Bianchi IX Dynamics in Dust Time

Based on MA & V. Husain [Phys. Rev. D 96, 044032 (2017), arXiv:1707.07098]

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Bianchi IX spacetime

$$ds^2 = -dt^2 + e^{-2\Omega(t)}(e^{2\beta(t)})_{ij}\omega^i\omega^j$$

$$\beta_{ij} = \mathrm{diag}\{\beta_+ + \sqrt{3}\beta_-, \beta_+ - \sqrt{3}\beta_-, -2\beta_+\}$$

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- BKL conjecture
- Excellent example of gravitational chaos
- Quantum problem is unsolved

Techniques to study Mixmaster GR dynamics

- Metric variable approach (Covariant Formalism)[BKL (1970-72)]
- Particle in a box (Hamiltonian formalism) [Misner (1969), Ryan & Shepley (1975) ..]
- Dynamical systems approach [Bogyavlenskii & Noviko, Ellis & MacCallum (1969), Wainwright & Hsu (1989), Ringström (2000),… Hewitt (2016)…]

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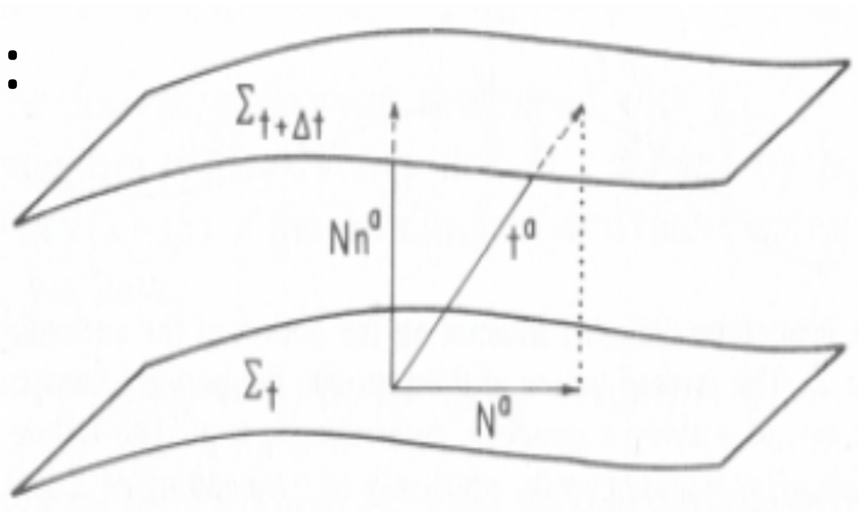
What is the dynamics in matter time?

Hamiltonian approach

Dynamical variables are:



$$(q_{ab}, \pi^{ab})$$

$$(\phi, p_\phi)$$

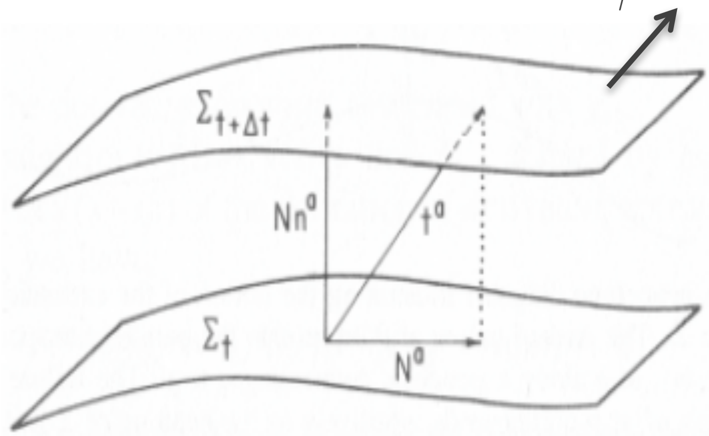
$$S = \int dt d^3x \, [\pi^{ab} \dot{q}_{ab} + p_\phi \dot{\phi} - \underbrace{N\mathcal{H} - N^a \mathcal{C}_a}_{\text{Hamiltonian}}]$$

Dust time

Husain & Pawlowski (2012)

Surfaces of constant time = level surfaces of dust

$\phi = const$



$$\lambda \equiv \phi - t = 0$$

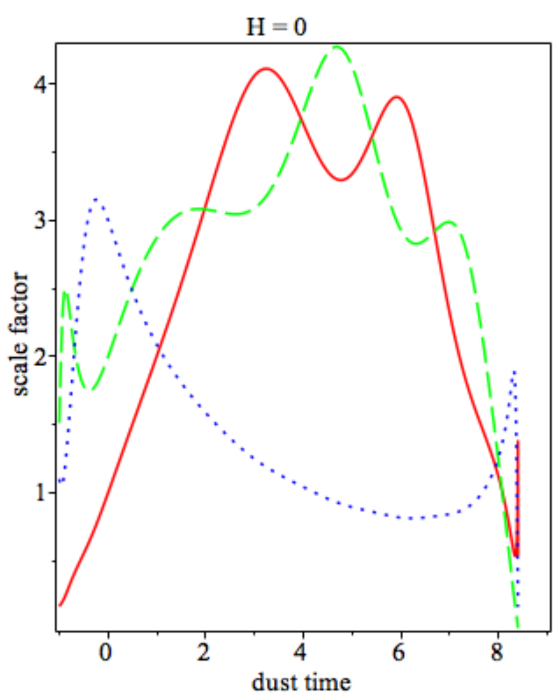
$$\implies N = -1$$

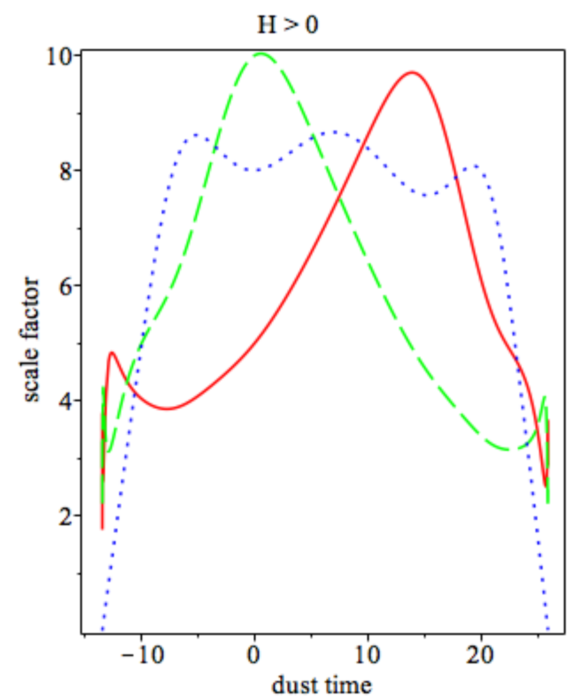
$$H_p = -\int d^3x \, (\mathcal{H}_G + \mathcal{H}_M + N^a \mathcal{C}_a)$$

Bianchi IX in dust time

$$H_p = \underbrace{-e^{3\Omega}(p_+^2 + p_-^2 - p_\Omega^2)}_{K(\text{scale factors, momenta})} + \underbrace{e^{-\Omega}v(\beta_+, \beta_-)}_{V(\text{scale factors})}$$

$$(\Omega, p_\Omega), \ (\beta_\pm, p_\pm)$$





“Free particle” = (Dust) Kasner

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Kasner Solution:

$$H_p = 0 : \ a_i = t^{p_i}$$

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$$\sum p_i = 1 \quad \sum p_i^2 = 1$$

Characterizing Dust-Kasner

Three relevant integration constants

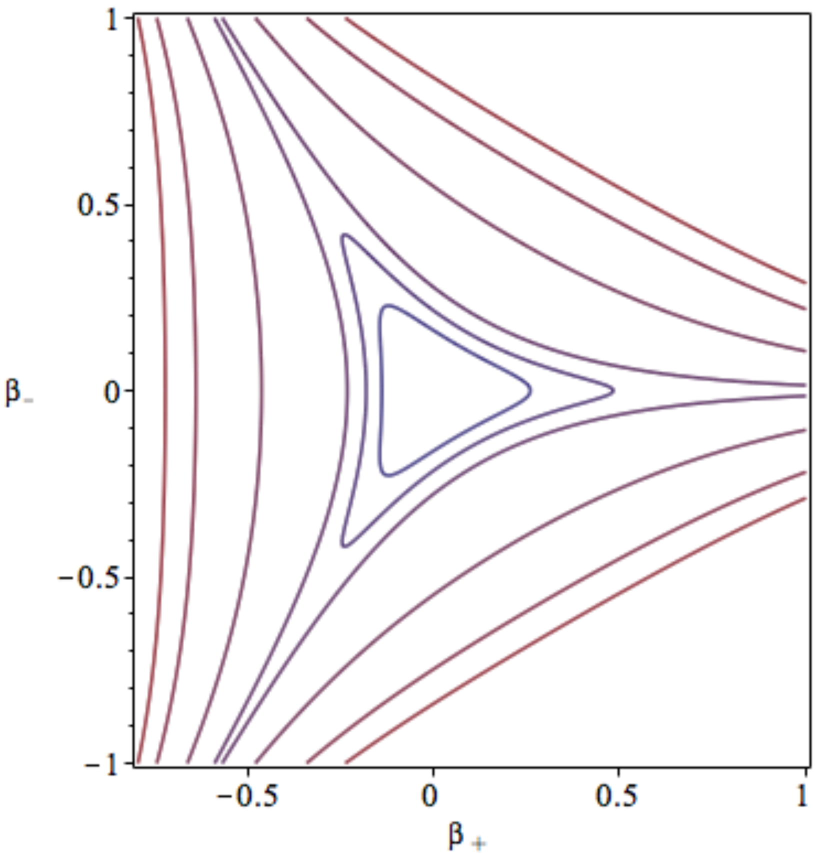
$$(p_\Omega^0, v^0, \theta)$$

Define:

$$\frac{p_-^0}{\Gamma} = \sin\theta$$

Particle in a box

$$H_p = -e^{3\Omega}(p_+^2 + p_-^2 - p_\Omega^2) + e^{-\Omega} v(\beta_+, \beta_-)$$



At each wall one term in V is dominant and perturbs the DK Hamiltonian

DK —Bounce→ DK' —Bounce→ DK''

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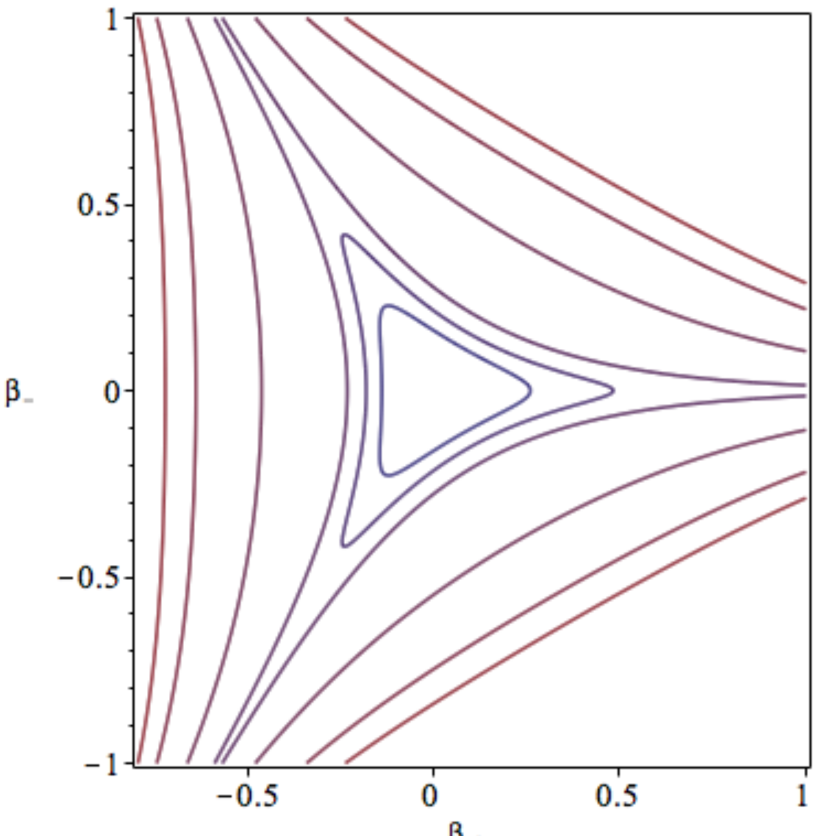

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Bianchi IX dynamics can be regarded as a series of bounces between Dust-Kasner solutions

Dynamics near one wall

$H_p = \text{constant} \qquad p_- = \text{constant} \qquad p_\Omega^0 - \frac{1}{2} p_+ = \text{constant}$

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$$\Gamma^{(i)} \sin\theta_i = \Gamma^{(f)} \sin\theta_f$$

$$\left(\frac{p_\Omega^0}{\Gamma}\right)^{(i)} \sin\theta_f - \left(\frac{p_\Omega^0}{\Gamma}\right)^{(f)} \sin\theta_i = \frac{1}{2}\sin(\theta_i + \theta_f)$$

$$\Delta(p_\Omega^0)^2 - 24 H_p \Delta v^0 = \Delta\Gamma^2$$

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Vacuum Case: $H_p = 0 \implies \Gamma = p_\Omega^0$

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Asymptotic Limit: $v^0 \to 0 \implies \Gamma \to p^0_\Omega$

Conclusion

- Dust time gives a **new** physical picture.
- Complementary to the other approaches.
- New transition law for dynamics away from the corners
- Bonus: The simple form of the Hamiltonian allows a straightforward construction of the path integral for Bianchi I with dust