Fluctuation Dynamo in Collisionless and Weakly Collisional Magnetized Plasmas

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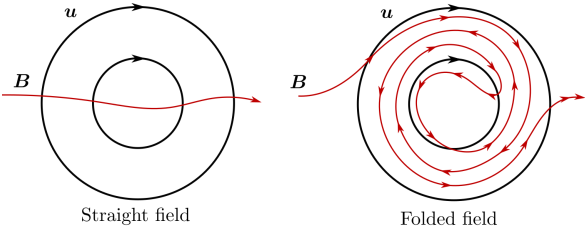
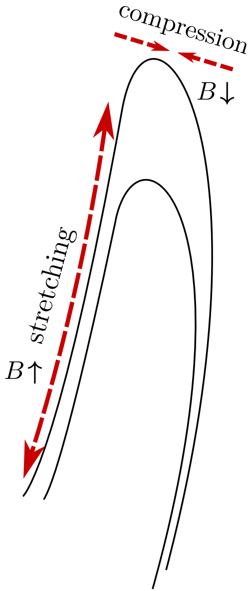
Fluctuation dynamo and folded magnetic fields





Random velocity shears stretch and twist a seed magnetic field:

$$\frac{\mathrm{d} \ln B}{\mathrm{d}t} = \hat{\boldsymbol{b}}\hat{\boldsymbol{b}} \boldsymbol{:} \boldsymbol{\nabla} \boldsymbol{u} - \boldsymbol{\nabla} \cdot \boldsymbol{u},$$

arranging the magnetic fields into long, thin folds.

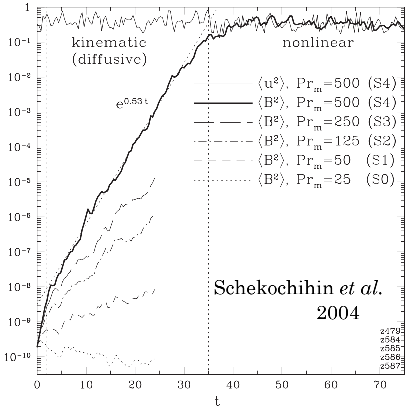
B anti-correlated with field-line curvature $\hat{\boldsymbol{b}} \cdot \boldsymbol{\nabla} \hat{\boldsymbol{b}}$

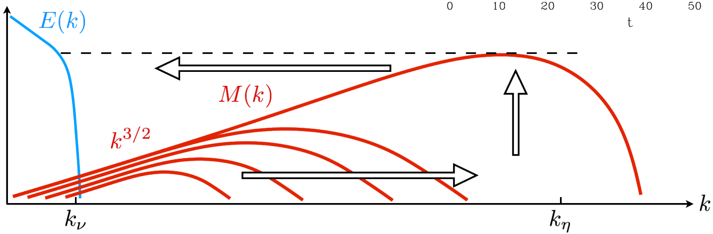
From Schekochihin *et al.*, Astrophy. J. **612**, 276 (2004).

The $\mathrm{Pm} \gg 1$ MHD fluctuation dynamo

Four phases:

1. Diffusion-free
2. Kinematic
   - Kazantsev $k^{3/2}$ spectrum.
3. Nonlinear
   - $\rho \boldsymbol{u} \cdot \nabla \boldsymbol{u} \sim \boldsymbol{B} \cdot \nabla \boldsymbol{B} / 4\pi$
   - smallest-scale stretching suppressed
   - Secular growth of $\langle B^2 \rangle$
4. Saturation
   - minimization of $\hat{\boldsymbol{b}}\hat{\boldsymbol{b}} : \nabla \boldsymbol{u}$
   - $v_{\mathrm{A}} \sim u_{\mathrm{rms}}$ (not scale-by-scale!)





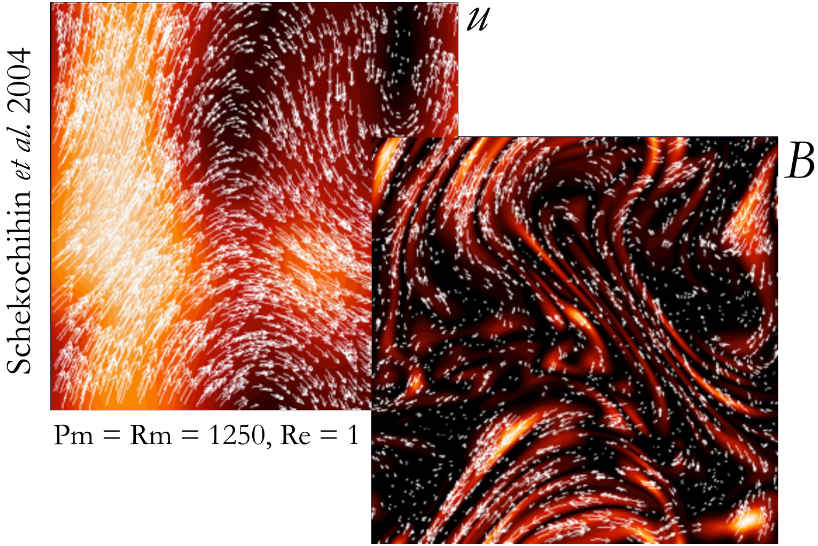


Figure: Simulation results of the $\mathrm{Pm} \gg 1$ turbulent MHD dynamo.

(See also Haugen *et al.* 2004, Beresnyak 2012, Beresnyak & Lazarian 2014)

Theoretical ingredients of the plasma dynamo

ICM only requires $B \sim 10^{-18}$ G to be magnetized (i.e. $\rho_\mathrm{i} \sim \lambda_\mathrm{mfp}$).
Conservation of magnetic moment $\mu \doteq w_\perp^2/B \longrightarrow \mathrm{d}_t(p_\perp/nB) = 0$.
Thus

1. As B increases, $p_\perp$ increases $\longrightarrow p_\perp \neq p_\parallel$ (*Bad for dynamo! – Helander et al. 2016*)
2. Estimate size of $\Delta p \doteq p_\perp - p_\parallel$ in weakly collisional plasmas using CGL equations and collisions:

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{p_\perp - p_\parallel}{p} \approx \underbrace{3\frac{\mathrm{d}\ln B}{\mathrm{d}t}}_{\text{adiabatic production}} - \underbrace{\nu_\mathrm{i}\frac{p_\perp - p_\parallel}{p}}_{\text{collisional relaxation}}$$

(Recall: $\mathrm{d}_t \ln B = \hat{\boldsymbol{b}}\hat{\boldsymbol{b}} \,\boldsymbol{:}\, \boldsymbol{\nabla u} - \boldsymbol{\nabla} \cdot \boldsymbol{u}$). So

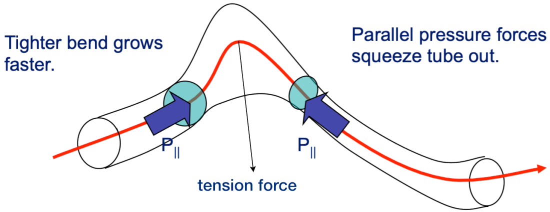
$$\boldsymbol{\nabla} \cdot (\boldsymbol{P} - \boldsymbol{I}p) \approx -\frac{3p}{\nu_i}\boldsymbol{\nabla} \cdot \left[\hat{\boldsymbol{b}}\hat{\boldsymbol{b}}\left(\hat{\boldsymbol{b}}\hat{\boldsymbol{b}} \,\boldsymbol{:}\, \boldsymbol{\nabla u}\right)\right]$$

Results in $\sim 1\%$ deviations from local thermodynamic equilibrium.

Mirror and firehose instabilities

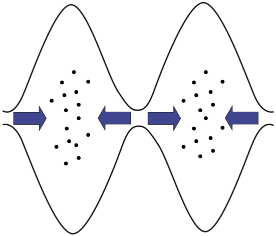
These instabilities arise in high-β ($\doteq 8\pi p/B^2$) plasmas.

Firehose ($\Delta \doteq p_\perp/p_{||} - 1 < -2/\beta$):



Rosenbluth 1956
Parker 1958

Mirror ($\Delta > 1/\beta$):



Perpendicular pressure forces blow out field lines.

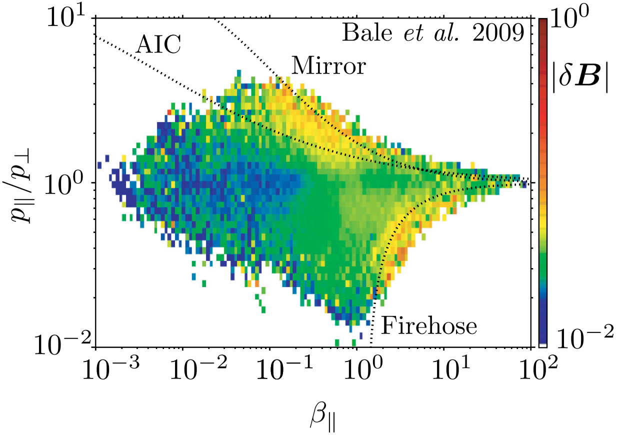
$$-\hat{\boldsymbol{b}}\,(p_\perp - p_{||})\nabla_{||}\delta B_{||}$$

Rudakov and Sagdeev 1961
Southwood & Kivelson 1993

Saturation at $\nu_{\rm eff} \sim |\hat{\boldsymbol{b}}\hat{\boldsymbol{b}}\boldsymbol{:}\boldsymbol{\nabla u}|\beta$ (Kunz+ 2014; Melville+ 2016).

Mirror and firehose instabilities

These instabilities set thresholds on the pressure anisotropy in the solar wind: $|\Delta| \doteq \left|\frac{p_\perp}{p_\parallel} - 1\right| \lesssim 1/\beta$ (see Chen *et al.* 2016)



Anisotropy can be tightly regulated if $\nu_{\rm eff} \sim |\hat{\boldsymbol{b}}\hat{\boldsymbol{b}} : \nabla \boldsymbol{u}|\beta$. (see also: MRI turbulence, Kunz, Stone & Quataert, 2016 PRL)

Three regimes for plasma dynamo

This physics suggests three dynamo regimes:

1. Unmagnetized regime ($\Omega_{\rm i} \ll \nu_{\rm i}$, see Rincon *et al.* 2016)
2. magnetized 'kinetic' regime ($\Omega_{\rm i} \ll |\hat{\boldsymbol{b}}\hat{\boldsymbol{b}}\boldsymbol{:}\boldsymbol{\nabla}\boldsymbol{u}|\beta$)
3. magnetized 'fluid' regime ($\Omega_{\rm i} \gg |\hat{\boldsymbol{b}}\hat{\boldsymbol{b}}\boldsymbol{:}\boldsymbol{\nabla}\boldsymbol{u}|\beta$)

I now present results from:

1. Hybrid-kinetic simulations (St-Onge & Kunz 2017)
   - How does the dynamo operate in a collisionless plasma?
   - *Ab initio* measurement of $\nu_{\rm eff}$ motivates...
2. Braginskii-MHD simulations (St-Onge+, JPP (in review).
   - *Given* a prescribed viscosity, how does the plasma self-organize itself to amplify the magnetic field?
3. Analytic Modeling
   - Predicting the dynamo in certain asymptotic regimes.

Hybrid-Kinetic simulations

- Full-f Hybrid Kinetics
  - kinetic Ions,

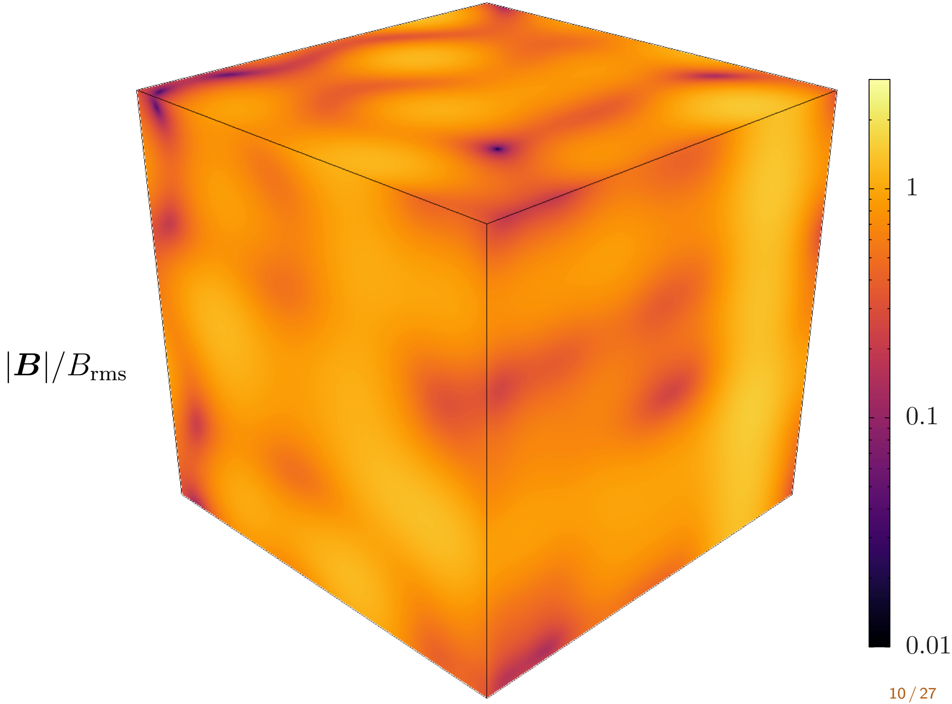
$$\frac{\partial f_{\rm i}}{\partial t} + \boldsymbol{v}\cdot\boldsymbol{\nabla} f_{\rm i} + \left[\frac{e}{m_{\rm i}}\left(\boldsymbol{E} + \frac{\boldsymbol{v}}{c}\times\boldsymbol{B}\right) + \frac{\boldsymbol{F}}{m_{\rm i}}\right]\cdot\boldsymbol{\nabla}_{\boldsymbol{v}} f_{\rm i} = 0.$$

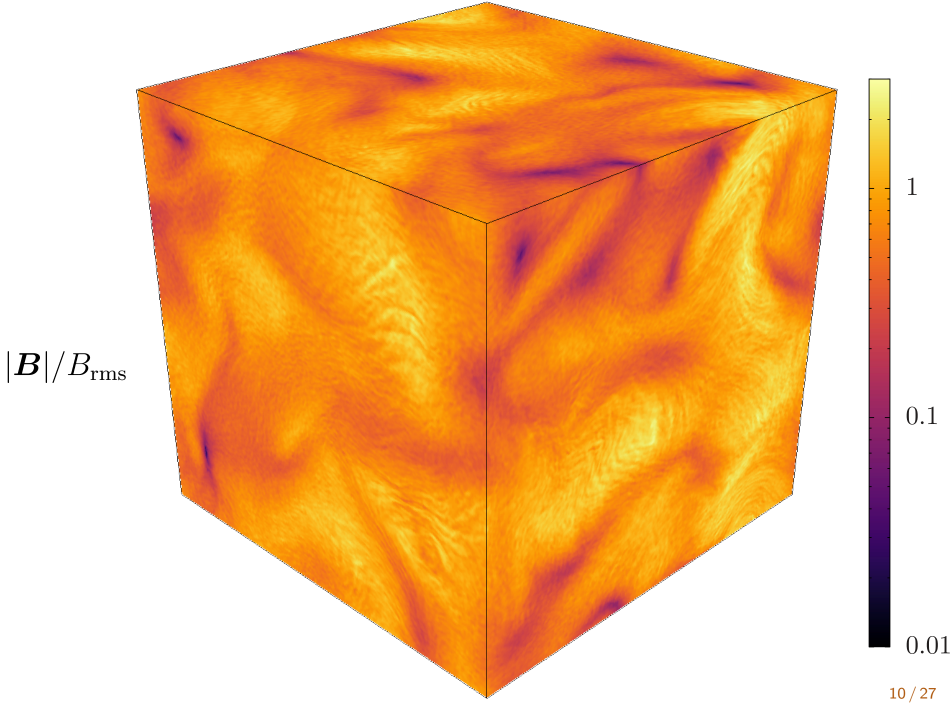
  - isothermal fluid electrons,

$$\boldsymbol{E} + \frac{1}{c}\boldsymbol{u}_{\rm i}\times\boldsymbol{B} - \frac{\eta}{c}\boldsymbol{\nabla}\times\boldsymbol{B} + \frac{\eta^{\rm hyper}}{c}\boldsymbol{\nabla}\times\nabla^2\boldsymbol{B} = -\frac{T_{\rm e}\boldsymbol{\nabla} n}{en} + \frac{(\boldsymbol{\nabla}\times\boldsymbol{B})\times\boldsymbol{B}}{4\pi en}.$$

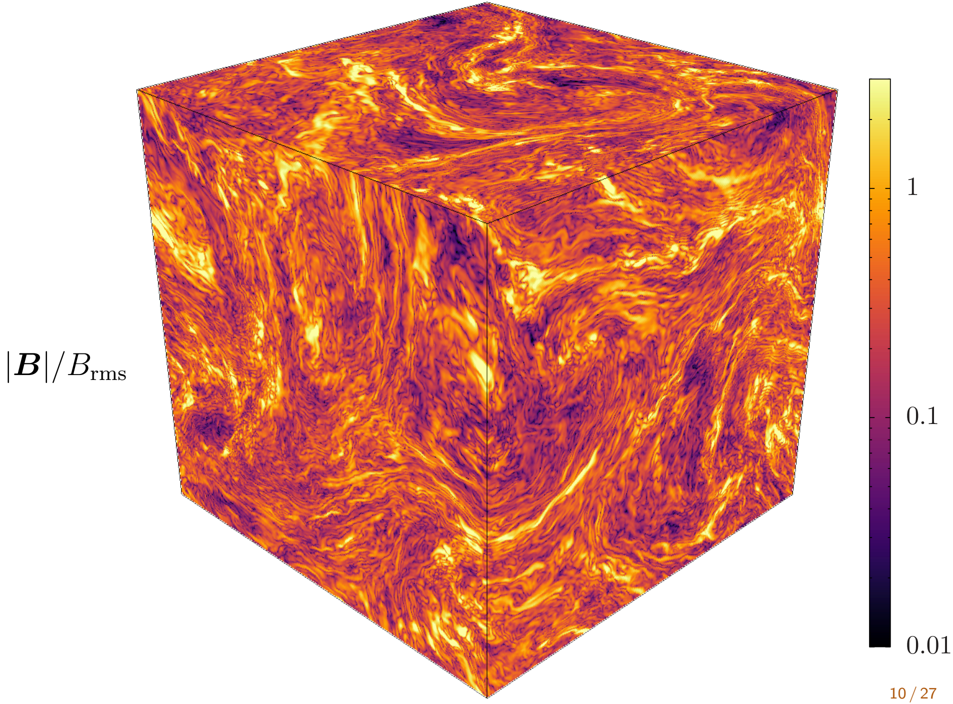
- Non-helical, incompressible, time-correlated ($\sim L/u_{\rm rms}$) forcing
- Focus on two specific runs:

$$\begin{array}{lllll} (1) & L = 16\rho_{\rm i0} & \beta_{\rm i0} = 10^6 & N = 504^3 & \mathrm{PPC} = 216 \\ (2) & L = 10\rho_{\rm i0} & \beta_{\rm i0} = 10^4 & N = 252^3 & \mathrm{PPC} = 216 \end{array}$$

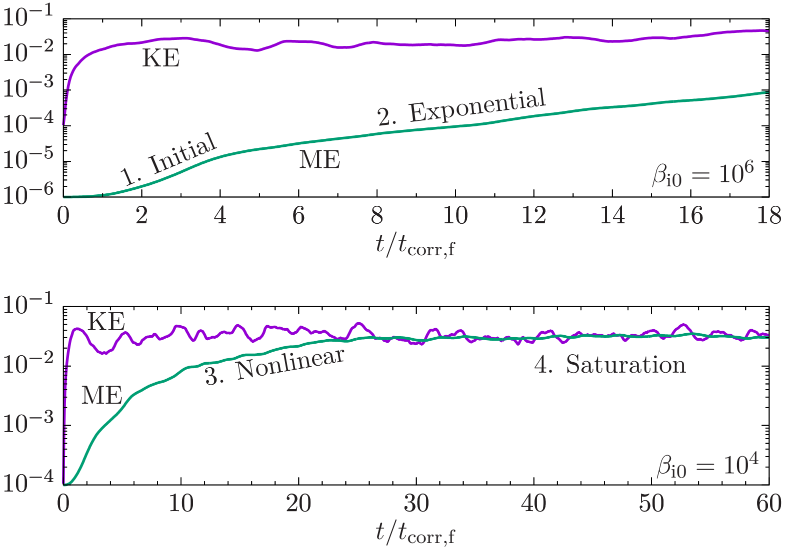

$|\boldsymbol{B}|/B_{\rm rms}$
1
0.1
0.01



$|\boldsymbol{B}|/B_{\rm rms}$
1
0.1
0.01


$|\boldsymbol{B}|/B_{\rm rms}$
1
0.1
0.01

The Punchline[1]...



[1] St-Onge & Kunz, ApJ Lett. 2018, **863** (2), L25

Mirror and firehose instabilities redux

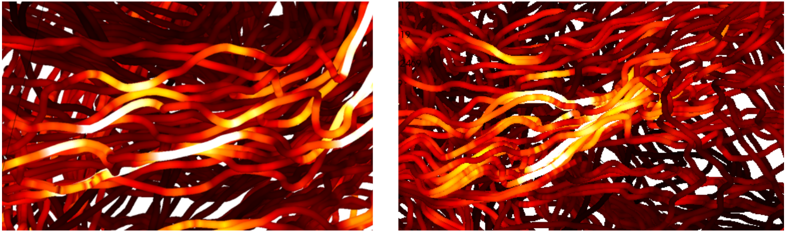
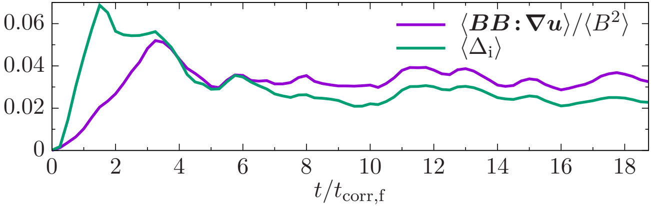


Figure: Visual evidence of mirror instabilities.

- Kinetic instabilities generate magnetic energy above $\rho_{\rm i}$.
- Evidence of firehose visually and in curvature PDFs.
- Plasma becomes Braginskii-like ($3\hat{\boldsymbol{b}}\hat{\boldsymbol{b}}\boldsymbol{:}\boldsymbol{\nabla u} \sim \nu_{\rm eff}\Delta_{\rm i}$):



Regulation of the pressure anisotropy is *imperfect*:

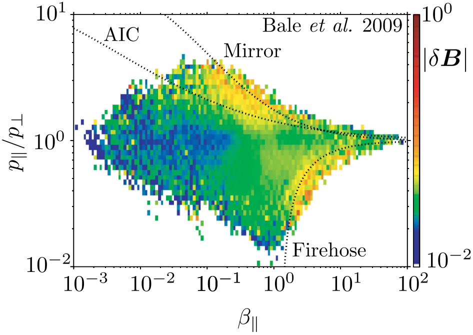


Figure: Solar wind measurements – *strong regulation*

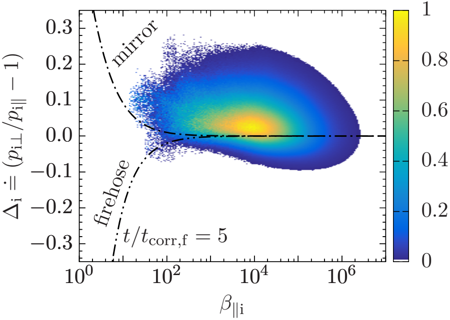


Figure: Plasma dynamo simulation – *weak regulation*

This suggests 'hard-wall' limiters may not be the ideal closure for kinetic microphysics.

Braginskii-MHD simulations[2]

- Dilute magnetized plasma ($\Omega_{\mathrm{i}} \gg \nu_{\mathrm{i}} \gg \omega$)
- Incompressible Braginskii MHD equations

$$\mathrm{d}_t \boldsymbol{u} = \boldsymbol{B} \cdot \boldsymbol{\nabla} \boldsymbol{B} - \boldsymbol{\nabla} p + \boldsymbol{\nabla} \cdot (\hat{\boldsymbol{b}}\hat{\boldsymbol{b}} \Delta p) + \mu \nabla^2 \boldsymbol{u},$$

$$\mathrm{d}_t \boldsymbol{B} = \boldsymbol{B} \cdot \boldsymbol{\nabla} \boldsymbol{u} + \eta \nabla^2 \boldsymbol{B}.$$

- Nonhelical, incompressible, time-correlated forcing
- Pressure anisotropy $\Delta p = 3\mu_{\mathrm{B}} \hat{\boldsymbol{b}}\hat{\boldsymbol{b}} \boldsymbol{:} \boldsymbol{\nabla} \boldsymbol{u}$, both:
  - unlimited (parameter scan on μ_{B})
  - hard-wall limited:

$$\Delta p = \begin{cases} \min\left(B^2/2,\, 3\mu_{\mathrm{B}} \hat{\boldsymbol{b}}\hat{\boldsymbol{b}} \boldsymbol{:} \boldsymbol{\nabla} \boldsymbol{u}\right), & \Delta p > 0 \\ \max\left(-B^2,\, 3\mu_{\mathrm{B}} \hat{\boldsymbol{b}}\hat{\boldsymbol{b}} \boldsymbol{:} \boldsymbol{\nabla} \boldsymbol{u}\right), & \Delta p < 0 \end{cases}$$

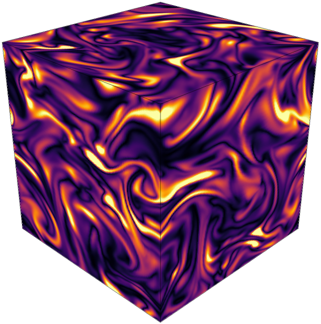


Figure: $|\boldsymbol{B}|/B_{\mathrm{rms}}$ of the unlimited Braginskii-MHD dynamo.

[2] Submitted to JPP

Hard-walled Braginskii looks like $\mathrm{Pm} \gtrsim 1$ MHD

(in box-averaged evolution)

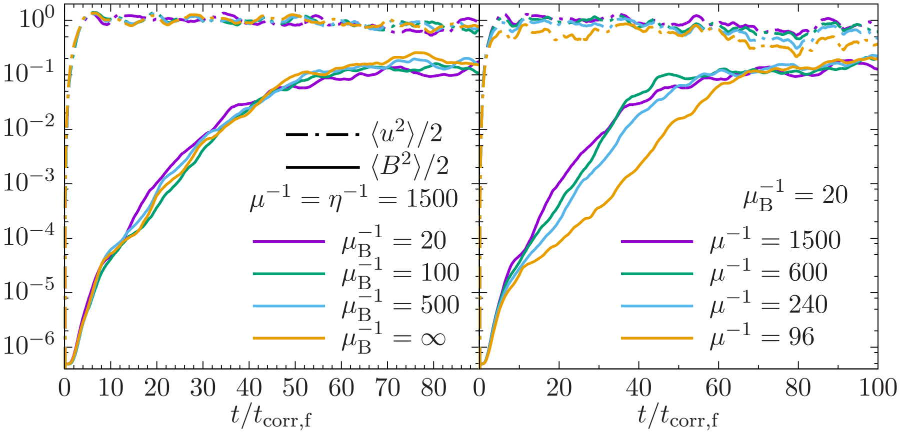


Figure: Evolution of magnetic energy

Hard-walled Braginskii looks like $\mathrm{Pm} \gtrsim 1$ MHD

(in spectra)

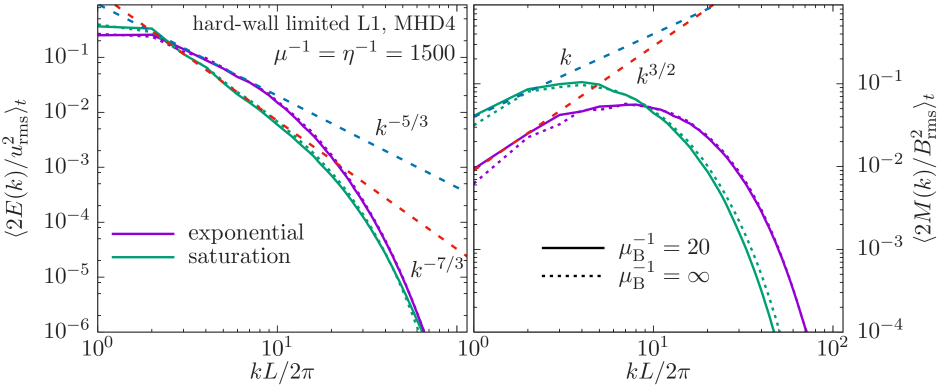


Figure: Kinetic and magnetic energy spectra

Unlimited Braginskii dynamo mimics saturated MHD

- Unlimited regime relevant to early stages of plasma dynamo
- Mimics saturated MHD in:
  - statistics of $\boldsymbol{\nabla u}$ and alignment with respect to $\hat{\boldsymbol{b}}$
  - magnetic spectrum
  - fold geometry (including PDF of $\hat{\boldsymbol{b}} \cdot \boldsymbol{\nabla} \hat{\boldsymbol{b}}$)
  - spectral anisotropy of turbulent velocity

Why is this? Compare

$$\boldsymbol{B} \cdot \boldsymbol{\nabla B} = \boldsymbol{\nabla} \cdot (\hat{\boldsymbol{b}}\hat{\boldsymbol{b}} B^2)$$

to

$$\boldsymbol{\nabla} \cdot (\hat{\boldsymbol{b}}\hat{\boldsymbol{b}} \Delta p) \propto \boldsymbol{\nabla} \cdot (\hat{\boldsymbol{b}}\hat{\boldsymbol{b}} \, \mathrm{d}_t \ln B).$$

Pressure anisotropy plays the role of magnetic-field strength in tension force.

A modified Kazantsev model for $\mathrm{Re}_\parallel/\mathrm{Re}_\perp \ll 1$

Consider a velocity field with prescribed statistics

$$\overline{u^i(t,\boldsymbol{x})} = 0, \qquad \overline{u^i(t,\boldsymbol{x})u^j(t',\boldsymbol{x}')} = \delta(t-t')\kappa^{ij}(\boldsymbol{x}-\boldsymbol{x}'),$$

which are anisotropic with respect to $\hat{\boldsymbol{b}}$:

$$\kappa^{ij}(\boldsymbol{k}) = \kappa^{(\mathrm{i})}(k,|\xi|)\big(\delta^{ij} - \hat{k}_i\hat{k}_j\big) + \kappa^{(\mathrm{a})}(k,|\xi|)\big(\hat{b}^i\hat{b}^j + \xi^2\hat{k}_i\hat{k}_j - \xi\hat{b}^i\hat{k}_j - \xi\hat{k}_i\hat{b}^j\big),$$

where $\xi \doteq \hat{\boldsymbol{k}}\cdot\hat{\boldsymbol{b}}$. We derive an equation for the joint PDF of B, $\boldsymbol{k}$ and $\hat{\boldsymbol{b}}$:

$$\mathcal{P}(B,\boldsymbol{k},\hat{\boldsymbol{b}}) = \delta(|\hat{\boldsymbol{b}}|^2-1)\delta(\hat{\boldsymbol{b}}\cdot\boldsymbol{k})(4\pi^2 k)^{-1}P(B,k).$$

This model was originally developed for the saturated state in MHD by Schekochihin (2004)

A modified Kazantsev model for $\mathrm{Re}_\parallel/\mathrm{Re}_\perp \ll 1$

We then derive an equation for the magnetic energy spectrum $M(k) \doteq (1/2)\int_0^\infty \mathrm{d}B\, B^2 P(B,k)$:

$$\frac{\partial M}{\partial t} = \underbrace{\frac{\gamma_\perp}{8}\frac{\partial}{\partial k}\left[(1+2\sigma_\parallel)k^2\frac{\partial M}{\partial k} - (1+4\sigma_\perp+10\sigma_\parallel)kM\right]}_{\textbf{drift–diffusion in } k\textbf{-space}}$$

$$+ \underbrace{2(\sigma_\perp+\sigma_\parallel)\gamma_\perp M}_{\textbf{growth}} - \underbrace{2\eta k^2 M}_{\textbf{decay}},$$

where

$$\gamma_\perp = \int \frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^3}\, k_\perp^2 \kappa_\perp(\boldsymbol{k}), \qquad \textbf{mixing}$$

$$\sigma_\perp = \frac{1}{\gamma_\perp}\int \frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^3}\, k_\parallel^2 \kappa_\perp(\boldsymbol{k}), \qquad \textbf{shearing}$$

$$\sigma_\parallel = \frac{1}{\gamma_\perp}\int \frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^3}\, k_\parallel^2 \kappa_\parallel(\boldsymbol{k}). \qquad \textbf{stretching}$$

A modified Kazantsev model for $\mathrm{Re}_\parallel/\mathrm{Re}_\perp \ll 1$

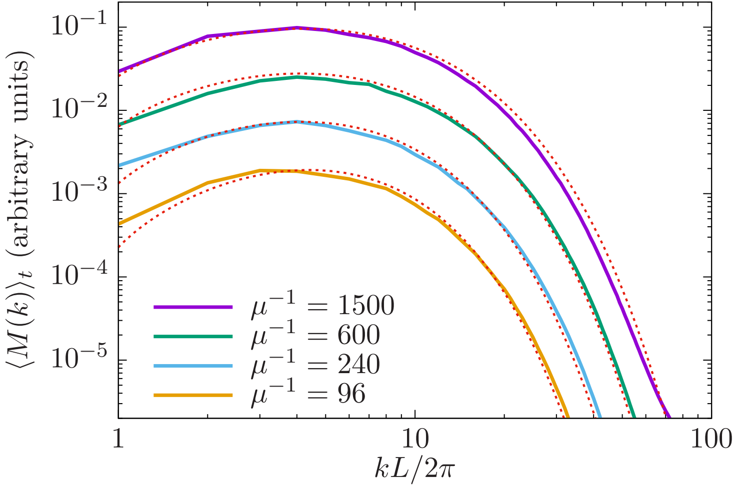


Figure: Comparison of predicted versus simulation magnetic energy spectra.

A modified Kazantsev model for $\mathrm{Re}_\parallel/\mathrm{Re}_\perp \ll 1$

In the limit $\mathrm{Rm} \to \infty$, the dynamo growth rate γ is given by

$$\frac{\gamma}{\gamma_\perp} = \frac{1}{8(1+2\sigma_\parallel)} \left[16(\sigma_\perp + \sigma_\parallel)(1+2\sigma_\parallel) - (1+2\sigma_\perp+6\sigma_\parallel)^2\right].$$

Sufficiently large $\gamma_\perp$ (mixing) or μ_B (parallel viscosity) kills the dynamo!

A new "Prandtl" number

Unlimited Braginskii MHD has two important dimensionless numbers:

$$\underbrace{\frac{\mu_\parallel}{\eta}}_{\textbf{MHD Pm}}, \qquad \underbrace{\frac{\mu_\parallel}{\mu_\perp}}_{\textbf{NEW!}}.$$

Ratio of stretching and mixing in the dynamo matters, and is controlled by $\mu_\parallel/\mu_\perp$.

Predictions of the model for $\mathrm{Re}_\perp \gg 1$, $\mathrm{Re}_\parallel \sim 1$

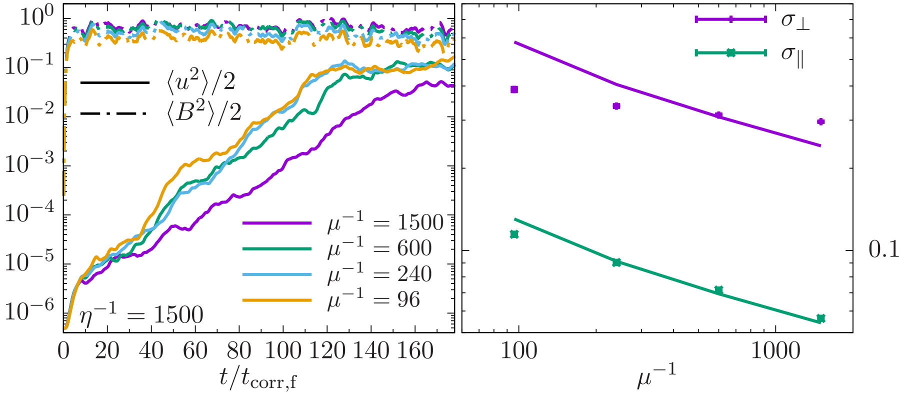


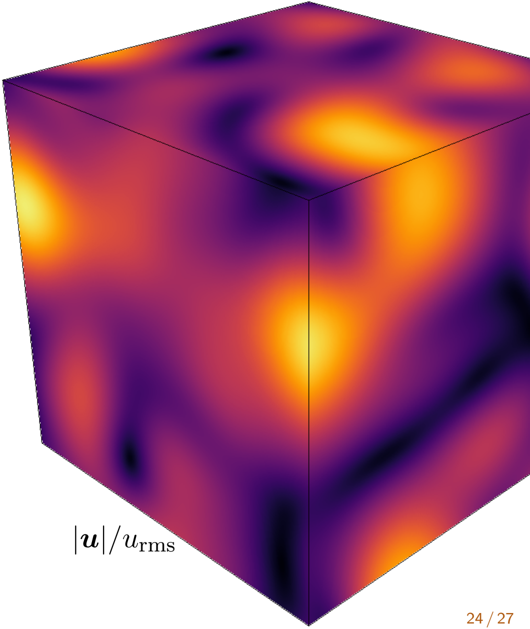
Figure: Evolution of magnetic energy (left) and $\sigma_\perp$, $\sigma_\parallel$ as a function of μ^{-1} (right).

Examine the opposite limit: $\mathrm{Re}_\parallel \ll 1$ Stokes flow

In the Stokes flow regime, viscosity is so large that the velocity is determined by a balance between dissipation and driving alone:

$$-\mu\nabla^2\boldsymbol{u} = \widetilde{\boldsymbol{f}}$$

As $\nu \to \infty$, flow becomes δ-correlated in time.



Unlimited Braginskii Dynamo for $\mathrm{Re}_\parallel \ll 1$ Stokes flow.

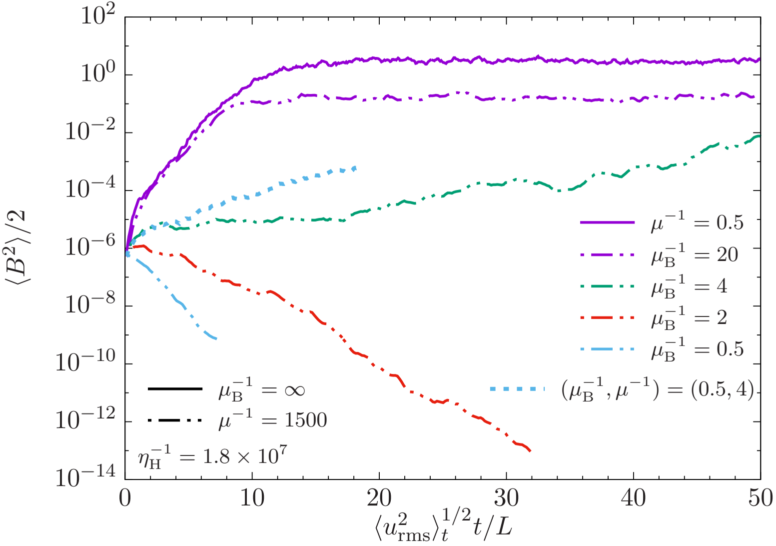


Figure: Evolution of the magnetic energy for MHD and unlimited Braginskii-MHD in the Stokes flow regime for fixed u_{rms}.

The take-away points

To summarize,

- **Dynamo exists in a collisionless *magnetized* plasma.** (See also Rincon *et al.* 2016 for unmagnetized regime)
- Larmor-scale instabilities play a crucial role.
- Many features appear MHD-like (Pm $\gtrsim 1$), despite collisionless plasma.
- Saturation at $u \sim v_{\mathrm{A}}$.

For weakly collisional plasmas,

- Too anisotropic a viscosity is deleterious for the dynamo *(controls ratio of mixing to stretching)*
- Perfect pressure-anisotropy regulation $\longrightarrow \mathrm{Pm} \sim 1$ MHD
- Weak pressure-anisotropy regulation $\longrightarrow$ saturated MHD

Future research directions

- Exact determination of ν_{eff} in the magnetized kinetic regime
- Other components of the Braginskii viscosity (i.e. gyro-viscosity)
- Kinetic electron effects
  - Dynamo relies on magnetized electrons (flux-freezing)!
  - Resistive scale (i.e. fold separation) set by electron physics
- Interplay between mean-field and fluctuation dynamos:
  - Historical anxiety about mean-field dynamo in the face of fluctuation dynamo
  - Fluctuation dynamo can lead to catastrophic α quenching!
  - Could kinetic effects alleviate these concerns?

Questions?

Questions?[3]

[3] questions?