The Black Hole Graviton Laser TIME BOMB

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Graviton Laser
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Gravitation and Quantum Mechanics

- There has not been very much work on the interplay of gravity and quantum mechanics.
- Here I do not mean quantum gravity, but the effect of gravity on a quantum mechanical system.
- The reason is probably that the effects are very weak. The gravitational coupling constant is $\frac{G}{c^4}$
- This combination has units of inverse Newtons, and in the MKS system it is numerically of the order $\sim 10^{-45}$
- However the gravitational potential is also proportional to the product of the two masses involved.
- The interaction of gravitation with a quantum mechanical system in the lab has only recently been observed.

Q-bounce

- The Q-bounce experiment, which stands for quantum bouncer, was proposed and carried out in the last ten years
- Here a system of ultra cold neutrons were observed.
- Ultra cold neutrons are normally defined to have a kinetic energy of less that 300 neV, and they are unable to penetrate into the solid material walls of a vessel, they bounce off the walls, and are in fact contained.
- If they are further distilled in energy so that the kinetic energy is in the few peV range, then they start to feel the gravitational potential due to the earth.
- The energy levels of the Schrödinger equation are easily found.

- The Schrödinger equation is:

$$-\frac{\hbar^2}{2m_N}\frac{d^2}{dz^2}\psi_E(z) + m_N g z \psi_E(z) = E\psi_E(z)$$

- The energy eigenfunctions are:

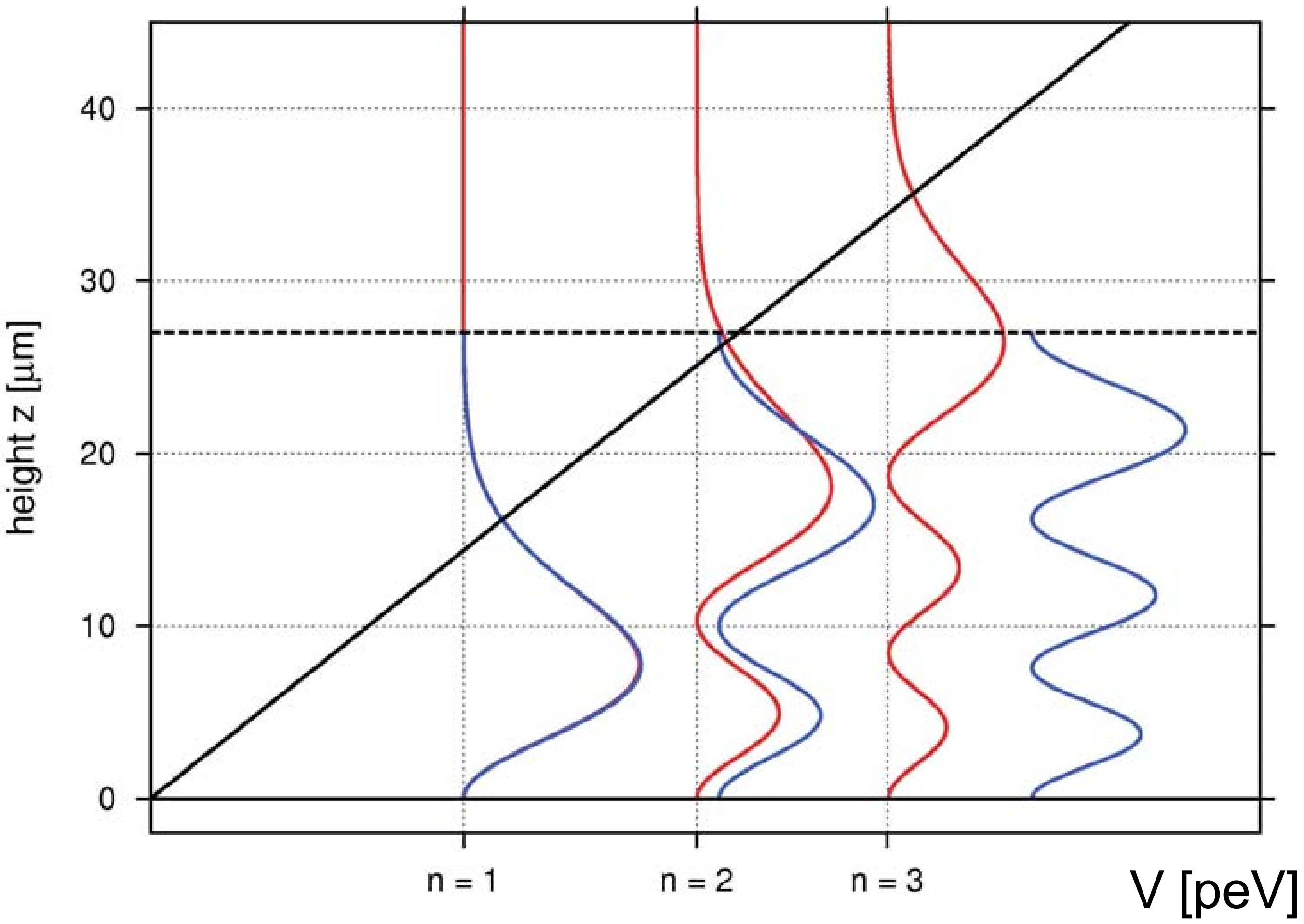
$$\psi_n(z) = \mathcal{N}_n Ai\left(\tfrac{z}{z_0} - \alpha_n\right) \qquad \mathcal{N}_n = \frac{1}{\sqrt{z_0}Ai'\left(-\alpha_n\right)}$$

$$E_n = m_N g z_0 \alpha_n \qquad z_0^3 = \frac{\hbar^2}{2gm_N^2}$$

- In numbers, the energy levels are approximately given by:

$$E_n = \left(\frac{9m_N h^2 g^2}{32}\left(n - \frac{1}{4}\right)^2\right)^{1/3} \times 10^{-12} eV = 1,69\left(n - \frac{1}{4}\right)^{2/3} peV$$

	E_n	E_n
1st state	1.41peV	1.41peV
2nd state	2.46peV	2.56peV
3rd state	3.32peV	3.97peV



Graviton Laser

- The idea of a laser which amplifies gravitational waves is very intriguing.
- For this possibility, one needs to believe in the existence of gravitons.
- For a laser, one needs a “lasing medium”, a quantum mechanical system which can exist for long enough time, in an excited state in the bulk, where the population is inverted.
- An ensemble of very cold UCN’s could satisfy these conditions, and could be imagined as providing the lasing medium.
- A laser works by the principle that in a lasing medium, spontaneously emitted “gravitons” can subsequently stimulate the emission of “gravitons” that are coherent with the original graviton. The process can cascade multiple times, creating in principle a the end, an avalanche of coherent gravitons.

Lasing medium

- If the gain in the lasing medium is sufficiently high, a single pass is adequate to obtain a significant amplification, there is no need for reflecting mirrors.
- With photons, there exist many examples of single pass lasing media, nitrogen gas lasers for example.
- Very cold UCN's, organize themselves according to the Schrödinger energy levels of a particle in a linear (gravitational) potential.
- Neutrons in an excited state can only decay through their interactions. They interact with the phonons in the base, and they interact with gravity.
- A neutron in an excited state can disintegrate, emit a phonon or emit a graviton. We can compute the rate of spontaneous (and stimulated) emission of a graviton for an excited neutron.

Gravitons

- A graviton corresponds to the non-interacting, free field quantum excitation of the linearized Einstein equations:

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \qquad \partial^\sigma \partial_\sigma h_{\mu\nu} = 0$$

$$h_{\mu\nu} = \int d^3k \left(\frac{e^{-ik_\mu x^\mu}}{\sqrt{V}} A(\vec{k}) \epsilon_{\mu\nu} + \frac{e^{ik_\mu x^\mu}}{\sqrt{V}} \epsilon^*_{\mu\nu} A^\dagger(\vec{k}) \right)$$

$$\left[A(\vec{k}), A^\dagger(\vec{k}') \right] = \frac{2\pi\hbar c^2}{\omega} \delta^3(\vec{k} - \vec{k}')$$

- The weak field metric is given by:

$$c^2 d\tau^2 = \left(1 - \frac{2GM_\oplus}{c^2 r}\right) c^2 dt^2 - \left(1 + \frac{2GM_\oplus}{c^2 r}\right) (\vec{x} \cdot \vec{dx})^2 / r^2$$
$$- h(x - ct) dy^2 + h(x - ct) dz^2 - r^2 d\Omega^2.$$

Gravitational Interaction

- Classically the neutrons interact with the gravitational field essentially through the geodesic equation.

$$\frac{d^2x^i}{d\tau^2} + \Gamma^i_{\mu\nu}\frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\tau} = 0$$

$$\Gamma^\lambda_{\mu\nu} = (1/2)\eta^{\lambda\sigma}(\partial_\mu h_{\sigma\nu} + \partial_\nu h_{\sigma\mu} - \partial_\sigma h_{\mu\nu})$$

$$h_{00} = -2\phi, \quad h_{0i} = w_i, \quad h_{ij} = -2\psi\delta_{ij} + 2s_{ij} \qquad \delta^{ij}s_{ij} = 0,$$

- For the wave in a Schwarzschild background, we have:

$$w_i = 0,\ \phi = GM_\oplus/r,\ \psi = GM_\oplus/3r \qquad s_{ij} = s^\oplus_{ij} + s^\approx_{ij}$$

$$s^\oplus_{ij} = (-2GM_\oplus/r^3)(x_ix_j - (r^2/3)\delta_{ij}) \quad s^\approx_{zz} = -s^\approx_{yy} = (1/2)h(x-t)$$

Lagrangian

- We obtain the equation of motion from a Lagrangian:

$$S[x^\mu(\tau)] = \int -m\left(-\eta_{\mu\nu}\frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\tau}\right)^{1/2} d\tau + \int \frac{m}{2}\, h_{\mu\nu}\frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\tau}\left(-\eta_{\mu\nu}\frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\tau}\right)^{-1/2} d\tau$$

- This comes from linearizing the free field action constructed with metric: $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$
- Changing the parametrization from τ to t and performing a Legendre transformation yields the Hamiltonian.

Schrödinger Equation and the Interaction Hamiltonian

- The Schrödinger equation governing the "free" part of the dynamics is given by:

$$\left(\frac{-\hbar^2}{2m_N}\frac{d^2}{dz^2} + m_N g z\right)\psi(z,t) = E\psi(z,t).$$

- While the interaction is given by:

$$H^{\approx}(x^i, \pi_j) = -\frac{\sqrt{G}}{c^2}\frac{h(x-ct)(\pi_z^2 - \pi_y^2)}{m_N}$$

- Having rescaled the field by:

$$h(x-ct) \rightarrow \frac{\sqrt{G}}{c^2}h(x-ct)$$

- to get a canonically normalized field.
- However, this is one for a graviton moving in the plane.

Interaction for arbitrary gravitons

- For a graviton moving in a direction $\hat{k} = (\sin\theta, 0, \cos\theta)$

$$H^{\approx}(x^i, \pi_j) = -\frac{\sqrt{G}}{c^2}\frac{h(\hat{k}\cdot\vec{x} - ct)((-\hat{k}_z\pi_x + \hat{k}_x\pi_z)^2 - \pi_y^2)}{m_N}$$

$$= -\frac{\sqrt{G}}{c^2}\frac{h(\hat{k}\cdot\vec{x} - ct)\sin^2\theta\,\pi_z^2}{m_N}$$

Spontaneous emission

- The amplitude for spontaneous emission of a graviton will be proportional to:

$$\langle\psi_{n'}| - \frac{\pi_z^2}{m_N}|\psi_n\rangle = \langle\psi_{n'}|(2m_N gz - 2E_n)|\psi_n\rangle$$
$$= \langle\psi_{n'}|2m_N gz|\psi_n\rangle.$$

- which gives:

$$\langle\psi_{n'}|2m_N gz|\psi_n\rangle = -\frac{4m_N g z_0}{(\alpha_{n'} - \alpha_n)^2}.$$

- Then the rate of spontaneous or stimulated emission, modifying a calculation in Baym, is given by:

$$d\Gamma^{\text{emm.}}_{n'\to n} = \frac{G}{c^4}\frac{4\pi^2 c^2}{ckV}(N_k + 1)\sin^4\theta |\langle\psi_{n'}|2m_N gz|\psi_n\rangle|^2 \delta(E_{n'} - E_n - \hbar ck)$$

- This can be used to compute the absorption cross section

Rate of graviton emission

- Then the number of states in d^3k of phase space is

$$\frac{1}{V}\sum_{\vec{k}} \to d\Omega \int \omega^2 d\omega/(2\pi c)^3$$

- Integrating over the energy of the emitted graviton gives:

$$\Gamma^{\text{emm.}}_{n'\to n} = d\Omega \int \frac{\omega^2 d\omega}{(2\pi c)^3}\left(\frac{2\pi\hbar c^2 N_k}{\omega}\right)\frac{2\pi}{\hbar}\frac{G}{c^4}\sin^4\theta |\langle\psi_{n'}|2m_N g z|\psi_n\rangle|^2 \delta(E_{n'} - E_n - \hbar\omega)$$

- Which should be integrated over all energies in a narrow angular spread. Then: $\sigma = \Gamma^{\text{emm.}}_{n'\to n}/\Phi$
- With $\Phi = \frac{1}{\hbar\omega}\int d\omega I(\omega)$ $\quad I(\omega) = d\Omega \frac{\omega^4}{(2\pi c)^4}\left(\frac{2\pi\hbar\omega c^2 N_k}{\omega^2}\right)$
- We get:

$$\sigma = \frac{4\pi^2\hbar G}{c^3}\frac{16}{(\alpha_{n'} - \alpha_n)^6}$$

Population inversion

- If we can induce the neutrons to excited states, in the long time limit, the occupation numbers asymptote to:

$$\mathcal{N}_{n'}(t) = \mathcal{N}_n(t) \rightarrow \tfrac{1}{2}(\mathcal{N}_n(0) + \mathcal{N}_{n'}(0))$$

- Thus if we pump between levels one and three, after a long time, we will have a population inversion between levels three and two.
- The neutrons in level three can only decay by emitting a graviton. The amplification factor is given by:

$$\kappa = \sigma(\mathcal{N}_{n'} - \mathcal{N}_n)$$

- This is the gain per meter. It turns out to be ridiculously small.

$$\sigma \;=\; 5.3 \times 10^{-68} \,\mathrm{per\ meter}^2$$

- This rate can be enhanced by imagining the lasing takes place on the surface of a neutron star. Here the atmosphere is Carbon, and the density is of the order of: 1.75×10^{29}

- But this still makes the gain:

$$\sigma \mathcal{N}_{carbon} \approx 9.4 \times 10^{-39} \text{ per meter}$$

- Which is still absurdly small.

- What can we do? The density cannot be appreciably increased nor can the distance through which the graviton can pass.
- But this is not true.
- Black holes admit unstable, circular, null, geodesics (orbits).

Unstable photon orbits

- A photon can scatter from infinity in almost any direction from a black hole. Only those photons heading directly towards the black hole are absorbed.
- From a finite radius, the set of directions from which the photon is absorbed is a cone of finite opening angle, pointing towards the black hole.
- As we descend down to the event horizon, the cone now completely flips over and closes in the outward direction. Only a photon pointing radially away from the black hole can escape.
- Therefore, in between, there exists a radius at which the cone has opening angle 180 degrees. Here there must exist an unstable circular orbit. The radius of this orbit is $3GM$.
- Thus a photon can orbit many times before finally spiralling out from the black hole.

- Quantum mechanically such states for the photon also exist.
- The effective potential for the radial motion in the Schwarzschild metric is:

$$V(r(r_*)) = \left(1 - \frac{2M}{r}\right)\left[\frac{l(l+1)}{r^2} + \frac{2M(1-s^2)}{r^3}\right] \qquad r_* = r + 2M\ln\left(\frac{r}{2M} - 1\right)$$

- With a Schrodinger like equation $\phi(t, r, \vartheta, \varphi) = e^{i\omega t}\frac{1}{r}\psi_{n\ell\omega}(r)Y_{\ell m}(\vartheta, \varphi)$

$$\left[-\frac{d^2}{dr_*^2} + V(r)\right]\psi_{n\ell\omega} = \omega^2\psi_{n\ell\omega} \qquad \frac{dr_*}{dr} = f(r)^{-1}$$

$$\psi(r_*) = T(\omega)e^{i\omega r_*}, \qquad r_* \longrightarrow -\infty,$$

$$\psi(r_*) = e^{i\omega r_*} + R(\omega)e^{-i\omega r_*}, \qquad r_* \longrightarrow +\infty$$

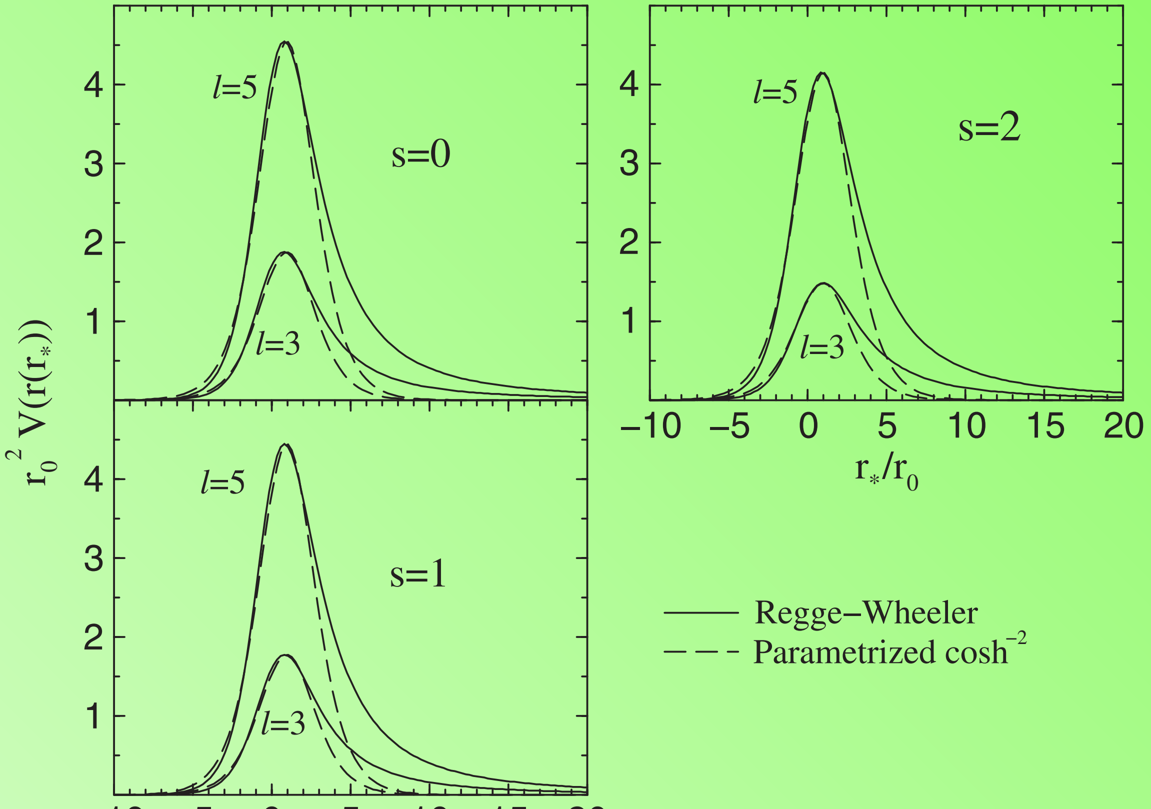
- The deflection is related to the real part of the phase shift:

$$\Theta(l) = 2\frac{d\delta_l^R}{dl}$$

- One finds:

$$\Theta(l) = \theta_1 + b\ln\left(\frac{l-l_C}{l_C}\right), \qquad l > l_C,$$

$$\Theta(l) = \theta_2 + 2b\ln\left(\frac{l_C-l}{l_C}\right), \qquad 0 \le l < l_C,$$



Lasing medium

- In the presence of a Kerr black hole, light massless particles, such as the axion can exhibit the phenomena of super-radiance.
- This is a process described by Penrose where the black hole loses angular momentum pumping states of the axion into highly occupied angular momentum states.
- A. Arvanitaki, with collaborators has described exactly such phenomena.

Nonrelativistic approximation $\alpha/l \ll 1$

$$v \sim \frac{\alpha}{\bar{n}}$$

$$\phi = e^{-i\omega t + im\varphi} Y(\theta) R(r) + \text{H.c.}.$$

$$R_{\text{far}}(r) = (2kr)^l e^{-kr} U(l + 1 - \frac{\alpha^2}{r_g k}, 2(l+1), 2kr),$$

$$R_{\text{near}}(r) = \left(\frac{r - r_+}{r - r_-}\right)^{-iP} {}_2F_1(-l, l+1, 1+2iP, \frac{r - r_-}{r_+ - r_-}),$$

Super radiance

- Is a kinematical phenomena, other examples include Cherenkov radiation and a rotating axisymmetric body.
- If the rotational velocity is faster than the angular phase velocity, super radiance occurs.

$$e^{im\varphi - i\omega_\gamma t} \qquad \frac{\omega_\gamma}{m} < \Omega_{\text{cylinder}}$$

- for black holes the angular velocity at the horizon is used
- this rotation rate is relativistic, which can give rise to significant super radiance rates
- gravity is universal, so the effect is universal for all particles
- the effect is maximal if the Compton wavelength of the particle is comparable with the size of the black hole. For astrophysical black holes, this requires masses between

$$10^{-20} \text{ and } 10^{-10} \text{ eV.}$$

Lasing gain

- The wave functions of the axions are essentially hydrogenic states, with nothing particularly specific.
- They can drop down in energy and emit a graviton. The probability that such a graviton is emitted is given by the total amplitude for spontaneous emission of a graviton. It will not behave any differently than our previous calculation.
- From this amplitude, the absorption cross section can also be computed, in the particle picture for the graviton.
- In the quantization of the gravitons we must use:

$$g_{\mu\nu} = \gamma_{\mu\nu} + h_{\mu\nu}$$

- With $\gamma_{\mu\nu}$ corresponding to the Schwarzschild metric.
- However we will still find $\sigma = \frac{\hbar G}{c^3} \times o(1) \quad \sim 10^{-68}$

- Thus the gain depends on the density of the axions. This is completely dependent on the model for the axions and dependent on the super radiance.
- It is calculated that the population of the axionic states quickly rise to

$$N_{\max} \simeq \frac{G_N M^2}{m} \Delta a_* \sim 10^{76} \left(\frac{\Delta a_*}{0.1}\right) \left(\frac{M}{10 M_\odot}\right)^2 \sim 10^{70}$$

- This gives rise to a density of approximately

$$\sim 10^{70}/R_s^3$$

- Then the graviton would amplify with gain per meter of

$$\sigma \times 10^{70}/R_s^3 = \frac{\hbar G}{c^3} 10^{70}/R_s^3 \times o(1)$$

$$\sim 10^2/R_s^3 \times o(1)$$

- For a solar mass black hole the radius is about 3000 meters, giving the gain per meter to be, for 1 to 10 solar masses

$$\sim 10^{-8} \quad \text{to} \quad 10^{-11}$$

- Thus the graviton is amplified once every second to one thousand seconds. Eventually this would be an intense, coherent graviton beam.
- For a super massive black hole of 10^7 solar masses, the gain per meter would be 10^{-19} .
- If the density of the lasing medium is not anywhere near the projected maximum density, it can also drastically lower the gain.
- It is clear observationally, we have not seen the existence of any intense, coherent beams of gravitational radiation, essentially death rays, thus the gains have to be smaller.

TIME BOMB

- Thus in principle, since the beginning of the universe, about 13 billion years or 13×10^{16} seconds, gravitons have been lasing in the vicinity of black holes with such super radiant axion clouds.
- They are continually emitting coherent, amplified beams of gravitons.
- There has not been enough time for the beams to be sufficiently amplified, to be a threat. The gain per meter has not been sufficiently high.
- But there is a ticking time bomb in the process of being created.
- For gains per meter of the order of 10^{-25} the amplification is order 1 in the time the universe has existed.
- If the axionic super radiant cloud prediction is valid, there is bad news on the horizon.