On the convection of ionospheric density features

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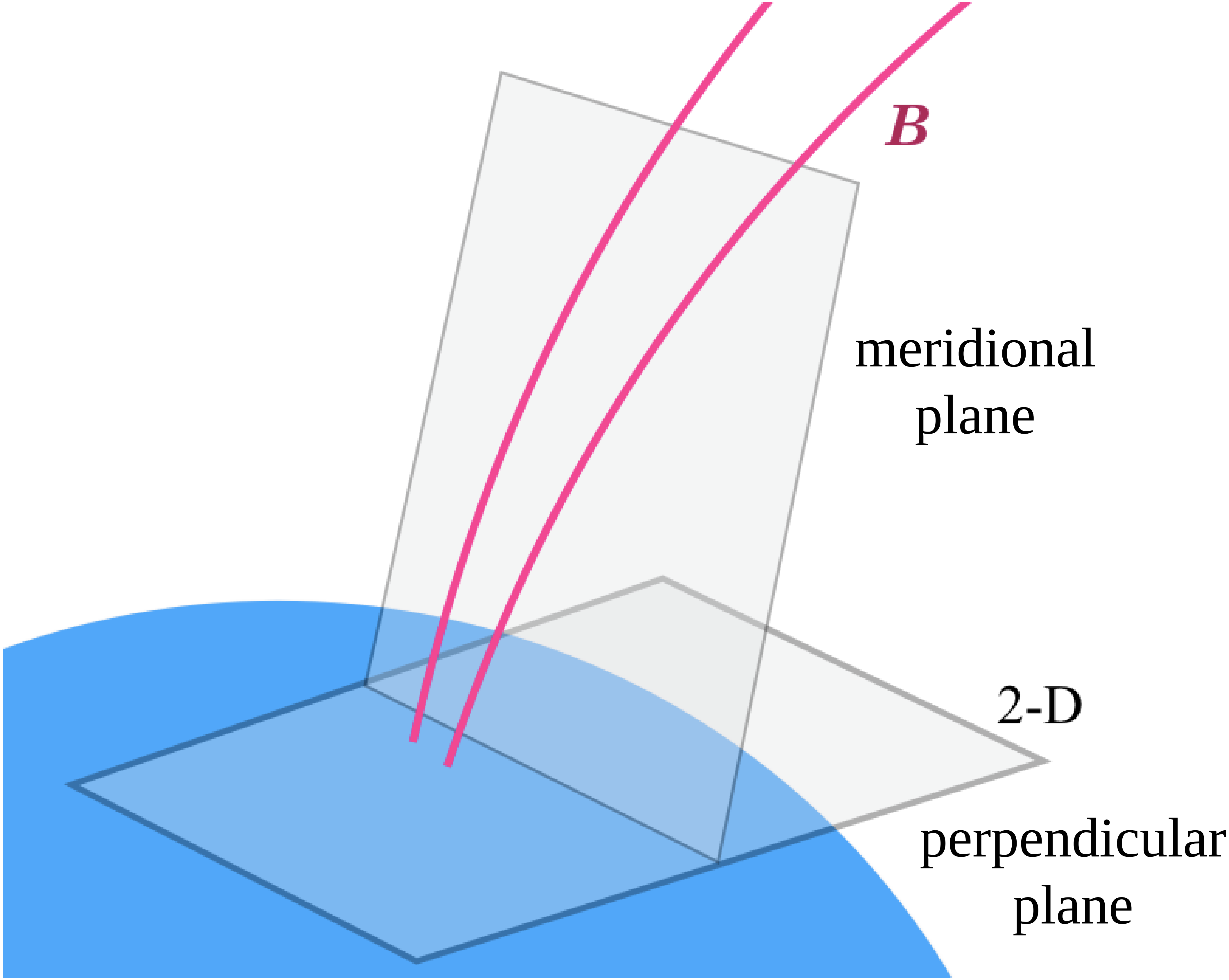
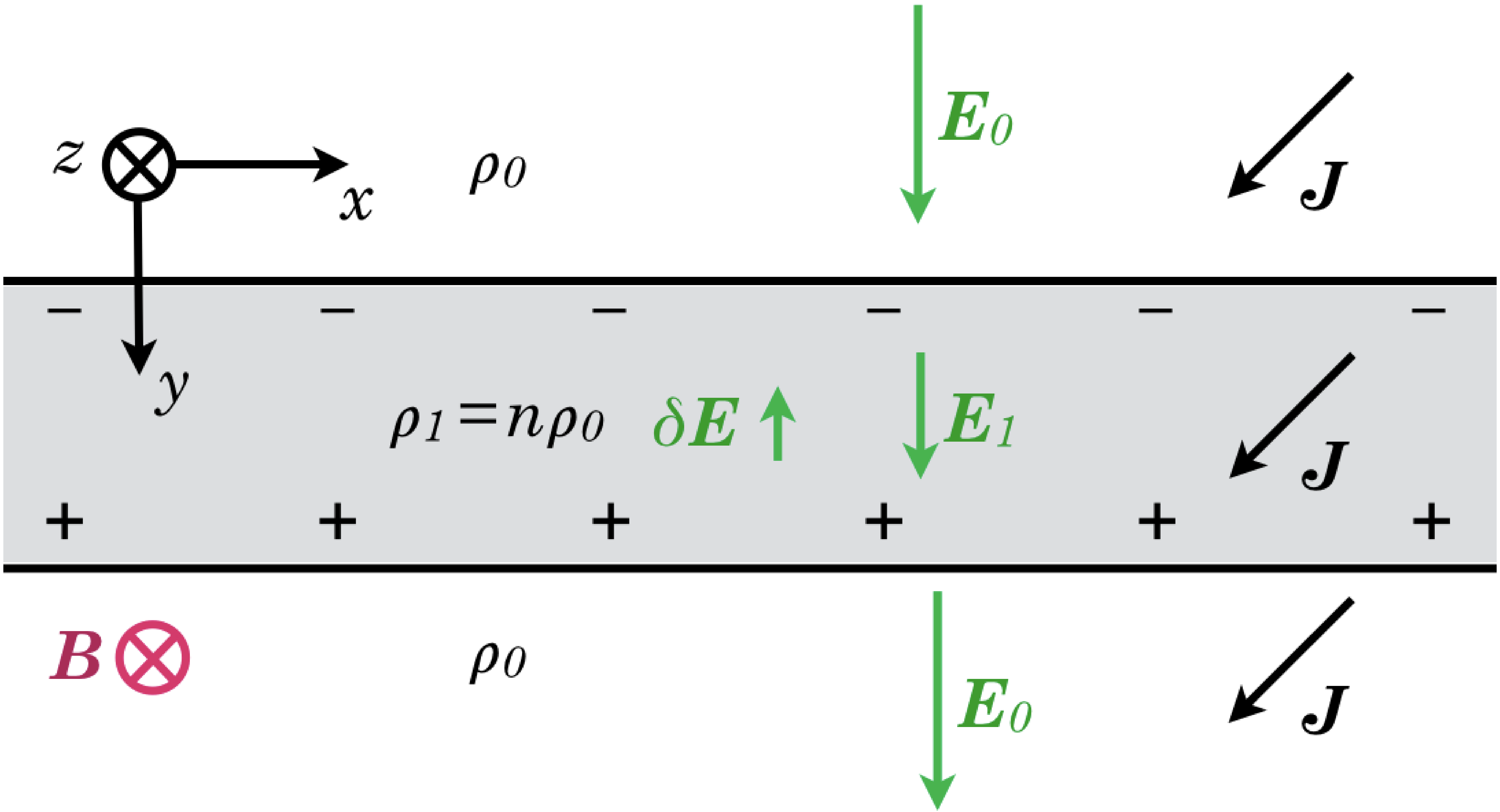
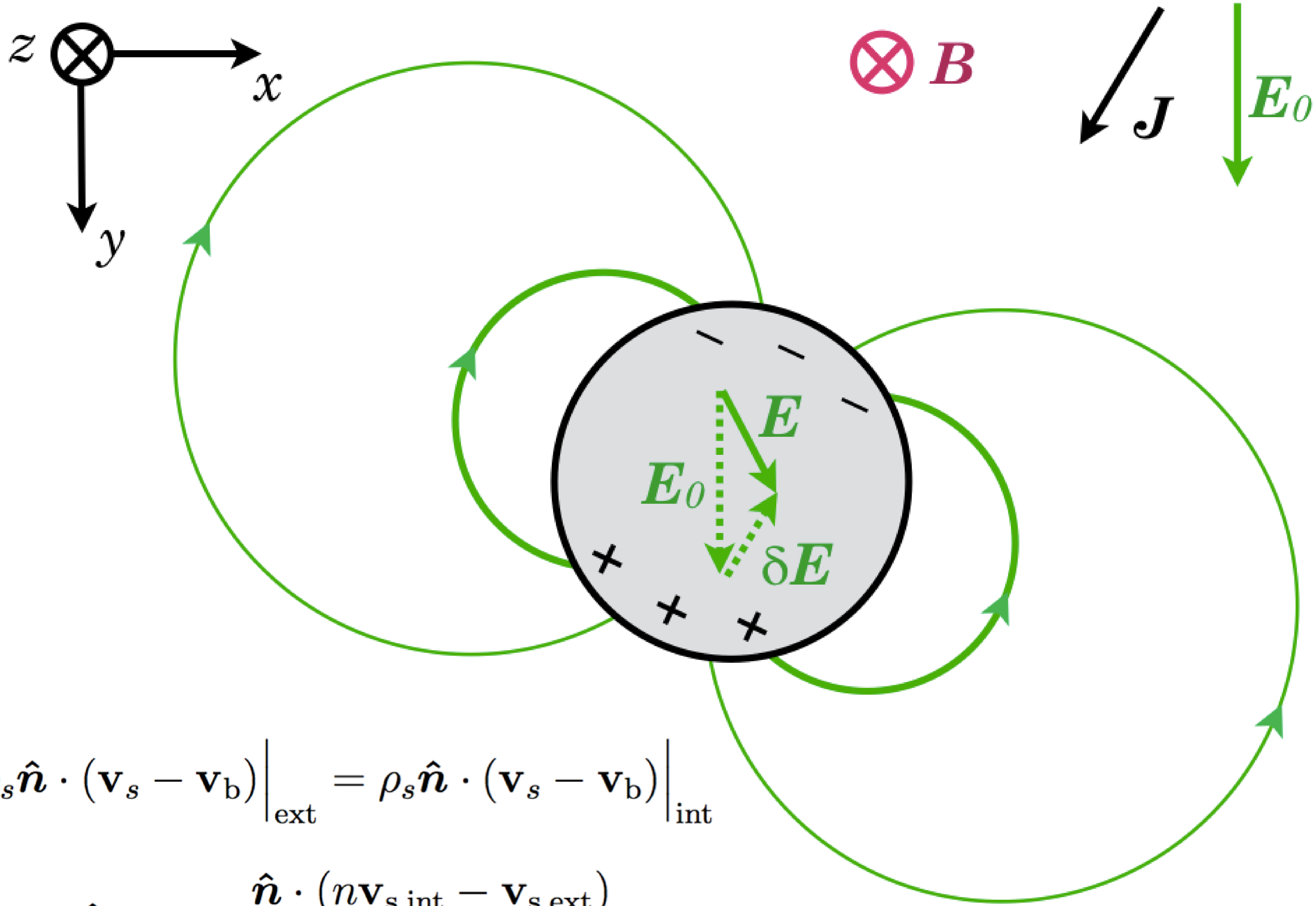
Assumptions

- 2-D plasma
- Closed field lines
- Steady state
- Cold plasma
- Currents don’t make significant changes in ***B***.
- Single ion species
- Fully-magnetised electrons
- No neutral drift (or ***E*** measured in neutral frame)


B
meridional
plane
2-D
perpendicular
plane



z
x
y
ρ_0
$\boldsymbol{E}_0$
$\boldsymbol{J}$
$-$ $-$ $-$ $-$ $-$ $-$
$\rho_1 = n\rho_0$
$\delta\boldsymbol{E}$
$\boldsymbol{E}_1$
$\boldsymbol{J}$
$+$ $+$ $+$ $+$ $+$ $+$
$\boldsymbol{B}$
ρ_0
$\boldsymbol{E}_0$
$\boldsymbol{J}$



$$\rho_s \hat{\boldsymbol{n}} \cdot (\mathbf{v}_s - \mathbf{v}_\mathrm{b})\Big|_\mathrm{ext} = \rho_s \hat{\boldsymbol{n}} \cdot (\mathbf{v}_s - \mathbf{v}_\mathrm{b})\Big|_\mathrm{int}$$

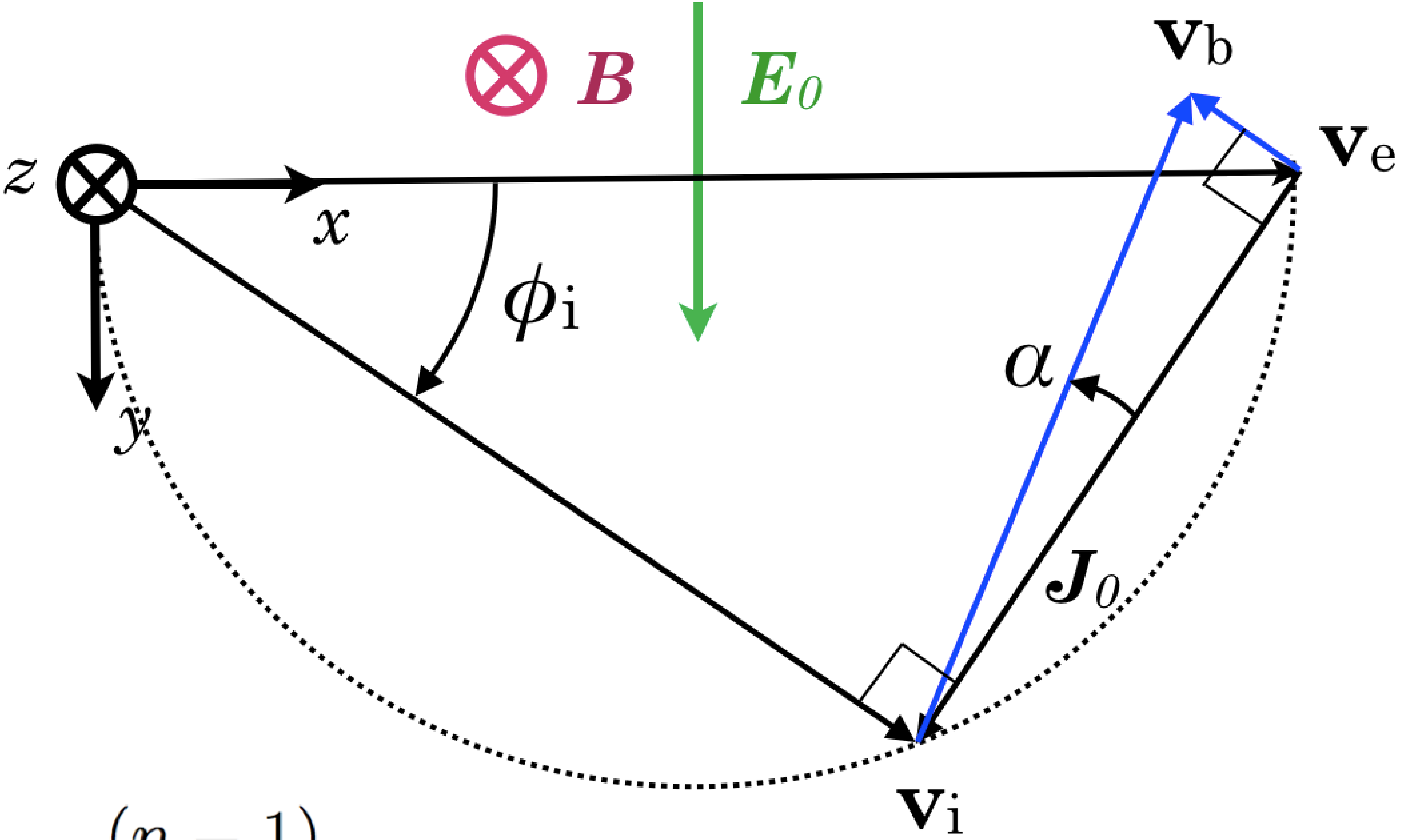
$$\hat{\boldsymbol{n}} \cdot \mathbf{v}_\mathrm{b} = \frac{\hat{\boldsymbol{n}} \cdot (n\mathbf{v}_\mathrm{s,int} - \mathbf{v}_\mathrm{s,ext})}{(n-1)}$$

$$
\begin{aligned}
\mathbf{v}_{\mathrm{b}} &= \frac{\cos\phi_s}{B} R_s \boldsymbol{E}_0 - \frac{(n+1)\eta\sin\phi_s\cos\phi_s}{(n-1)B\sigma_{\mathrm{P}}} M_s H^{-1}[\sigma]\boldsymbol{E}_0 \\
&= \mathbf{v}_{s,0} - \frac{\sin\phi_s\cos\phi_s}{B\sigma_{\mathrm{P}}} M_s H^{-1}\boldsymbol{J}_0
\end{aligned} \tag{14}
$$

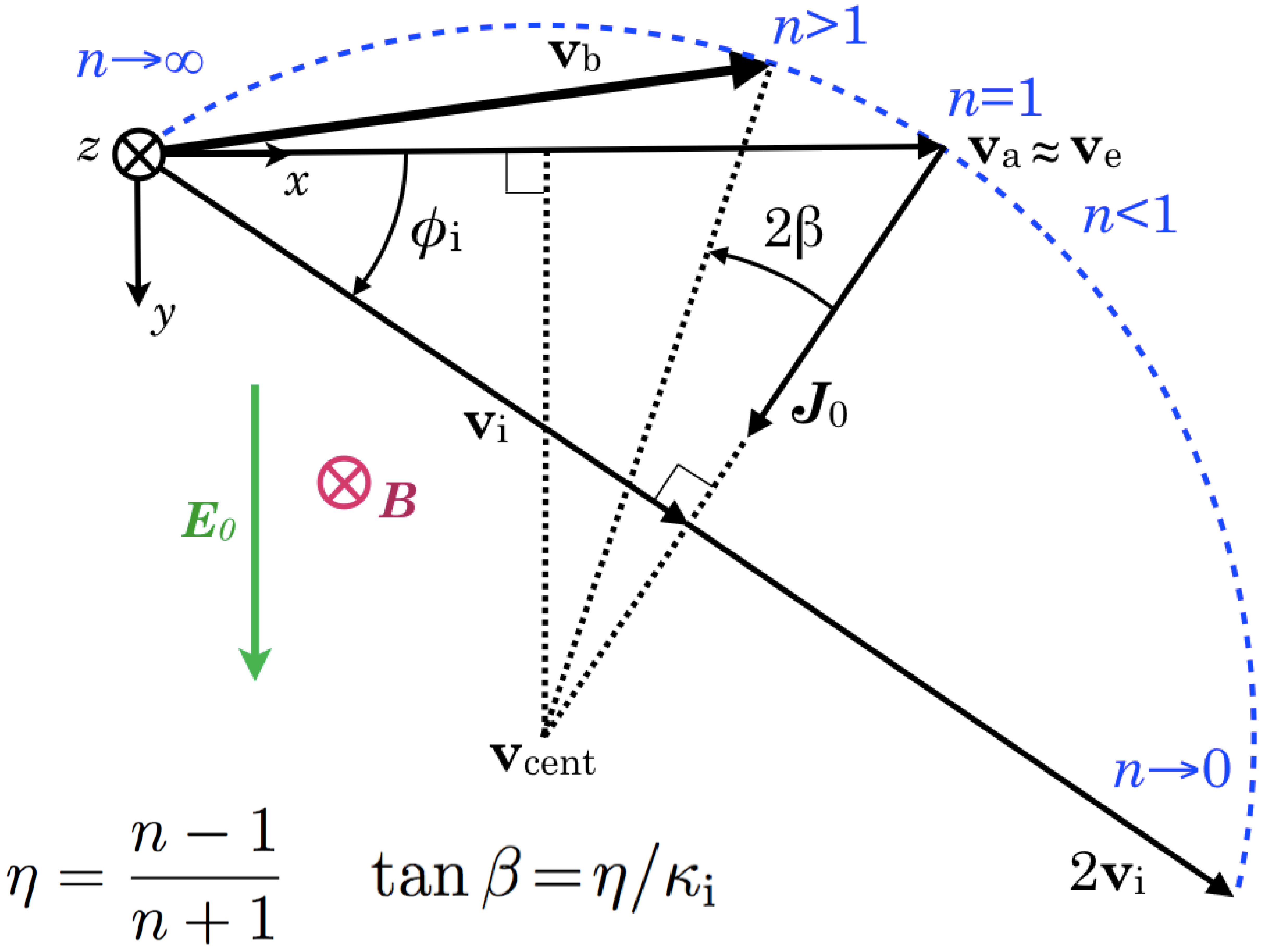
where the first term on the RHS is the background drift of the species, and we have used $\boldsymbol{J}=[\sigma]\boldsymbol{E}$. We now use $\boldsymbol{J}=q_{\mathrm{i}}n_{\mathrm{i}}(\mathbf{v}_{\mathrm{i}}-\mathbf{v}_{\mathrm{e}})$; $\sigma_{\mathrm{P}}=q_{\mathrm{i}}n_{\mathrm{i}}\kappa_{\mathrm{i}}/B(1+\kappa_{\mathrm{i}}^2)$; and $\kappa_{\mathrm{i}}=\cot\phi_{\mathrm{i}}$ to get (dropping the subscript 0 now since all quantities except η are background values)

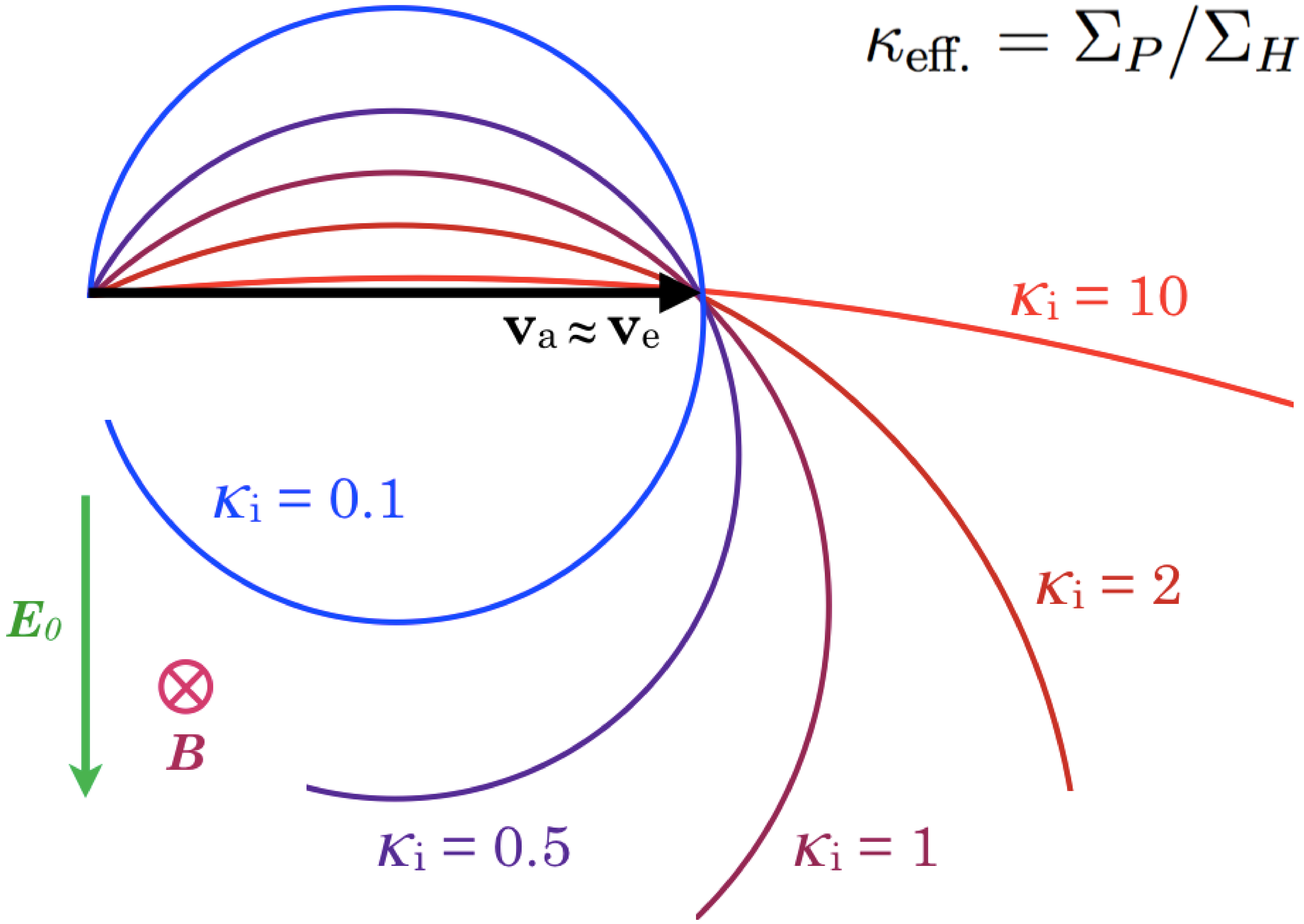
$$
\begin{aligned}
\mathbf{v}_{\mathrm{b}} &= \mathbf{v}_s - \frac{\sin\phi_s\cos\phi_s}{\sin\phi_{\mathrm{i}}\cos\phi_{\mathrm{i}}} M_s H^{-1}(\mathbf{v}_{\mathrm{i}}-\mathbf{v}_{\mathrm{e}}) \\
&= \mathbf{v}_s + \frac{\sin\phi_s\cos\phi_s}{\sin\phi_{\mathrm{i}}\cos\phi_{\mathrm{i}}} \begin{bmatrix} 1 & \eta\kappa_s \\ -\eta\kappa_s & 1 \end{bmatrix} \begin{bmatrix} 1 & -\eta/\kappa_{\mathrm{i}} \\ \eta/\kappa_{\mathrm{i}} & 1 \end{bmatrix}^{-1} (\mathbf{v}_{\mathrm{e}}-\mathbf{v}_{\mathrm{i}}) \\
&= \mathbf{v}_s + \frac{\sin\phi_s\cos\phi_s}{\sin\phi_{\mathrm{i}}\cos\phi_{\mathrm{i}}} \begin{bmatrix} 1 & \eta\kappa_s \\ -\eta\kappa_s & 1 \end{bmatrix} \frac{\kappa_{\mathrm{i}}}{\kappa_{\mathrm{i}}^2+\eta^2} \begin{bmatrix} \kappa_{\mathrm{i}} & \eta \\ -\eta & \kappa_{\mathrm{i}} \end{bmatrix} (\mathbf{v}_{\mathrm{e}}-\mathbf{v}_{\mathrm{i}}) \\
&= \mathbf{v}_s + \frac{\sin\phi_s\cos\phi_s}{\sin^2\phi_{\mathrm{i}}(\kappa_{\mathrm{i}}^2+\eta^2)} \begin{bmatrix} 1 & \eta\kappa_s \\ -\eta\kappa_s & 1 \end{bmatrix} \begin{bmatrix} \kappa_{\mathrm{i}} & \eta \\ -\eta & \kappa_{\mathrm{i}} \end{bmatrix} (\mathbf{v}_{\mathrm{e}}-\mathbf{v}_{\mathrm{i}})
\end{aligned} \tag{15}
$$

This equation ought to yield the same answer for either ions or electrons. We first demonstrate this for $|\eta| \ll 1$.



$$\alpha \approx \frac{(n-1)}{\sin 2\phi_\mathrm{i}}$$


$n\to\infty$
$\mathbf{v}_\mathrm{b}$
$n>1$
$n=1$
z
x
y
$\mathbf{v}_\mathrm{a} \approx \mathbf{v}_\mathrm{e}$
$n<1$
ϕ_i
2β
$\boldsymbol{J}_0$
$\mathbf{v}_\mathrm{i}$
$\boldsymbol{E}_0$
$\otimes \boldsymbol{B}$
$\mathbf{v}_\mathrm{cent}$
$n\to 0$
$2\mathbf{v}_\mathrm{i}$
$\eta = \frac{n-1}{n+1}$
$\tan\beta = \eta/\kappa_\mathrm{i}$

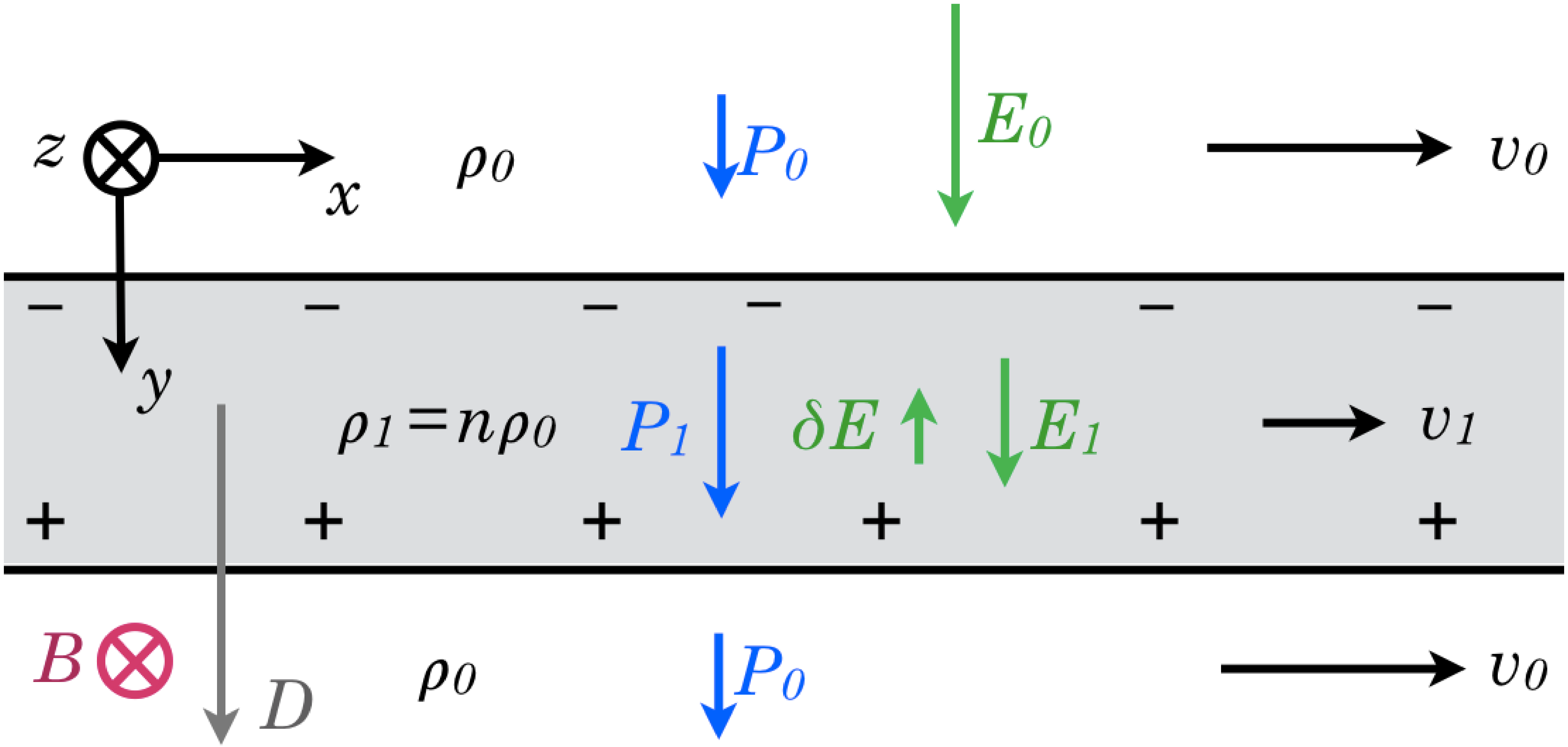

$\kappa_{\text{eff.}} = \Sigma_P / \Sigma_H$
$\mathbf{v}_{\mathrm{a}} \approx \mathbf{v}_{\mathrm{e}}$
$\kappa_{\mathrm{i}} = 10$
$\kappa_{\mathrm{i}} = 0.1$
$\kappa_{\mathrm{i}} = 2$
$\boldsymbol{E}_0$
$\boldsymbol{B}$
$\kappa_{\mathrm{i}} = 0.5$
$\kappa_{\mathrm{i}} = 1$

Removing neutral collisions

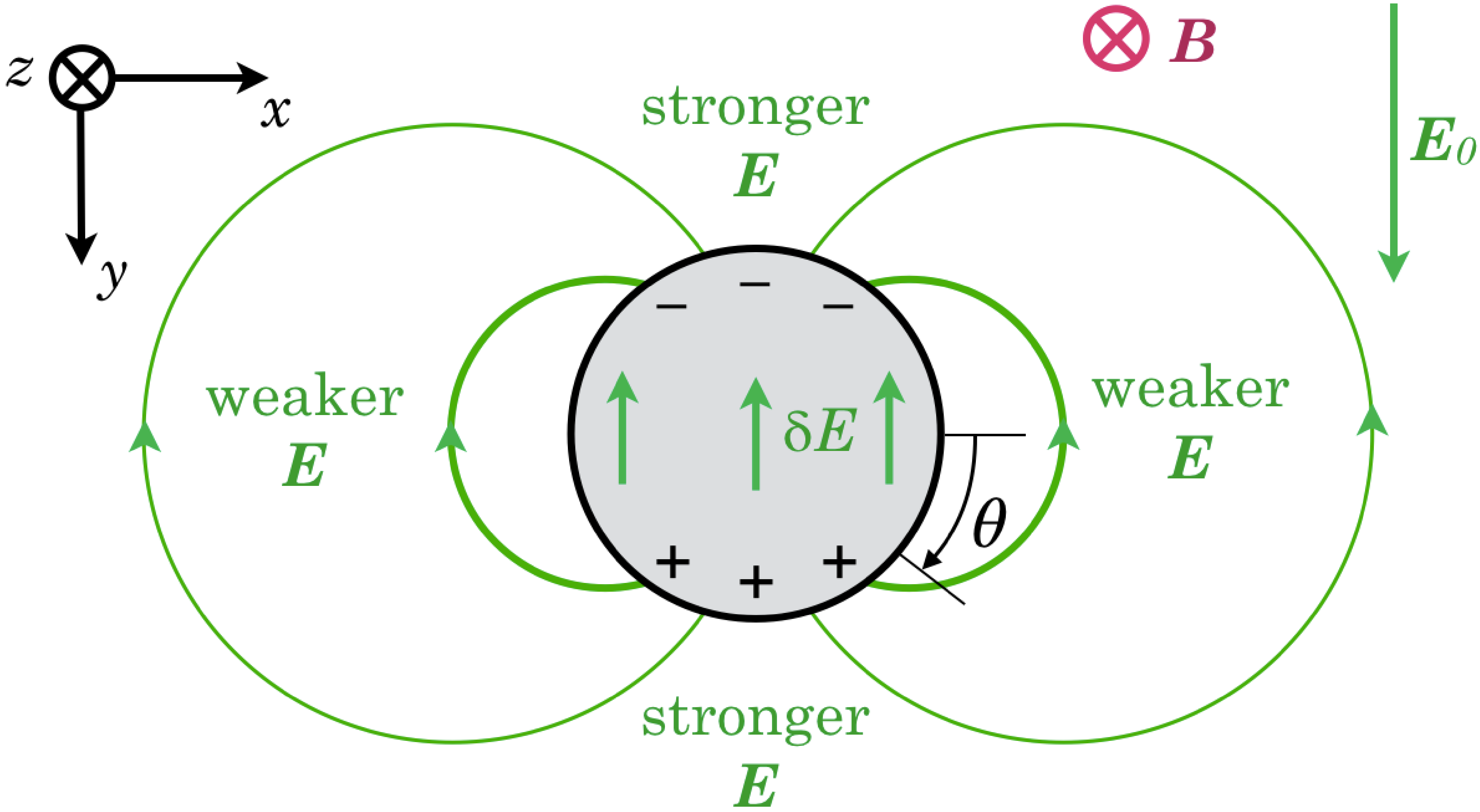
- Next two figures:

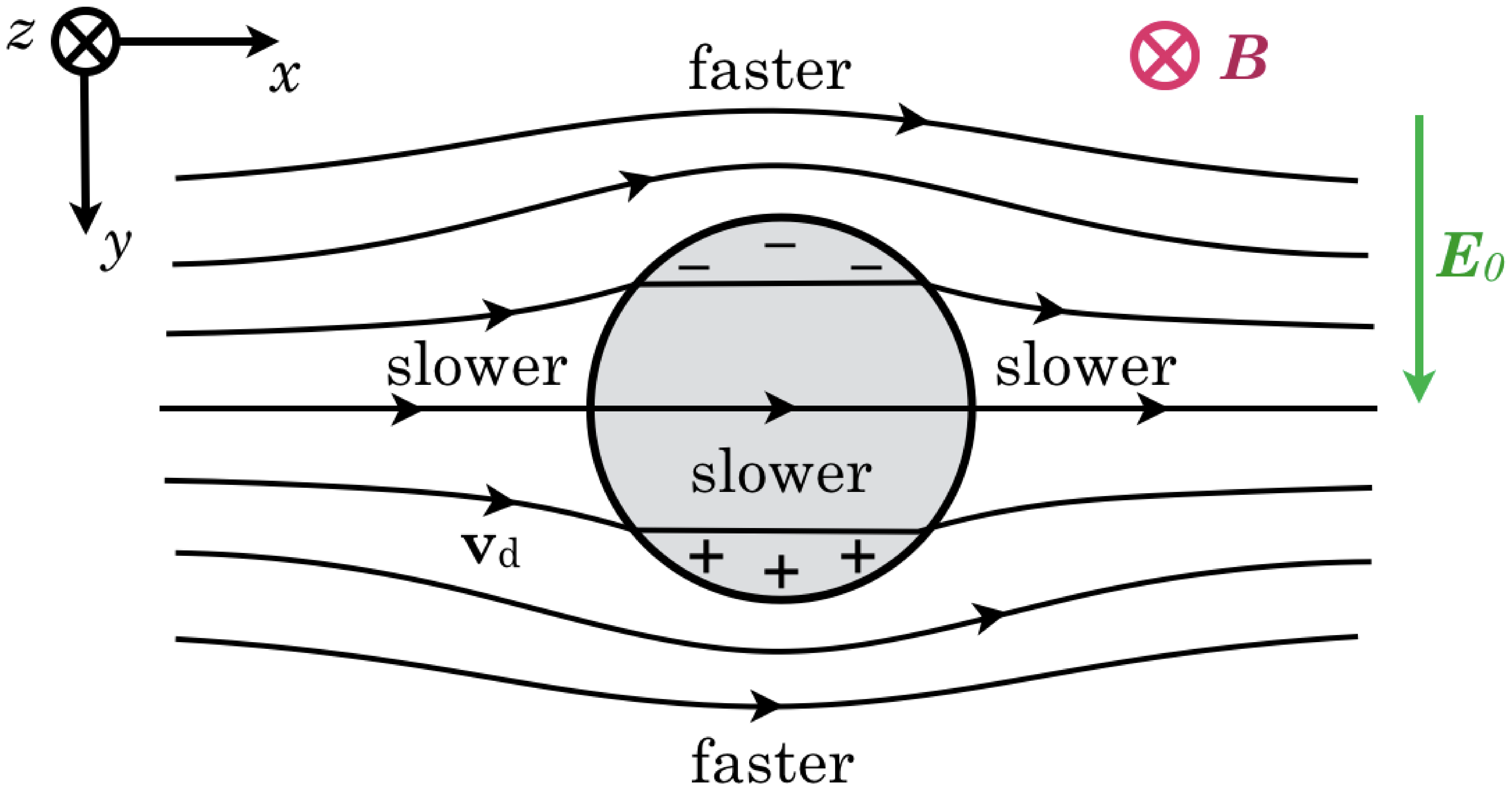
- Slab geometry: same result for ***E*** as before, although for different reasons.

- Circular geometry: the patch (or depletion) still suffers a polarisation, but the dipole axis is exactly aligned with ***E***. Otherwise identical results for patch drift.



$$\chi_{\mathrm{e}} = \frac{\rho_{\mathrm{m}}}{\varepsilon_0 B^2} = \frac{c^2}{\mathrm{v}_{\mathrm{A}}^2}$$

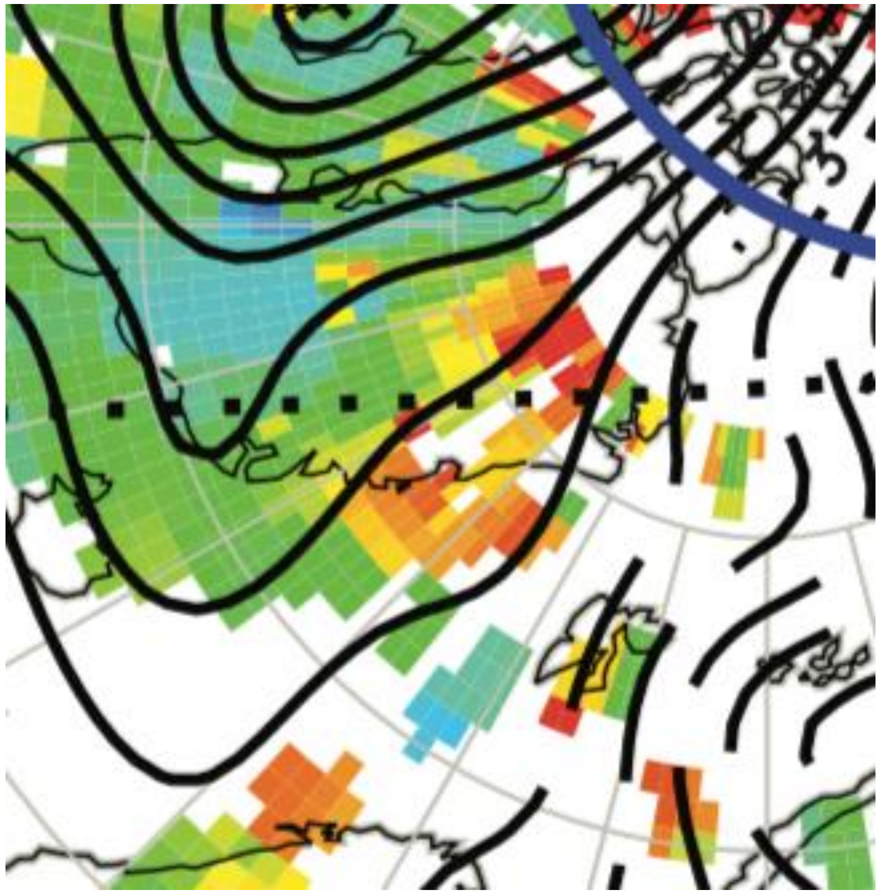

z
x
y
B
E_0
stronger
E
weaker
E
δE
θ
−
+
weaker
E
stronger
E


z
x
y
faster
B
E_0
slower
slower
slower
v_d
faster

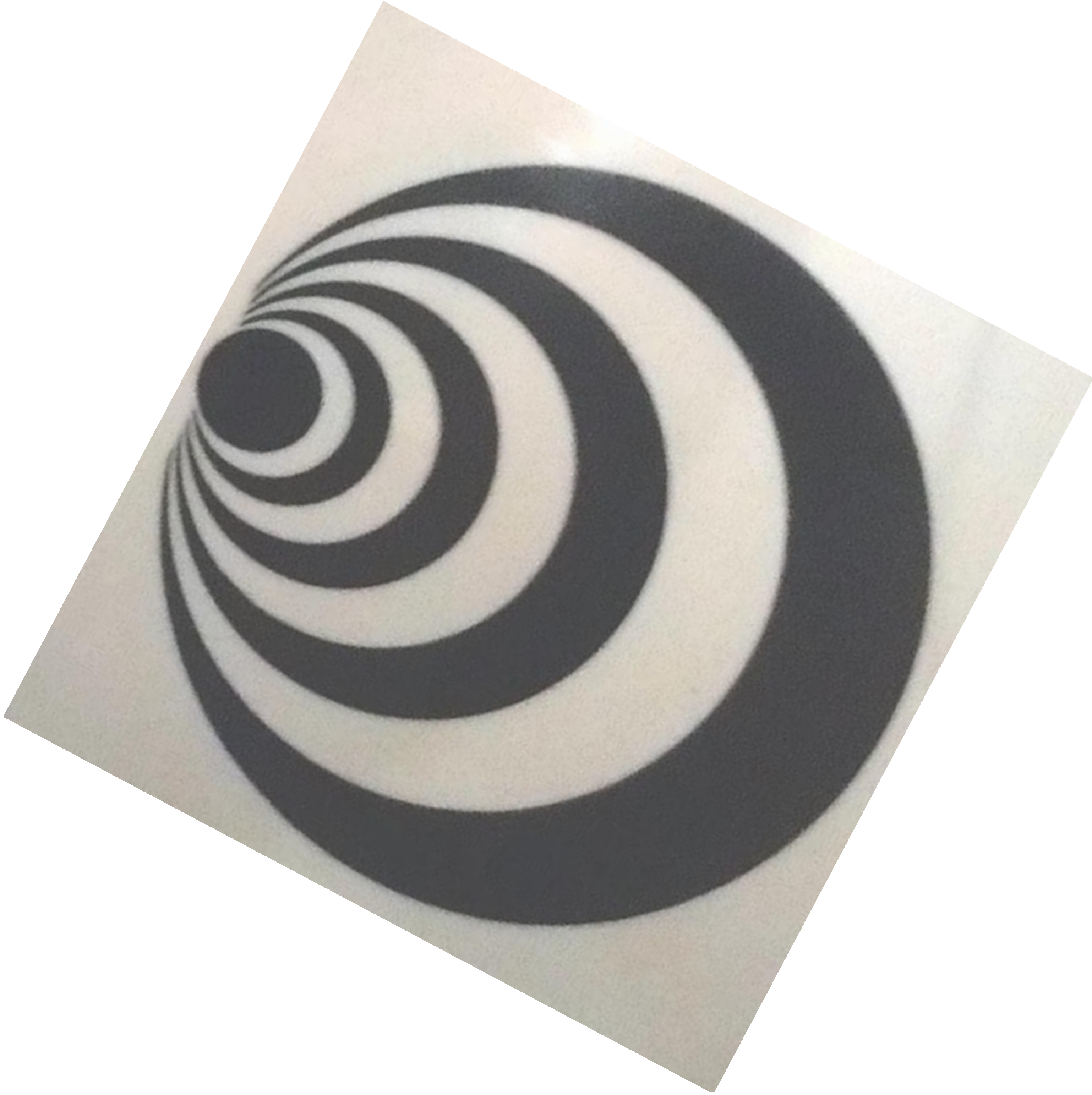
Conclusions

- The treatment of a collisionless plasma as having an electric susceptibility yields results consistent with the $\kappa \to \infty$ limit of a conducting plasma.
- While an open flux tube may have a uniform electric field “imposed” on it, plasma on closed flux tubes will experience a structuring of the electric field that depends on mass density features.
- A non-conducting plasma can have a free-charge distribution, concomitant with an arbitrary initial 2-D flow field. But a conducting plasma has a unique steady state.
- For a circular density feature, a dipolar surface charge with appropriate magnitude and orientation yields a divergence-free current field.
- The boundary of a circular density feature retains a circular shape, and its electrons, although it does not “own” a particular parcel of ions.
- The boundary should convect with a velocity given by the expression found – always slower than ambipolar for an enhancement and usually faster for a depletion.

Questions?



detail from Zhang *et al.* 2013 Fig. 2.C.

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