Plasma Acceleration In The Magnetic Nozzle

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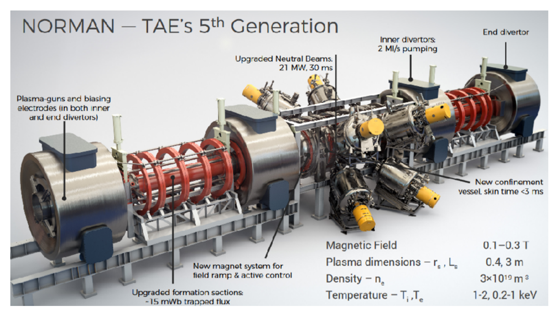
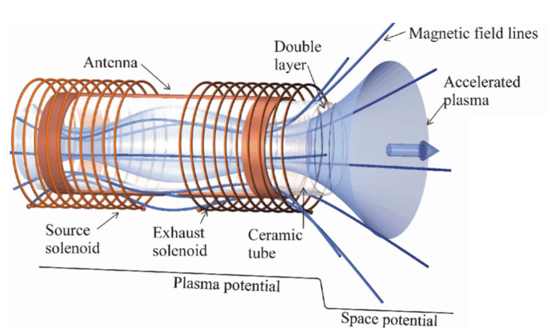
Motivation

Plasma acceleration in the magnetic nozzle is an essential effect in a number of plasma devices

- Tri-Alpha mirror machine for fusion research

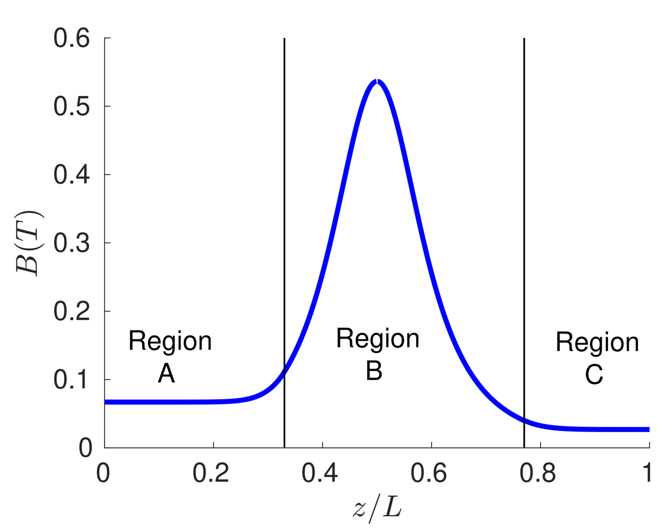


- Helicon Thruster for space propulsion



Physical Model

The flow of plasma in the magnetic nozzle is studied in the 1D paraxial approximation using a two fluid MHD model. Ions are assumed to be magnetized with anisotropic pressure. Electrons are isothermal with $T_e = 200\ eV$ and have isotropic pressure. The flow is considered for both collissionless plasma and also in the presence of ionization and charge-exchange.The magnetic field has no time-dependence. The mirror ratios are B_0/B_l =8.0 and B_0/B_r =20.0 at the left and right end of the nozzle respectively.



Flow along the magnetic nozzle

The equations that describe the flow of plasma were derived from the continuity, the CGL equations in the absence of heat fluxes and the electron momentum equation with electron inertia being ignored.

$$\frac{\partial n}{\partial t} + \nabla\cdot(nV_\parallel \vec{b}) = \nu_z n \quad (1) \qquad 0 = en\frac{\partial \phi}{\partial z} - T_e\frac{\partial n}{\partial z} \quad (2)$$

$$\frac{d}{dt}\left(\frac{p_\parallel B^2}{n^3}\right) = 0 \quad (3) \qquad \frac{d}{dt}\left(\frac{p_\perp}{nB}\right) = 0 \quad (4)$$

The time-dependent flow of plasma is described by the following non-linear, coupled, partial differential equations

$$\frac{\partial n}{\partial t} = nV_\parallel\frac{\partial \ln B}{\partial z} - V_\parallel\frac{\partial n}{\partial z} - n\frac{\partial V_\parallel}{\partial z} + \nu_z n \quad (5)$$

$$\frac{\partial p_\parallel}{\partial t} = p_\parallel V_\parallel\frac{\partial \ln B}{\partial z} - V_\parallel\frac{\partial p_\parallel}{\partial z} - 3p_\parallel\frac{\partial V_\parallel}{\partial z} - \nu_{cx}p_\parallel \quad (6)$$

$$\frac{\partial p_\perp}{\partial t} = 2p_\perp V_\parallel\frac{\partial \ln B}{\partial z} - V_\parallel\frac{\partial p_\perp}{\partial z} - p_\perp\frac{\partial p_\perp}{\partial z} - \nu_{cx}p_\perp \quad (7)$$

$$m_i n\left(\frac{\partial V_\parallel}{\partial t} + V_\parallel\frac{\partial V_\parallel}{\partial z}\right) = -en\frac{\partial \phi}{\partial z} - \frac{\partial p_\parallel}{\partial z} + (p_\parallel - p_\perp)\frac{\partial \ln B}{\partial z} - \nu_{cx}m_i nV_\parallel \quad (8)$$

Stationary State and Sonic Point Singularity

In the absence of ionization and charge-exchange ($\nu_z = \nu_{cx} = 0$), the stationary state equations (in normalized units) that describe ion dynamics are

$$\frac{\partial n}{\partial z} = n\left(\frac{\partial \ln B}{\partial z} - \frac{1}{M}\frac{\partial M}{\partial z}\right) \quad (9) \qquad \frac{\partial p_\parallel}{\partial z} = p_\parallel\left(\frac{\partial \ln B}{\partial z} - \frac{3}{M}\frac{\partial M}{\partial z}\right) \quad (10)$$

$$\frac{\partial p_\perp}{\partial z} = p_\perp\left(2\frac{\partial \ln B}{\partial z} - \frac{1}{M}\frac{\partial M}{\partial z}\right) \quad (11) \qquad \phi = T_e \ln\left(\frac{n}{n_L}\right) \quad (12)$$

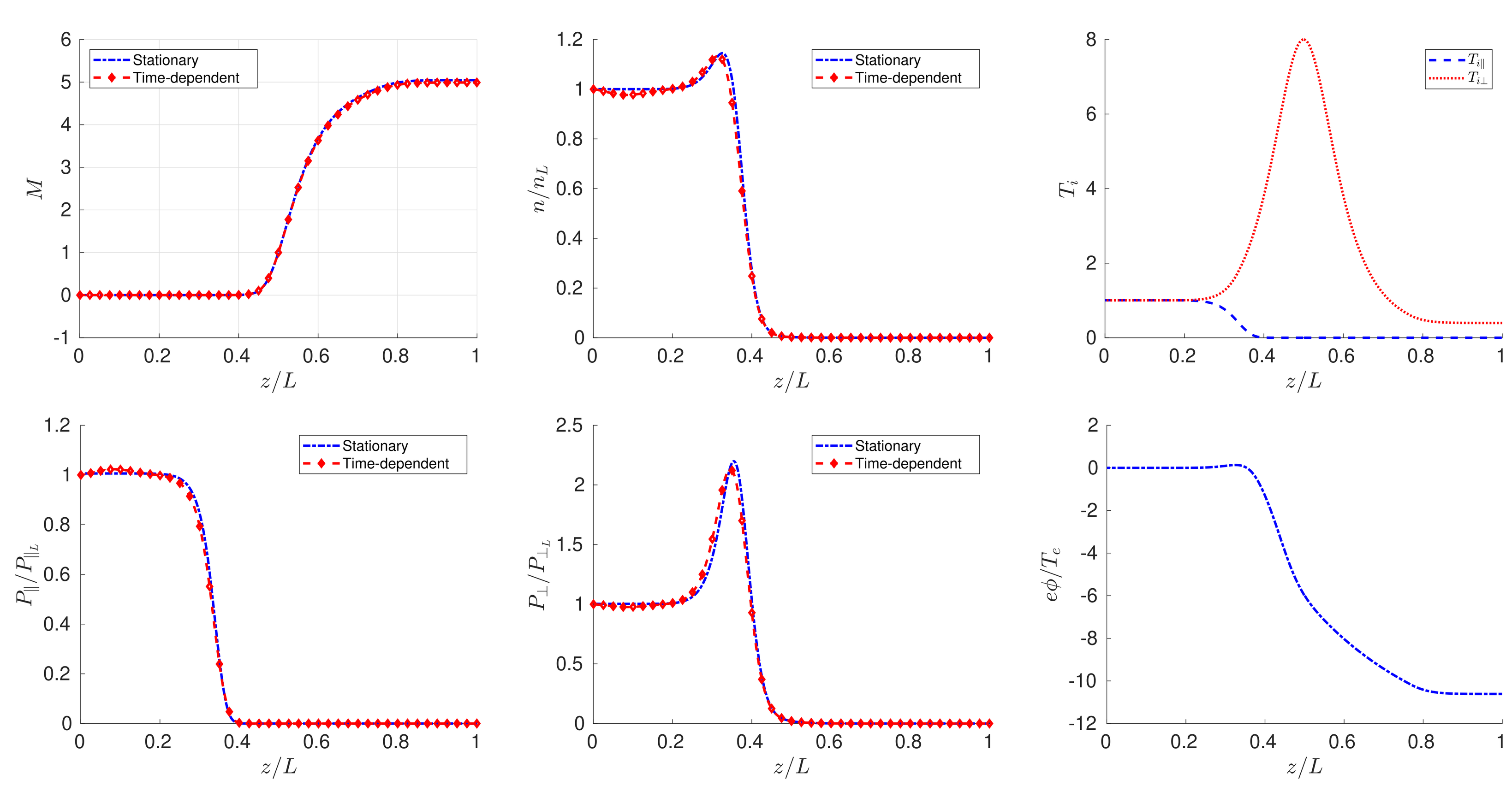
$$\left(M^2 - 1 - \frac{3p_\parallel}{nT_e}\right)\frac{\partial M}{\partial z} = -\left(1 + \frac{p_\perp}{nT_e}\right)M\frac{\partial \ln B}{\partial z}. \quad (13)$$

Here the plasma velocity $V_\parallel$ is normalized to the speed of sound c_s, $M = V_\parallel/c_s$ where $c_s = 9.784\times 10^4 m/s$. The entire nozzle had a length $L = 4m$, with the center at $z_0 = 0$ where the sonic point singularity occurred. At the left end it was assumed that $T_{i\parallel_L} = T_{i\perp_L} = 200\ eV$ since the plasma was thermalized with a density value $n_L = 1.0\times 10^{19}m^{-3}$. A singularity occurs in eq. 13 for values of $M = M_{cr} = \sqrt{1+3p_\parallel/nT_e}$ and this singularity corresponds to the transition from sub-sonic to super-sonic flow. The singularity can be resolved if eq. 13 is expanded in a Taylor series around the singular point $z = z_0$ to obtain values for $\partial M/\partial z$.

$$\left(\frac{\partial M}{\partial z}\right)^2 = -\frac{(1+p_\perp/nT_e)(1+3p_\parallel/nT_e)}{2(1+6p_\parallel/nT_e)}\frac{\partial^2 \ln B}{\partial z^2}\Bigg|_{z=z_0} = -\frac{(1+T_\perp/T_e)(1+3T_\parallel/T_e)}{2(1+6T_\parallel/T_e)}\frac{\partial^2 \ln B}{\partial z^2}\Bigg|_{z=z_0}. \quad (14)$$

This expression illustrates that the condition $\partial^2\ln B/\partial z^2 < 0$ for $z = z_0$ is required for the existence of a smooth (regular) accelerating solution. Equations 9 - 13 can then be integrated numerically to the left $z < z_0$ and right $z > z_0$ using the initial values for $\partial M/\partial z$ from eq. 14 Integration of equations 9 – 13 to the left $z < z_0$ using the Shooting Method will allow for the condition$T_{i\parallel_L} = T_{i\perp_L} = 200\ eV$ at the left end of the nozzle to be satisfied. Equations 5-8 that describe time-dependent flow were integrated numerically using BOUT++. It was found that for large time scales, the time-dependent solutions approach the stationary state solutions

Results

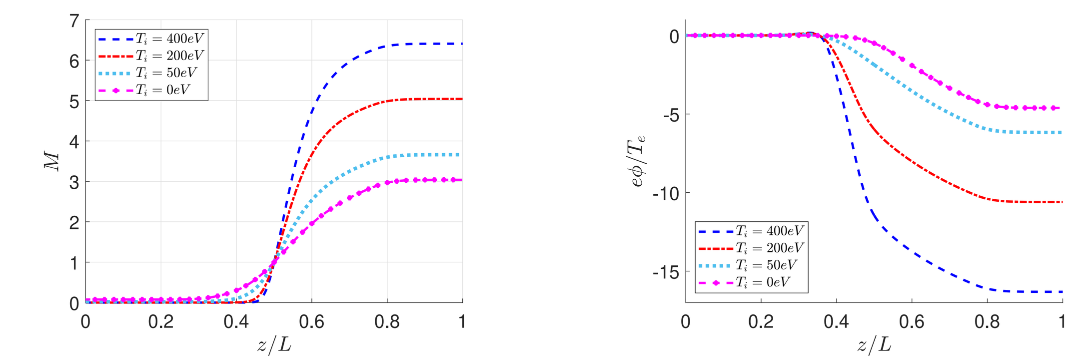


The Case of Cold Ions

For cold ions with $T_{i\parallel_L} = T_{i\perp_L} = 0\ eV$ it was found that the value of M at both ends of the nozzle depend only on the mirror ratio $R = \frac{B(z_0)}{B(z)}$, regardless of the geometry of the nozzle. Ion acceleration is described by the equation

$$\frac{M^2}{2} - \frac{1}{2} = -\ln\left(\frac{B(z)}{M(z)B_0}\right). \quad (15)$$

For a specific mirror R at the left end of the nozzle, a minimum value for M is required in order for the ions to become accelerated. There is no acceleration for cold ions in regions where $B(z) = const.$, unlike for hot ions where there is a small increase in M due to thermal motion. Higher values of M are obtained at the right end of the nozzle for higher initial T_i values. There is a greater potential difference across the nozzle if $T_i > 0$.



Ionization and Charge Exchange

The stationary state equations (in normalized units) that describe ion dynamics are

$$\frac{\partial n}{\partial z} = n\left(\frac{\partial \ln B}{\partial z} - \frac{1}{M}\frac{\partial M}{\partial z} + \frac{\nu_z}{M}\right) \quad (16) \qquad \frac{\partial p_\parallel}{\partial z} = p_\parallel\left(\frac{\partial \ln B}{\partial z} - \frac{3}{M}\frac{\partial M}{\partial z} - \frac{\nu_{cx}}{M}\right) \quad (17)$$

$$\frac{\partial p_\perp}{\partial z} = p_\perp\left(2\frac{\partial \ln B}{\partial z} - \frac{3}{M}\frac{\partial M}{\partial z} - \frac{\nu_{cx}}{M}\right) \quad (18) \qquad \phi = T_e\ln\left(\frac{n}{n_L}\right) \quad (19)$$

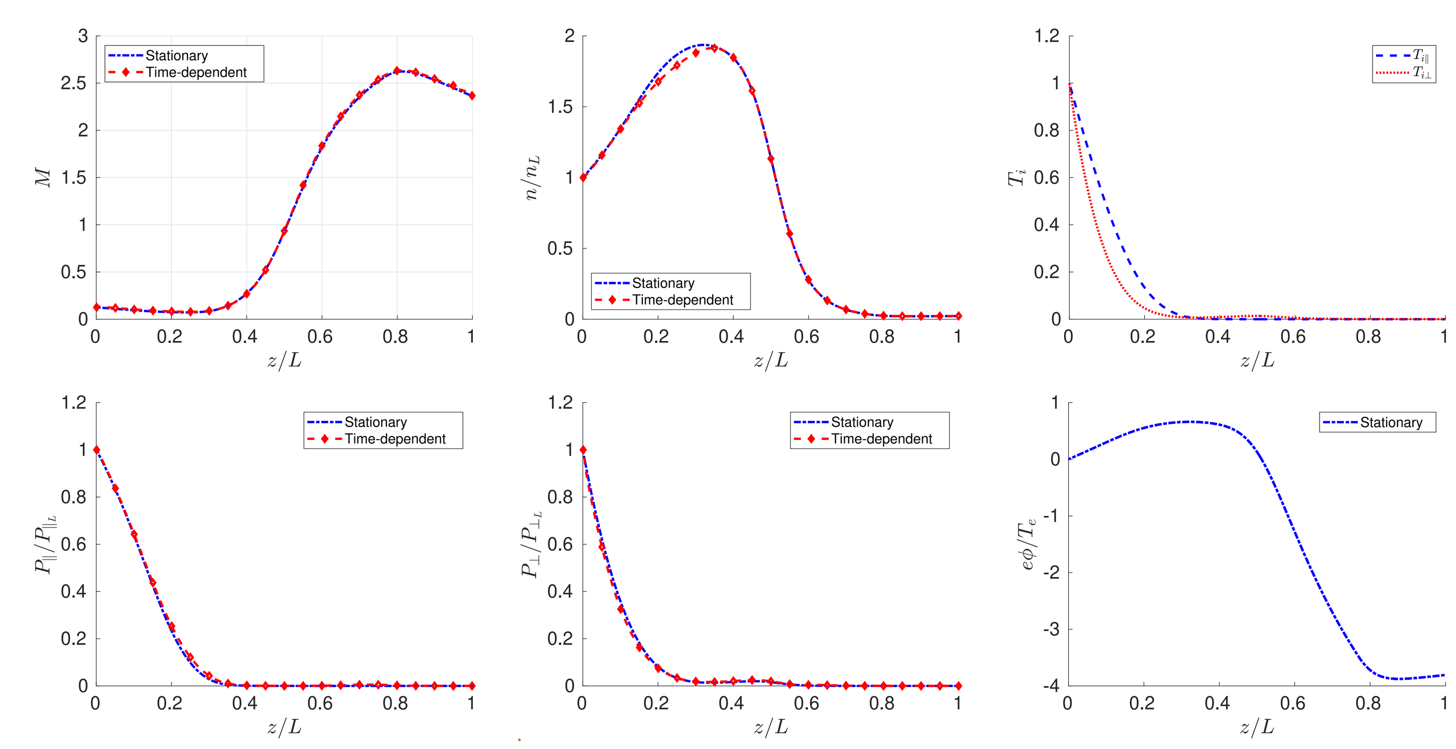
$$\left(M^2 - 1 - \frac{3p_\parallel}{nT_e}\right)\frac{\partial M}{\partial z} = -\left(1 + \frac{p_\perp}{nT_e}\right)M\frac{\partial \ln B}{\partial z} + \left(\frac{p_\parallel}{nT_e} - M^2\right)\nu_{cx} - \nu_z \quad (20)$$

Applying L'Hopitals's rule to eq. 20 in order to obtain a value for $\frac{\partial M}{\partial z}$ at the singular point results in the quadratic equation

$$a\left(\frac{\partial M}{\partial z}\right)^2 + b\left(\frac{\partial M}{\partial z}\right) + c = 0 \quad (21)$$

where the coefficients a, b and c depend on the sonic point values of $T_{i\parallel}$, $T_{i\perp}$, $\frac{\partial \ln B}{\partial z}$ and $\frac{\partial^2 \ln B}{\partial z^2}$. The sonic point no longer occurs at the initial $z = z_0$ but is shifted in the $z > z_0$ direction, with larger values of ν_z and ν_{cx} resulting in a greater shift. Equations 16-20 can be integrated numerically using the value of $\frac{\partial M}{\partial z}$ from eq. 21 by using the Shooting Method similar to the previous case. The time-dependent equations were solved using BOUT++ and it was found that after a long time interval, the time-dependent solutions approached the stationary state solutions.

Results for Ionization and Charge Exchange



The value of M at the right end of the nozzle was lower than when ionization and charge exchange were absent. This result is due to charge exchange acting similar to a frictional force on plasma flow. The potential difference across the nozzle was lower when ionization and charge exchange were present.

Conclusion

In order for a plasma to be accelerated to super sonic speeds it is required that $\frac{\partial \ln B}{\partial z} = 0$ at the sonic point. The main mechanism for plasma acceleration is the ambipolar potential created by the separation of electrons from ions due to electron diffusion. Larger values for M at the right end of the nozzle can be obtained if

1. The nozzle has a high mirror ratio R in order for $\frac{\partial \ln B}{\partial z}$ to be large.
2. A high T_e value to allow for the creation of a larger ambipolar potential.
3. Higher $T_{i\perp}$ values at the left end of the nozzle where the plasma is injected.
4. A low rate of charge-exchange in order to minimize the drag force acting on plasma flow.