Measuring The $\bar{\nu}_\mu$-induced Charged-Current Coherent Pion Production Cross Section Using The T2K Near Detector ND280

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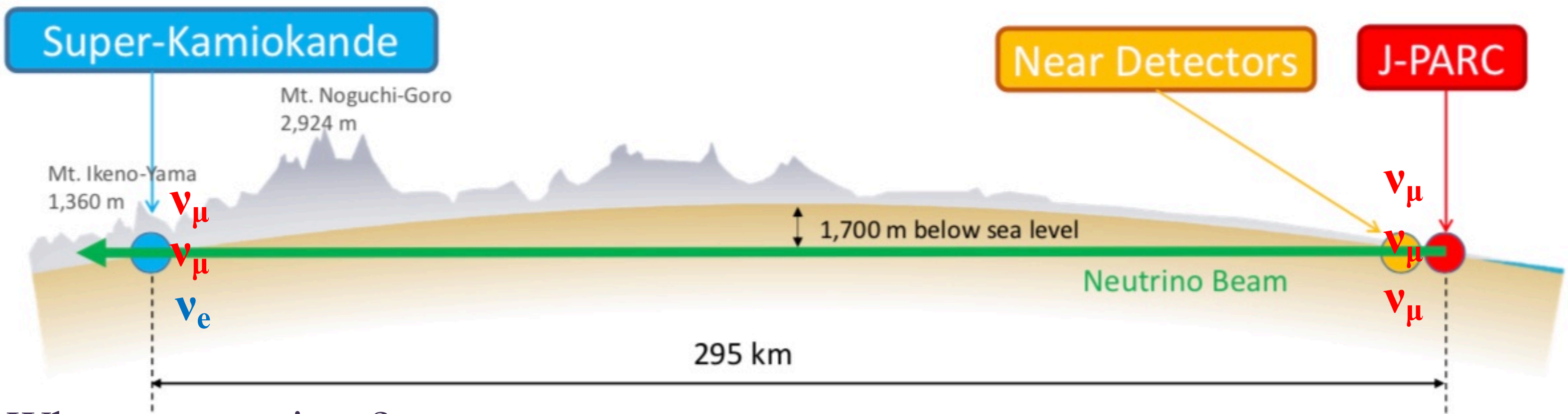
CAP 2019 Congress

Overview

- What is the T2K experiment?
- How do neutrinos interact?
- What is neutrino coherent pion production?
- How to measure neutrino coherent pion production?
  - What is the selection?
  - How to remove the background events?
  - How to the extract cross-section measurement?

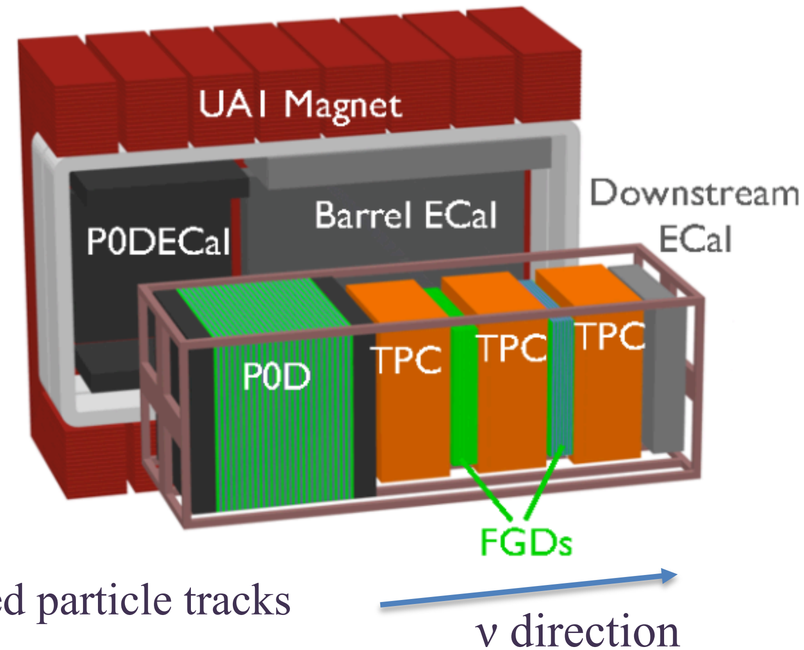
The T2K Experiment



- What are neutrinos?
  - The most abundant massive particles that rarely interact
  - Neutrinos can also change identities while they propagate through space (neutrino oscillation)
- The Tokai-to-Kamioka (T2K) experiment is a long baseline neutrino oscillation experiment (See M. Hartz's talk R2-10 Thursday @ 13:15)
  - Muon (anti)neutrinos are produced by the J-PARC proton accelerator facility
  - The near detector complex measures (un-oscillated) muon neutrino properties
  - The far detector Super-Kamiokande (SK) measures the the appearance of electron neutrinos (neutrino oscillation)
- T2K provides rich neutrino interaction programs at its near detector (this talk)
  - To understand neutrino interactions

T2K Off-axis Near Detector ND280

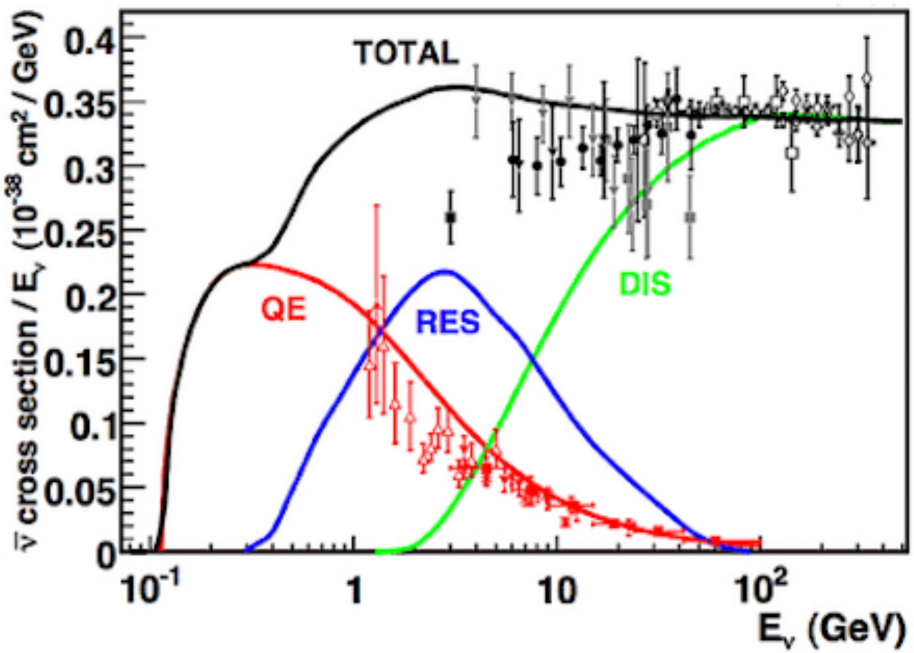
- Magnet
  - All the detector components are surrounded a magnet (0.2 T)
- P0D
  - Dedicated π^0 detector
  - Provide π^0 background constraints on Oxygen for Super-K (main background)
- TPC
  - 3 time projection chambers (Argon gas)
  - Momentum reconstruction (track curvature)
  - Particle identification (dE/dx)
- FGD
  - 2 fine-grained detectors
  - Carbon and Oxygen target mass
  - FGD1: plastic scintillator layers
  - FGD2: alternating plastic and water layers
  - Also provide particle identification for stopped particle tracks

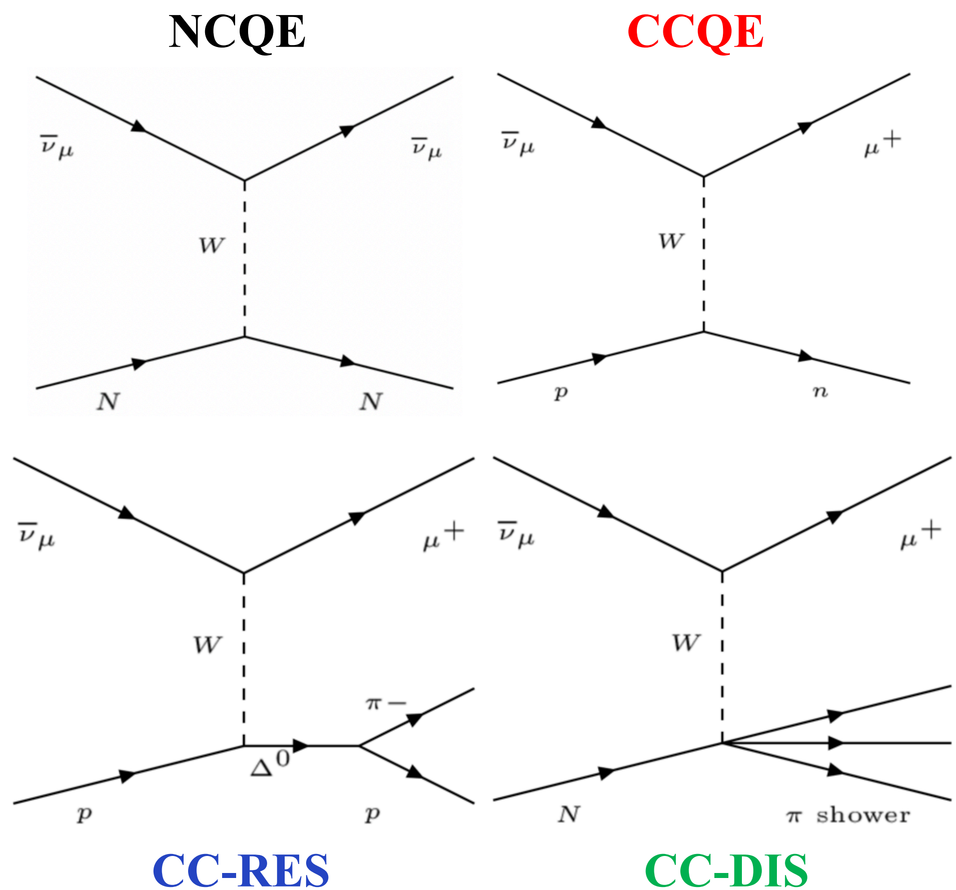


- ECal
  - Various Electromagnetic Calorimeters (ECal) detectors surrounding
  - Reconstruction of neutral particles (i.e. photons)

Neutrino interactions

- Neutrinos (anti-neutrinos) can only interact via weak interactions
  - Neutral-current (NC) mediated by the Z^0 boson
  - Charged-current (CC) mediated by the $W^{\pm}$ boson
- CC and NC interactions can be subcategorized based on the outgoing particles of the interactions
  - e.g. a neutrino interaction with a nucleon producing an outgoing charged lepton is called Charged-Current Quasi-Elastic (CCQE) scattering

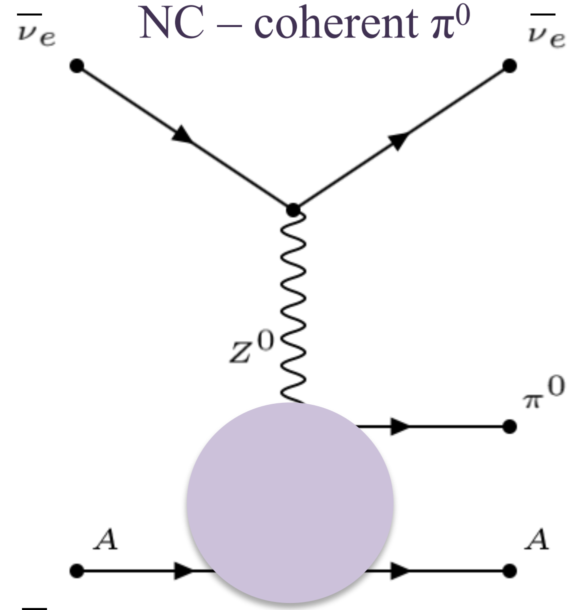


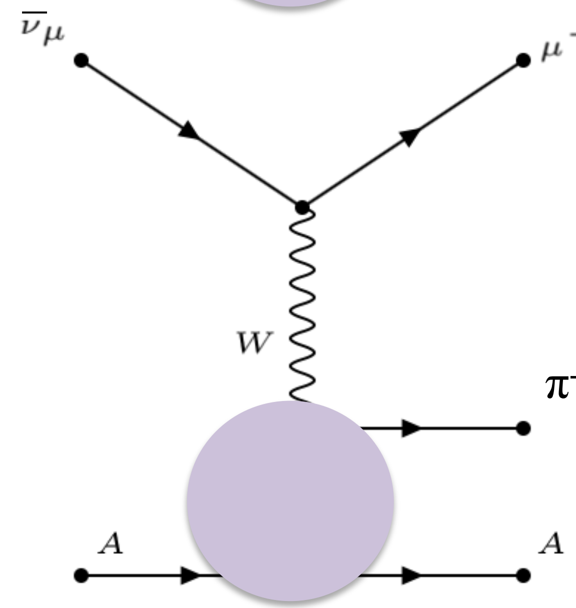


Anti-neutrino coherent pion production

- Neutrino (anti-neutrino) coherent pion production
  - A neutrino scatters off an entire nucleus
  - Produces 1 lepton and 1 pion
    - Small angle w.r.t. neutrino direction
  - no fragmentation of the nucleus
    - small four-momentum transfer
- Neutrino coherent pion production
  - NC-coherent (NC-COH) has an outgoing π^0
  - CC-coherent (CC-COH) has an outgoing π^-
- Why do we care about such an interaction?
  - This is not well understood theoretically - interesting measurement
  - π^0 from NC-COH can also mimic electron neutrino appearance in the T2K oscillation analysis
    - 3% of the electron neutrino background
    - 30%-100% COH model uncertainty
- Measurements of NC-COH and CC-COH would help to constrain the physics model used in neutrino interaction generators
  - Underlying physics model for NC-COH and CC-COH are the same

NC – coherent π^0

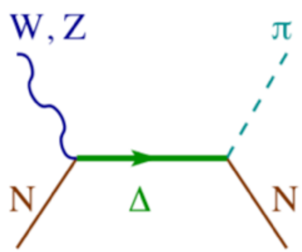




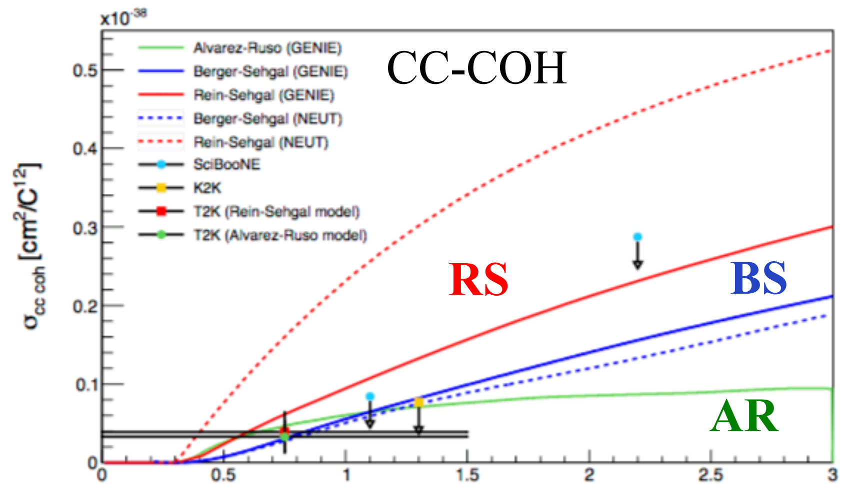
CC – coherent π^-

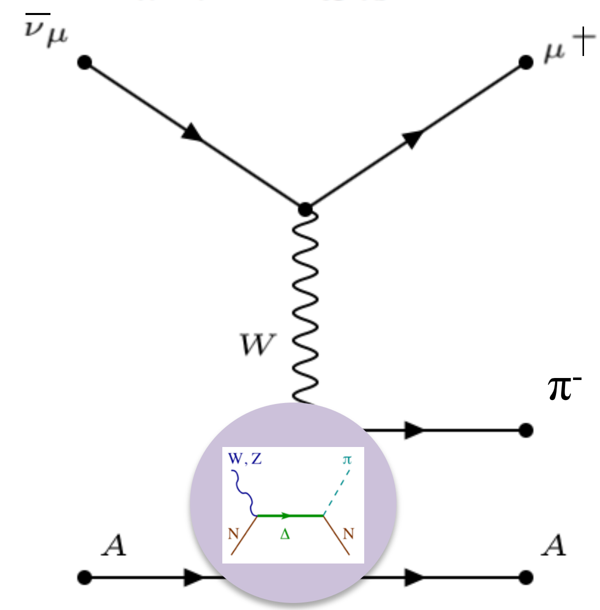
Neutrino Coherent Pion Production Models

- Wildly varying predictions for different models
- Rein and Sehgal (1983, 2007)
  - Partially conservation of the axial current (PCAC)
- Berger and Sehgal (2009)
  - PCAC
  - External pion-carbon scattering data
- Alvarez-Ruso (2007)
  - Microscopic
  - Nucleon level process



  - Pions are produced from (virtual) Δ resonances or N*
  - Nucleon must remain in the same quantum state



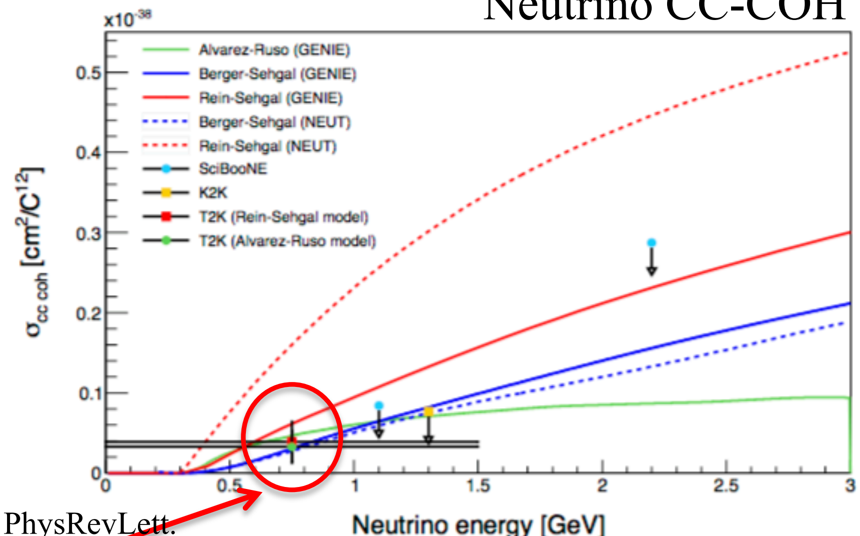


CC – coherent π^-

Current Status of CC-COH Measurements

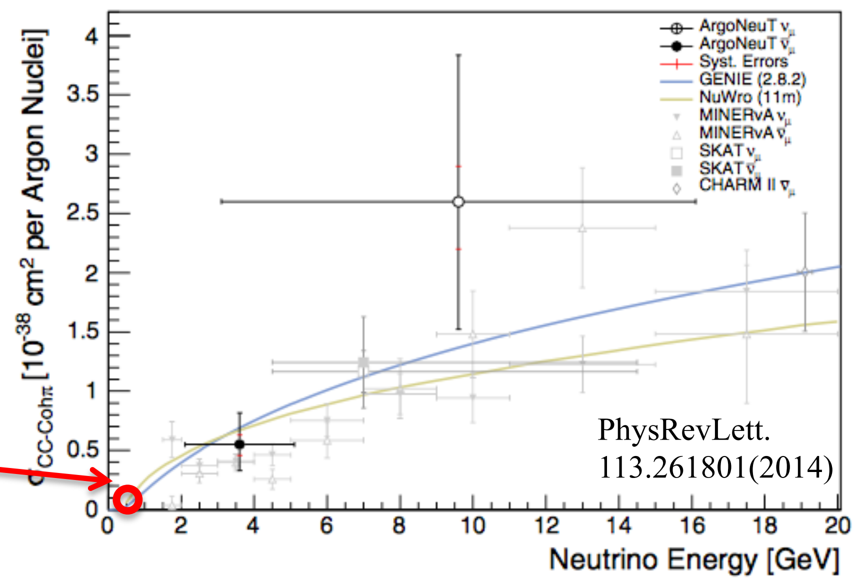
- ν_μ CC-coherent π^+ on C12
  - Has been observed by MINERvA with mean neutrino energy from 2 GeV
  - Not observed by SciBooNe (1.1 GeV and 2.2 GeV) and K2K (1.3 GeV)
    - upper limits for cross section set
  - T2K (0.6 GeV) measures CC-coherent π^+ on C12 for the first time
- $\bar{\nu}_\mu$ CC-coherent π^- on C12
  - Not observed yet at sub-GeV region
  - T2K has collected roughly equal neutrino and anti-neutrino data
  - An observation of the process at sub-GeV region is possible (this talk)

Neutrino CC-COH



PhysRevLett. 117.192501(2016)

Anti-neutrino CC-COH



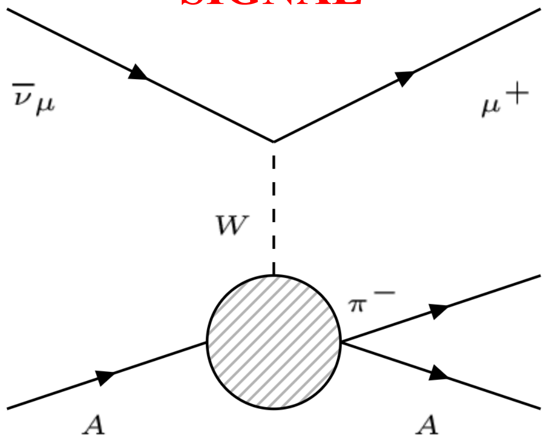
PhysRevLett. 113.261801(2014)

Analysis Strategy

- Physics goal: CC-COH cross section
    - Dependent on resonant pion production (RES) background modelling
    - Dependent on the detector modelling of FGD reconstruction
- T2K is trying to promote more model independent cross-section measurements
    - The background can be measured together with the signal
    - Introducing a new topology for a double differential cross-section measurement
- Topology definition
    - CC1Pi-0rP
    - Signal: CC-COH, or CC-RES with no reconstructable protons
        - FGD has low proton reconstruction efficiency below 450 MeV/c
        - These signals can be calculated and compared to by theorists
- Extract (model dependent) CC-COH cross section from the CC1Pi-0rP result
    - Can switch to different RES and COH models for additional (model dependent) CC-COH cross sections

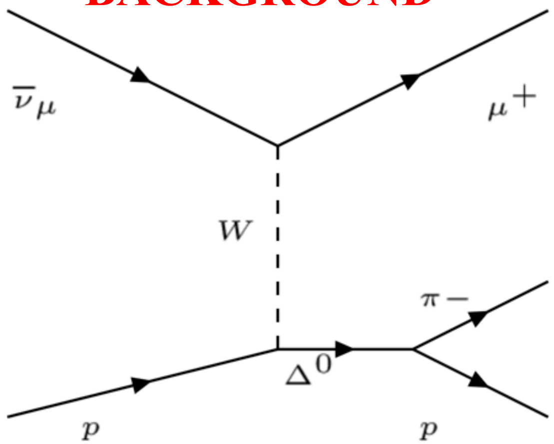
Anti-Neutrino CC-COH

SIGNAL



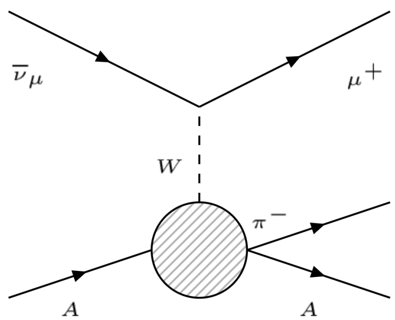
Anti-Neutrino CC-RES

BACKGROUND

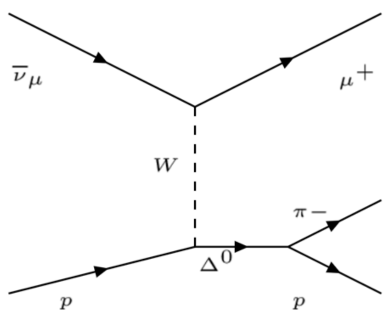


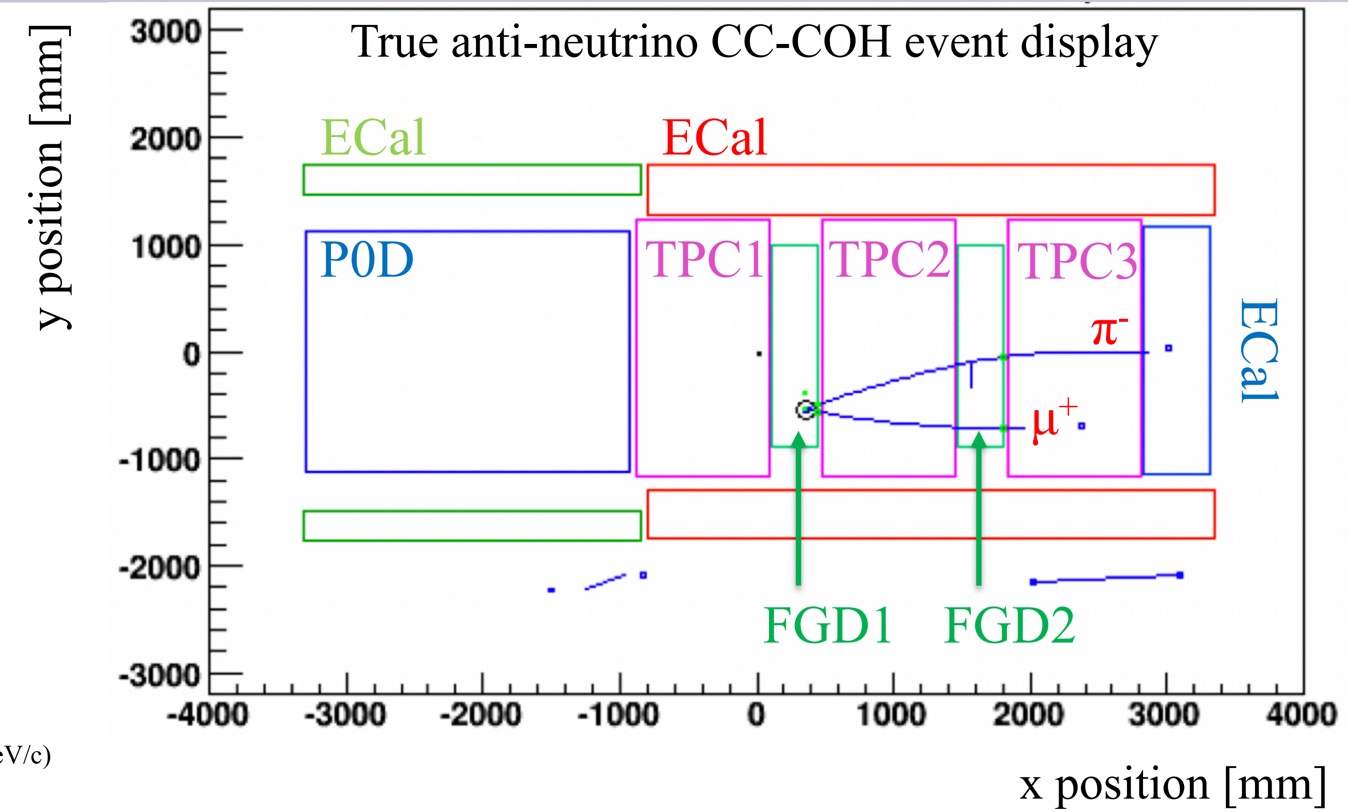
CC1Pi-0rP Topology Selection

Anti-Neutrino CC-COH



Anti-Neutrino CC-RES

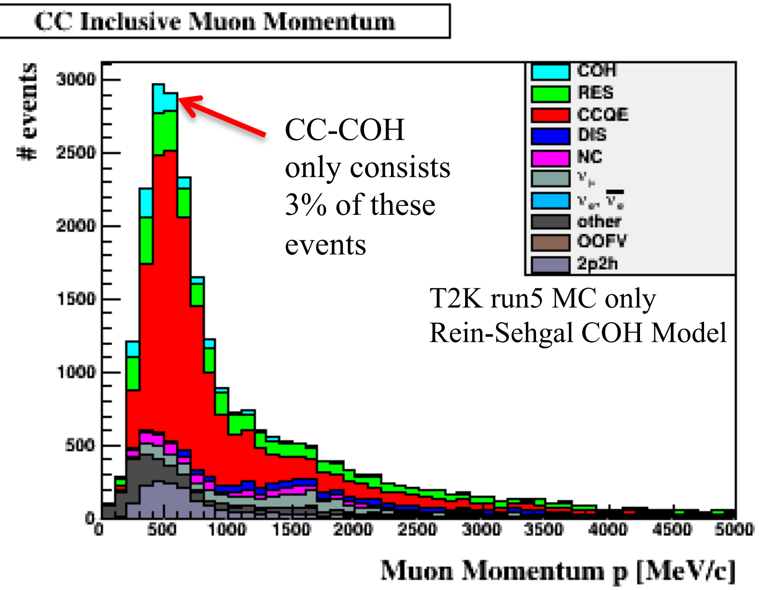




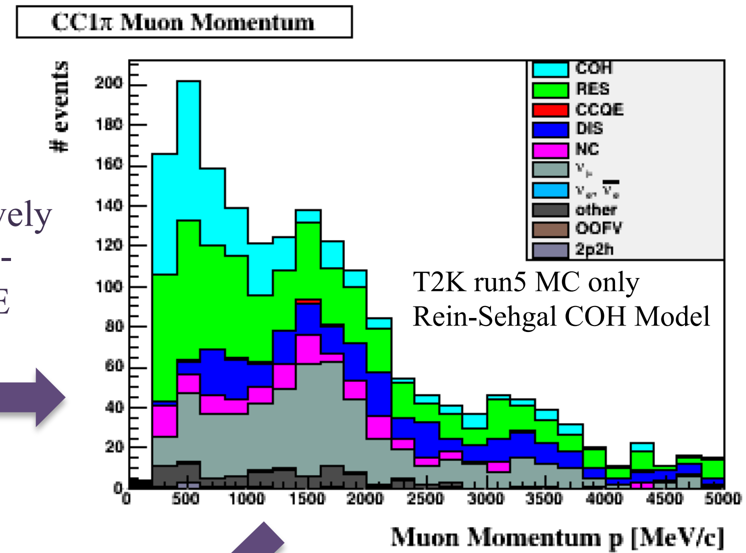
- Selection Step

1. Highest momentum particle in an event to be positively charged and μ-like (particle likelihood based on dE/dx in TPC)
2. Additional negatively charged π-like (and not proton-like) particle
3. Exactly 2 FGD1 particle tracks
   - CC1Pi-0rP should have only 2 outgoing particles reconstructed

CC1Pi-0rP Topology Selection



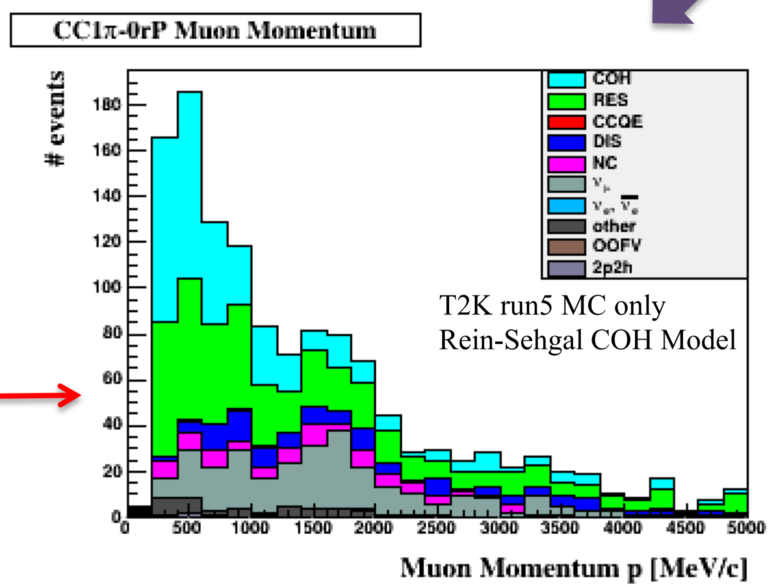
2. Additional negatively charged π-like track - removes most CCQE background events



1. Positively charged μ-like track - selects all the $\bar{\nu}_\mu$ CC interaction events

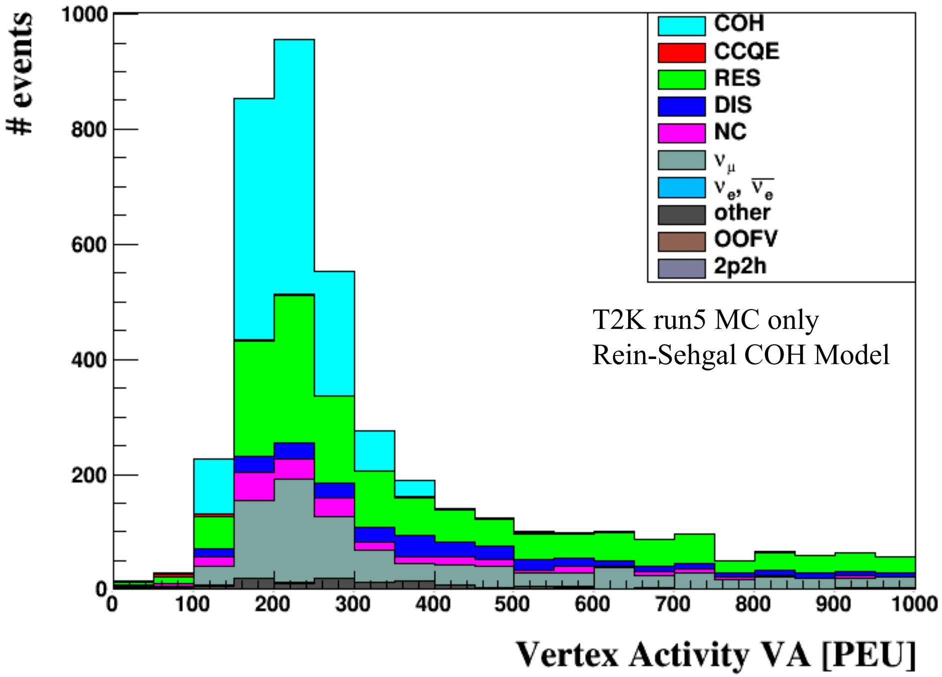
3. Exactly 2 FGD1 particle tracks requirement - removes DIS or RES background events with reconstructed protons

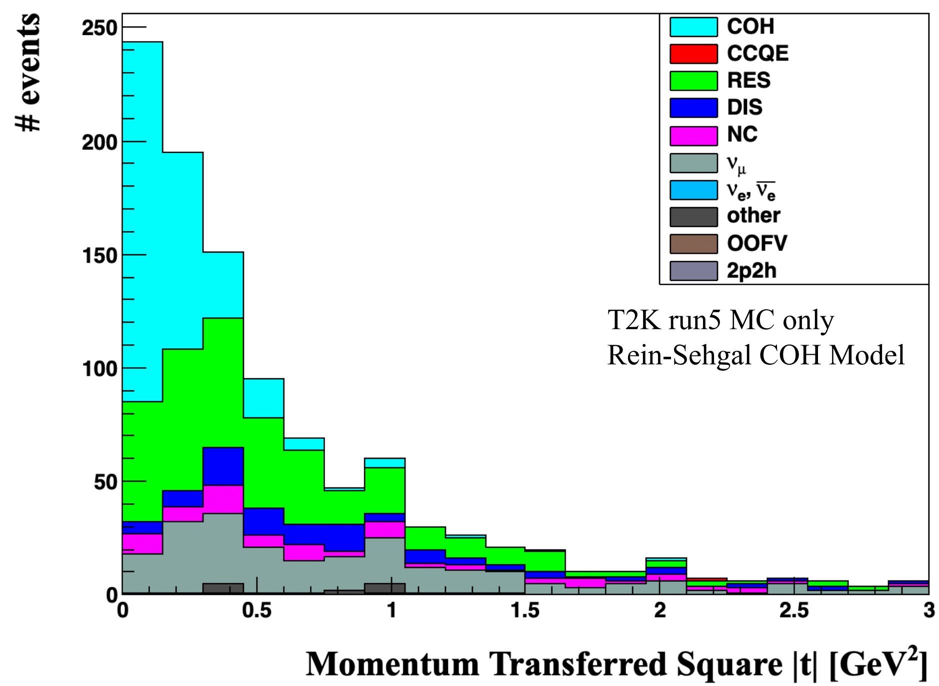
Selected CC1Pi-0rP Events



CC-COH Event Selection

- Characteristics of coherent events
  - Low energy deposition (vertex activity) around the neutrino interaction vertex
  - Low four momentum transfer to the nucleus

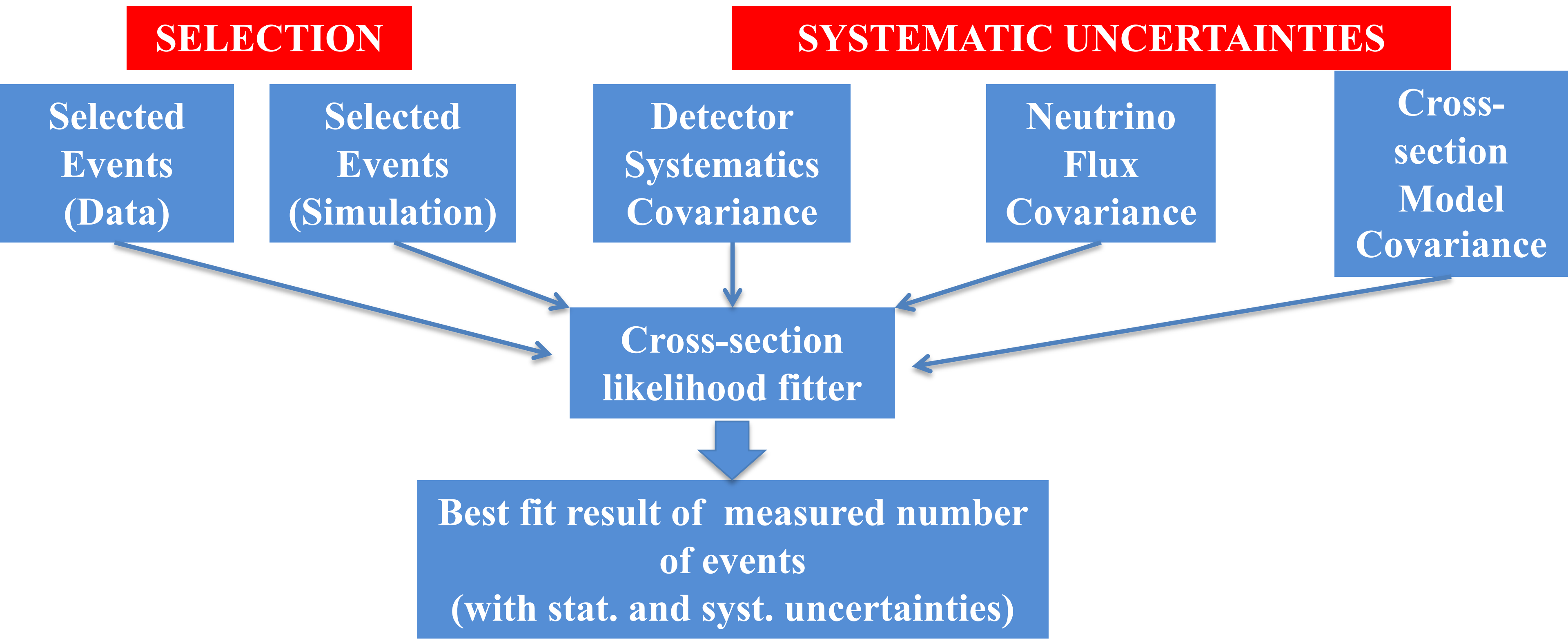




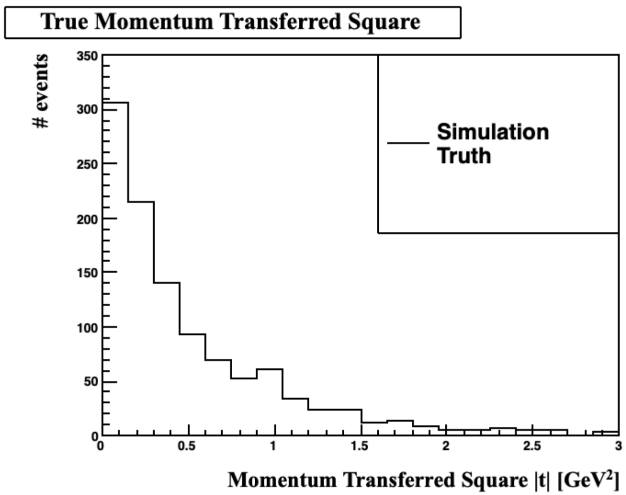
Cross-section Likelihood Fitter

- Cross section
  - Likelihood of two particles to interact under certain conditions
  - $\sigma = \frac{N_{\text{event}}}{\epsilon \cdot T \cdot \Phi}$, $N_{event:}$ measured number of events, ε: efficiency of the selection, T: number of target nucleus, Φ: number of incident neutrino
- Likelihood fitter
  - Finds the best fit value of measured number of events with a binned likelihood fit
  - Adjust simulation prediction to data (best fit)
  - Maximize (log)likelihood: minimizes the chi-square

$$\chi^2 = \chi^2_{\text{stat.}} + \chi^2_{\text{syst.}} = -2\ln\mathfrak{L}_{\text{stat.}} - 2\ln\mathfrak{L}_{\text{syst.}}$$

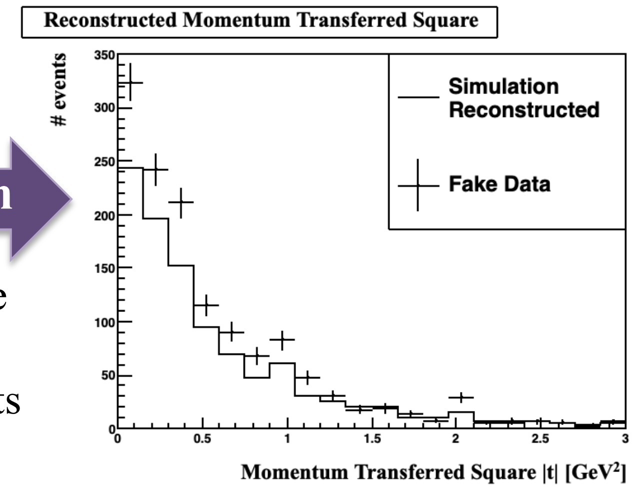


Cross-section Likelihood Fitter

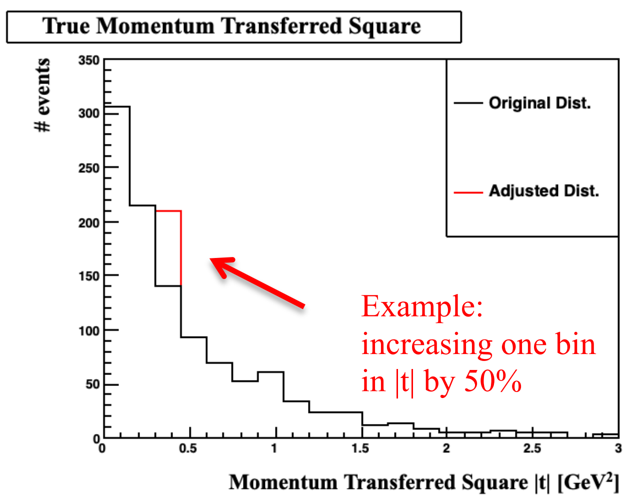


reconstruction

smearing of the distribution – detector effects

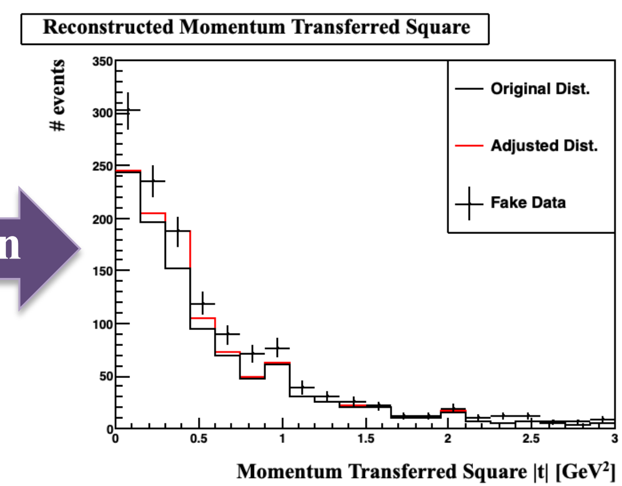


- Likelihood fitter adjusts parameters (analysis bins, other variables such as detector systematics, cross-section models) in the true distribution, to achieve best fit (to the data) in the reconstructed distribution

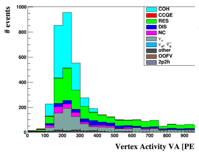


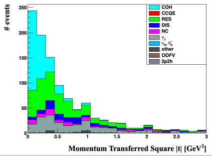
reconstruction

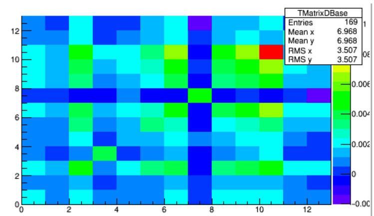
Notice multiple bins in the reco. dist. are affected

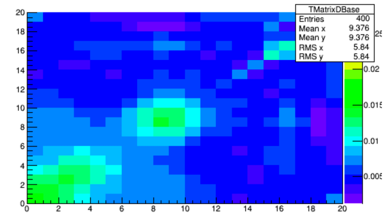


Analysis Status and Outlook









waiting to reveal real data

Selected Events (Data)

finalizing truth information

Selected Events (Simulation)

ready

Detector Systematics Covariance

ready

Neutrino Flux Covariance

finalizing results

Cross-section Model Covariance

Cross-section likelihood fitter

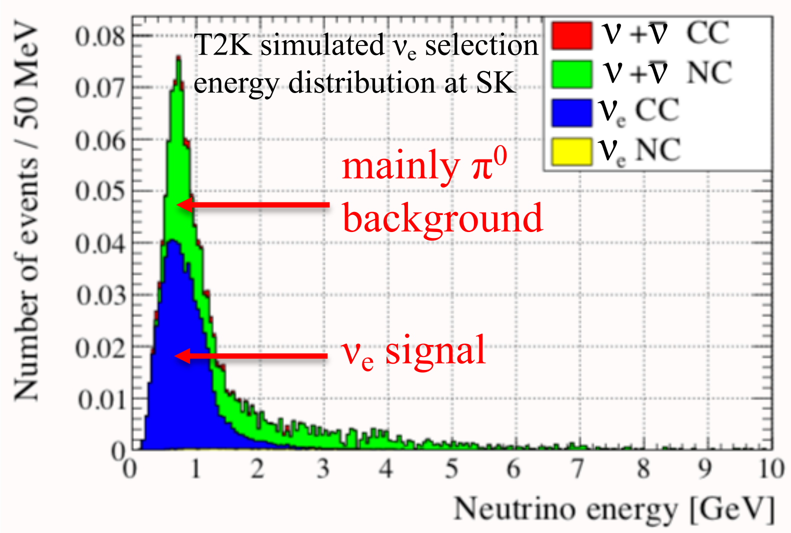
Best fit result of measured number of events (with stat. and syst. uncertainties)

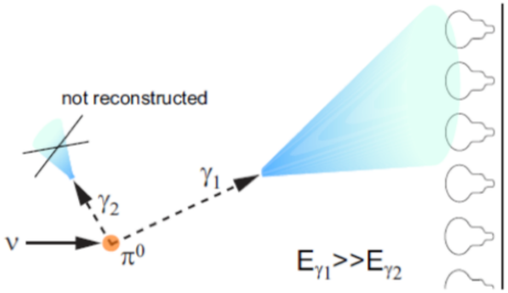
- Next Steps
  - Expect analysis to be finalized by end of July
  - Reveal real data and extract cross sections afterwards
  - Aiming to have results (paper) by the end of the year

Back up

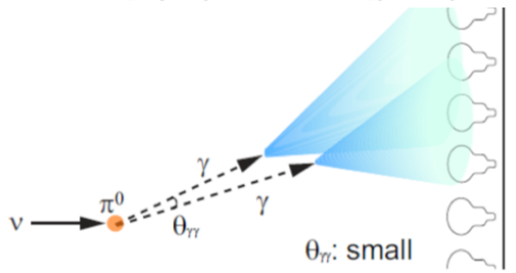
Motivations for measuring coherent pion production

- Measurements of NC-COH and CC-COH would help to constrain the physics model used in neutrino interaction generators
  - Underlying physics model for NC-COH and CC-COH are the same
- NC-coherent π^0
  - π^0 production is the main background to the water Cherenkov detector (such as SK) detecting appearance of ν_e (neutrino oscillation)
  - π^0 can be confused with an electron in water Cherenkov detectors
    - The π^0 decay is asymmetric resulting one low energy photon, thus not reconstructed
    - The two photons would be boosted in the π^0 direction, causing the rings to overlap with each other





(a) Highly asymmetric energy decay



(b) Small opening angle

Images:G. D. Lopez Thesis (2012)

Motivations for measuring coherent pion production

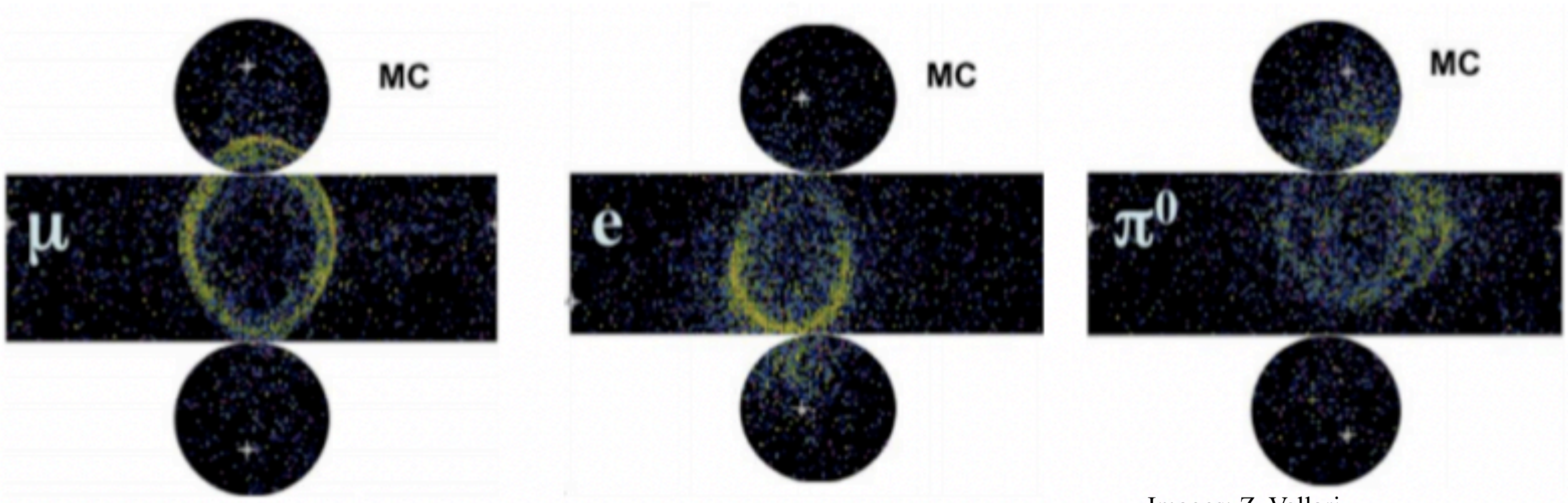
Table 17: Event rate table for 1Re, SKMC 14a with tuned RUN5c-9 RHC flux 13av3 and with BANFF/NIWG postfit reweight, 1.6346×10^{21} POT.

Event Type	$\nu_\mu \to \nu_\mu$	$\nu_e \to \nu_e$	$\bar{\nu_\mu} \to \bar{\nu_\mu}$	$\bar{\nu_e} \to \bar{\nu_e}$	$\nu_\mu \to \nu_e$	$\bar{\nu_\mu} \to \bar{\nu_e}$	Total
CCQE	0.0279892	1.10719	0.0522963	1.8216	2.22137	6.2014	11.4318
CCMEC	0.00631841	0.307697	0.00484409	0.244552	0.498493	0.723547	1.78545
CC 1π	0.0165998	0.219172	0.0160315	0.309088	0.311413	0.728695	1.601
CC coh.	5.15725e-05	0.00213968	0.000907666	0.0227318	0.00246954	0.0676121	0.0959124
CC other	0.00835359	0.0450377	0.00253352	0.0241857	0.0223188	0.014319	0.116748
NC 1π	0.354494	0.0143979	0.423351	0.0115915	0	0	0.803834
NC coh.	0.104348	0.00480559	0.440894	0.00677661	0	0	0.556825
NC 1γ	0.173845	0.00959142	0.38085	0.00821777	0	0	0.572504
NC other	0.123736	0.00476625	0.0575639	0.0021206	0	0	0.188187
Subtotal	0.815735	1.71479	1.37927	2.45086	3.05607	7.73557	
Total			17.1523				

- NC-COH is a 3% background to the electron neutrino appearance measurement
- 30% uncertainty assigned to NC-COH

	Parameter	Nominal	Error	Prior	Type	Comment
CC0π	CCQE model	2	N/A	N/A	D	MDLQE = 2 corresponds to the RFG model.
	SF$\to$RFG	1	N/A	N/A	D	
	M_A^{QE} (GeV)	1.21	0-10	flat	B	
	M_A^{QE} $\bar{\nu}$-H (GeV)	1.03	0	N/A	D	
	p_F ^{12}C (MeV)	217	31	flat	B	
	p_F ^{16}O (MeV)	225	31	flat	B	
	2p2h norm. ν–^{12}C	1	1	flat	B	
	2p2h norm. $\bar{\nu}$–^{12}C	1	1	flat	B	
	2p2h norm. ^{12}C to ^{16}O	1	0.2	flat	B	
	2p2h shape ν–^{12}C	0	1	flat	B (or N)	
	2p2h shape ν–^{16}O	0	1	flat	B (or N)	30% correlated with 2p2h shape ν–^{12}C
	BeRPA A	0.59	0.118	gauss.	B	
	BeRPA B	1.05	0.21	gauss.	B	
	BeRPA C	1.13	0.17	gauss.	B	
	BeRPA D	0.88	0.352	gauss.	B	
	BeRPA U	1.2	N/A	N/A	D	
RES	$M_A^{1\pi}$	1.07	0.15	gauss.	B	correlations in Fig.48
	C_5^A	0.96	0.15	gauss.	B	correlations in Fig.48
	$I = 1/2$ Bkg.	0.96	0.40	gauss.	B	correlations in Fig.48
Other	CC coh. norm. ^{12}C	reweight	0.3	gauss.	B	E_π weights are applied according to Table 12.
	CC coh. norm. ^{16}O	E_π weights	0.3	gauss.	B	Fully correlated with ^{12}C
	CC other shape	0.0	0.4	gauss	B	
	NC coh. norm.	1.0	0.3	gauss.	B	
	NC 1γ	2.0	2.0	gauss.	B	
	NC Other	1.0	0.3	gauss.	N,F	The NC other dial is treated separately at the near and far detectors.
	CC ν_e/ν_μ	1.00	$\sqrt{2} \times 0.02$	gauss.	F	2% anticorrelation between these two dials Sec.7
	CC $\bar{\nu}_e/\bar{\nu}_\mu$	1.00	$\sqrt{2} \times 0.02$	gauss.	F	
Pion FSI					N,F	see Sec. 5

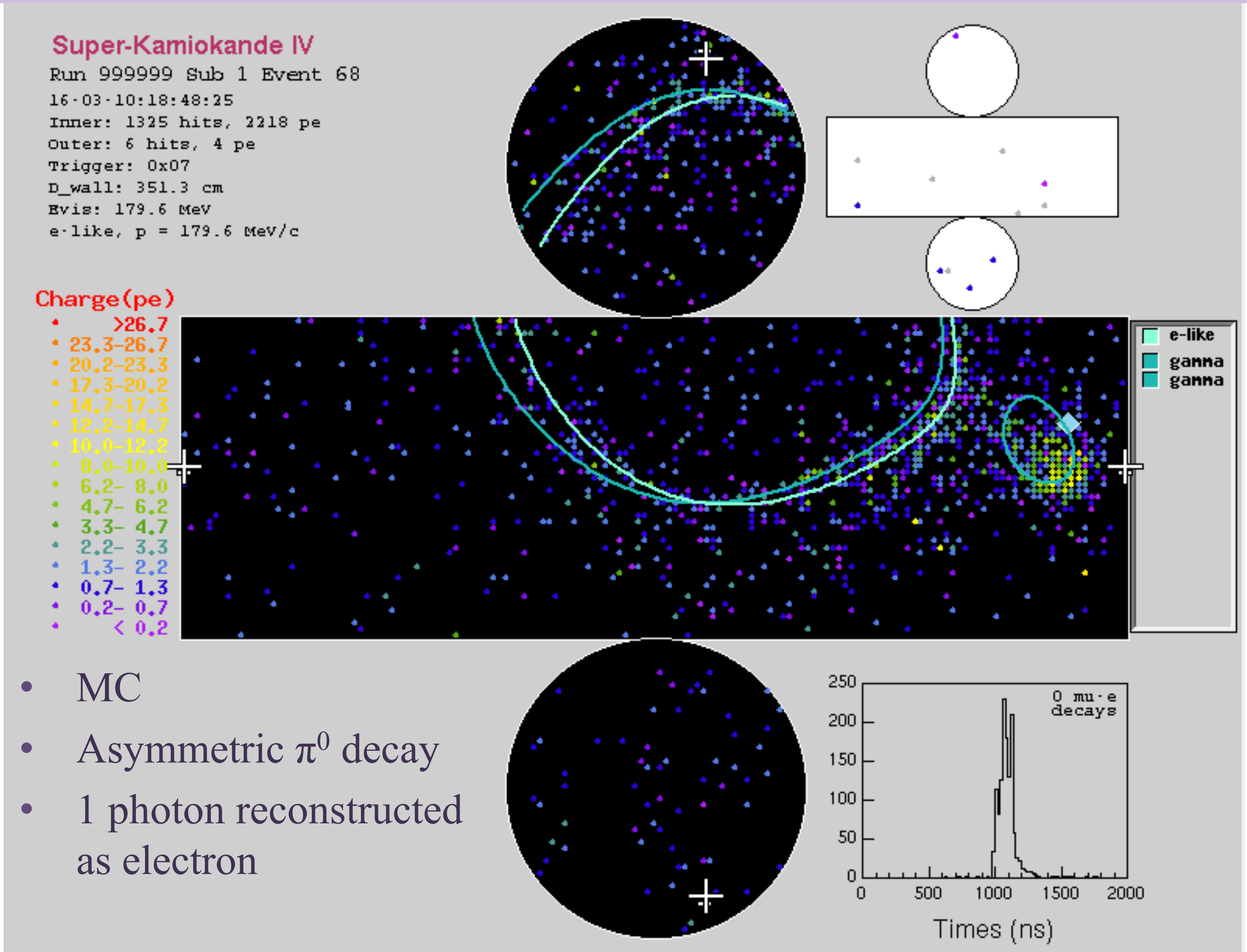
SK Event Displays



Images: Z. Vallari Thesis (2018)

- Muon ring: sharp
- Electron ring: fuzzy
- Neutral pion ring: stacked 2 electron rings, can mimic electron signal if one is not reconstructed

SK Event Displays

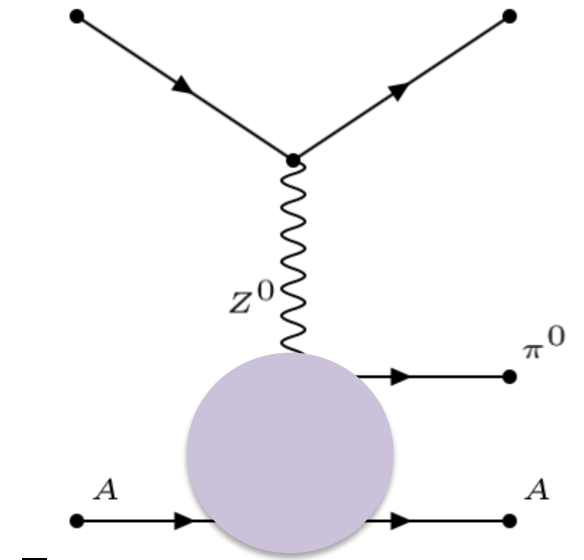


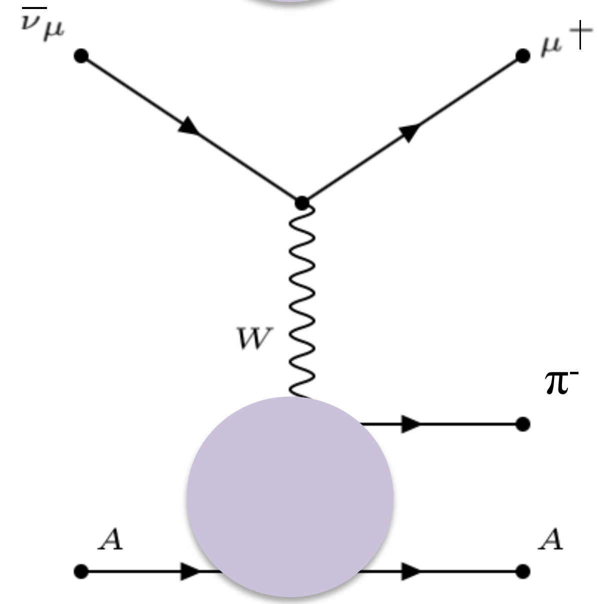
- MC
- Asymmetric π^0 decay
- 1 photon reconstructed as electron

Neutrino coherent pion production

- Neutrino (anti-neutrino) coherent scattering
  - A neutrino scatters off an entire nucleus
  - nucleus unchanged, cannot be excited
    - no quantum number exchange
  - no fragmentation of the nucleus
    - small four-momentum transfer
  - outgoing lepton and pion has small angle w.r.t. neutrino direction
- Neutrino coherent pion production
  - NC-coherent (NC-COH) has an outgoing π^0
  - CC-coherent (CC-COH) has an outgoing π^-
- Modeling
  - Rein and Sehgal (1983, 2007)
    - Partially conservation of the axial current (PCAC)
  - Berger and Sehgal (2009)
    - PCAC
  - Alvarez-Ruso (2007)
    - Microscopic

NC – coherent π^0

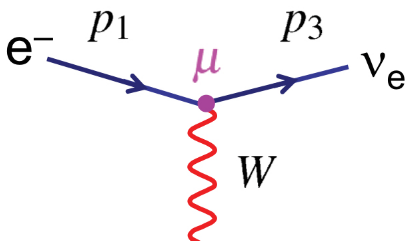




CC – coherent π^-

CVC and PCAC Hypothesis

- Weak interactions are experimentally determined to have the form of Vector – Axial-Vector (V-A)



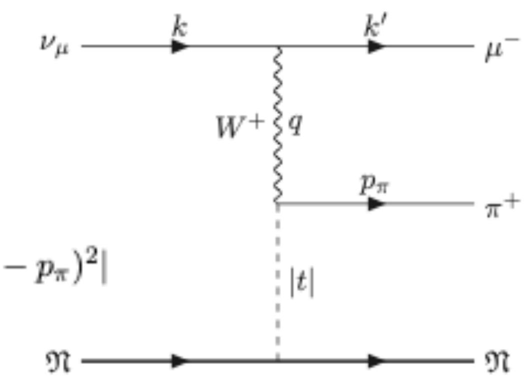
$$j^{\mu} \propto \bar{u}_{\nu_e}(\gamma^{\mu} - \gamma^{\mu}\gamma^{5})u_e$$

Vector Axial-Vector

- In pure vector, or axial-vector interactions parity is conserved
- Weak interactions do not conserve parity due to the linear combination of vector and axial-vector
  - γ^{μ} - $\gamma^{\mu}\gamma^{5}$ → $c_v\gamma^{\mu}$ - $c_A\gamma^{\mu}\gamma^{5}$
  - c_v – correction to the vector ”weak charge”
  - c_A – correction to the axial vector “weak charge”
- Conserved Vector Current (CVC) hypothesis
  - Experimentally, $c_v = 1.000$
- Partially Conserved Axial Current (PCAC) hypothesis
  - Experimentally, $c_A = 1.270 \pm 0.003$ → ”Almost” conserved

Rein and Sehgal (1983)

- From Adler's theorem,



$$\frac{d^2\sigma(\nu+\mathfrak{N}\to\ell^-+\mathfrak{N}')}{dQ^2dW}=\frac{G_F^2W}{2\pi^2M_{\mathcal{N}}}\frac{E_\ell}{E_\nu(E_\nu-E_\ell)}f_\pi^2\sigma(\mathfrak{N}+\pi\to\mathfrak{N}')$$

$$x_B=\frac{Q^2}{2M_{\mathcal{N}}(E_\nu-E_\ell)}\quad,\quad y_B=\frac{E_\nu-E_\ell}{E_\nu}\qquad |t|=|(q-p_\pi)^2|=|(k-k'-p_\pi)^2|$$

$$\left(\frac{d\sigma}{dx_B\,dy_B\,d|t|}\right)_{Q^2=0}=\frac{G_F^2\,M_{\mathcal{N}}\,E_\nu}{\pi^2}\,\frac{1}{2}\,f_\pi^2\,(1-y_B)\,\frac{d\sigma(\pi\mathfrak{N}\to\pi\mathfrak{N})}{d|t|}\bigg|_{E_\nu y=E_\pi}$$

- Adding nucleus dependencies

$$\frac{d\sigma(\pi\mathfrak{N}\to\pi\mathfrak{N})}{d|t|}=A^2\,|F_{\mathfrak{N}}(t)|^2\,\frac{d\sigma(\pi\mathcal{N}\to\pi\mathcal{N})}{d|t|}\qquad \frac{d\sigma(\pi\mathcal{N}\to\pi\mathcal{N})}{d|t|}=\frac{1}{16\pi}\left[\sigma_{\rm tot}^{\pi\mathcal{N}}\right]^2(1+r^2)$$

$$|F_{\mathfrak{N}}(t)|^2=e^{-b|t|}F_{\rm abs},\quad \text{with } b=\frac{R_0^2}{3}A^{2/3}\qquad r=\mathbf{Re}[f_{\pi N}(0)]/\mathbf{Im}[f_{\pi N}(0)]$$

- Rein-Sehgal triple differential coherent cross section

$$\frac{\mathrm{d}\sigma^{\rm NC}}{\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}|t|}=\frac{G_F^2\,M_{\mathcal{N}}\,E_\nu}{4\,\pi^2}\,f_{\pi^0}^2\,(1-y_B)\left(\frac{m_A^2}{m_A^2+Q^2}\right)^2A^2\,F_{abs}\,e^{-b|t|}\,\frac{1}{16\pi}\left[\sigma_{tot}^{\pi^0N}(Ey)\right]^2(1+r^2)$$

- For charged-current cross section:
  - Substitute pion decay constant: $f_{\pi^+}^2=2\,f_{\pi^0}^2$

Rein and Sehgal (2007)

- Following the original RS formulization

$$\frac{\mathrm{d}\sigma^{\mathrm{NC}}}{\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}|t|} = \frac{G_F^2\,M_{\mathcal{N}}\,E_\nu}{4\,\pi^2}\,f_{\pi^0}^2\,(1-y_B)\left(\frac{m_A^2}{m_A^2+Q^2}\right)^2 A^2\,F_{abs}\,e^{-b|t|}\,\frac{1}{16\pi}\left[\sigma_{tot}^{\pi^0 N}(Ey)\right]^2(1+r^2)$$

- For CC-COH, deficit was found in forward going muon direction
  - Correction factor was added to reduce the CC phase space

$$\mathcal{C} = \left(1-\frac{1}{2}\frac{Q^2_{\min}}{Q^2+m^2_{\pi^+}}\right)^2 + \frac{1}{4}y_B\frac{Q^2_{\min}(Q^2-Q^2_{\min})}{(Q^2+m_\pi^2)^2} \qquad Q^2_{\min} = m^2_{\mathrm{lep}}y_B/(1-y_B)$$

  - The modified RS cross section:

$$\frac{\mathrm{d}\sigma^{CC}}{\mathrm{d}x_B\,\mathrm{d}y_B\,\mathrm{d}|t|} = \frac{\mathrm{d}\sigma^{NC}}{\mathrm{d}x_B\,\mathrm{d}y_B\,\mathrm{d}|t|} \times 2\mathcal{C}\theta(Q^2-Q^2_{\min})\theta(y_B-y_{\mathrm{B,\,min}})\theta(y_{\mathrm{B,\,max}}-y_B)$$

  - The reduced phase space is:

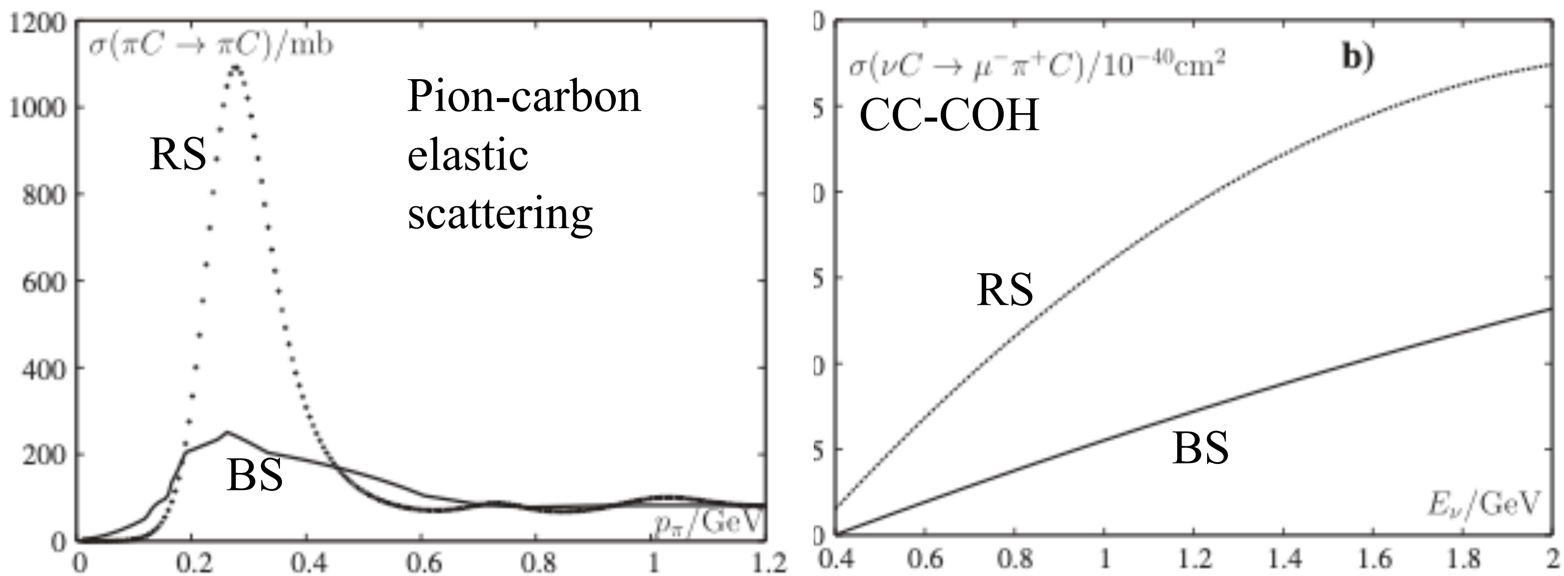
$$y_{\mathrm{B,\,min}} = m_\pi/E \qquad y_{\mathrm{B,\,max}} = 1-m_{\mathrm{lep}}/E$$

Berger and Sehgal (2009)

- The original and modified RS does not describe experimental measurements in the sub-GeV to few-GeV region
- 2 further modification was added by Berger and Sehgal
- Approximation of the kinematic term 1-y_B is replaced by the complete derived term

$$1 - y_B + \frac{y_B^2}{4}\left(1 - \left|\frac{Q^2}{(E_\nu - E_\ell)^2} + 1\right|\right)$$

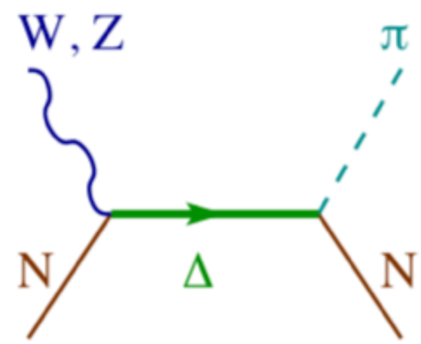
- BS used external pion-carbon scattering data to constrain the pion-nucleus cross section
  - RS tries to model the nuclear processes for the pion-nucleus elastic differential cross section used inside the model

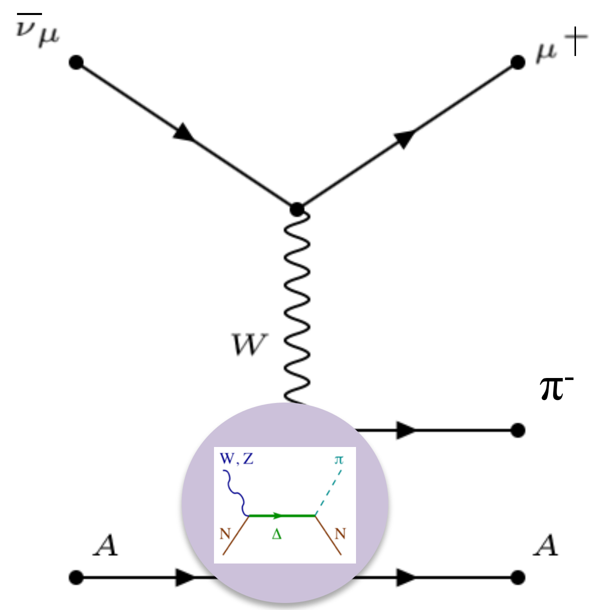


Alvarez-Ruso (2007)

- Microscopic model approach
  - Nucleon level process
  - Pions are produces of Δ resonances or N*
  - Outgoing pion's wave function is distorted by the nuclear potential
  - Nucleon must remain in the same quantum state
- Alvarez-Ruso's triple differential coherent pion production cross section

$$\frac{d\sigma}{dE_\ell d\Omega_\ell d\Omega_\pi} = \frac{1}{8(2\pi)^5}\frac{|\vec{k'}|.|\vec{p_\pi}|}{|\vec{k}|}|\mathcal{A}|^2$$



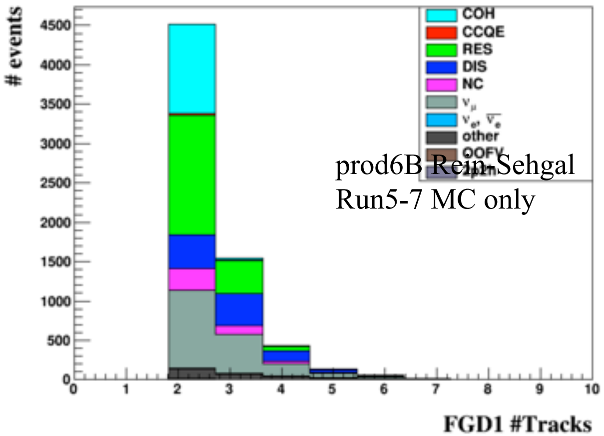


CC – coherent π-

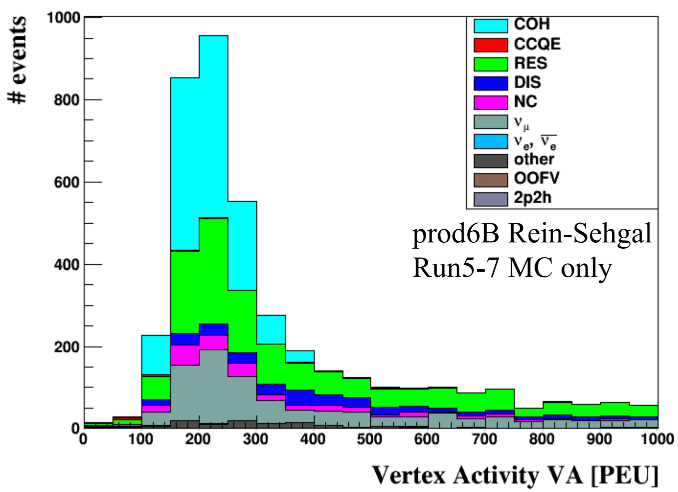
CC1Pi-0rP Selection

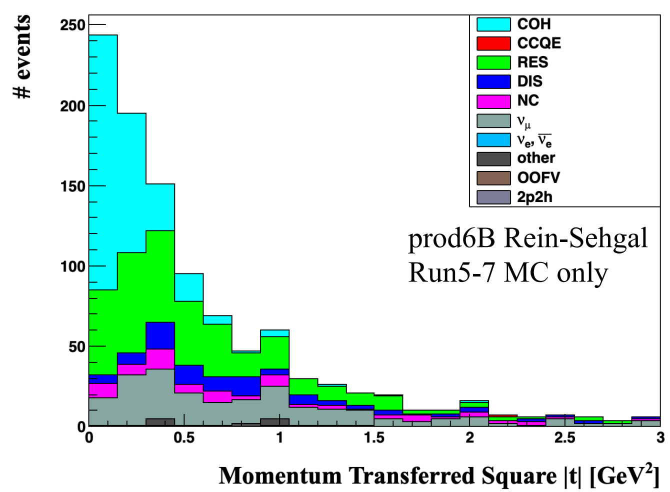
- CC1Pi0rP (0 reconstructable proton)
  - CCCOH or CCRES with proton momentum less than 450MeV/c
- Selection is the same as the CCCOH selection shown before

1. Highest momentum track in an event to be positively charged and μ-like
2. Additional negatively charged π-like (and not proton-like) track
3. Exactly 2 FGD1 particle tracks
   - CC-COH should have only 2 outgoing particles
   - CCRES with proton momentum less than 450MeV/s (not reconstructable) should have only 2 tracks



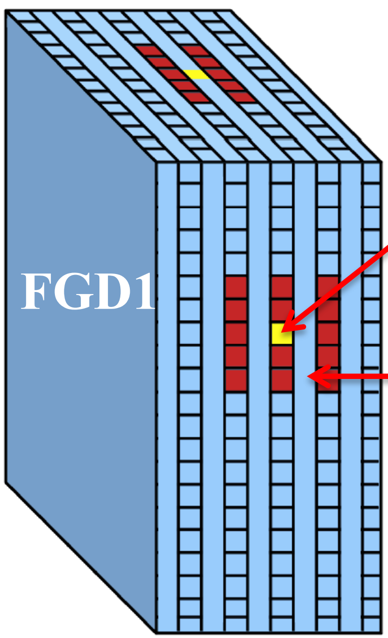
- Analysis variables
  - Vertex activity (VA) and momentum transferred |t|
  - These two variables are chosen to since they are also the selection variables for the CCCOH measurement





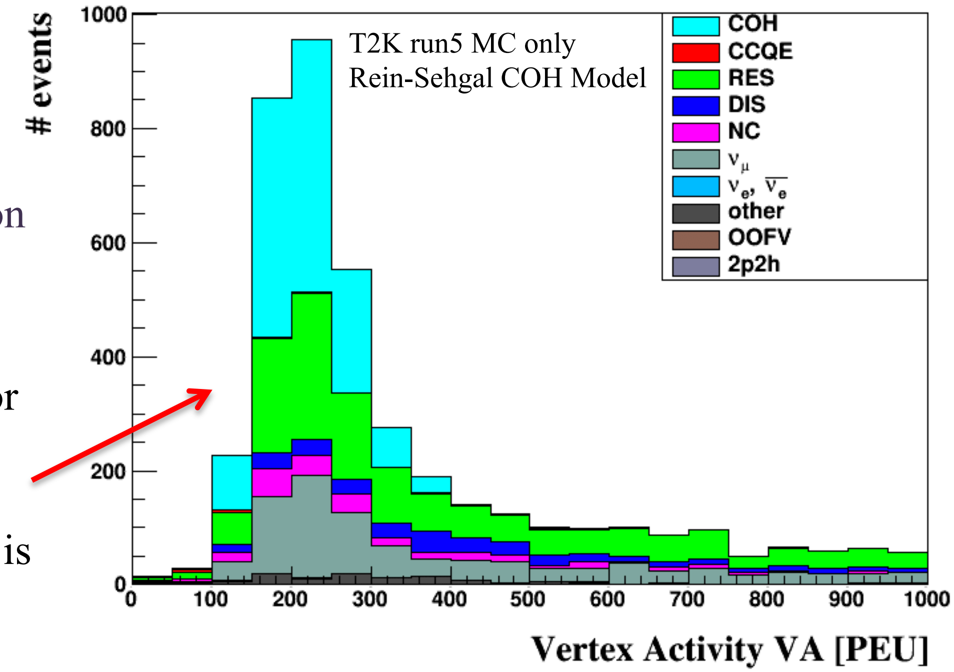
Event Selection: Vertex Activity

- FGD1: plastic scintillator layers
- Vertex Activity (VA)
  - energy deposition around the neutrino interaction vertex
    - measured in photon-equivalent unit (PEU)
  - VA low for CC-coherent π^- events
    - Only generates $1\mu + 1\pi$
    - No protons to deposit additional energy near the vertex



- Reconstructed neutrino interaction vertex

- e.g. 5X5 scintillator layers volume
- Energy deposition inside this volume is VA



Event Selection: Momentum Transferred Square

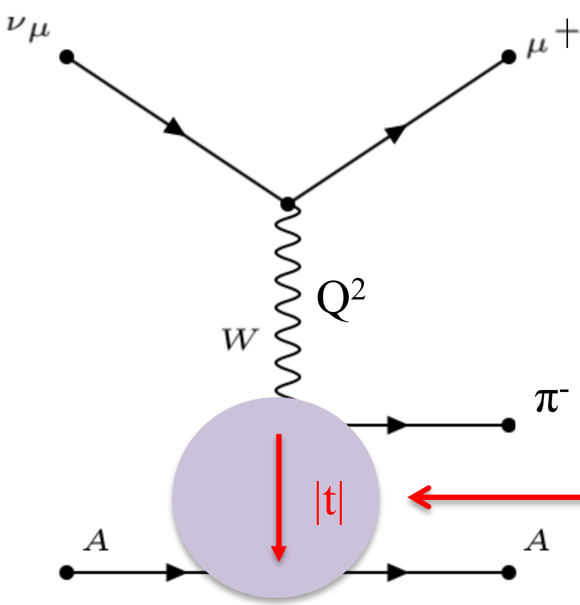
$$|t| = \left|(q - P_\pi)^2\right| = \left|\left((P_\nu - P_\mu) - P_\pi\right)^2\right|$$

$$Q^2 = -q^2$$

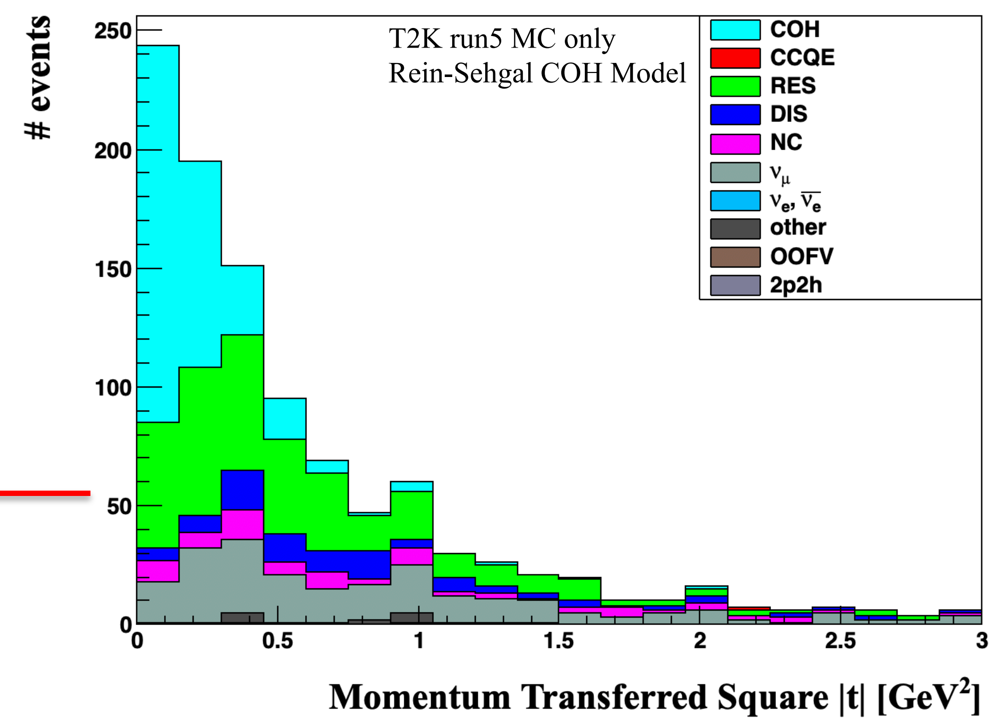
$$P_\nu = (E_\nu,\ 0,\ 0,\ E_\nu) = (E_\mu + E_\pi,\ 0,\ 0,\ E_\mu + E_\pi)$$

$$|t| = \Big(\sum_{i=\mu,\pi} (E_i - p_i^z)\Big)^2 + \Big(\sum_{i=\mu,\pi} p_i^x\Big)^2 + \Big(\sum_{i=\mu,\pi} p_i^y\Big)^2$$

- Momentum transferred square
  - Note this is not the standard |t| since the existence of the extra pion



CC – coherent π^-



Cross-section Likelihood Fitter

- Likelihood fitter
  - Finds the best fit value of measured number of events with a binned likelihood fit
  - Adjust simulation prediction to data (best fit)
  - Maximize (log)likelihood minimizes the chi-square

$$\chi^2 = \chi^2_{\mathrm{stat.}} + \chi^2_{\mathrm{syst.}} + \chi^2_{\mathrm{reg.}} = -2\ln\mathfrak{L}_{\mathrm{stat.}} - 2\ln\mathfrak{L}_{\mathrm{syst.}} - 2\ln\mathfrak{L}_{\mathrm{reg.}}$$

$$\chi^2_{\mathrm{stat.}} = \sum_i^N 2(N_i^{\mathrm{MC}} - N_i^{\mathrm{Data}} + N_i^{\mathrm{Data}}\ln(\frac{N_i^{\mathrm{Data}}}{N_i^{\mathrm{MC}}}))$$

  - $\chi^2_{\mathrm{syst.}} = \mathbf{\Delta}^T\mathbf{V}_{syst.}\mathbf{\Delta}$, $V_{\mathrm{syst.}}$: Systematics covariance matrix, **Δ**: difference between data and predicted value
  - Extra regularization term to ensure the smoothness of the best fit distribution (cross section should be continuous)

$$\chi^2_{\mathrm{reg.}} = \lambda\sum_i^N |c_i - c_{i-1}|^2$$