

A universal holographic wavefunction for hadrons

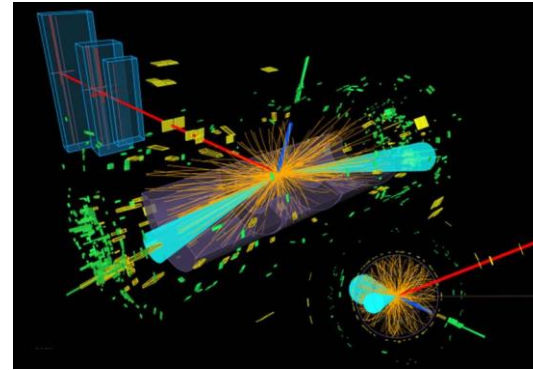
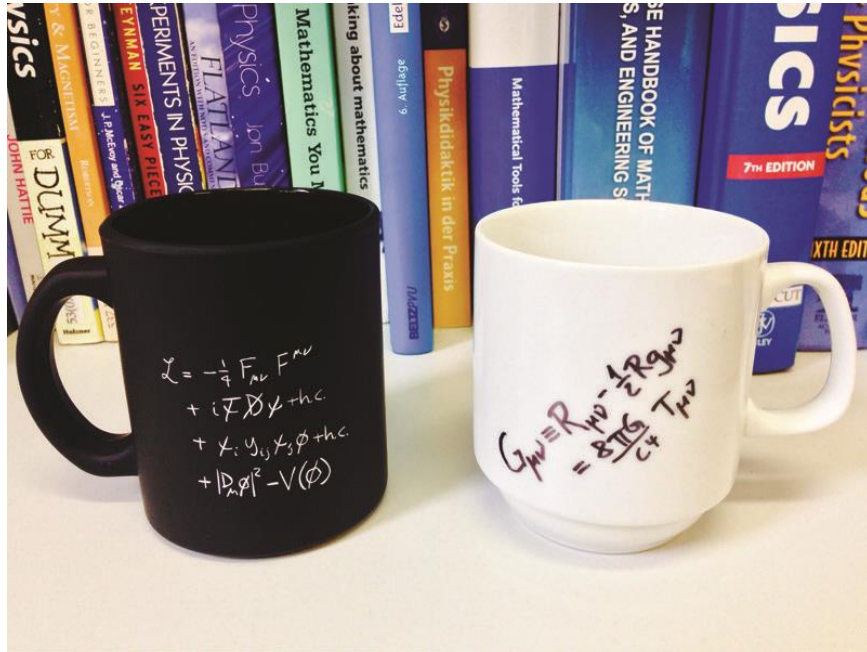
Ruben Sandapen



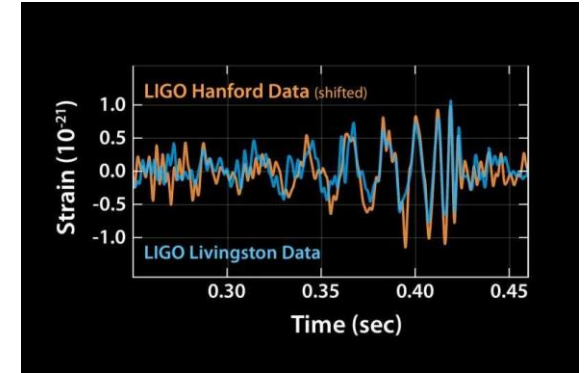
June, 2021

CAP Virtual Congress

Standard Model and General Relativity



Higgs boson: 1964
Discovery: 2012 at LHC



Gravity waves: 1915
Discovery: 2016 at LIGO

- Gravity: Einstein's General Relativity
- Strong, weak and EM interactions: Standard Model
- GR seems incompatible with SM
- Caveat: in same number of spacetime dimensions

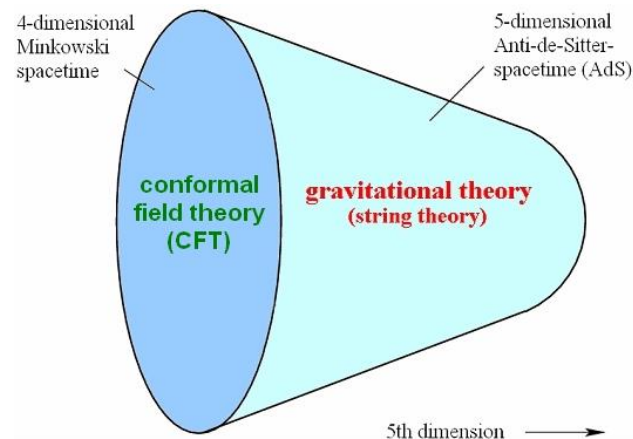
The holographic principle

Maldacena: AdS=CFT

Conformal Field Theory in 4-dim spacetime dual to string theory in higher dimensional curved spacetime

AdS₅ metric

$$ds^2 = \frac{R^2}{z_5^2} \left(h_{mn} dx^m dx^n - dz_5^2 \right)$$



Weak-strong duality

- Conformal symmetry implies scale invariance: no mass scale in theory
- When CFT is strongly coupled, gravity dual is weakly coupled and vice-versa

Anti de Sitter spacetime

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

spacetime geometry

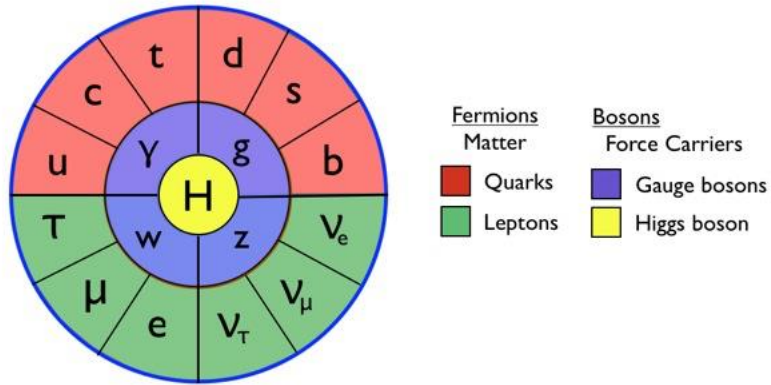
matter-energy

Cosmological constant: Λ

$$T_{\mu\nu} = 0$$

- $\Lambda > 0$: de Sitter spacetime (4-dim de Sitter is what our Universe looked like during inflation)
- $\Lambda < 0$: Anti de Sitter (AdS) spacetime
- $\Lambda = 0$: Minkowski (flat) spacetime

Quantum Chromodynamics

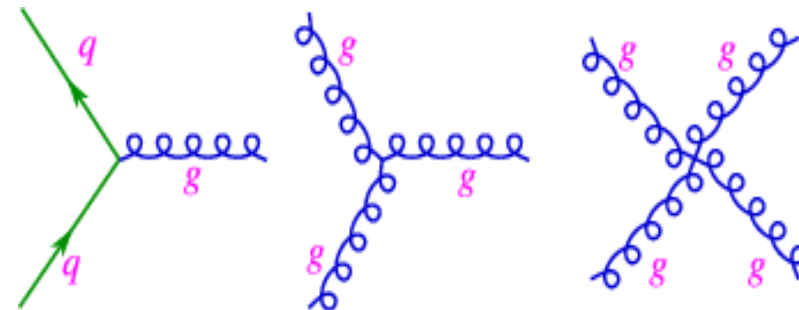


Particles of the Standard Model

QCD: Exact SU(3) gauge symmetry

- Quarks and gluons have color charge
- Quarks interact by exchanging gluons
- Gluons interact by exchanging gluons

QCD is part of the SM and is the theory for the strong interaction

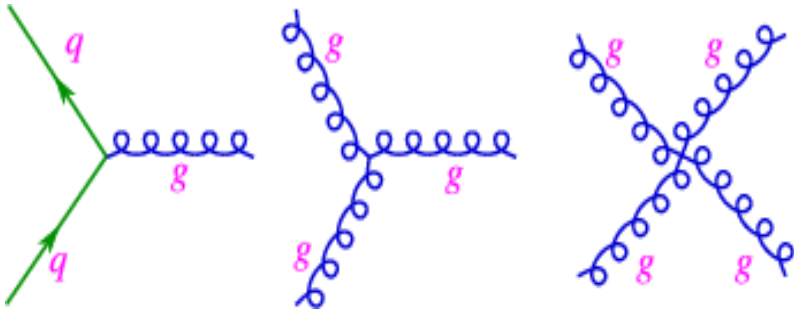


Asymptotic freedom to confinement

$$\mathcal{L}_{\text{QCD}} = \bar{\Psi}(i\gamma^\mu D_\mu - m)\Psi - \frac{1}{4}G_{\mu\nu}^a G^{a\mu\nu}$$

$$D_\mu = \partial_\mu - ig_s A_\mu^a T^a$$

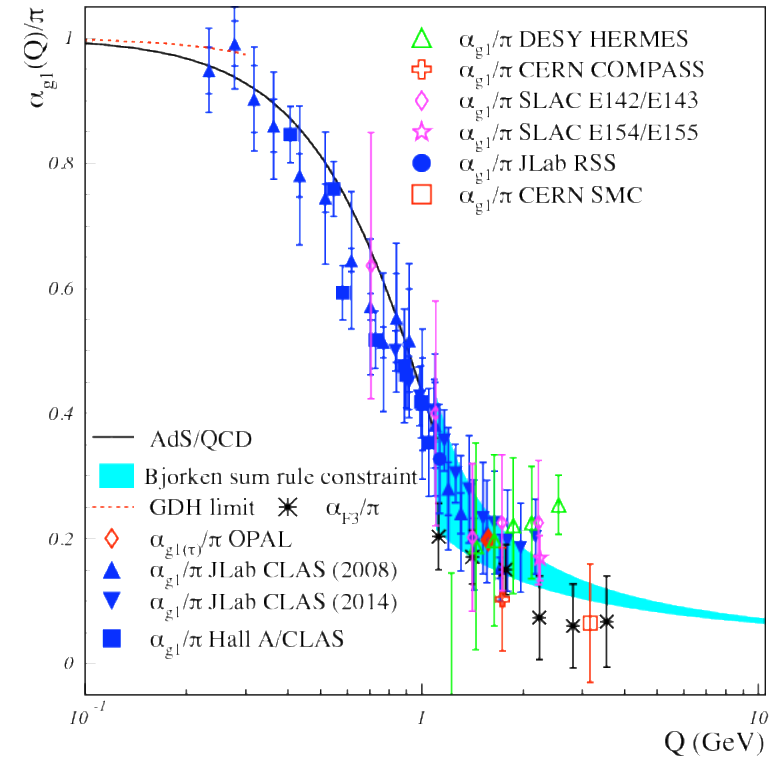
$$G_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g_s c^{abc} A_\mu^b A_\nu^c$$



Nobel Prize 2004 (Politzer, Gross, Wilczek)

- Weak coupling (asymptotic freedom): perturbation theory is very successful
- Strong coupling (confinement): perturbation theory fails, no exact solutions

Open problem

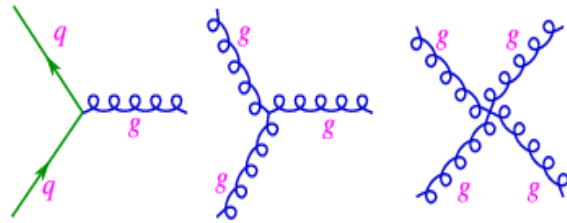


QCD has an underlying conformal symmetry

- Current quark masses: m

$$\mathcal{L}_{\text{QCD}} = \bar{\Psi}(i\gamma^\mu D_\mu - m)\Psi - \frac{1}{4}G_{\mu\nu}^a G^{a\mu\nu}$$

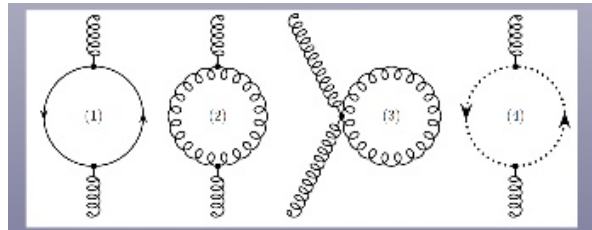
Current quark mass



- Another mass scale appears upon perturbative renormalization scale of loops: Λ_{QCD}

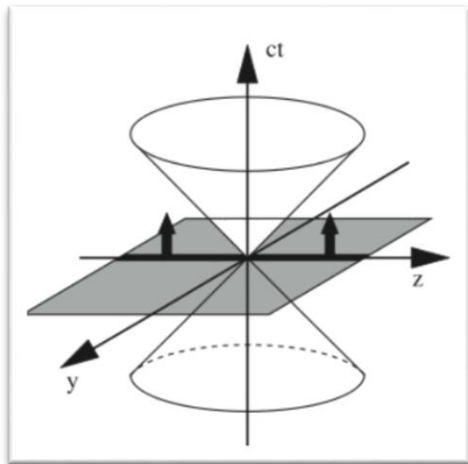
$$\Lambda_{\text{QCD}} \approx 200 \text{ MeV}$$

$$m_{u/d}(2 \text{ GeV}) \approx 4 \text{ MeV}$$

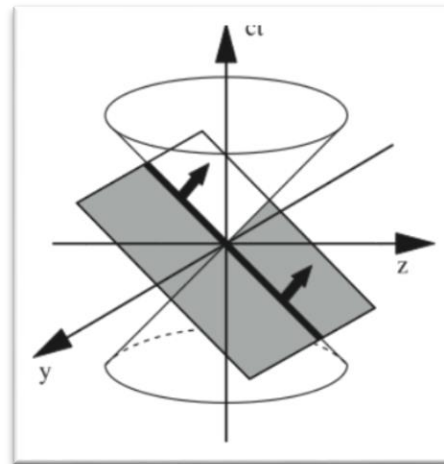


- Massless quarks: chiral symmetry
- Massless quarks and no loops: conformal symmetry

Light-front QCD



Ordinary time



Light-front time

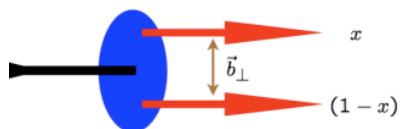
$$x^+ = x^0 + x^3$$

LF Schrodinger-like Equation in the conformal limit

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1 - 4L^2}{4\zeta^2} + U_{\text{eff}}(\zeta) \right) \phi(\zeta) = M^2 \phi(\zeta)$$

$$\zeta = \sqrt{\frac{x}{(1-x)}} \left| \sum_{j=1}^{N-1} x_j \mathbf{b}_{\perp,j} \right|$$

$$x = \frac{k^+}{P^+}$$



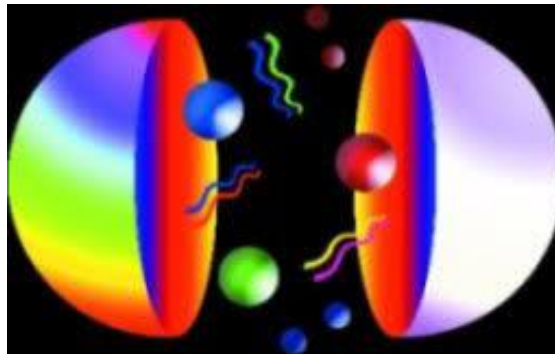
Holographic dictionary

Light front transverse distance maps onto 5th dimension of AdS

$$\zeta \leftrightarrow z_5$$

(Orbital angular momentum)² maps onto (AdS mass parameter x radius)² and spin

$$L^2 = (\mu R)^2 + (2 - J)^2$$



Unique confinement potential

- The dilaton field distorts the pure AdS geometry and drives confinement in physical spacetime
- Underlying conformal symmetry requires the dilaton to be quadratic
- The confinement potential is uniquely fixed by conformal symmetry and holographic mapping

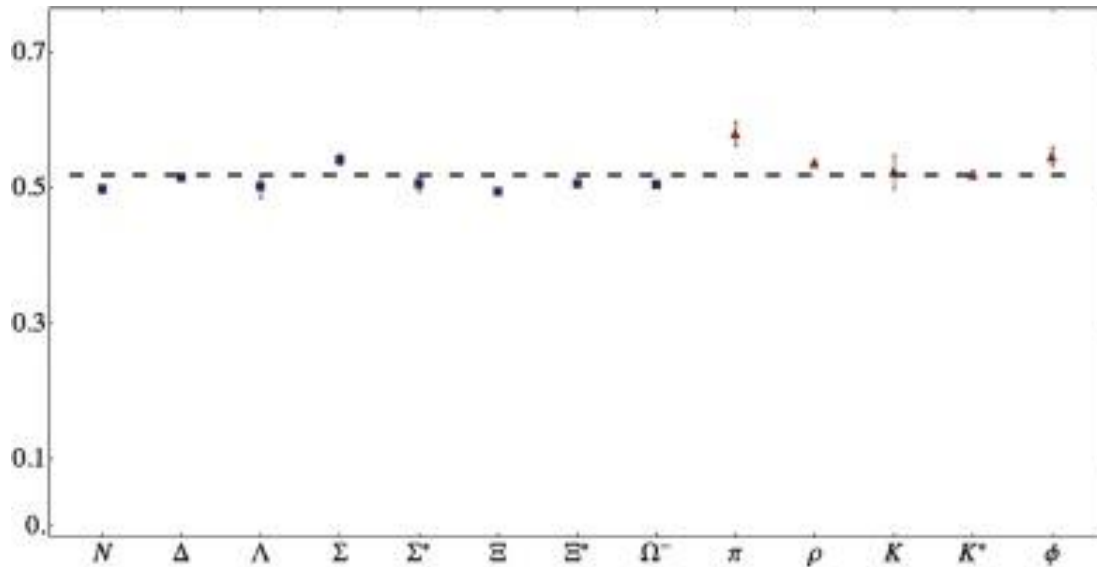
$$U(\zeta) = \frac{1}{2}\varphi''(\zeta) + \frac{1}{4}\varphi'(\zeta)^2 + \frac{2J-3}{2\zeta}\varphi'(\zeta)$$

$$\varphi = \kappa^2 z_5^2$$

$$U(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2(J-1)$$

κ : emerging mass scale !

A universal holographic mass scale



$$\kappa = 523 \pm 24 \text{ MeV}$$

Brodsky, de Teramond, Dosch, Lorce (2013)

Universal holographic wavefunction for ground state

$$\Psi(x, k_{\perp}^2) \propto \frac{1}{\sqrt{x\bar{x}}} \exp\left(-\frac{M^2}{2\kappa^2}\right) \quad M^2 = k_{\perp}^2/x\bar{x} \quad \text{Fourier conjugate to } \zeta$$

Quark masses and spins

- For a successful phenomenology, we need to account for dynamical effects of quark masses and spins

$$\Psi_{h,\bar{h}}^{\mathcal{P},\mathcal{V}}(x, \mathbf{k}) = S_{h,\bar{h}}^{\mathcal{P},\mathcal{V}}(x, \mathbf{k})\Psi(x, k_{\perp}^2),$$

Mesons (quark-antiquark)

$$S_{h_q h_{\bar{q}}}^{V(\lambda)}(x, \mathbf{k}) \propto \frac{\bar{v}_{h_{\bar{q}}}((1-x)P^+, -\mathbf{k})}{\sqrt{\bar{x}}} [\epsilon_V^\lambda \cdot \gamma] \frac{u_{h_q}(xP^+, \mathbf{k})}{\sqrt{x}};$$

Nucleons (quark-diquark)

$$S_{h_N h_q}^{N(\lambda)}(x, \mathbf{k}) \propto \frac{\bar{u}_{h_q}(xP^+, \mathbf{k})}{\sqrt{x}} [(\epsilon_D^\lambda \cdot \gamma)\gamma^5] \frac{u_{h_N}(P^+, \mathbf{0})}{\sqrt{1}}$$

$$S_{h_q h_{\bar{q}}}^P(x, \mathbf{k}) \propto \frac{\bar{v}_{h_{\bar{q}}}(\bar{x}P^+, -\mathbf{k})}{\sqrt{\bar{x}}} [\gamma^5] \frac{u_{h_q}(xP^+, \mathbf{k})}{\sqrt{x}}$$

□

$$S_{h_N h_q}^N(x, \mathbf{k}) \propto \frac{\bar{u}_{h_q}(xP^+, \mathbf{k})}{\sqrt{x}} [\mathbb{1}] \frac{u_{h_N}(P^+, \mathbf{0})}{\sqrt{1}}$$

EM transition form factors

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Light-front holographic radiative transition form factors for light mesons

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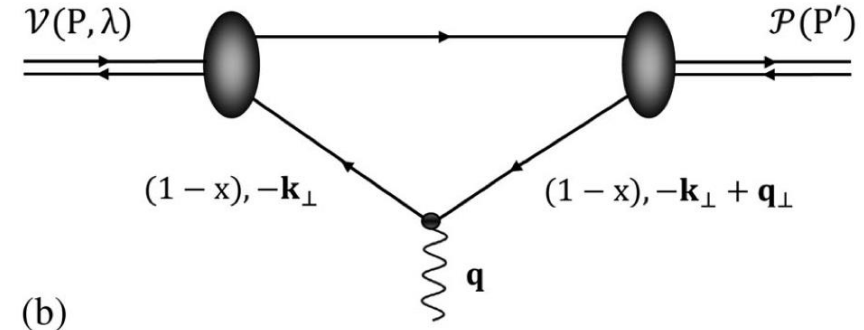
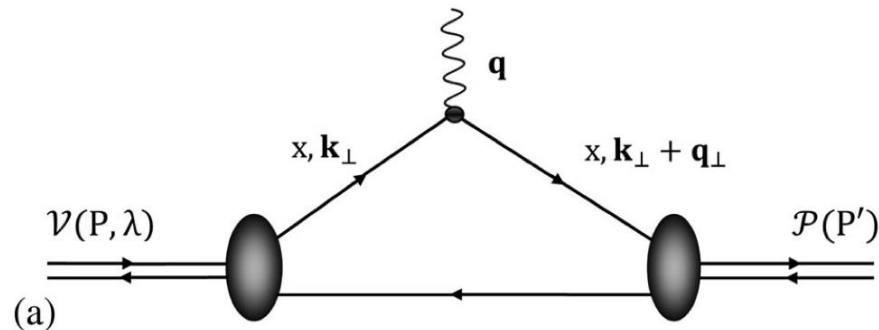
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$$V \rightarrow P + \gamma^*$$

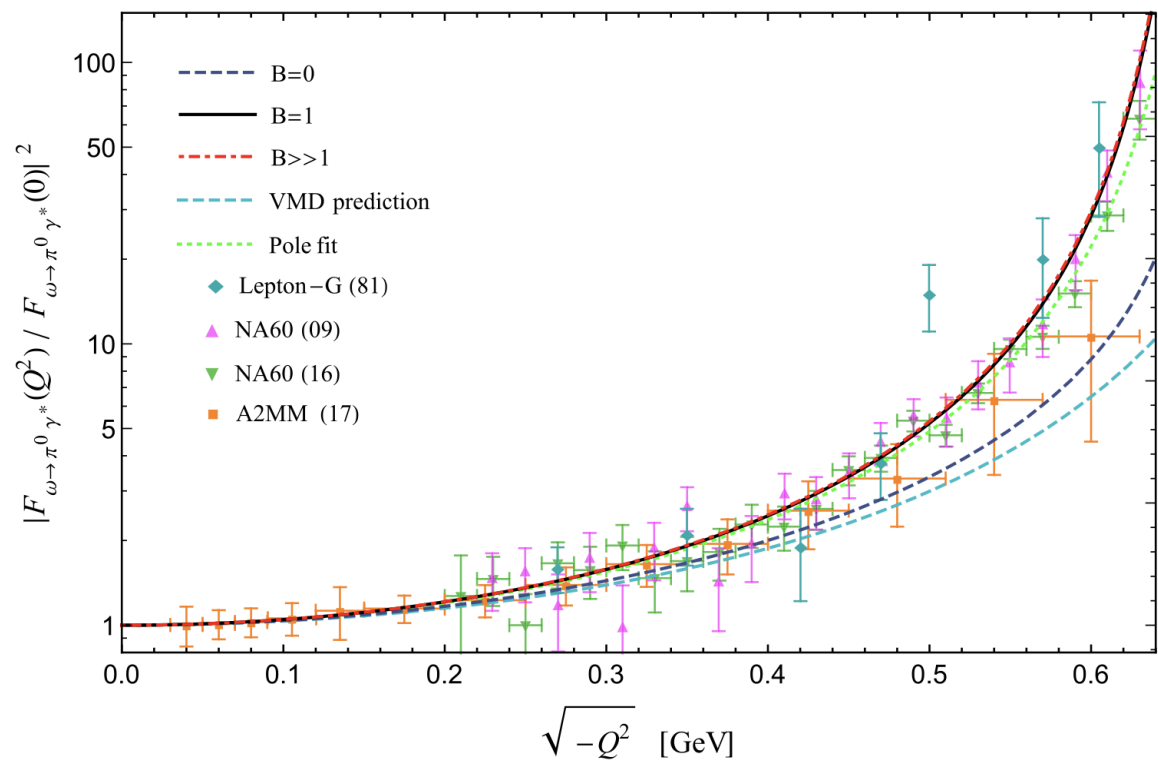


Predictions for radiative decay widths

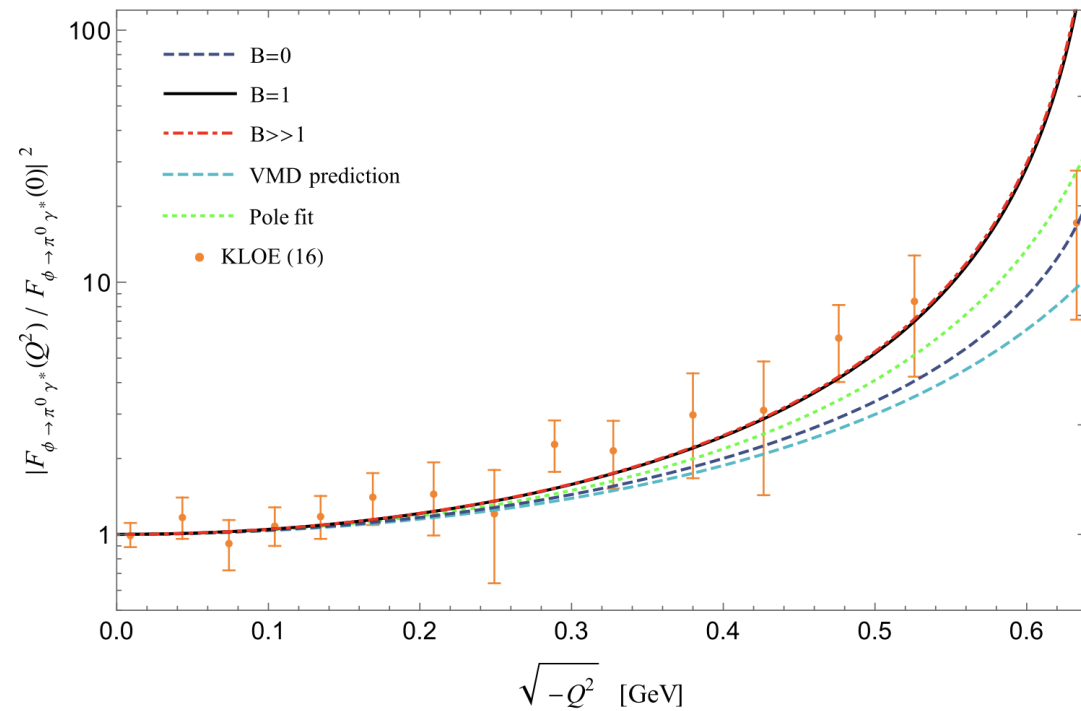
TABLE I. Our predictions for the $(\rho, \omega, \phi) \rightarrow \pi\gamma$ decay widths, compared to the PDG averages [2].

Decay widths	Spin-improved LFH [keV]			PDG (2018) [keV]
	B = 0	B = 1	B \gg 1	
$\Gamma(\rho^\pm \rightarrow \pi^\pm\gamma)$	23.46 ± 3.12	64.52 ± 6.94	66.37 ± 7.00	67.10 ± 7.82
$\Gamma(\rho^0 \rightarrow \pi^0\gamma)$	23.46 ± 3.12	64.52 ± 6.94	66.37 ± 7.00	70.08 ± 9.32
$\Gamma(\omega \rightarrow \pi^0\gamma)$	221.03 ± 29.90	607.96 ± 65.44	625.38 ± 66.03	713.16 ± 25.40
$\Gamma(\phi \rightarrow \pi^0\gamma)$	1.84 ± 0.33	5.06 ± 0.80	5.21 ± 0.82	5.52 ± 0.22

Predictions for the transition form factors

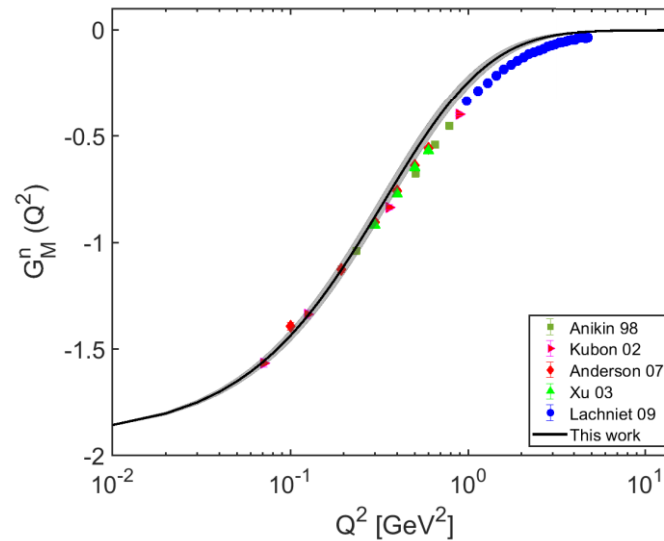
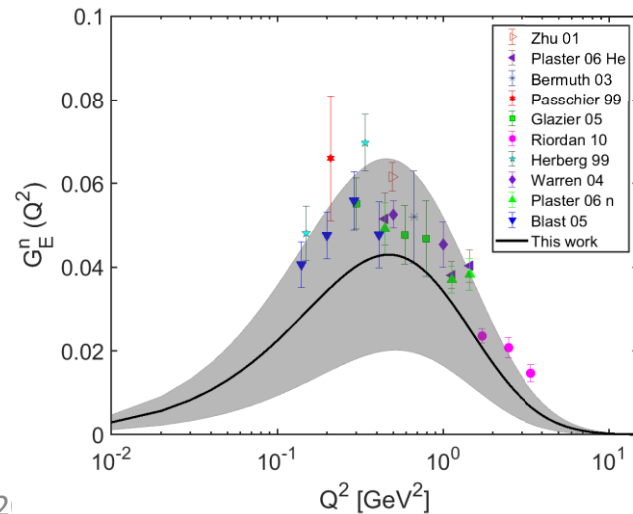
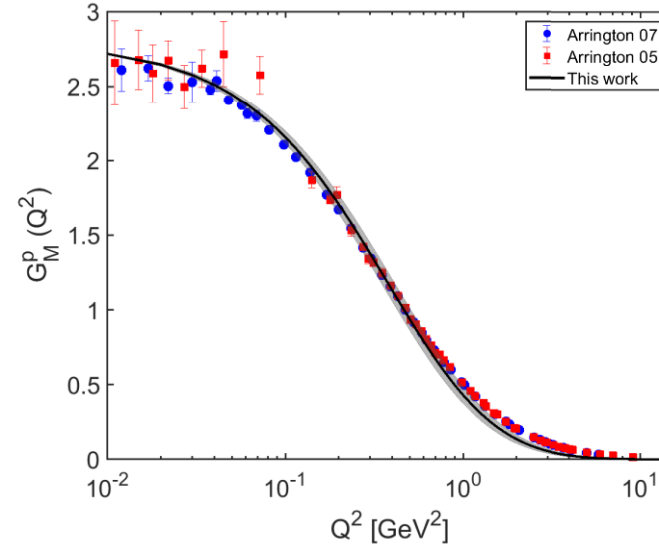
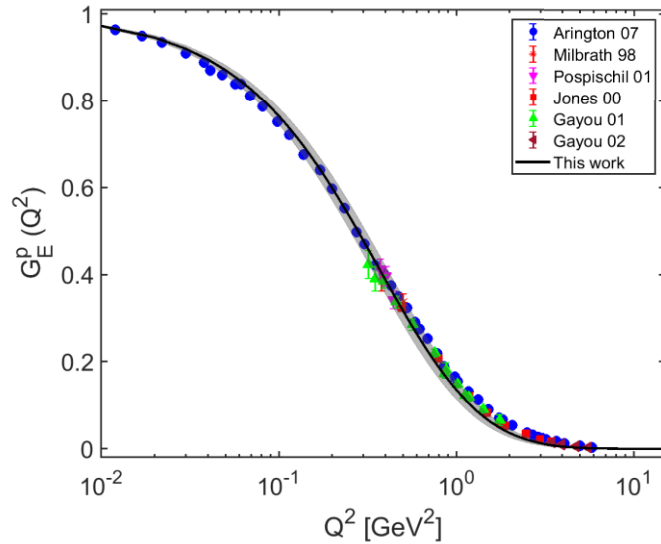


$\omega \rightarrow \pi + \gamma^*$



$\phi \rightarrow \pi + \gamma^*$

Nucleon EM elastic form factors



•M. Ahmady, D. Chakraborti, C. Mondal, R. Sandapen, [E-print: 2105.02213](#) [hep-ph]

•Excellent agreement at low momentum transfer

•Large uncertainties for neutron where LO contributions tend to cancel out

Predictions for the EM radii of nucleons

Radius	Our prediction	Experimental data
$\langle r_E \rangle_p$ fm	0.833 ± 0.010	0.833 ± 0.010 [48]; 0.831 ± 0.019 [50] ; 0.841 ± 0.084 [49]
$\langle r_M \rangle_p$ fm	0.7985 ± 0.0313	0.851 ± 0.026 [52]
$\langle r_E^2 \rangle_n$ fm ²	-0.0704 ± 0.0434	-0.1161 ± 0.0022 [52]; -0.110 ± 0.008 [53]
$\langle r_M \rangle_n$ fm	0.8388 ± 0.0288	$0.864_{-0.008}^{+0.009}$ [52]

Conclusions & Acknowledgements

- Light hadrons share a universal holographic wavefunction which is modified differently by their spin structures
- Thanks to
 - Collaborators: M. Ahmady, D. Chakraborti, S. Kaur, C. Mondal
 - NSERC and Acadia University for funding

