TRIUMF

First Principles Calculations of $p + {}^7\mathrm{Li}$ Radiative Capture (and the X17 anomaly)

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CAP Congress: June 6, 2022

Discovery, accelerated

Radiative Capture $A + B \to C + \gamma$

- Notation: $B(A,\gamma)C$
- A nuclear reaction that often occurs in astrophysics:
  - Stellar burning: $d(p,\gamma)^3\mathrm{He}$, $^3\mathrm{He}(\alpha,\gamma)^7\mathrm{Be}$, $\ldots$
  - Big Bang Nucleosynthesis: $d(p,\gamma)^3\mathrm{He}$, $^4\mathrm{He}(d,\gamma)^6\mathrm{Li}$, $\ldots$
  - Search for new physics: $^7\mathrm{Li}(p,\gamma)^8\mathrm{Be}$, $^3\mathrm{H}(p,\gamma)^4\mathrm{He}$

More Notation:
$d = {}^2\mathrm{H}$
$\alpha = {}^4\mathrm{He}$

adapted from:
Feng PRD **95**, 035017 (2017)

Calculating Radiative Capture

To calculate the rate of reaction (cross section) we need:

- initial wavefunction: $|\Psi_i\rangle$ $(A + B)$
- final wavefunction: $|\Psi_f\rangle$ (C)
- photon interaction (electromagnetic operator): $\hat{O}_\gamma$

We need to calculate the square of the transition matrix elements:

$$\sigma \sim \sum_{if} |\langle\Psi_f|\,\hat{O}_\gamma\,|\Psi_i\rangle|^2$$

Bound States: $|\Psi_f\rangle = \left| J_f^{\pi_f} T_f \right\rangle$

Eigenstate of the nuclear Hamiltonian:

$$H^A |\Psi_k\rangle = E_k |\Psi_k\rangle \text{, where } H^A = \sum_i^A T_i + \sum_{i<j} V_{ij}^{NN} + \sum_{i<j<f} V_{ijf}^{3N}$$

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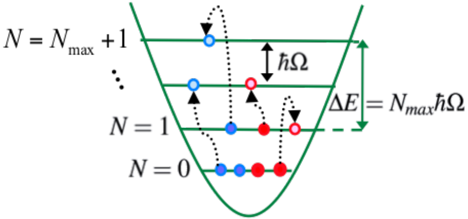
The No-Core Shell Model (NCSM)

Expand in anti-symmetrized products of harmonic oscillator single-particle states:

$$|\Psi_k\rangle = \sum_{N=0}^{N_{max}} \sum_j c_{Nj}^k |\Phi_{Nj}\rangle$$



Convergence to exact as $N_{max} \to \infty$

Unbound (Continuum) States: $|\Psi_i\rangle = \left|[|\Psi_A\rangle\,|\Psi_B\rangle\,\psi(\vec{r}_A - \vec{r}_B)]^{(J_i^{\pi_i} T_i)}\right\rangle$

- The incoming state is made of distinct clusters with relative motion
- Harmonic oscillator states cannot describe the tail of the wavefunction (long-range physics)
- A method beyond the NCSM is needed for scattering and reactions

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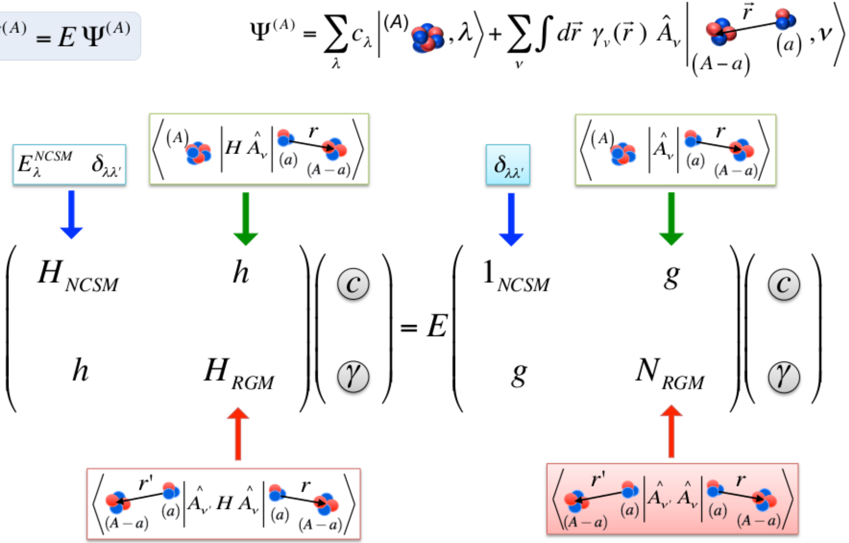
No-Core Shell Model with Continuum (NCSMC)

- Solution: extend the NCSM basis!

$$\Psi^{(A)} = \sum_\lambda c_\lambda \left|^{(A)} , \lambda \right\rangle + \sum_\nu \int d\vec{r} \, \gamma_\nu(\vec{r}) \, \hat{A}_\nu \left| \underset{(A-a)}{} \overset{\vec{r}}{\longleftarrow} \underset{(a)}{} , \nu \right\rangle$$

NCSMC Equations

$$H\,\Psi^{(A)} = E\,\Psi^{(A)}$$



NCSMC for $p + {}^7\mathrm{Li}$ (${}^8\mathrm{Be}$)

$$\Psi^{(8)}_{\text{NCSMC}} = \sum_\lambda c_\lambda \left| {}^8\mathrm{Be}, \lambda \right\rangle + \sum_\nu \int \mathrm{d}r \gamma_\nu(r) \hat{A}_\nu \left| {}^7\mathrm{Li} + p, \nu \right\rangle + \sum_\mu \int \mathrm{d}r \gamma_\mu(r) \hat{A}_\mu \left| {}^7\mathrm{Be} + n, \mu \right\rangle$$

Process:

- Solve NCSM for each constituent nucleus: ${}^8\mathrm{Be}$, ${}^7\mathrm{Li}$ and ${}^7\mathrm{Be}$
  - 30 eigenstates from ${}^8\mathrm{Be}$
  - 5 eigenstates each from ${}^7\mathrm{Li}$ and ${}^7\mathrm{Be}$
- Solve NCSMC for $c_\lambda(E)$, $\gamma_\nu(r, E) \to \Psi(E)$
- Cross-section depends on transition matrix elements e.g. $\langle \Psi(E_0) | M1 | \Psi(E) \rangle$

Results

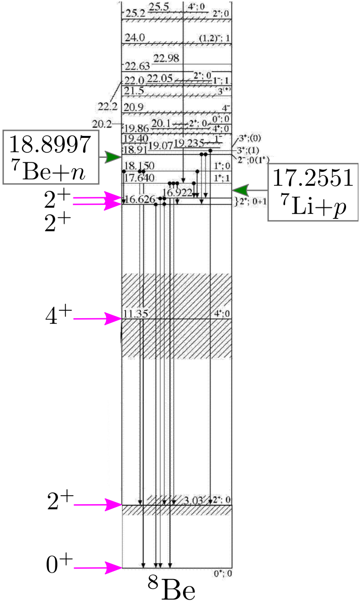
- ■ $^{8}\mathrm{Be}$ Structure
- □ Scattering: $^{7}\mathrm{Li}(p,p)^{7}\mathrm{Li}$
- □ Transfer Reactions: $^{7}\mathrm{Li}(p,n)^{7}\mathrm{Be}$, $^{7}\mathrm{Be}(n,p)^{7}\mathrm{Li}$
- ■ Radiative Capture: $^{7}\mathrm{Li}(p,\gamma)^{8}\mathrm{Be}$
- ■ Search for new physics: $^{7}\mathrm{Li}(p,X)^{8}\mathrm{Be}$

^{8}Be Structure

Calculations of ^{8}Be “bound” states (w.r.t. ^{7}Li $+ p$ threshold) are improved by inclusion of the continuum ($N_{max} = 9$)

State	Energy [MeV]		
	NCSM	NCSMC	Experiment
0^+	-15.96	-16.13	-17.25
2^+	-12.51	-12.72	-14.23
4^+	-3.97	-4.31	-5.91
2^+	$+0.76$	-0.10	-0.63
2^+	$+1.09$	$+0.31$	-0.33

- Energies likely too high due to neglected $\alpha + \alpha$ breakup
- Matches experiment well, except the 3rd 2^+ is still slightly above the ^{7}Li $+ p$ threshold



TUNL Nuclear Data Evaluation Project

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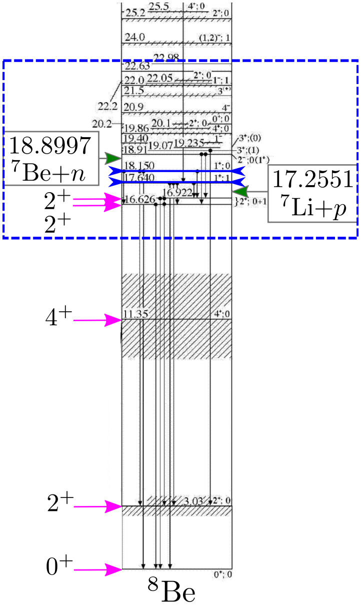

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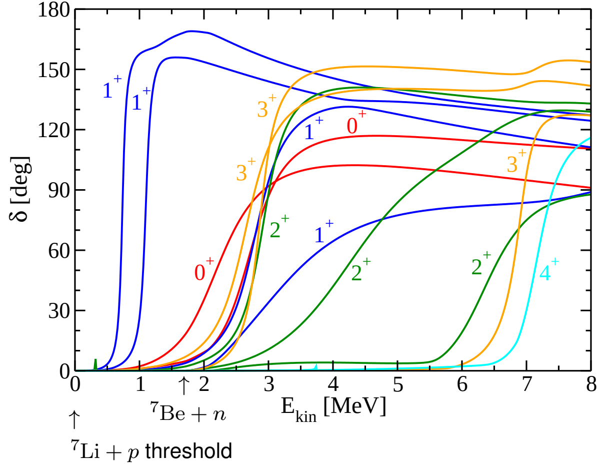
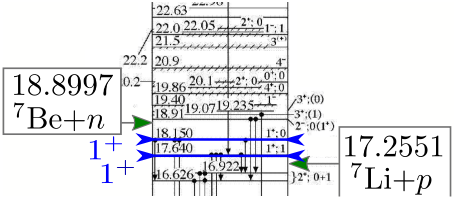
Eigenphase-shift Results (positive parity)

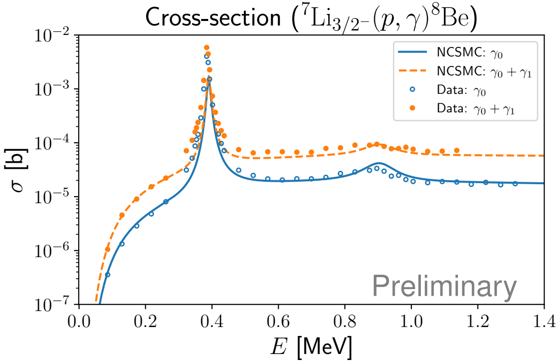


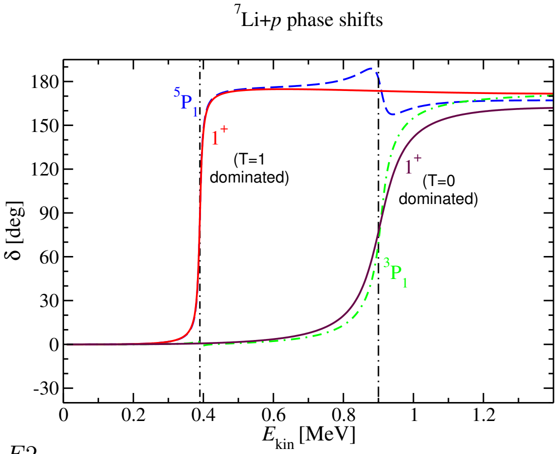


Additional resonances are seen compared to TUNL data

Radiative Capture

$$^{2S+1}P_J : \left[\left(\left| ^7\mathrm{Li} \right\rangle |p\rangle \right)^{(S)} Y_L(\hat{r}) \right]^J$$
$$P : L = 1$$





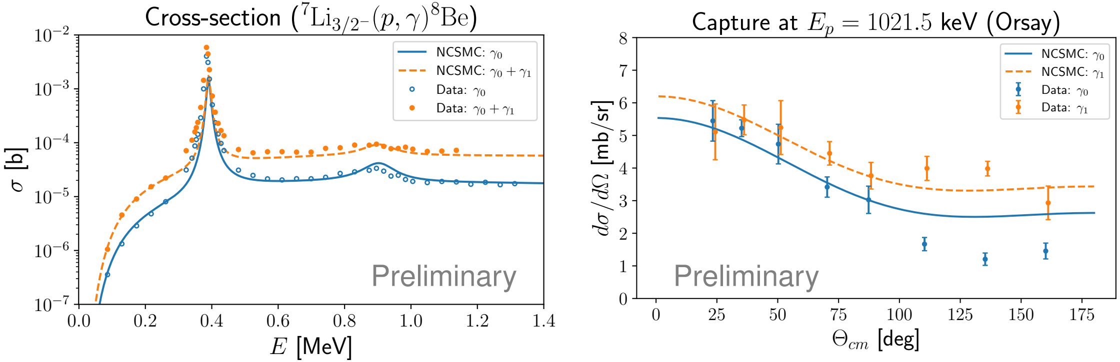
γ_0: decay to ground state (0^+)
γ_1: decay to first excited (2^+)

$$\hat{O}_\gamma = E1 + M1 + E2$$

Phenomenological adjustment: fit threshold and resonance positions to match experiment

Data: Zahnow et al
Z.Phys.A **351** 229-236 (1995)

Radiative Capture (cont.)



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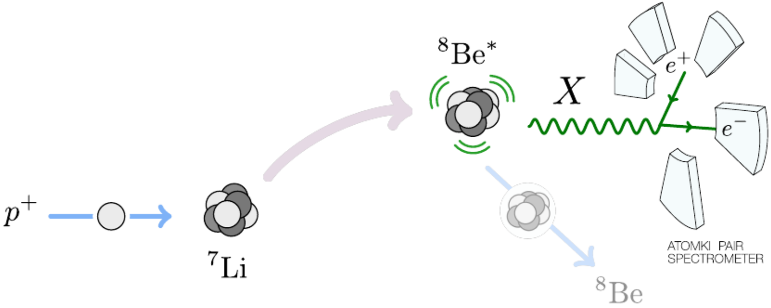
The X17 Anomaly in $p + {}^{7}\mathrm{Li} \to {}^{8}\mathrm{Be} + e^{+}e^{-}$

- ${}^{7}\mathrm{Li}(p, e^{+}e^{-})^{8}\mathrm{Be}$ @ATOMKI (Hungary)
- The decay of ${}^{8}\mathrm{Be}$ 1^{+} excited states produces electron-positron pairs



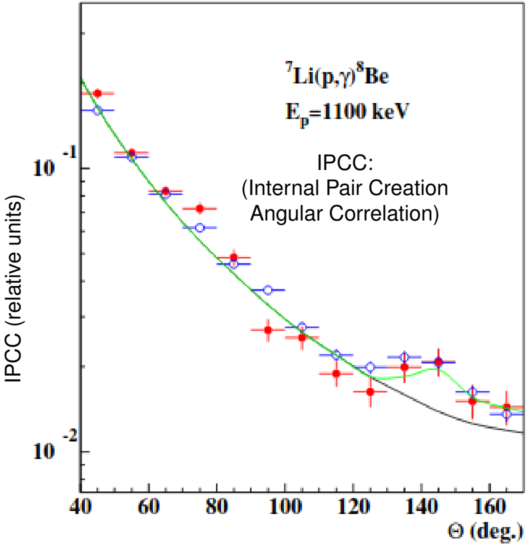
Feng PRD **95**, 035017 (2017)

The X17 Anomaly in $p + {}^{7}\mathrm{Li} \to {}^{8}\mathrm{Be} + e^{+}e^{-}$

Firak, Krasznahorkay, et al
EPJ Web of Conferences **232 04005 (2020)**

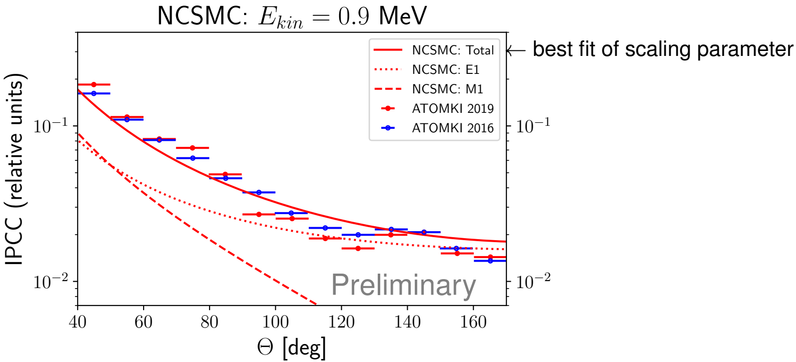
- The angle θ between the electron and positron was measured
- Minimum angle from a massive intermediate particle: $\theta \simeq \sin^{-1}(\frac{m_X}{E_X})$
- Bump could be explained by 17 MeV bosons decaying to $e^{+}e^{-}$

Can *ab initio* nuclear physics help interpret the anomaly?

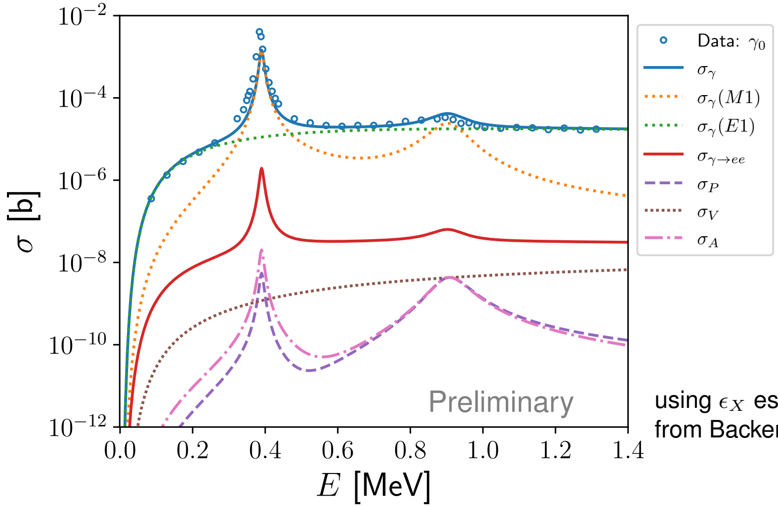


Pair Production Distribution

- Our calculations (based on 2106.06834) under(over)-predict low(high) angles (possible background contamination or missing E1-M1 interference)
- Ongoing and planned experiments at Orsay and Montreal will provide an independent verification of the anomaly
- New ATOMKI data just published (2205.07744), analysis in progress



Preliminary $^7\mathrm{Li}(p,X)^8\mathrm{Be}$ Cross-sections



Pseudo-scalar (0^-)
Vector (1^-)
Axial-vector (1^+)

using ϵ_X estimates
from Backens [2110.06055]

Summary

- NCSMC successfully describes the spectrum of $^{8}\mathrm{Be}$ including the 1^{+} resonances
- Calculations of $^{7}\mathrm{Li}(p,\gamma)^{8}\mathrm{Be}$ radiative capture match data

Outlook

- Compare $^{7}\mathrm{Li}(p,e^{+}e^{-})^{8}\mathrm{Be}$ to data with $\gamma \to e^{+}e^{-}$ operator and various $X \to e^{+}e^{-}$ operators (e.g. axions, vector bosons, axial vector bosons)
- Calculations of $^{3}\mathrm{H}(p,e^{+}e^{-})^{4}\mathrm{He}$ are also relevant to the X17 anomaly
- Explore charge-exchange reactions relevant for nucleosynthesis: $^{7}\mathrm{Be}(n,p)^{7}\mathrm{Li}$, $^{7}\mathrm{Li}(p,n)^{7}\mathrm{Be}$

Backup Slides

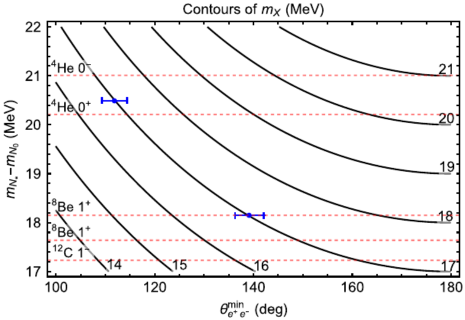
Constraints on m_X

Feng PRD **95**, 035017 (2017)

In the frame of the X boson the electron and positron momenta are anti-parallel. Boosted to a minimum separation angle:

$$\theta = 2\sin^{-1}\left(\frac{m_X}{E_X}\right)$$

- $^8\mathrm{Be}$ anomaly occurs in the isoscalar transition (decay of 1^+0 resonance)
- In-between resonances in $^4\mathrm{He}$
- Bumps could be explained by 17 MeV bosons decaying to e^+e^-



Exclusion Plot

Current (gray) and projected sensitivities of future experiments

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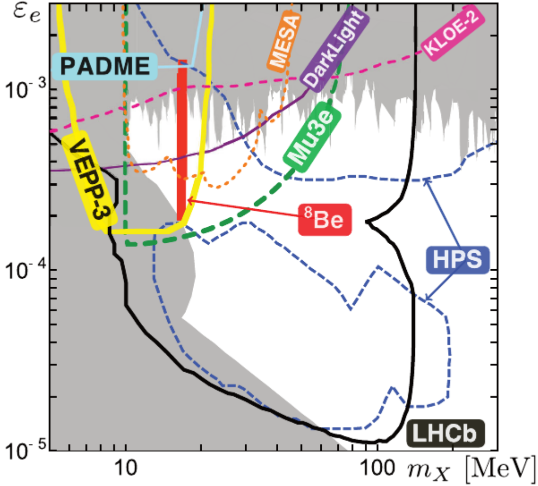
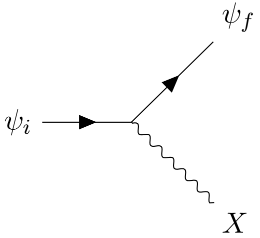


X17 Candidate Bosons

($m_X \simeq 17$ MeV, $\Delta E \geq 17.2251$ MeV [$^7\mathrm{Li} + p$],
$k_X = \sqrt{\Delta E^2 - m_X^2}$, $k_\gamma = \Delta E$)

Operators for $1^+ \to 0^+$ decay (in the long-wavelength approximation)

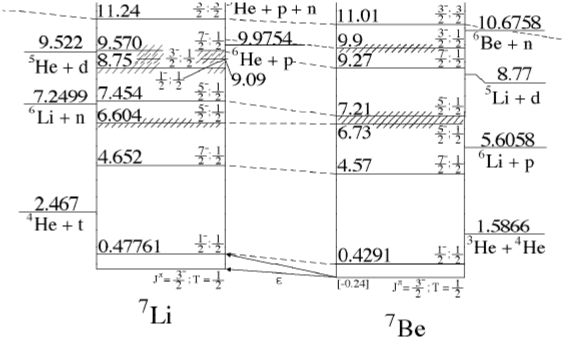
- Pseudo-scalar (0^-):$\langle X_P \rangle \sim \epsilon_P \left\langle \hat{S} \right\rangle k_X$
- Axial-vector (1^+): $\langle X_A \rangle \sim \epsilon_A \left\langle \hat{S} \right\rangle \sqrt{2 + \frac{m_X^2}{\Delta E^2}}$
- Vector (1^-): $\langle X_V \rangle \sim \epsilon_V \left\langle \hat{O}_\gamma \right\rangle \frac{k_X}{k_\gamma}$
- For comparison: γ (E1 (1^-), M1 (1^+), E2 (2^+), etc)
  $\langle E1 \rangle \sim \langle r Y_1 \rangle k_\gamma$
  $\langle M1 \rangle \sim \left(g_l \left\langle \hat{L} \right\rangle + g_s \left\langle \hat{S} \right\rangle \right) k_\gamma$



Input States from NCSM

$$\Psi^{(8)}_{\text{NCSMC}} = \sum_{\lambda} c_\lambda \left|{}^8\text{Be}, \lambda\right\rangle + \sum_{\nu} \int \mathrm{d}r \gamma_\nu(r) \hat{A}_\nu \left|{}^7\text{Li} + p, \nu\right\rangle + \sum_{\mu} \int \mathrm{d}r \gamma_\mu(r) \hat{A}_\mu \left|{}^7\text{Be} + n, \mu\right\rangle$$

- 3 NCSM calculations: ${}^7\text{Li}$, ${}^7\text{Be}$ and ${}^8\text{Be}$
- $\{\frac{3}{2}^-, \frac{1}{2}^-, \frac{7}{2}^-, \frac{5}{2}^-, \frac{5}{2}^-\}$ ${}^7\text{Li}$ and ${}^7\text{Be}$ states in cluster basis
- 15 positive and 15 negative parity states in ${}^8\text{Be}$ composite state basis



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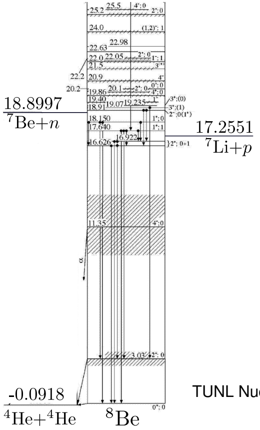

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