}p)$ Scattering Cross Sections versus Scattering angle θ

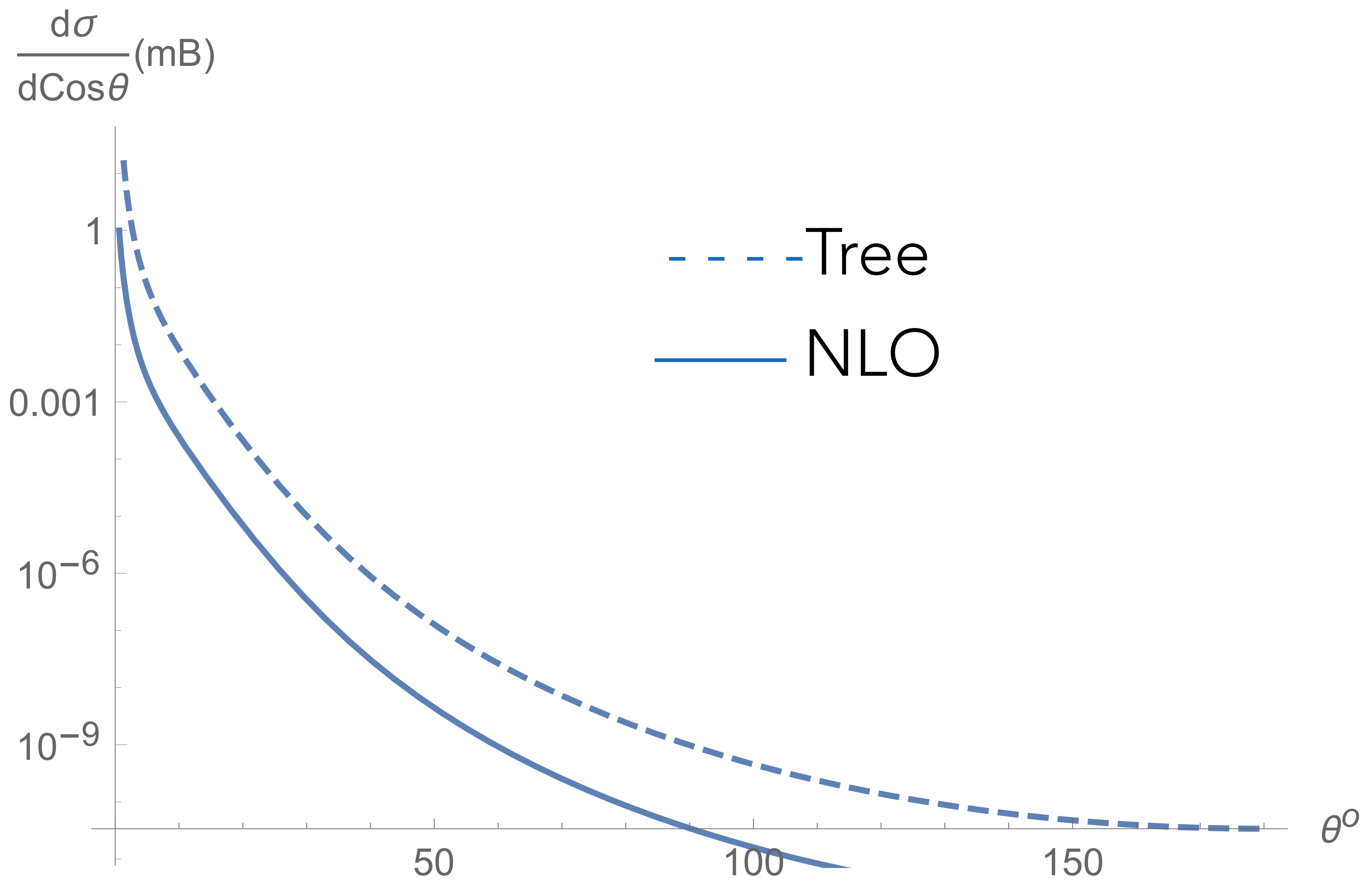


$(e^{-}p)$ Tree level and NLO $\frac{d\sigma}{dCos\theta}$

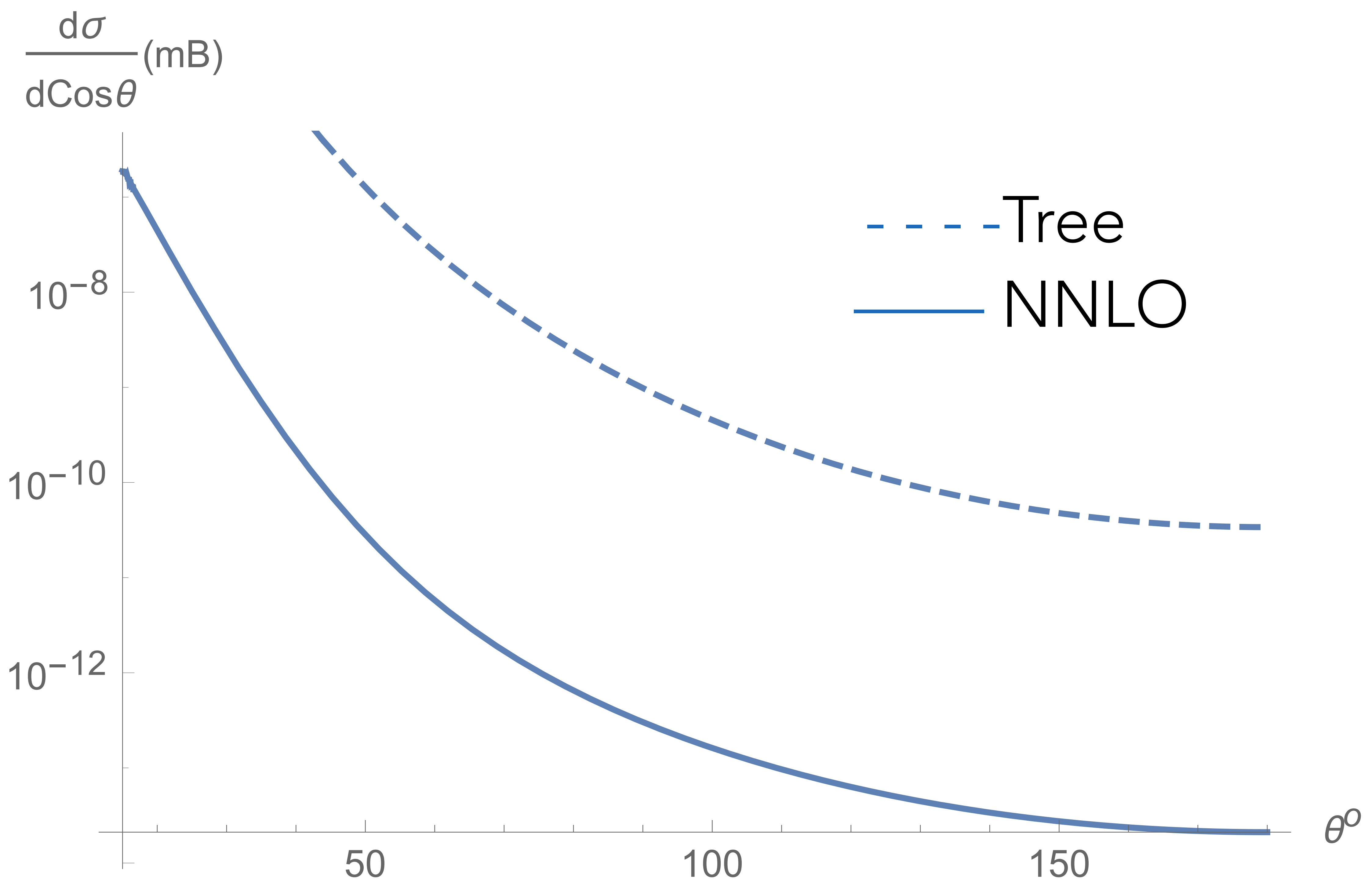


$(e^{-}p)$ Tree level and NNLO level $\frac{d\sigma}{dCos\theta}$

Graphs for Tree level, NLO and Quadratic level Differential ($\mu^- p$) Scattering Cross Sections versus Scattering angle θ



($\mu^- p$) Tree level and NLO $\frac{d\sigma}{dCos\theta}$



($\mu^- p$) Tree level and NNLO level $\frac{d\sigma}{dCos\theta}$

NLO AND NNLO LEVEL CORRECTION FACTORS

- The correction factors depend upon the scattering angle θ which appears in momentum transfer as

$$t = (k_2 - k_1)^2 = -2\, p_{in}^2\, [1 - Cos(\theta)] \qquad \text{where, } p_{in} = p_{out}$$

- For an arbitrary angle $\theta = 50^0$, NLO and Quadratic correction factors are

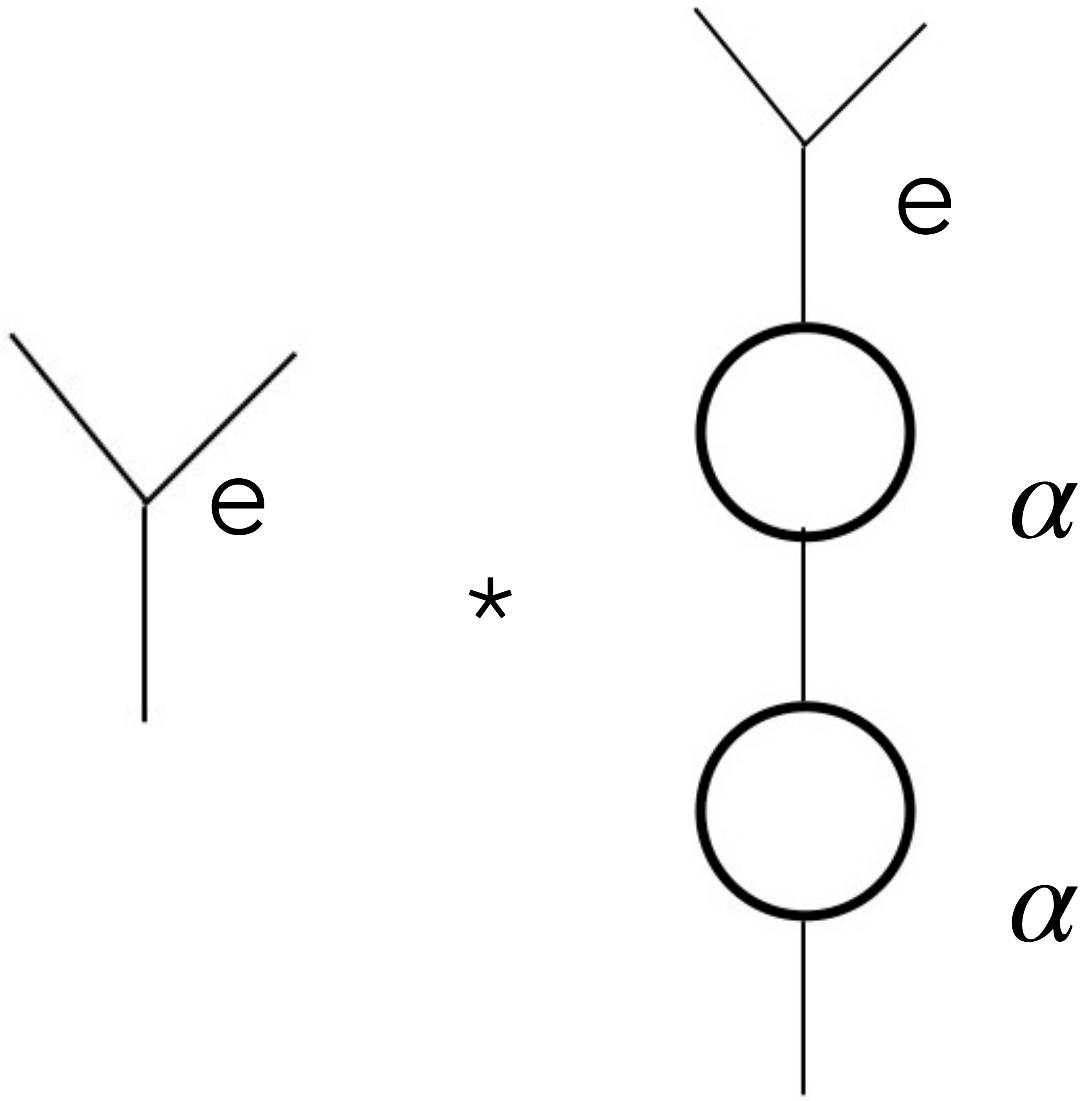
$$\delta_{e\mu}^{(1)} = \frac{2\Re[M_0 M_{1L}^\dagger]}{|M_0|^2} \sim 31\% \qquad \delta_{ep}^{(1)} = \frac{2\Re[M_0 M_{1L}^\dagger]}{|M_0|^2} \sim 17\% \qquad \delta_{\mu p}^{(1)} = \frac{2\Re[M_0 M_{1L}^\dagger]}{|M_0|^2} \sim 3.5\%$$

$$\delta_{e\mu}^{(2)} = \frac{|M_{qad}|^2}{|M_0|^2} \sim 2.4\% \qquad \delta_{ep}^{(2)} = \frac{|M_{qad}|^2}{|M_0|^2} \sim 0.69\% \qquad \delta_{\mu p}^{(2)} = \frac{|M_{qad}|^2}{|M_0|^2} \sim 0.02\%$$

ADVANTAGES OF USING COVARIANT APPROACH

- This is a general approach and can be used to calculate any scattering process with a distinguishable target.

- A good approach to calculate higher order effects by squaring one loop level diagrams e.g. our Quadratic leptonic tensor (α^3).



where $e^2 = 4\pi\alpha$

RESULTS:

- We have produced results for the QED quadratic leptonic tensor which were not calculated previously. We cross checked results using non covariant approach.
- Our $e^-\mu^-$ differential cross section results could be helpful in the background analysis for muon-hydrogen scattering experiments.
- We make predictions for the e^-p and μ^-p NNLO (quadratic) correction. Our e^-p results are particularly useful in background analysis for the proposed Electron-Ion Collider experiment.

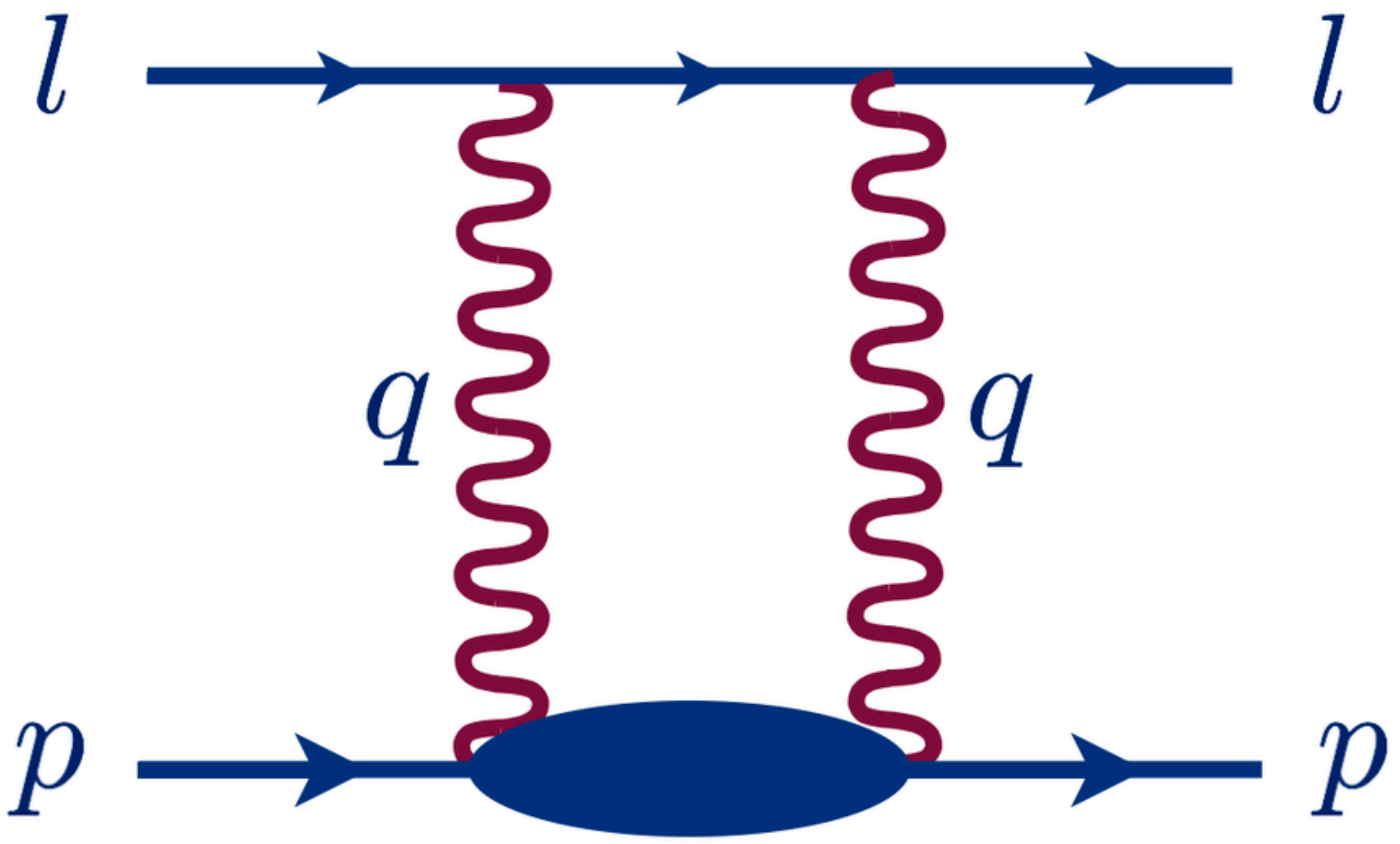
FUTURE GOALS

- For completeness, our next goal is to also include soft and hard photon bremsstrahlung cross sections in the results.
- We are planning to calculate full electroweak leptonic tensor by calculating NLO and quadratic tensor structure functions as we did in case of QED.
- These theoretical predictions will be important for many experimental programs such as MUSE, MOLLER (background studies), EIC etc. searching for physics beyond the Standard Model at the precision frontier.

REFERENCES FOR BOX DIAGRAMS

[1] M. Gorchtein, Phys. Rev. C **73**, 055201 (2006)

[2] Peter G. Blunden et al., Physical Review Letters 91(14)



Thank you for listening :)

QUESTIONS!!