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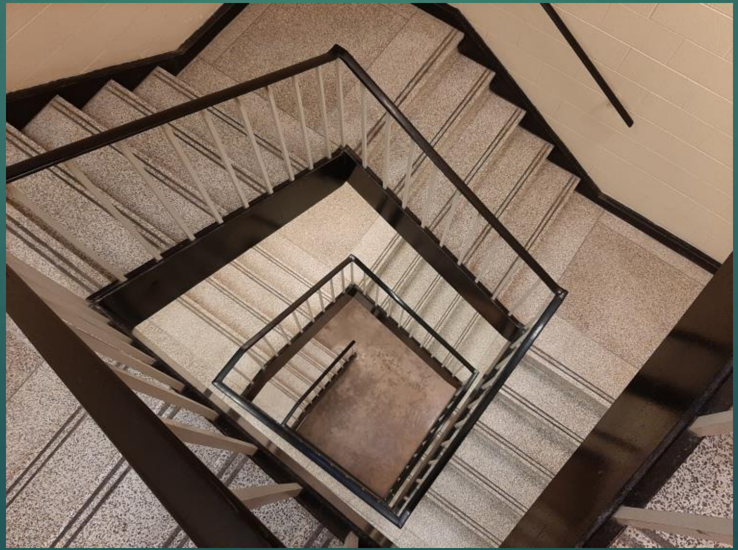

Introduction

- Noisy Intermediate-Scale Quantum (NISQ) era
  - Quantum computers
  - Up to few hundred qubits
  - Still noisy qubits

- D-wave

~ 5000*

- IBM

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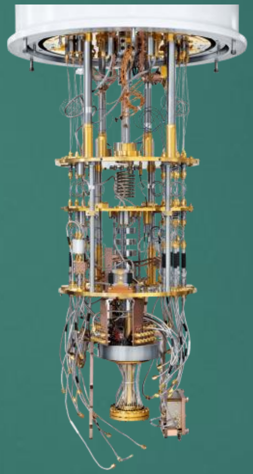

- Rigetti

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Google

XANADU
Photonic Quantum Computing

intel

and many others…

- Used in many applications with results and performances comparable to classical computers

SU(2) pure gauge lattice theory on a quantum computer

[S. A Rahman, R. Lewis, E. Mendicelli, and S. Powell, (Mar. 2021), arXiv: 2103.08661 [hep-lat]]

$$\hat{H} = \frac{g^2}{2}\left(\sum_{i=\text{links}} \hat{E}_i^2 - 2x \sum_{i=\text{plaquettes}} \hat{\square}_i\right)$$

- Using the standard angular momentum base:

$$|\psi\rangle = |j_A, m_A, m'_A\rangle\, |j_B, m_B, m'_B\rangle \ldots |j_L, m_L, m'_L\rangle$$

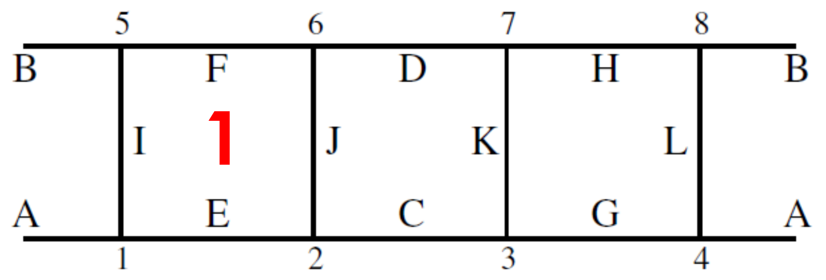
- The plaquette operator is:

$$\square_1 = \sum_{s_1}\sum_{s_2}\sum_{s_6}\sum_{s_5}(-1)^{s_1+s_2+s_6+s_5} U^E_{-s_1,s_2} U^J_{-s_2,s_6} U^F_{s_5,-s_6} U^I_{s_1,-s_5}$$

- The chromoelectric field contibution is:

$$\langle\psi|\sum_i \hat{E}_i^2\,|\psi\rangle = \sum_{i=A}^{L} j_i(j_i+1)$$

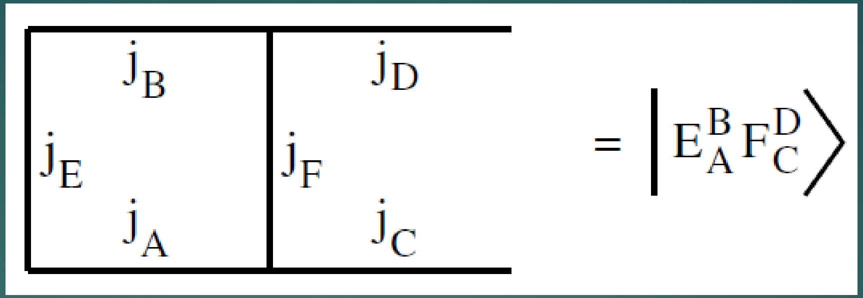
- Ring-shape lattice of plaquettes



- Periodic boundary condition

$$x \equiv \frac{2}{g^4}$$

The smallest lattice: 2-plaquette



- Using the angular momentum base and summing over all the projections of J, mL and mR, highly simplify the calculation of the matrix representation.

  - The lattice is symmetric under vertical and horizontal reflection and spatial translation.

$$H = \frac{g^2}{2}\begin{pmatrix} 0 & -2x & -2x & 0 \\ -2x & 3 & 0 & -\frac{x}{2} \\ -2x & 0 & 3 & -\frac{x}{2} \\ 0 & -\frac{x}{2} & -\frac{x}{2} & 3 \end{pmatrix}$$

- Block diagonalizing →

$$H = \frac{g^2}{2}\left(\begin{array}{ccc|c} 0 & -2\sqrt{2}x & 0 & 0 \\ -2\sqrt{2}x & 3 & -\frac{x}{\sqrt{2}} & 0 \\ 0 & -\frac{x}{\sqrt{2}} & 3 & 0 \\ \hline 0 & 0 & 0 & 3 \end{array}\right)$$

$$x \equiv \frac{2}{g^4}$$

- <u>This is essential for reducing the number of needed qubits</u>

D-Wave

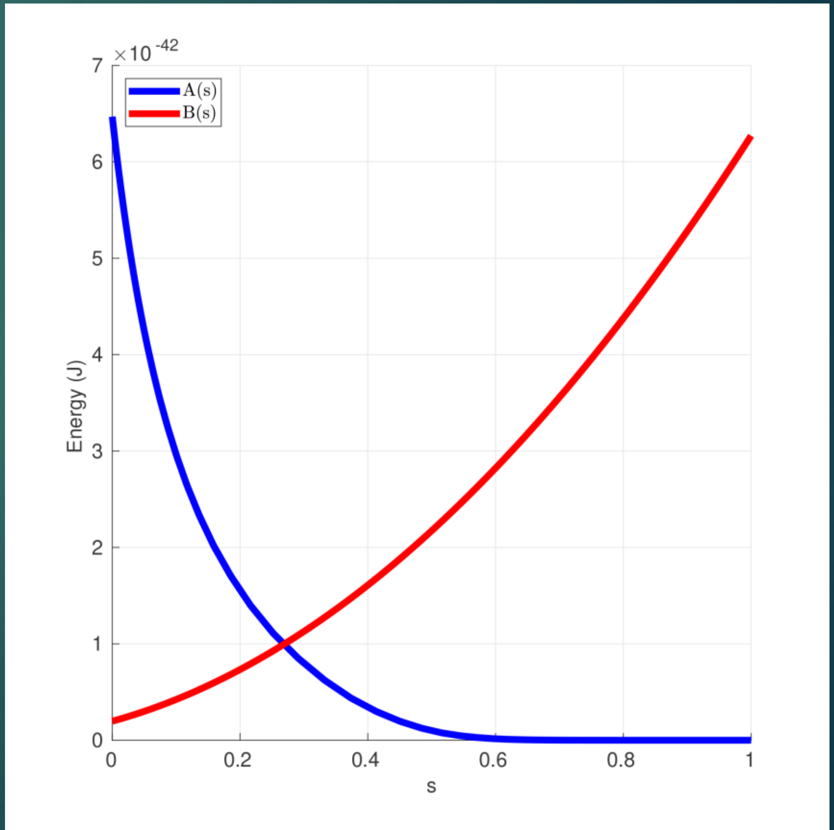
- D-Wave finds the ground state of any Ising-like Hamiltonian:

$$H(q) = \sum_{i=1}^{N} h_i q_i + \sum_{i=1}^{N} \sum_{j=i+1}^{N} J_{ij} q_i q_j$$

$$H(q,s) = A(s)\left[\sum_{i=1}^{N} q_i\right] + B(s)\left[\sum_{i=1}^{N} h_i q_i + \sum_{i=1}^{N} \sum_{j=i+1}^{N} J_{ij} q_i q_j\right]$$

Initial DW Ham. — Problem to solve, final Ham.

- Performing the Annealing



- Adiabatic theorem:

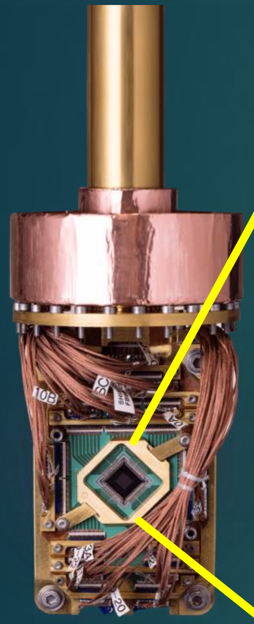
If the Hamiltonian is varied slowly enough, the ground state will stay close to the instantaneous ground state of the Hamiltonian at each time t. [Phys. Rev. A. 65.042308]

$$|\langle G_{state}(T)|\Psi(T)\rangle|^2 \geq 1 - \left(\frac{\min_{0\leq t\leq T}[IE_{state}(t) - G_{state}(t)]}{(\max_{0\leq t\leq T}|\langle IE_{state}(t)|dH/dt|G_{state}(t)\rangle|)^2}\right)^2$$

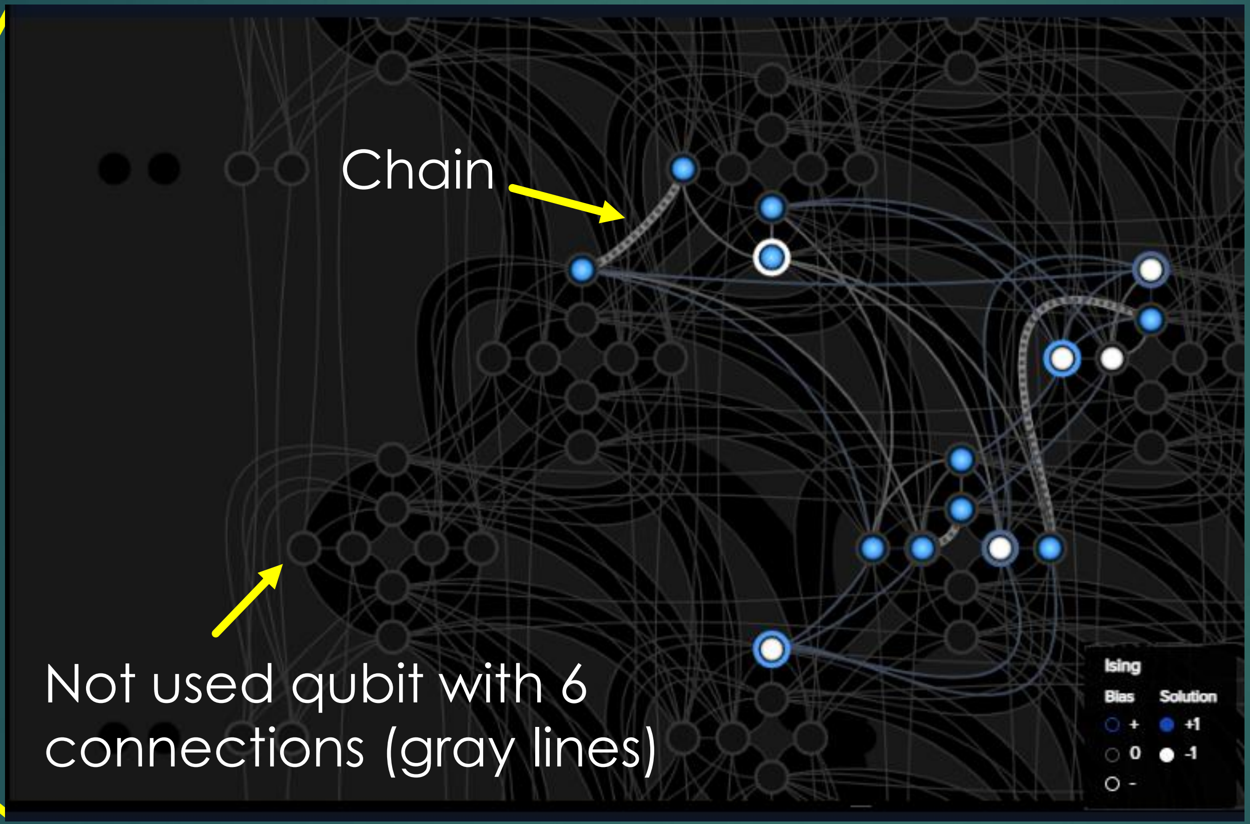
D-Wave Chain Strength

- The Chain is a set of qubits needed to represent the same variable.

- Problem embedded on the hardware



QPU



- Chain Broken \ Un-Broken

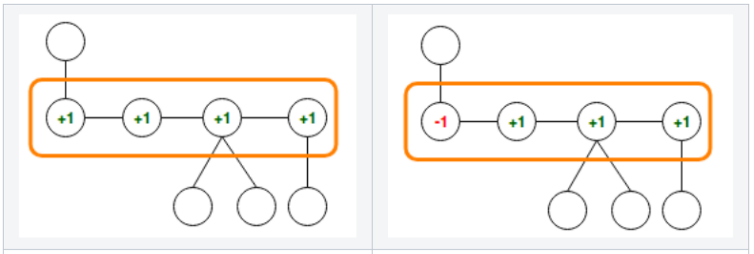


Fig 4: Un-broken chain

Fig 5: Broken chain

[https://support.dwavesys.com/]

- How to choose the chain value:

- The numerical value of the chain should be big enough to avoid that the chains break easily and small enough to not change the physics of the problem.

Quantum Annealer Eigensolver (QAE)*

- QAE introduces an extra Lagrange multiplier λ, to avoid the trivial minimum, the null vector to appear:

$$\langle\psi| H |\psi\rangle \rightarrow \langle\psi| H |\psi\rangle - \lambda \langle\psi|\psi\rangle$$

- Uses a fixed point representation, K qubits are used to represent a real number: $a_i = -q_K^i + \sum_{k=1}^{K-1} \frac{q_k^i}{2^{K-k}}$

(K=1) -1 0

(K=2) -1 $-\frac{1}{2}$ 0 $\frac{1}{2}$

(K=4) -1 $-\frac{7}{8}$ $-\frac{3}{4}$ $-\frac{5}{8}$ $-\frac{1}{2}$ $-\frac{3}{8}$ $-\frac{1}{4}$ $-\frac{1}{8}$ 0 $\frac{1}{8}$ $\frac{1}{4}$ $\frac{3}{8}$ $\frac{1}{2}$ $\frac{5}{8}$ $\frac{3}{4}$ $\frac{7}{8}$

- Each matrix element can only assume the values allowed by K. (Limits the precision)

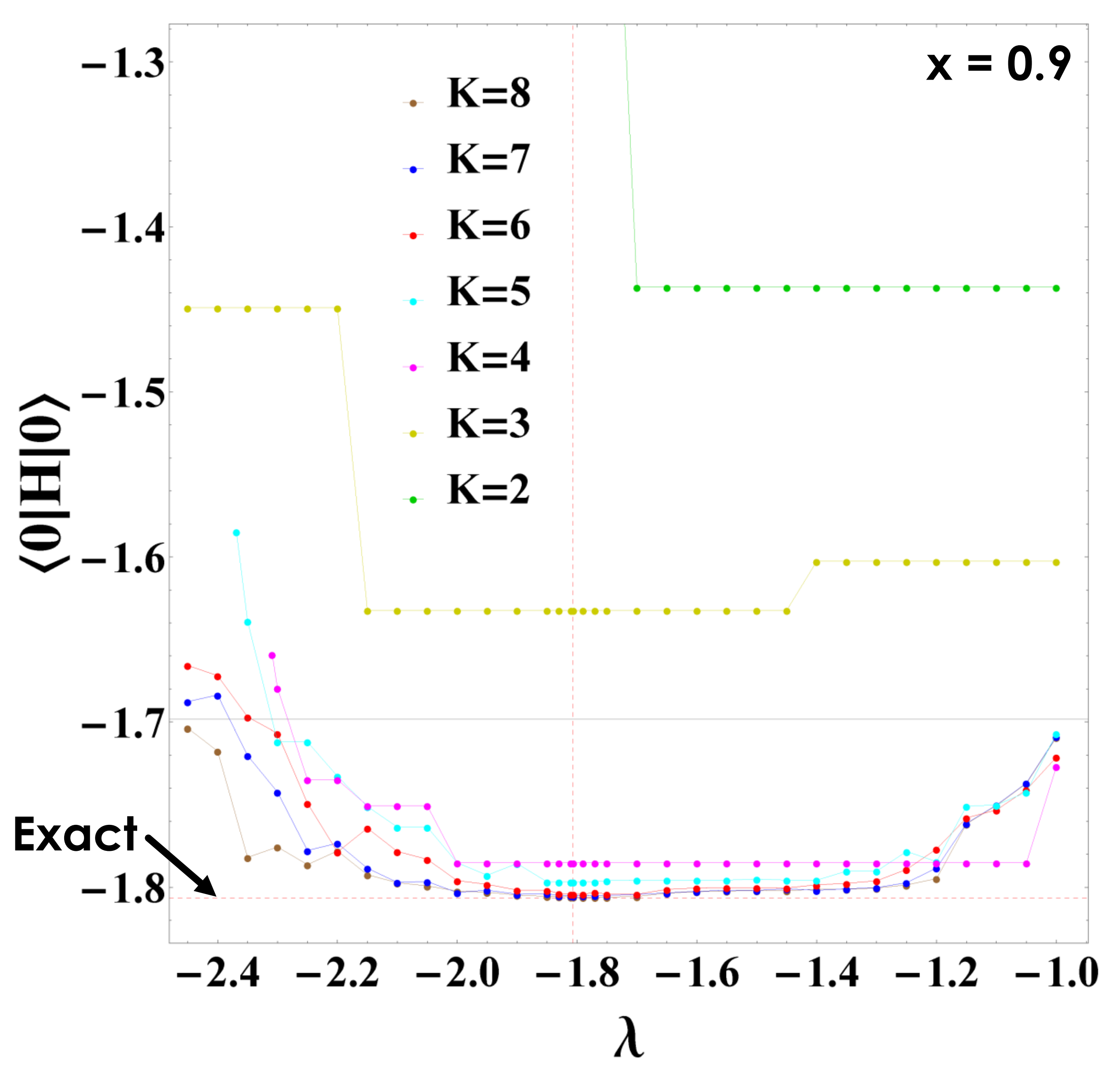
*[A. Teplukhin, B. K. Kendrick, and D. Babikov, J. Chem. Theory & Comp. 15, 4555(2019), doi: 10.1021/acs.jctc.9b00402.]

Algorithm Steps

- Steps to extract the ground state for a single value of the gauge coupling:

1°

Find <u>λ optimum</u> and the number of qubits needed (K) on the quantum simulator.



Algorithm Steps

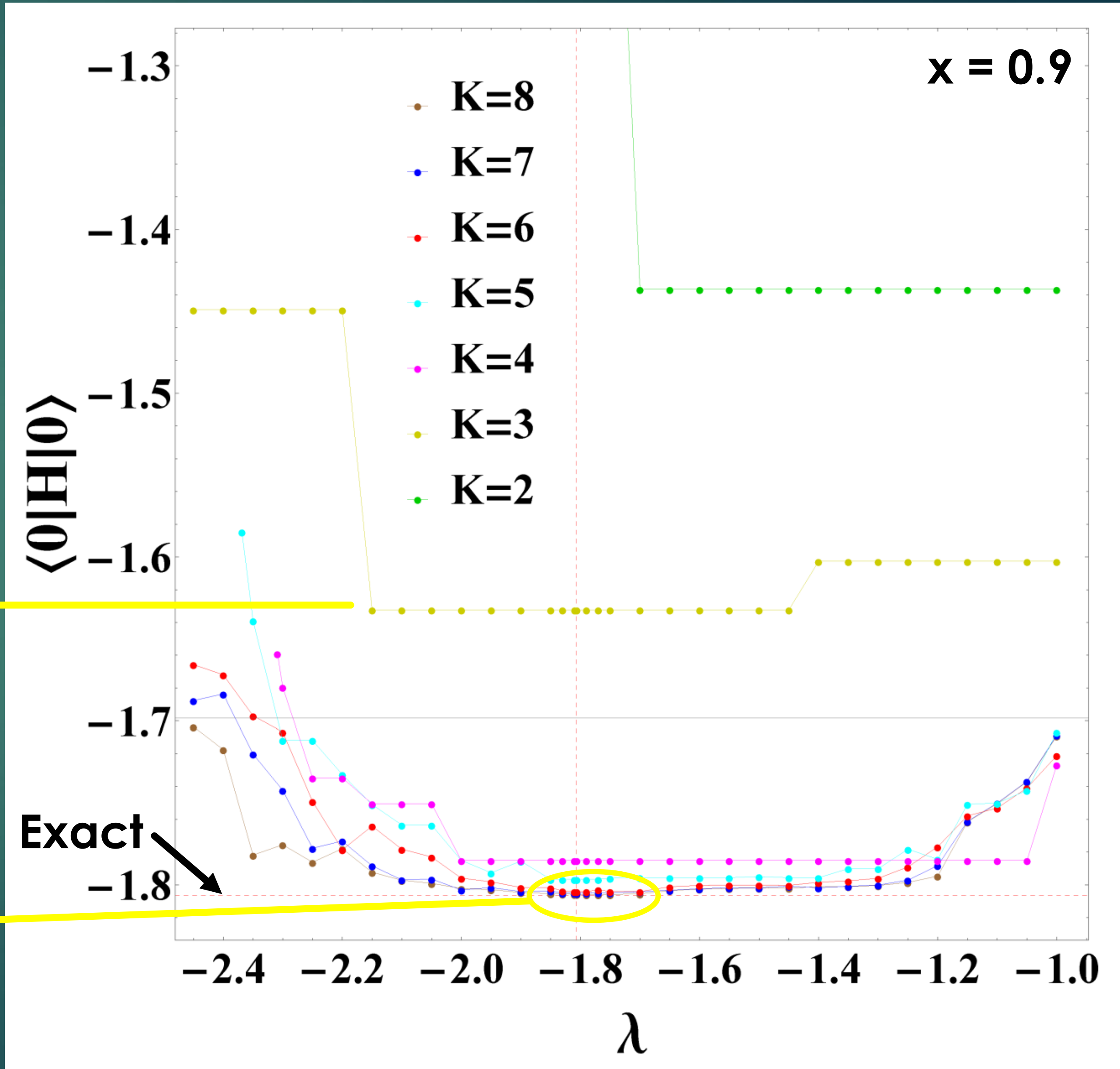
- Steps to extract the ground state for a single value of the gauge coupling:

1°

Find λ optimum and the number of qubits needed (K) on the quantum simulator.

- Using K = 2 and 3 is not enough

- For K = 6, 7 and 8 there is a range of optimum values



Algorithm Steps

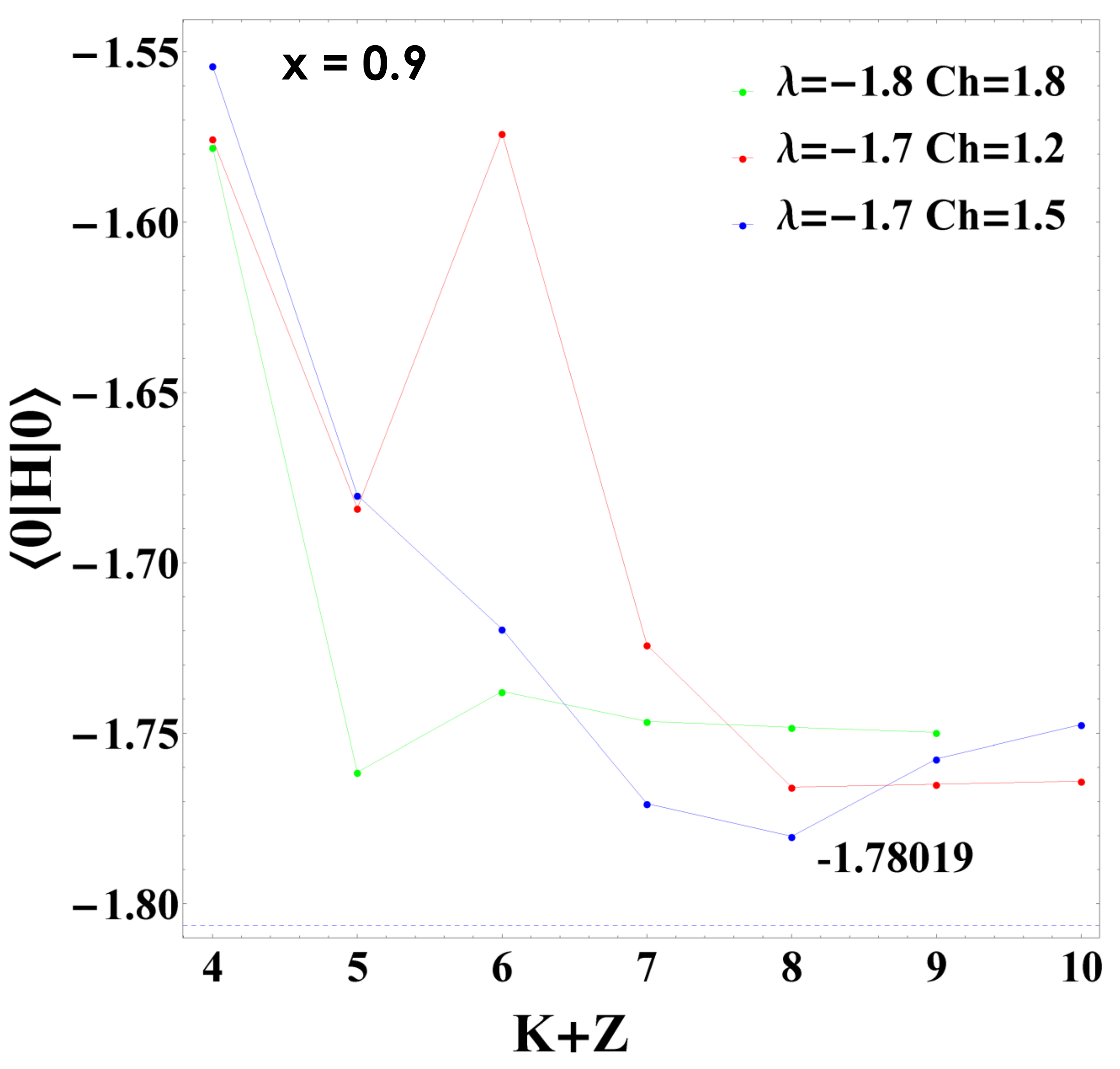
- Steps to extract the ground state for a single value of the gauge coupling

2°

Tune the Chain strength

3°

Accept or re-do it increasing the number of qubits (K)



The Adaptive QAE algorithm (AQAE)

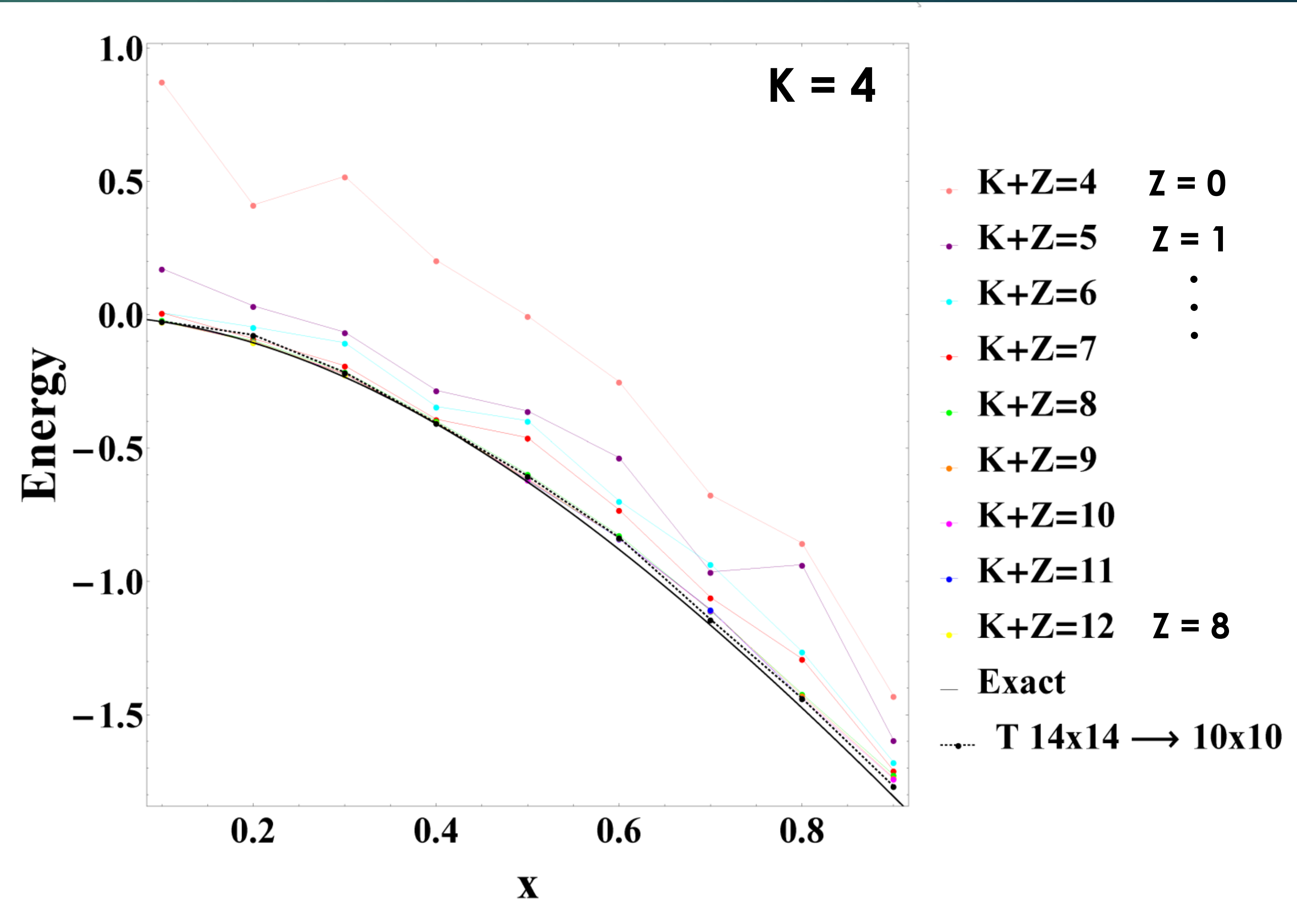
- Short explanation of the AQAE:

- Run the QAE several time (Z) and use as a starting state (vector) the one previously found.

- The search for the new vector is centered on the previous one and now its allowed values are more finely spaced.

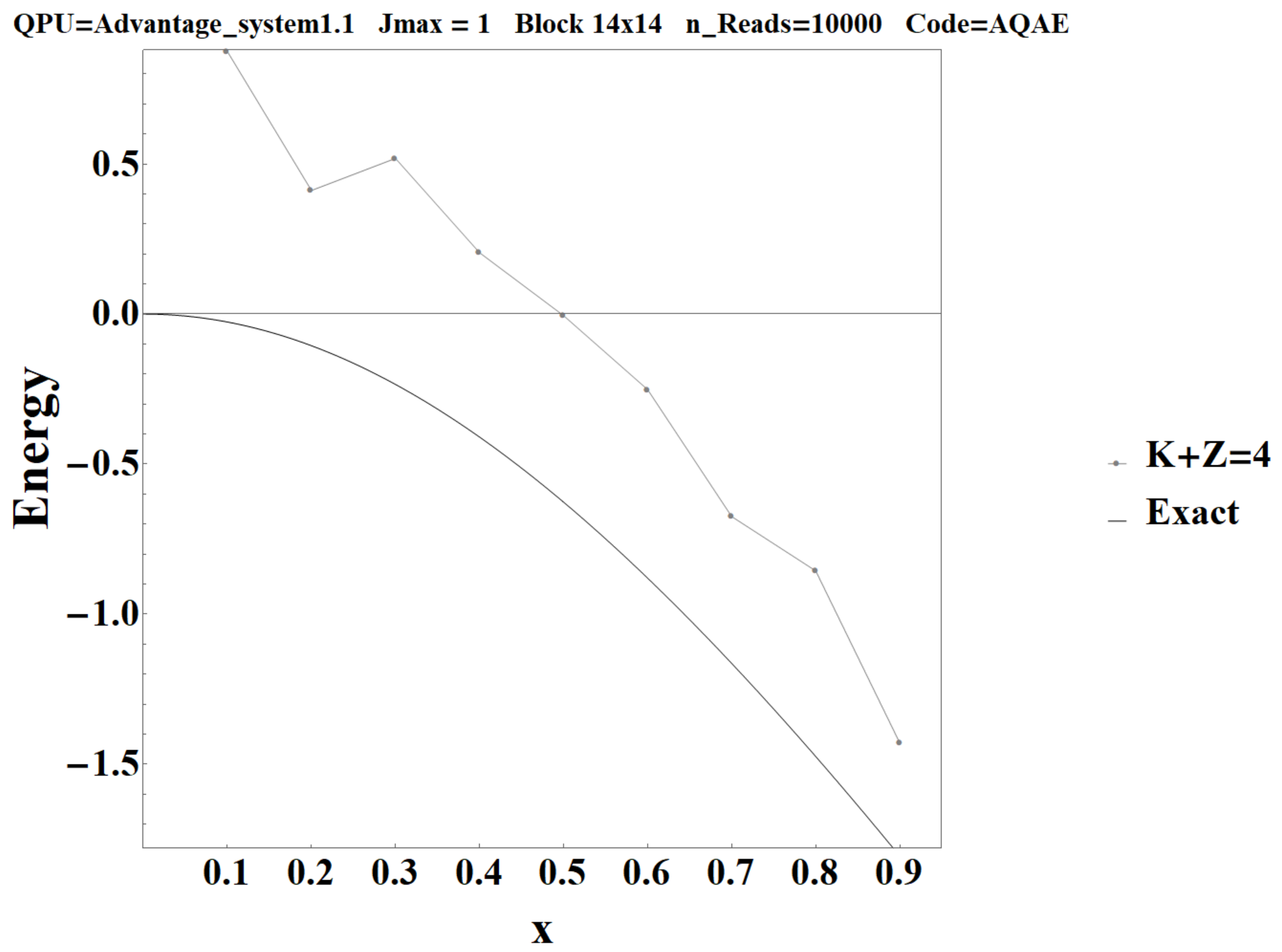
- Therefore it converges faster with fewer qubits.

- AQAE results

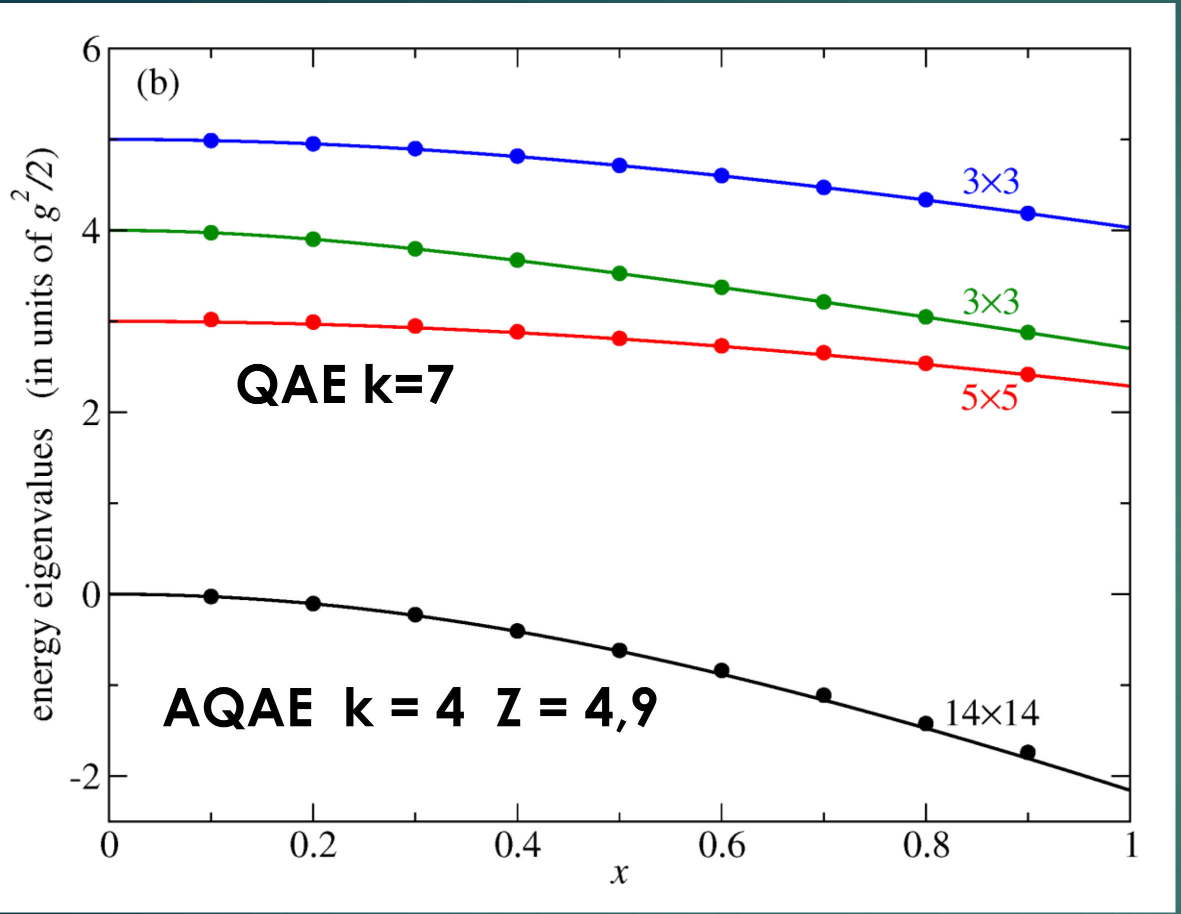


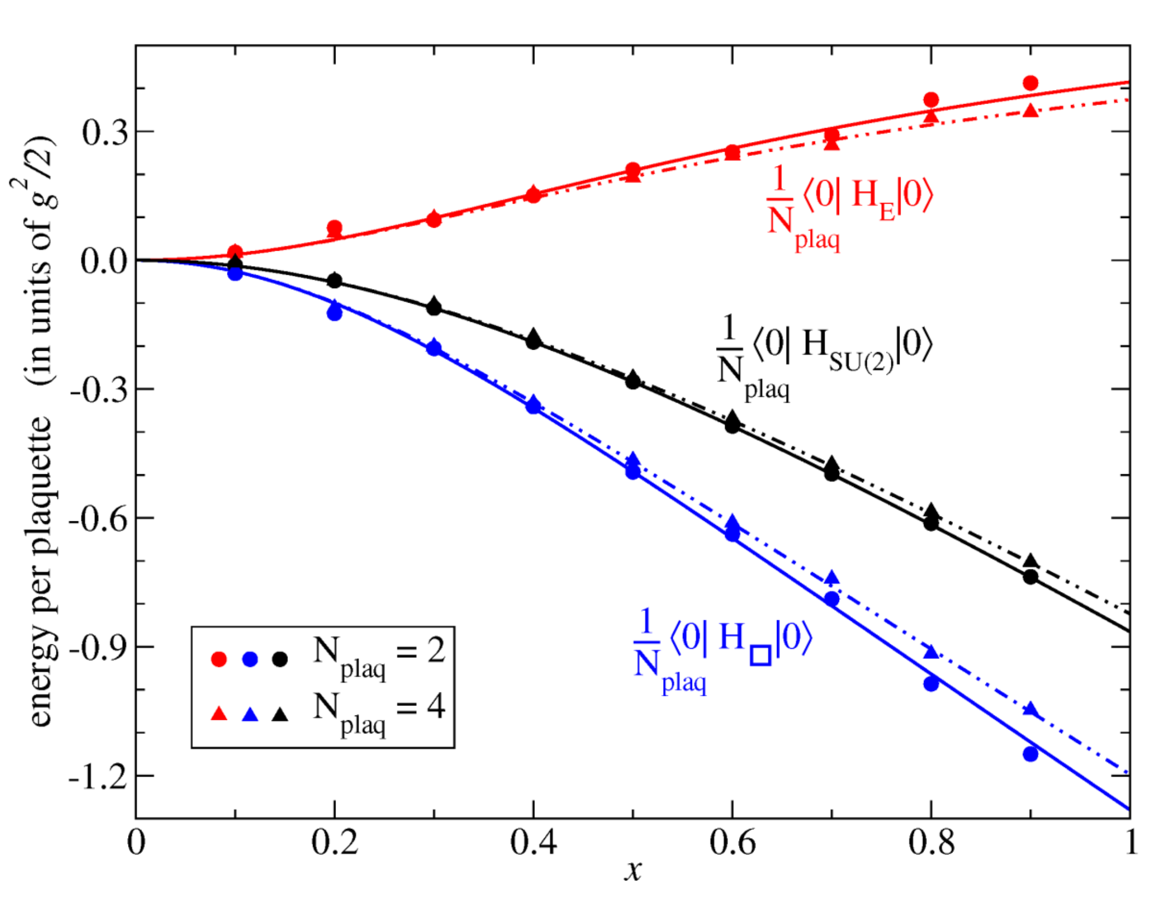
2-plaquettes jmax=1 block (14x14)

The AQAE in action



Some Final Results





- Satisfactory results for the ground state and the excited states.
  2-plaquettes and jmax=1.
- The data points are in acceptable agreement with the exat values (solid lines).
  [jmax=1/2]

Summary and view for the future

- In the NISQ era we cannot simulate lattice gauge theory on small standard lattice sizes (10^3) using a quantum computer.
- We have shown that the D-Wave quantum computers can be used with a bit of effort to extract the ground states of a lattice gauge theory.
- A further study of our algorithm is in [B. Krakoff, S. M. Miniszewski, and C. Negre, (Apr. 2021), arXiv:2104.11311v1 [cs.ET]]

- The present goal is mastering the use of quantum computers.
- Conceptually quantum computers can solve unsolvable problems for classical computers.
  - It is a technological problem as well as conceptual!

~ 1980

2021

Commodore 1987

Foldable pc 2020

Thank you for your time