The Nuclear Delta Force in Quadrupole Deformed Nuclei

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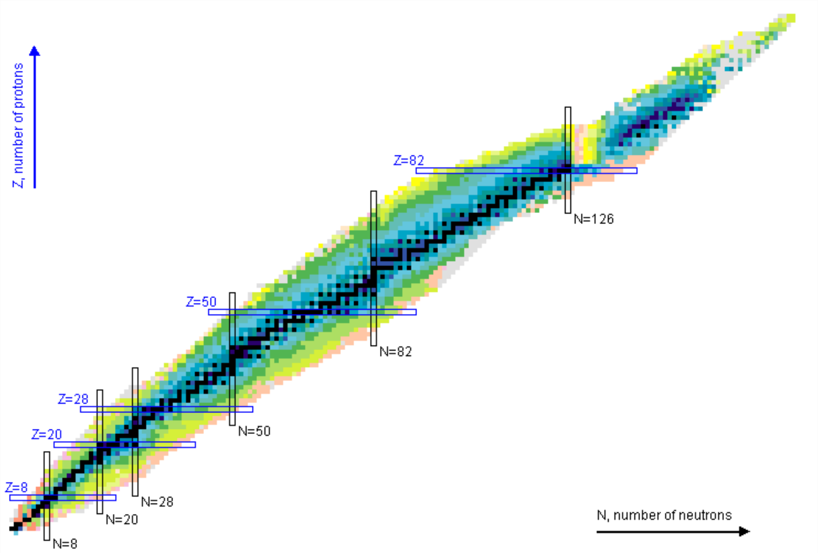
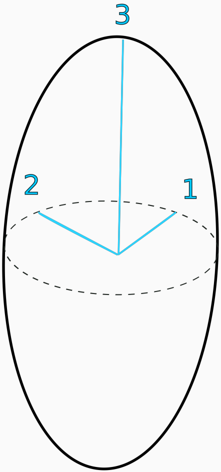
Chart of Nuclides



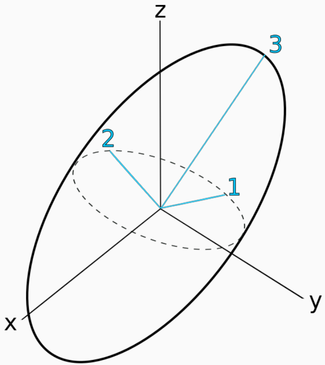
Collective Degrees of Freedom

Quadrupole Deformation: Axial Symmetry



Rotor Wavefunction

$$|RMK\rangle = \frac{1}{N} D^R_{MK}(\phi_E, \theta_E, \psi_E)$$



Applying the Axial Symmetry to the Rotor Wavefunction

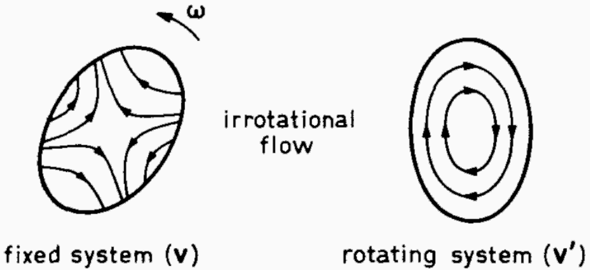
The characteristic symmetry elements is the π rotation about the short axis and the identity element:

$$\hat{S} = \hat{R}(\mathbf{e}_1, \pi) + \hat{\mathbb{I}}.$$

Applying this operator to $|RMK\rangle$ and normalizing, we obtain the rotor wavefunction:

$$|\Psi RMK\rangle = \sqrt{\frac{2R+1}{16\pi^2(1+\delta_{K,0})}} \left(D^R_{MK} + (-1)^R D^R_{M\bar{K}}\right).$$

Liquid Drop Model



Left: The surface flow patterns are visualized from the fixed lab frame for a liquid drop.
Right: The surface flow patterns are visualized in the intrinsic rotating frame for a liquid drop†.

$$\mathscr{I}_{33} = 0, \mathscr{I}_{11} = \mathscr{I}_{22} = \frac{3}{4}\mathscr{I}_0$$

†A. Bohr and B. Mottelson, *Nuclear Structure, Vol. II*, W. A. Benjamin Inc., **1975**

Axial Rotor Hamiltonian

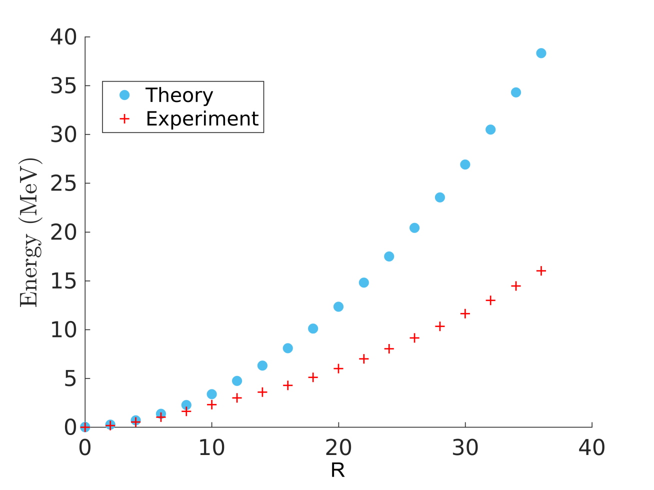
$$H_{Rot} = \frac{1}{2}\left[\frac{\hat{R}_1^2}{\mathscr{I}_{11}} + \frac{\hat{R}_2^2}{\mathscr{I}_{22}} + \frac{\hat{R}_3^2}{\mathscr{I}_{33}}\right]$$

But, $\mathscr{I}_{11} = \mathscr{I}_{22} = \frac{3}{4}\mathscr{I}_0$ and $\mathscr{I}_{33} = 0 \implies R_3 = 0$

$$H_{Rot} = \frac{2}{3}\left[\frac{\hat{R}^2}{\mathscr{I}_o}\right]$$

$$H_{Rot}|\Psi RM0\rangle = \frac{2\hbar^2}{3\mathscr{I}_o} R(R+1)|\Psi RM0\rangle$$

Comparison To Data: ^{126}Ce

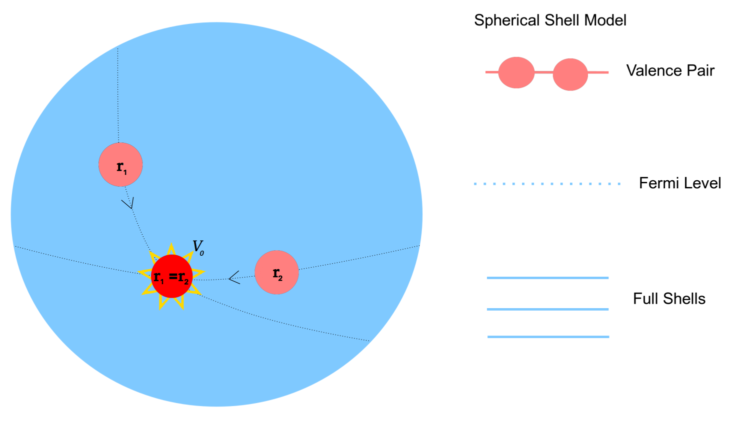


The parameter $\mathscr{I}_0$ was estimated using the $R = 2$ state, to be 23.6 $\hbar^2/$ MeV.

Data extracted using the NNDC On-Line Data Service from the ENSDF database.

Single Particle Degrees of Freedom

Nuclear Delta Force



$$H = H_{Spherical} + H_\delta$$

$$E = (E_1 + E_2)\langle \Psi J'\Omega' | \Psi J\Omega \rangle + \langle \Psi J'\Omega' | V_\delta \delta(\mathbf{r}_2 - \mathbf{r}_1) | \Psi J\Omega \rangle$$

Wavefunction

$$|\Psi J\Omega\rangle \triangleq \frac{1-(-1)^{j_1+j_2-J}}{2}|\mathscr{R}\rangle|J\Omega\rangle$$

where

$$|J\Omega\rangle = \sum_{\Omega_1,\Omega_2}\langle j_1\Omega_1 j_2\Omega_2|J\Omega\rangle|j_1\Omega_1\rangle|j_2\Omega_2\rangle$$

and

$$|\mathscr{R}(r_1)\rangle|\mathscr{R}(r_2)\rangle \triangleq |\mathscr{R}\rangle$$

Nuclear Delta Force

$$H_\delta = V_\delta \delta(\mathbf{r}_2 - \mathbf{r}_1) = V_\delta \delta(r_2 - r_1)\delta(\theta_2 - \theta_1)\delta(\phi_2 - \phi_1)$$

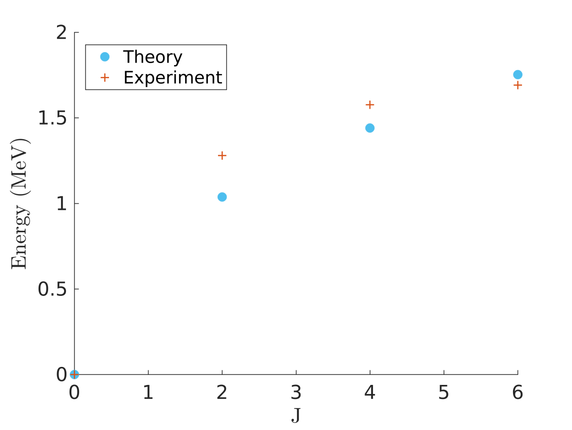
Applying this, we derive

$$\langle \Psi J' \Omega' | H_\delta | \Psi J \Omega \rangle = V_\delta \langle \mathscr{R}' | \delta(r_2 - r_1) | \mathscr{R} \rangle \langle J' Q' | \delta(\theta_2 - \theta_1)\delta(\phi_2 - \phi_1) | J Q \rangle$$

where

$$V_\delta \langle \mathscr{R}' | \delta(r_2 - r_1) | \mathscr{R} \rangle \triangleq \xi$$

Comparison to Data: ^{134}Te



By fitting the model predictions to the data, ξ was found to be -7.0 MeV for the two $g_{9/2}$ valence protons of ^{134}Te.

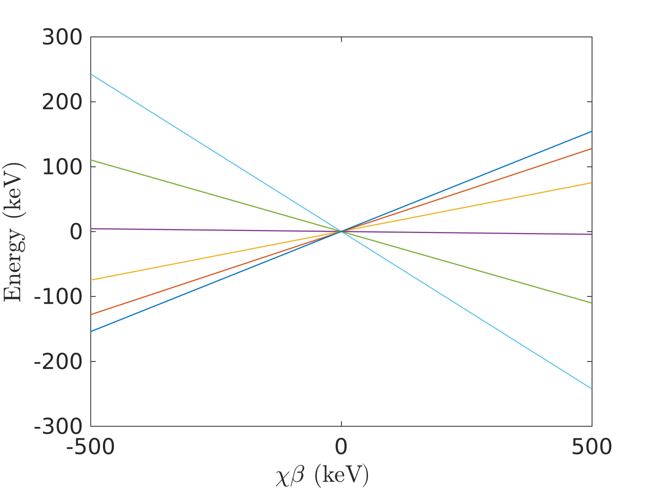
Data extracted using the NNDC On-Line Data Service from the ENSDF database.

Two Coupled Nucleons in an Axially-Deformed Potential

$$H = H_{spherical} + H_\beta$$

$$H_\beta = \pm\chi\beta\left[Y_{20}(\theta_1, \phi_1) + Y_{20}(\theta_2, \phi_2)\right]$$

Model

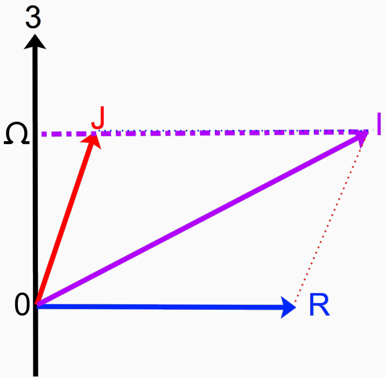


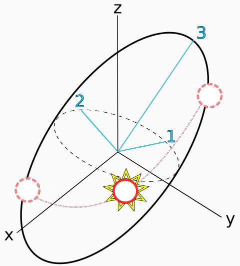
For two $h_{11/2}$ protons, the resultant energies from H_β are shown for the $\Omega = 0$ state.

Pair Rotor Coupling Model with Delta Force Interaction

The Basis Wavefunction

$$|IMj_1j_2J\Omega\rangle = \frac{1-(-1)^{j_1+j_2-J}}{2}\sqrt{\frac{2I+1}{16\pi^2(1+\delta_{\Omega,0})}}\left[D^I_{M\Omega}|J\Omega\rangle + (-1)^{I-J}D^I_{M\bar{\Omega}}|J\bar{\Omega}\rangle\right]$$





Model

$$H = H_{Rotor} + H_{\delta} + H_{\beta}$$

Rotor Hamiltonian

$$H_{Rotor} = \frac{2}{3}\left[\frac{\mathbf{R}^2}{\mathscr{I}_o}\right]$$

$$H_{Rotor} = \frac{2}{3\mathscr{I}_0}\left[\mathbf{I} - \mathbf{J}\right]^2$$

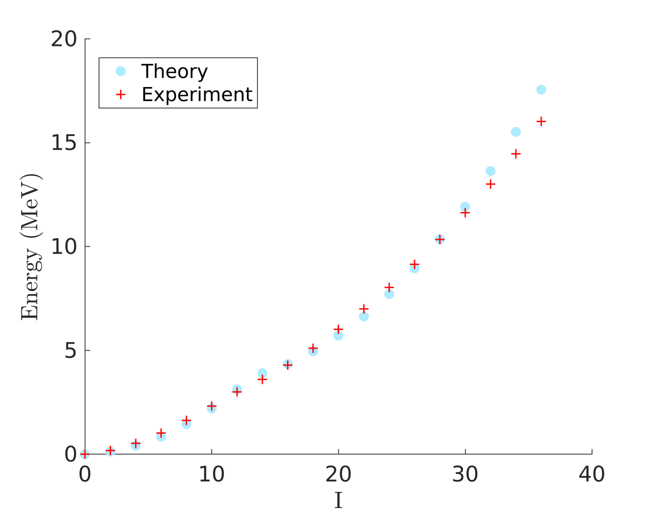
$$H_{Rotor} = \frac{2\hbar^2}{3\mathscr{I}_0}\left[\hat{I}^2 - \hat{I}_3^2 + \hat{J}^2 - \hat{J}_3^2\right] + \frac{4\hbar^2}{3\mathscr{I}_0}\left[\hat{I}_{+1}\hat{J}_{-1} + \hat{I}_{-1}\hat{J}_{+1}\right]$$

$$H_{Rotor} = H_{Diagonal} + H_{Coriolis}$$

Model

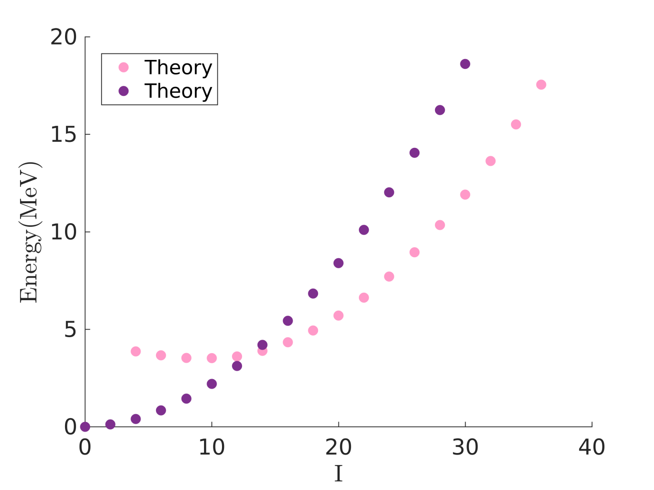
$$H = H_{Diagonal} + H_{Coriolis} + H_{\delta} + H_{\beta}$$

Comparison to Data: ^{126}Ce



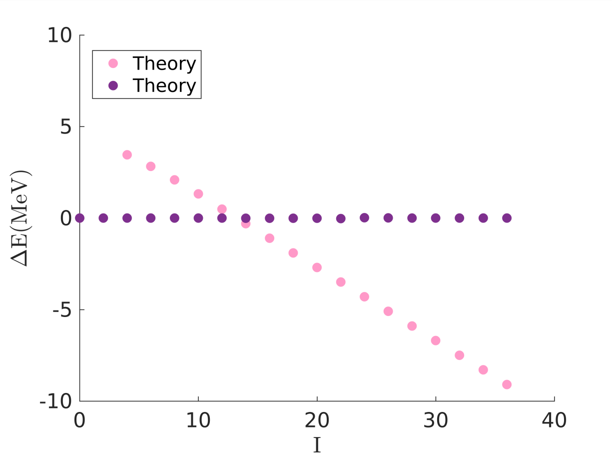
Two $h_{11/2}$ protons taken as the valence particles for ^{126}Ce.
$\chi\beta$ =0.2 MeV, ξ=-8.4 MeV, $\mathscr{I}_0$=33.2 $\hbar^2$/MeV

Band Crossing

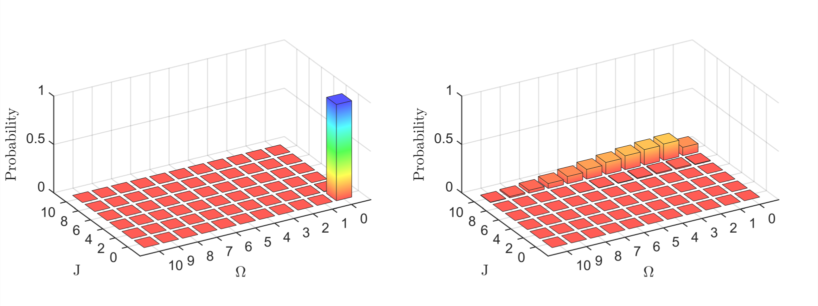


Single Particle Effects Vs Collective Effects

$$\Delta E = E - \frac{2\hbar^2}{3\mathscr{I}_0} I(I+1)$$



Probability Distribution

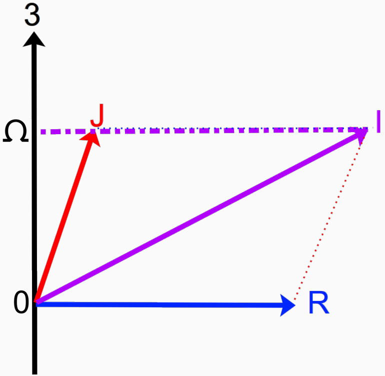


Left: The probability distribution over the angular momentum states for $I = 12$.
Right: The probability distribution over the angular momentum states for $I = 14$.

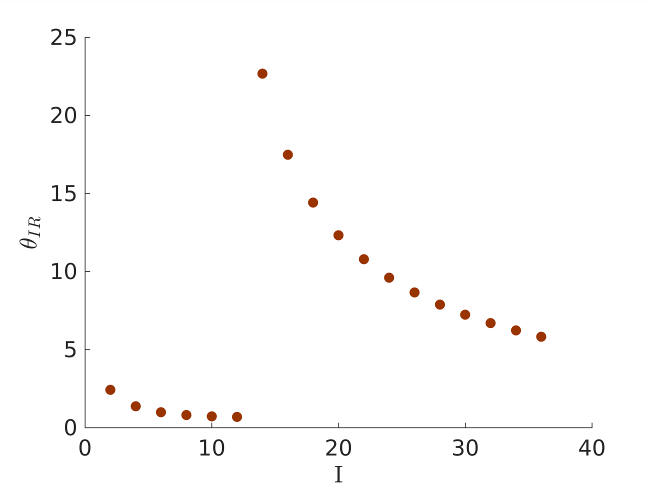
Angles

What if we want to find the angles between the vectors I and R?

$$\cos(\theta_{IR}) = \frac{\langle \mathbf{I} \cdot \mathbf{R} \rangle}{\sqrt{\langle I^2 \rangle}\sqrt{\langle R^2 \rangle}}$$



θ_{IR}



Using the lowest energy wavefunction for a given spin, the expectation value of the angle between **I** and **R** is calculated as a function of I.

G-Factor

$$\boldsymbol{\mu} = g\frac{\mu_N}{\hbar}\mathbf{I}$$
$$\boldsymbol{\mu} = \frac{\mu_N}{\hbar}\left(g_R\mathbf{R} + g_p\mathbf{J}\right)$$

Equating both sides and taking the dot product with $\vec{I}$,

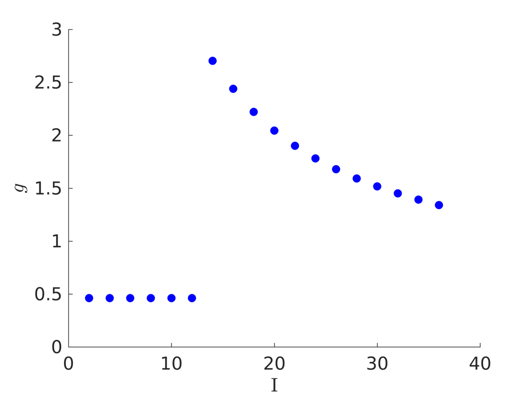
$$g = \frac{g_R\mathbf{I}\cdot\mathbf{R} + g_p\mathbf{I}\cdot(\mathbf{I}-\mathbf{R})}{I^2},$$

where we can estimate

$$g_R = \frac{Z}{A}$$
$$g_p = a \times g_{\text{Free Proton}}.$$

G-Factor

$$g = \frac{(g_R - g_p)\langle \mathbf{I} \cdot \mathbf{R} \rangle + g_p \langle I^2 \rangle}{\langle I^2 \rangle}$$



The expectation value of g predicted for a given spin, I, using the lowest energy wavefunction in the model. Here, $a = 0.65$, $g_{\text{Free Proton}} = 5.59$, $g_R = \frac{58}{126}$.

Summary, Future Work, and Thanks

1. The model shows that the single particle degrees of freedom are of comparable impact with the collective degrees of freedom.
2. Transition rates and B(E2) to be calculated.
3. New deformations to be explored : Triaxial quadrupole, octupole, hexadecapole, etc.

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Questions?