Dynamical spin effects in pseudoscalar mesons

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Overview

1. Light-front holographic QCD
2. Dynamical spin effects
3. Predicting observables
4. Conclusions

M. Ahmady, C. Mondal, R. Sandapen, arXiv: 1805.08911 (2018)

The holographic light-front Schrödinger Equation

Brodsky & de Téramond (PRL, 09) | Brodsky, de Téramond, Dosch & Erlich (Phys. Rep. 15)

$$\left(-\frac{\mathrm{d}^2}{\mathrm{d}\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U_{\mathrm{eff}}(\zeta)\right)\phi(\zeta) = M^2\phi(\zeta)$$

- $\zeta^2 = x(1-x)b_\perp^2$: key light-front variable for bound states
- $x = k^+/P^+$: light-front momentum fraction of quark
- $b_\perp$: transverse distance between quark and antiquark
- U_{eff} : effective (includes coupling to higher Fock sectors) $q\bar{q}$ potential

Meson light-front wavefunction :

$$\Psi(x,\zeta,\varphi) = \frac{\phi(\zeta)}{\sqrt{2\pi\zeta}}\sqrt{x(1-x)}e^{iL\varphi}$$

Assumptions behind the holographic LFSE

The hLFSE is derived in LFQCD with

- No quantum loops (no Λ_{QCD})
- Massless quarks (chiral limit)

} semiclassical approximation

- Conformal invariance $\rightarrow$ Gravity dual in anti-de Sitter space (Maldacena: AdS=CFT)

Brodsky, de Téramond, Dosch & Erlich (Phys. Rep. 15)

A unique confinement potential

$$U_{\text{eff}}(\zeta) = \overbrace{\kappa^4\zeta^2}^{\text{Conformal sb}} + \overbrace{2\kappa^2(J-1)}^{\text{AdS/QCD}}$$

- Harmonic Oscillator (HO) potential comes out uniquely in the way conformal symmetry is broken (allowing emergence of a mass scale κ) in semiclassical LFQCD
- AdS/QCD mapping : $\zeta \leftrightarrow z_5$ (z_5 is 5^{th} dimension of AdS) gives the spin term

Brodsky, de Téramond & Dosch (Phys. Lett. 13)

Massless pseudoscalars

Solving the holographic LF Schrödinger Equation

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U_{\rm eff}(\zeta)\right)\phi(\zeta) = M^2\phi(\zeta)$$

with

$$U_{\rm eff}(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(J-1)$$

gives

$$M^2 = (4n + 2L + 2)\kappa^2 + 2\kappa^2(J-1)$$

- Predicts Regge trajectories with massless ground states $(n = L = J = 0)$
- Identify with the Goldstone bosons (π, K, η) of spontaneous chiral symmetry breaking in QCD

Fixing the fundamental AdS/QCD scale

Regge slopes global fits

Reference	Fit	κ[MeV]
Brodsky et al. [Phys. Rep. 15]	Mesons	540–590
Brodsky et al. [IJMD, 16]	Mesons & Baryons	523

Universal $\kappa \sim 500$ MeV successfully predicts

- Diffractive ρ production Forshaw & Sandapen (PRL, 12)
- Diffractive ϕ production Ahmady, Sandapen, Sharma (PRD, 16)
- $\Lambda_{\rm QCD}$ to 5-loops in $\overline{\rm MS}$ Brodsky, Deur, de Téramond (J.Phys, 17)

Restoring dependence on quark masses

- In momentum space, 'complete' invariant mass of $q\bar{q}$ pair

$$M_{q\bar{q}}^2 = \frac{\mathbf{k}_\perp^2}{x(1-x)} \to \frac{\mathbf{k}_\perp^2 + m_f^2}{x(1-x)}$$

Brodsky, de Téramond, Subnuclear Series Proc. (09)

- Pseudoscalar meson wavefunction becomes

$$\Psi^P(x,\zeta^2) = \mathcal{N}\sqrt{x(1-x)}\exp\left[-\frac{\kappa^2\zeta^2}{2}\right]\exp\left[-\frac{m_f^2}{2\kappa^2 x(1-x)}\right]$$

- m_f : effective quark masses (because of coupling of valence sector to higher Fock sectors)

Restoring dependence on quark spins

- In semiclassical approximation :

$$\Psi^P(x,\zeta^2,\lambda,\lambda') = \Psi^P(x,\zeta^2) \times S^{\text{non dynamical}}_{\lambda,\lambda'}$$

with

$$S^{\text{non dynamical}}_{\lambda,\lambda'} = \lambda \delta_{\lambda,-\lambda'}$$

- Normalization

$$\sum_{\lambda,\lambda'} \int \mathrm{d}^2\mathbf{b}\mathrm{d}x |\Psi^\pi(x,\zeta^2,\lambda,\lambda')|^2 = 1$$

Dynamical spin effects

$$\Psi(x, \mathbf{k}_\perp) \to \Psi(x, \mathbf{k}_\perp, \lambda, \lambda') = \Psi(x, \mathbf{k}_\perp) \times S_{\lambda\lambda'}(x, \mathbf{k}_\perp)$$

- For pseudoscalar meson :

$$S^{\text{dynamical}}_{\lambda\lambda'}(x, \mathbf{k}_\perp) = \frac{\bar{v}_{\lambda'}(x, \mathbf{k}_\perp)}{\sqrt{1-x}} \left[\frac{M_P^2}{2P^+}\gamma^+\gamma^5 + BM_P\gamma^5 \right] \frac{u_\lambda(x, \mathbf{k}_\perp)}{\sqrt{x}}$$

instead of

$$S^{\text{non dynamical}}_{\lambda\lambda'} = \frac{1}{\sqrt{2}}\lambda\delta_{\lambda,-\lambda'}$$

- B is the dynamical spin parameter:
  1. $B = 0$: no dynamical spin effects
  2. $B \neq 0$: dynamical spin effects
  3. $B \gg 1$: maximal dynamical spin effects

AdS/QCD scale, quark masses and mixing angle

To generate predictions for all observables, we use

- $\kappa = (523 \pm 24)$ MeV
- $m_{u/d} = (330 \pm 30)$ MeV
- $m_s = (500 \pm 30)$ MeV
- $\theta = (-14.1 \pm 2.8)^\circ$ for η–η' mixing in singlet-octet flavour basis:

$$|\eta_1\rangle = \frac{1}{\sqrt{3}}(|u\bar{u}\rangle + |d\bar{d}\rangle + |s\bar{s}\rangle)$$

$$|\eta_8\rangle = \frac{1}{\sqrt{6}}(|u\bar{u}\rangle + |d\bar{d}\rangle - 2|s\bar{s}\rangle)$$

$$|\eta\rangle = \cos\theta|\eta_8\rangle - \sin\theta|\eta_1\rangle$$

$$|\eta'\rangle = \sin\theta|\eta_8\rangle + \cos\theta|\eta_1\rangle$$

Charge radii

$$\sqrt{\langle r_P^2 \rangle} = \left[\frac{3}{2} \int \mathrm{d}x \mathrm{d}^2 \mathbf{b}_\perp [b_\perp (1-x)]^2 |\Psi_P(x, \mathbf{b}_\perp)|^2 \right]^{1/2}$$

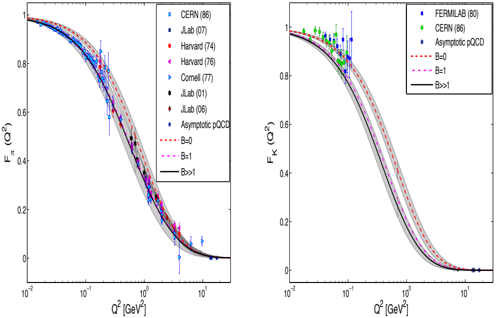
- $\pi^{\pm}$: maximal dynamical spin preferred
- $K^{\pm}$: no dynamical spin preferred

P	B	$\sqrt{\langle r_P^2 \rangle}_{\text{Th.}}$ [fm]	$\sqrt{\langle r_P^2 \rangle}_{\text{Exp.}}$ [fm]
$\pi^{\pm}$	0	0.543 ± 0.022	
	1	0.673 ± 0.034	0.672 ± 0.008
	$\gg 1$	0.684 ± 0.035	
$K^{\pm}$	0	0.615 ± 0.038	
	1	0.778 ± 0.065	0.560 ± 0.031
	$\gg 1$	0.815 ± 0.070	

Spacelike EM form factors

Drell & Yan (PRL, 70); West (PRL, 70)

$$F_{\rm EM}(Q^2) = 2\pi \int {\rm d}x{\rm d}b_\perp \; b_\perp \; J_0[(1-x)b_\perp Q] \; |\Psi^P(x,\mathbf{b}_\perp)|^2$$



$\pi^\pm$: maximal dynamical spin. $K^\pm$: no dynamical spin

Decay constants

$$\langle 0|\bar{\Psi}\gamma^{\mu}\gamma_5\Psi|P\rangle = f_P p^{\mu}$$

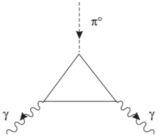
$$f_P = 2\sqrt{\frac{N_c}{\pi}}\int \mathrm{d}x\{((x(1-x)M_\pi^2) + Bm_f M_\pi\}\frac{\Psi^\pi(x,\zeta)}{x(1-x)}\Big|_{\zeta=0}$$

- $\pi^{\pm}$: maximal dynamical spin preferred
- $K^{\pm}$: no dynamical spin preferred

P	B	$f_P^{\text{Th.}}$ [MeV]	$f_P^{\text{Exp.}}$ [MeV]
$\pi^{\pm}$	0	162 ± 8	$130 \pm 0.04 \pm 0.2$
	1	138 ± 5	
	$\gg 1$	135 ± 6	
$K^{\pm}$	0	156 ± 8	156 ± 0.5
	1	142 ± 7	
	$\gg 1$	135 ± 6	

Radiative decay widths

- Using Adler-Bell-Jackiw anomaly relations



- π^0: maximal dynamical spin favoured
- η: no dynamical spin preferred
- η': maximal dynamical spin preferred

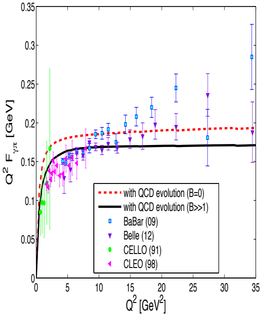
P	B_l	B_s	$\Gamma^{\mathrm{Th.}}_{P\to 2\gamma}$[KeV]	$\Gamma^{\mathrm{Exp.}}_{P\to 2\gamma}$[KeV]
π^0	0	-	$(5.62 \pm 0.57) \times 10^{-3}$	$(7.82 \pm 0.22) \times 10^{-3}$
	1	-	$(7.72 \pm 0.62) \times 10^{-3}$	
	$\gg 1$	-	$(8.13 \pm 0.68) \times 10^{-3}$	
η	0	0	0.542 ± 0.082	0.516 ± 0.018
	1	0	0.600 ± 0.056	
	1	1	0.622 ± 0.055	
	$\gg 1$	0	0.663 ± 0.061	
	$\gg 1$	$\gg 1$	0.710 ± 0.059	
η'	0	0	3.51 ± 0.48	4.28 ± 0.19
	1	0	3.88 ± 0.49	
	1	1	3.94 ± 0.49	
	$\gg 1$	0	4.51 ± 0.56	
	$\gg 1$	$\gg 1$	4.73 ± 0.57	

$\pi \to \gamma$ transition form factor

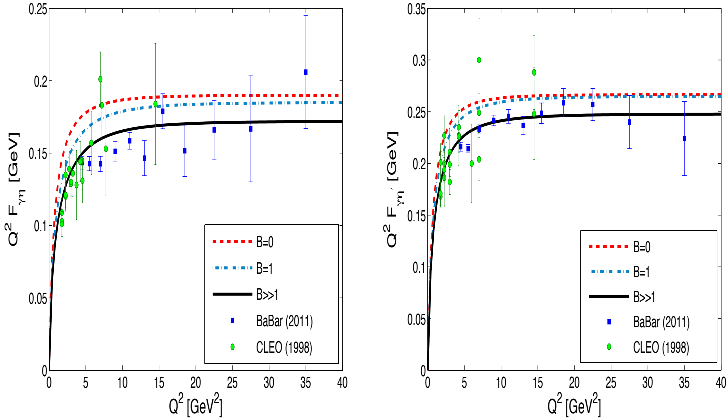
Lepage & Brodsky, PRD (80)

$$F_{\gamma P}(Q^2) = \frac{\sqrt{2}}{3} f_P \int_0^1 \mathrm{d}x \frac{\overbrace{\varphi_P(x, xQ)}^{\text{meson DA}}}{Q^2 x}$$

- Maximal dynamical spin preferred for π^0
- Cannot accommodate BaBar (2009) anomaly



$\eta^{(\prime)} \to \gamma$ transition form factors



Maximal dynamical spin effects preferred for both η and η'

Conclusions

Dynamical spin effects are

- maximal in pion, negligible in kaon and probably important in the η–η' system.
- more correlated to the quark flavour content of the pseudoscalar mesons rather than to their masses.

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