Universality of low-energy Rashba scattering

Joel Hutchinson
Joseph Maciejko

Theoretical Physics Institute
TPI
Est. 1960
University of Alberta

NSERC
CRSNG

UNIVERSITY OF
ALBERTA

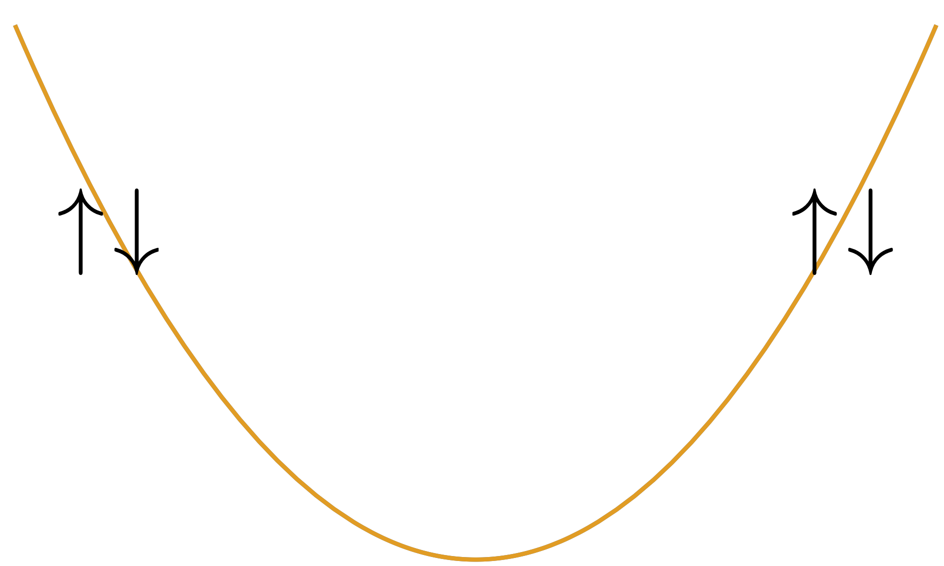
Rashba spin-orbit coupling

- Spin degeneracy is a consequence of time-reversal + <u>inversion</u> symmetry

$$E(\mathbf{k}\uparrow) = E(-\mathbf{k}\downarrow) \qquad \text{Time-reversal}$$

$$E(\mathbf{k}\uparrow) = E(-\mathbf{k}\uparrow) \qquad \text{Inversion}$$

$$\Rightarrow E(\mathbf{k}\uparrow) = E(\mathbf{k}\downarrow)$$



Rashba spin-orbit coupling

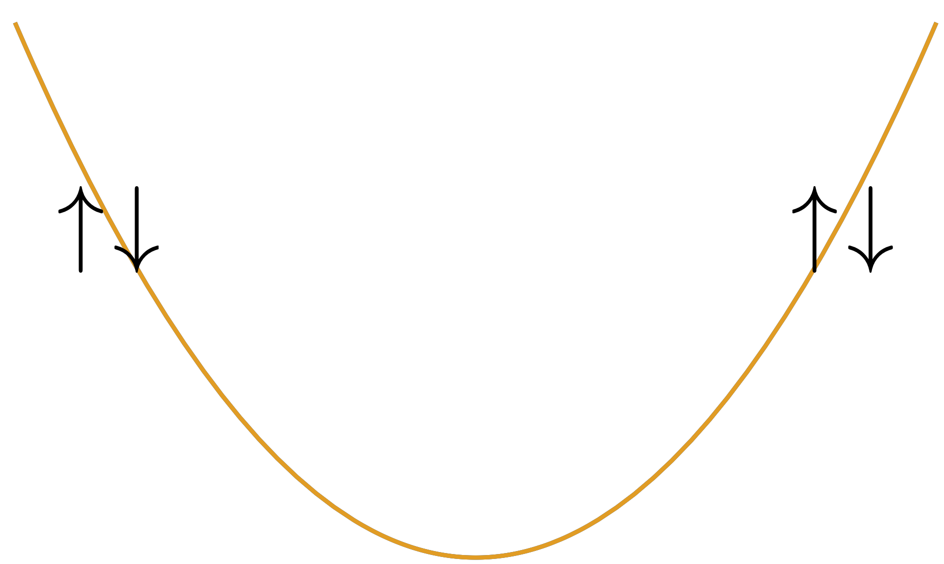
- Spin degeneracy is a consequence of time-reversal + <u>inversion</u> symmetry

$$E(\mathbf{k}\uparrow) = E(-\mathbf{k}\downarrow) \qquad \text{Time-reversal}$$

$$E(\mathbf{k}\uparrow) = E(-\mathbf{k}\uparrow) \qquad \text{~~Inversion~~}$$

$$\Rightarrow E(\mathbf{k}\uparrow) \neq E(\mathbf{k}\downarrow)$$

- Inversion asymmetry causes "spin-split" dispersion



e.g. surfaces, interfaces, quantum well with confining potential

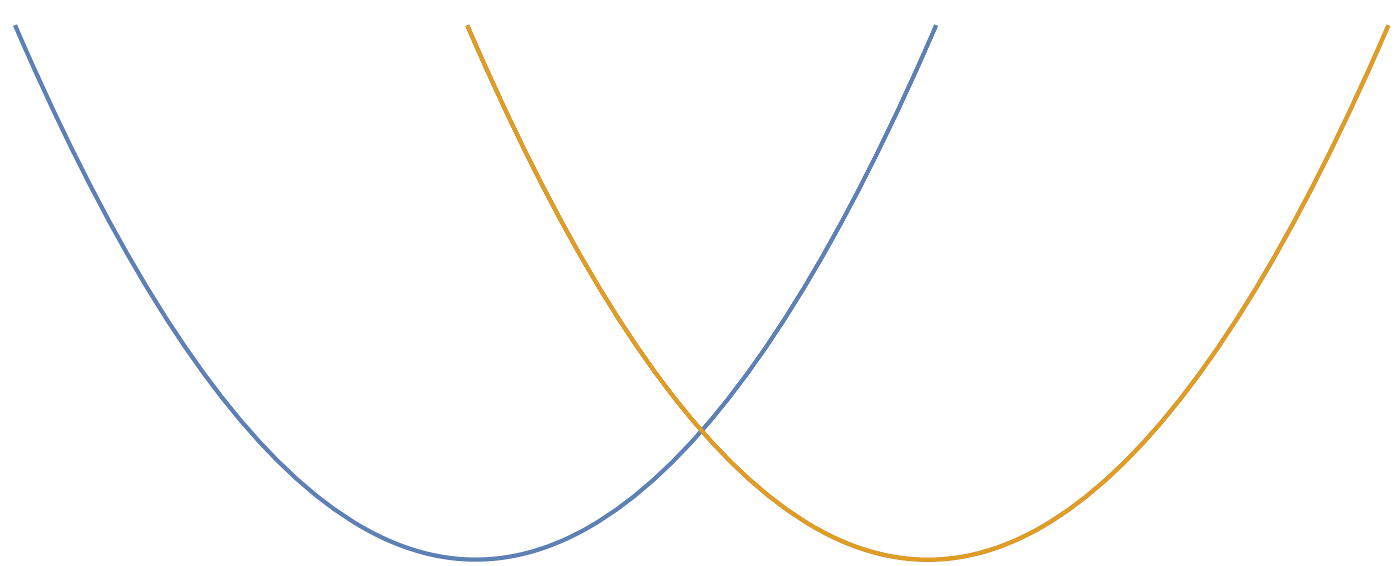
Rashba spin-orbit coupling

- Spin degeneracy is a consequence of time-reversal + <u>inversion</u> symmetry

$$E(\mathbf{k}\uparrow) = E(-\mathbf{k}\downarrow) \qquad \text{Time-reversal}$$
$$E(\mathbf{k}\uparrow) = E(-\mathbf{k}\uparrow) \qquad \text{~~Inversion~~}$$

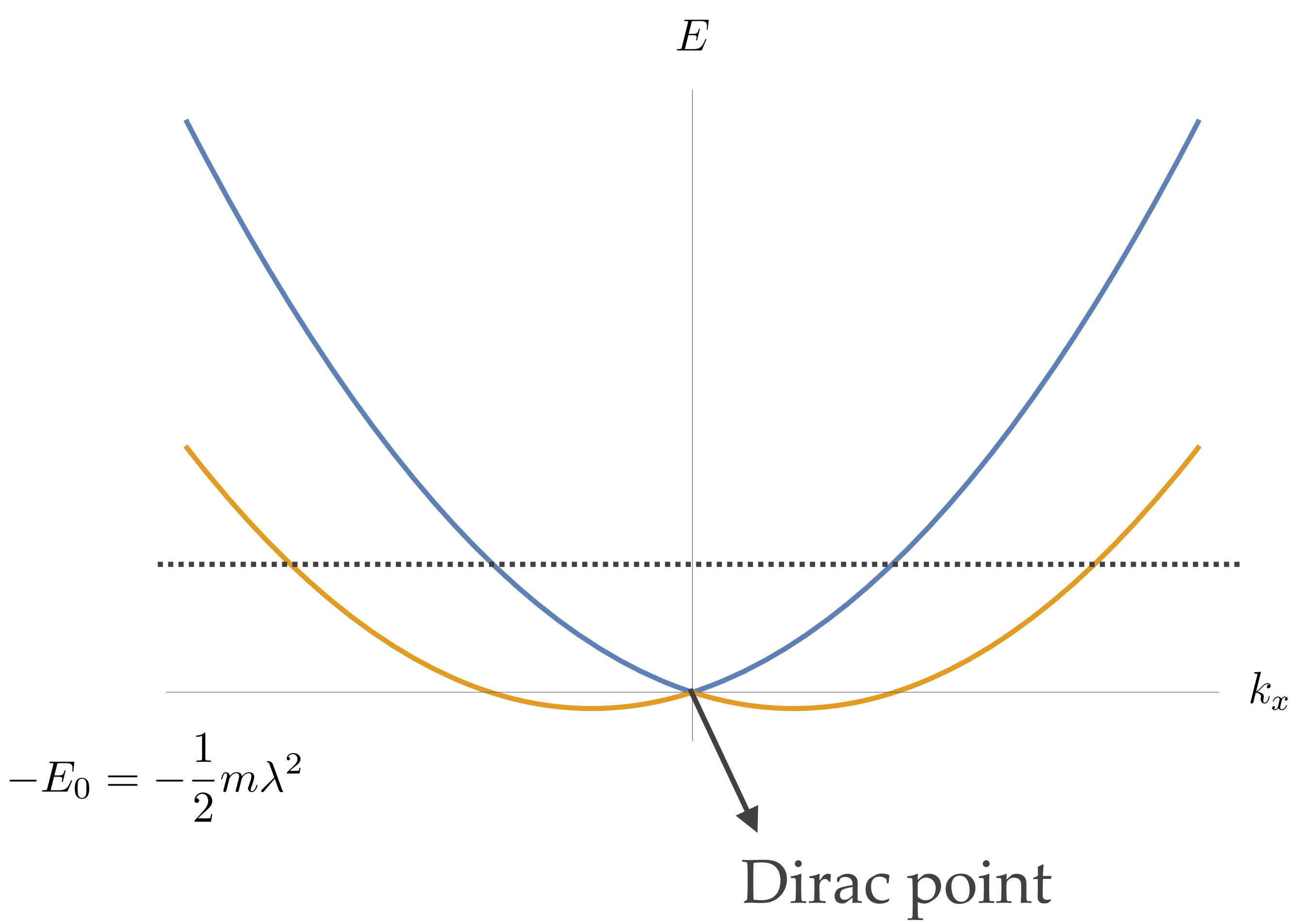
$$\Rightarrow E(\mathbf{k}\uparrow) \neq E(\mathbf{k}\downarrow)$$

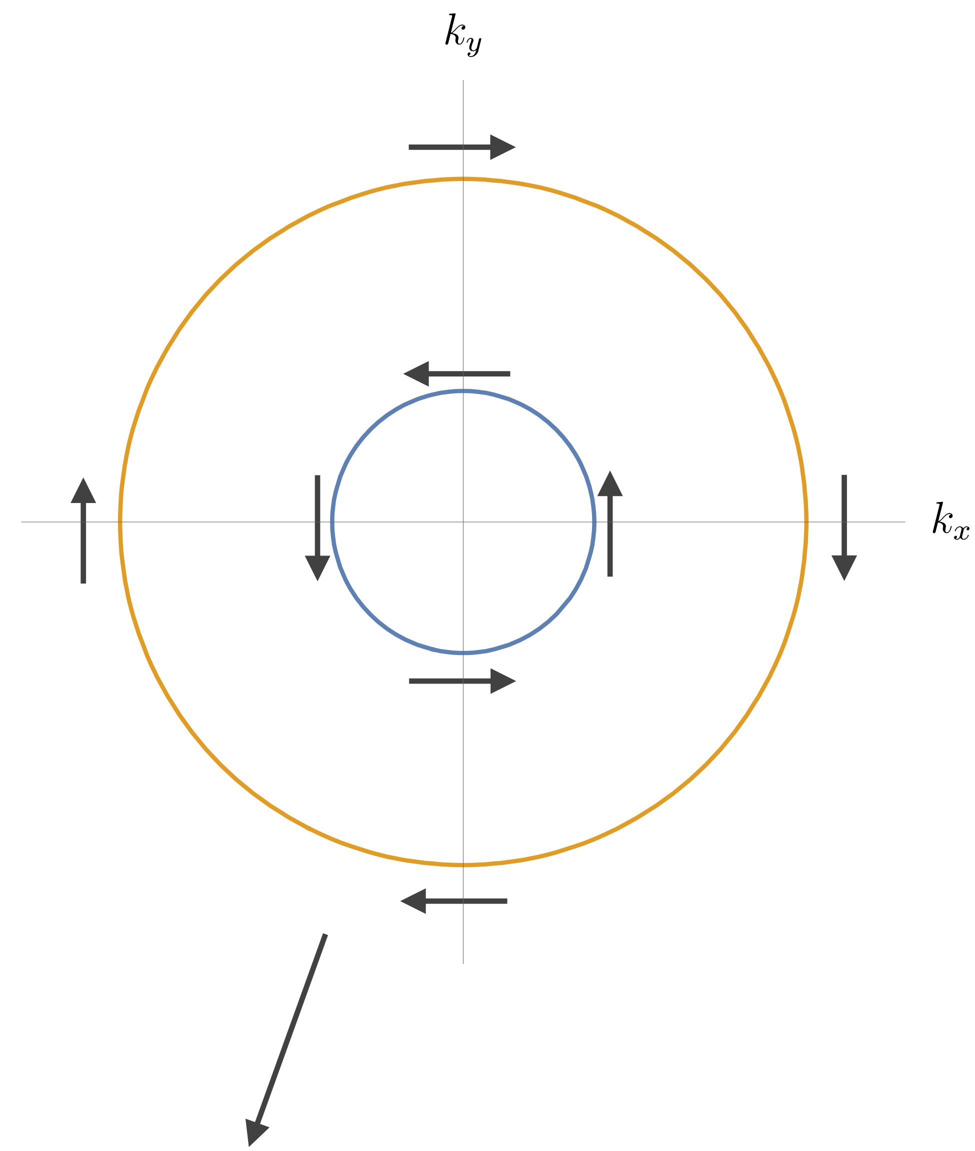
- Inversion asymmetry causes "spin-split" dispersion



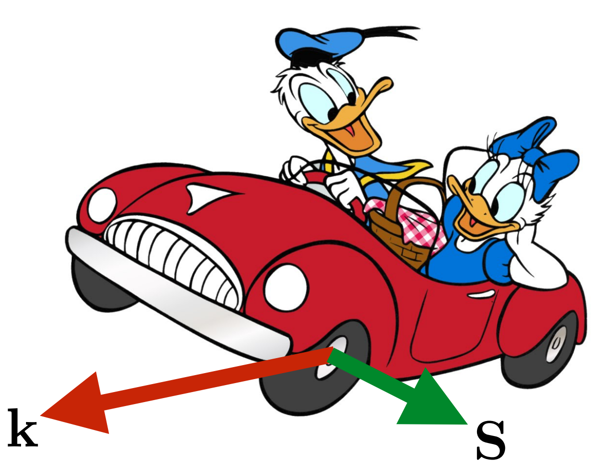
e.g. surfaces, interfaces, quantum well with confining potential

Rashba spin-orbit coupling



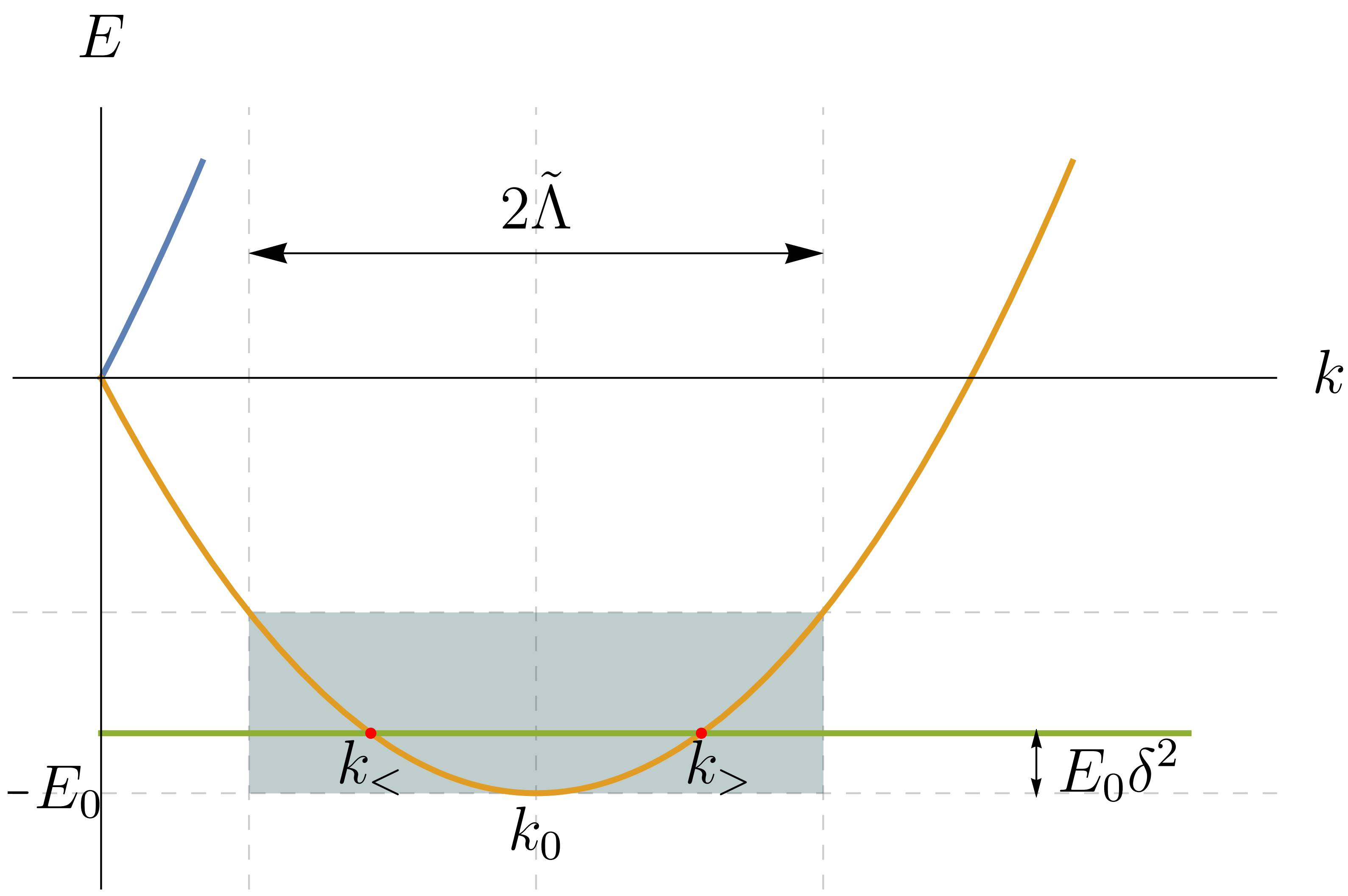


Spin and momentum are locked. Lots of potential applications!



Low-energy Rashba

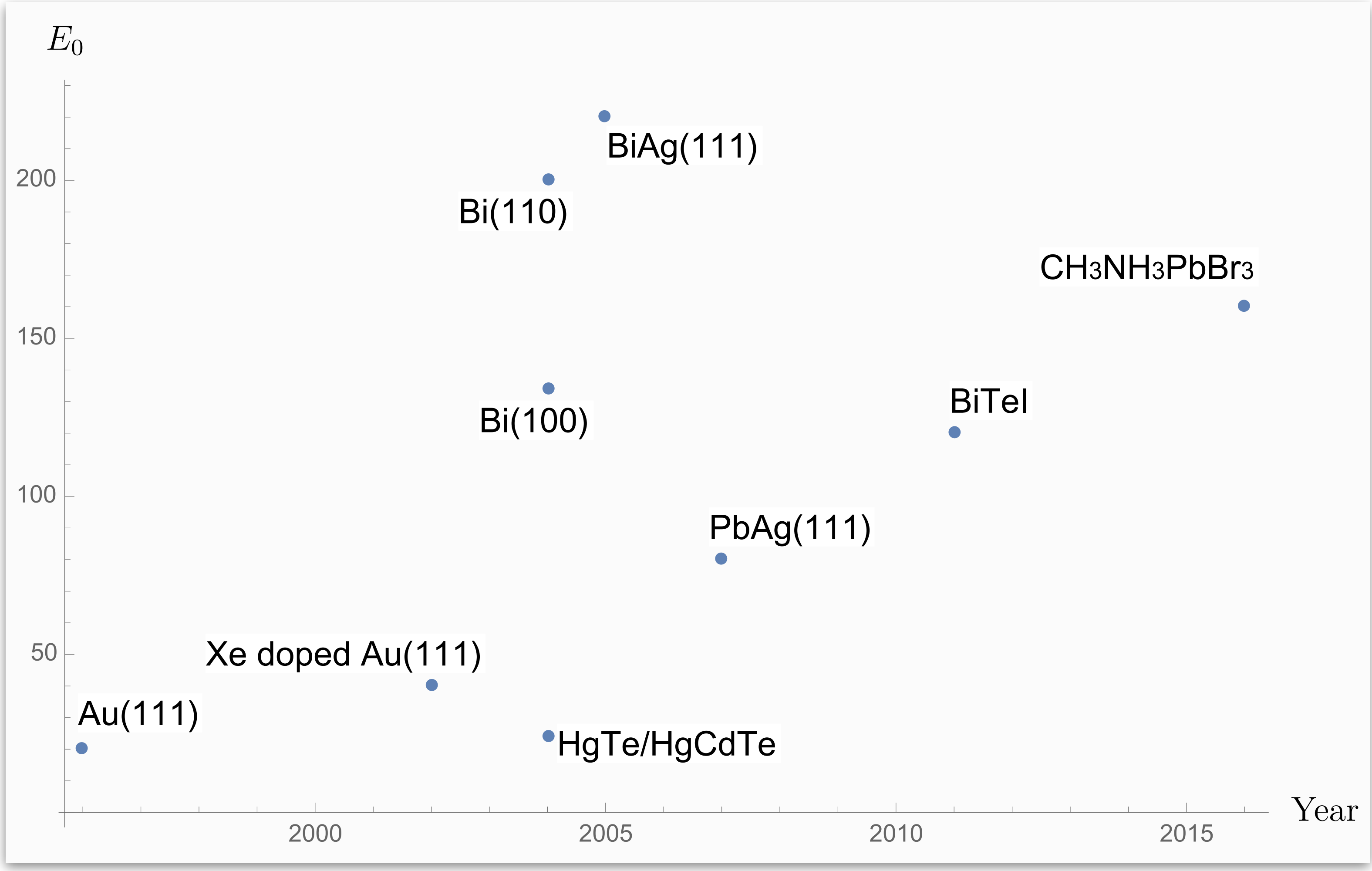
- 2D Hamiltonian: $$H(\boldsymbol{k}) = \frac{\boldsymbol{k}^2}{2m} + \lambda \hat{\boldsymbol{z}} \cdot (\boldsymbol{\sigma} \times \boldsymbol{k})$$



There are two different scattering states at each angle.

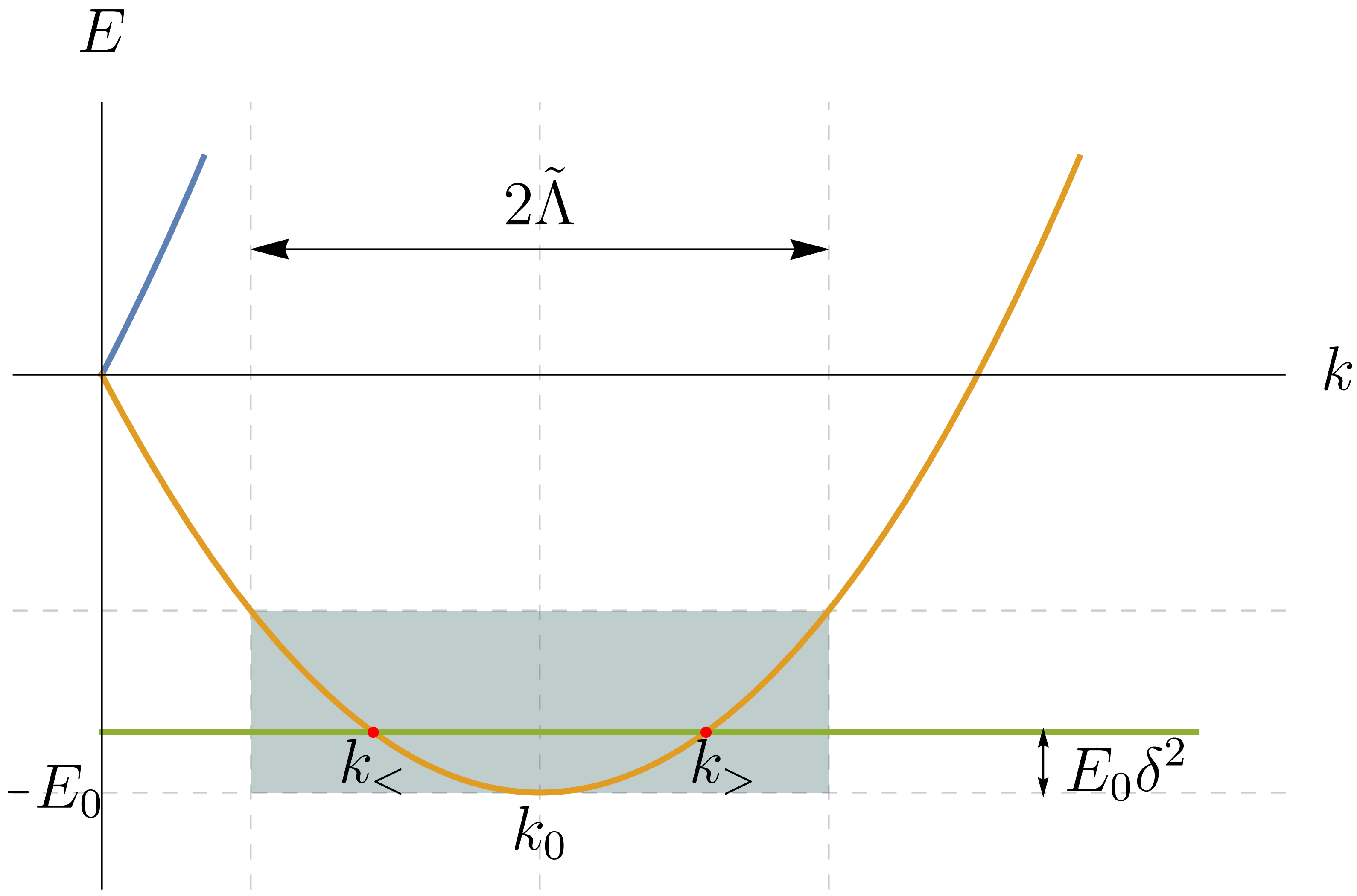
$$k_{\gtrless} \equiv k_0(1 \pm \delta) \qquad \delta \equiv \sqrt{1 - |E|/E_0} \qquad E_0 = \frac{1}{2} m\lambda^2$$

Low-energy Rashba



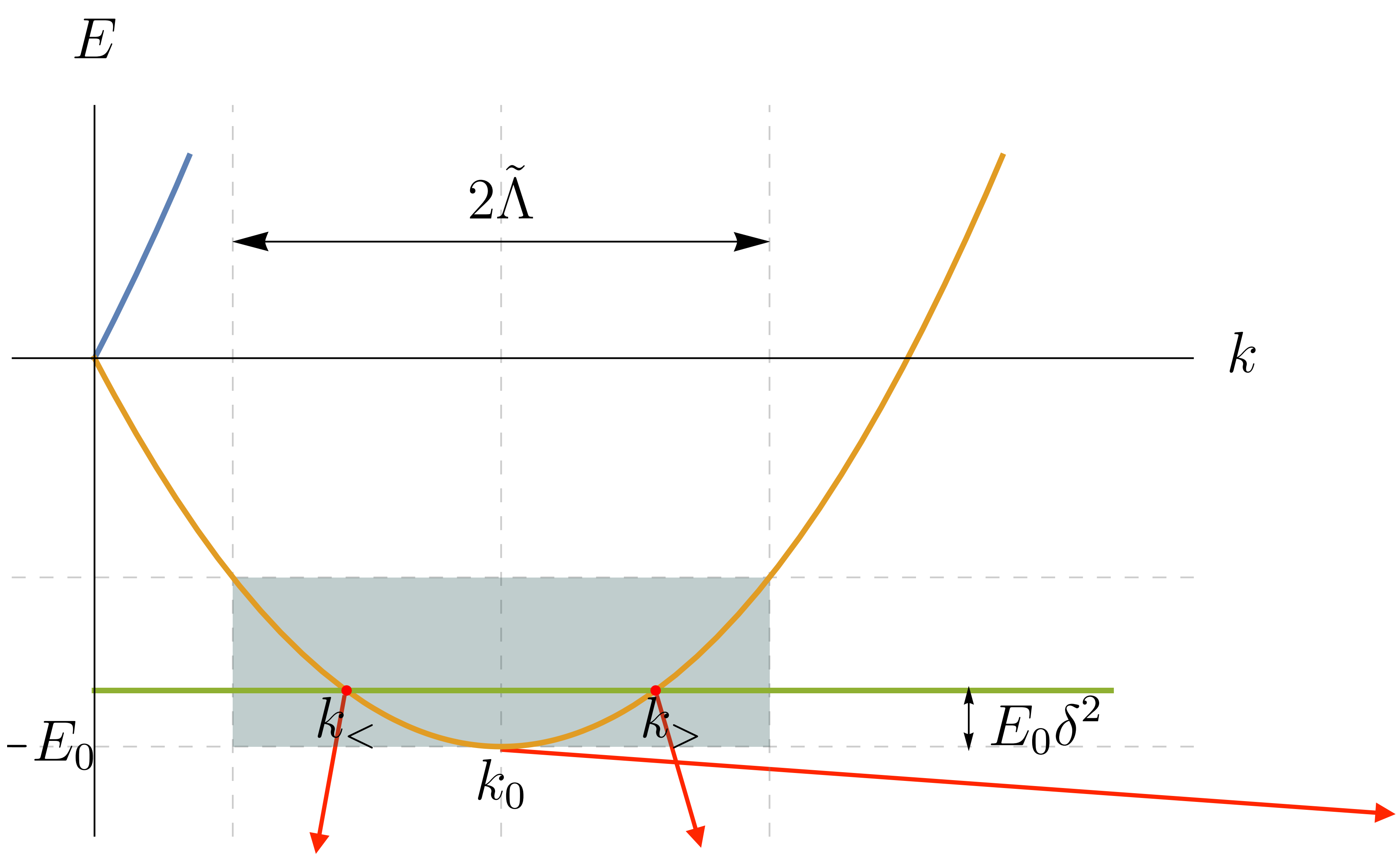
Low-energy Rashba

- Basic question: Is there anything fundamentally different about Rashba scattering in this regime, independent of interactions and many-body physics?



Low-energy Rashba

- <u>Basic question</u>: Is there anything fundamentally different about Rashba scattering in this regime, independent of interactions and many-body physics?



Van Hove singularity in DOS

Antiparallel group velocities

Example: Hard Disk

$$V = \begin{cases} \infty & r < R \\ 0 & r > R \end{cases}$$

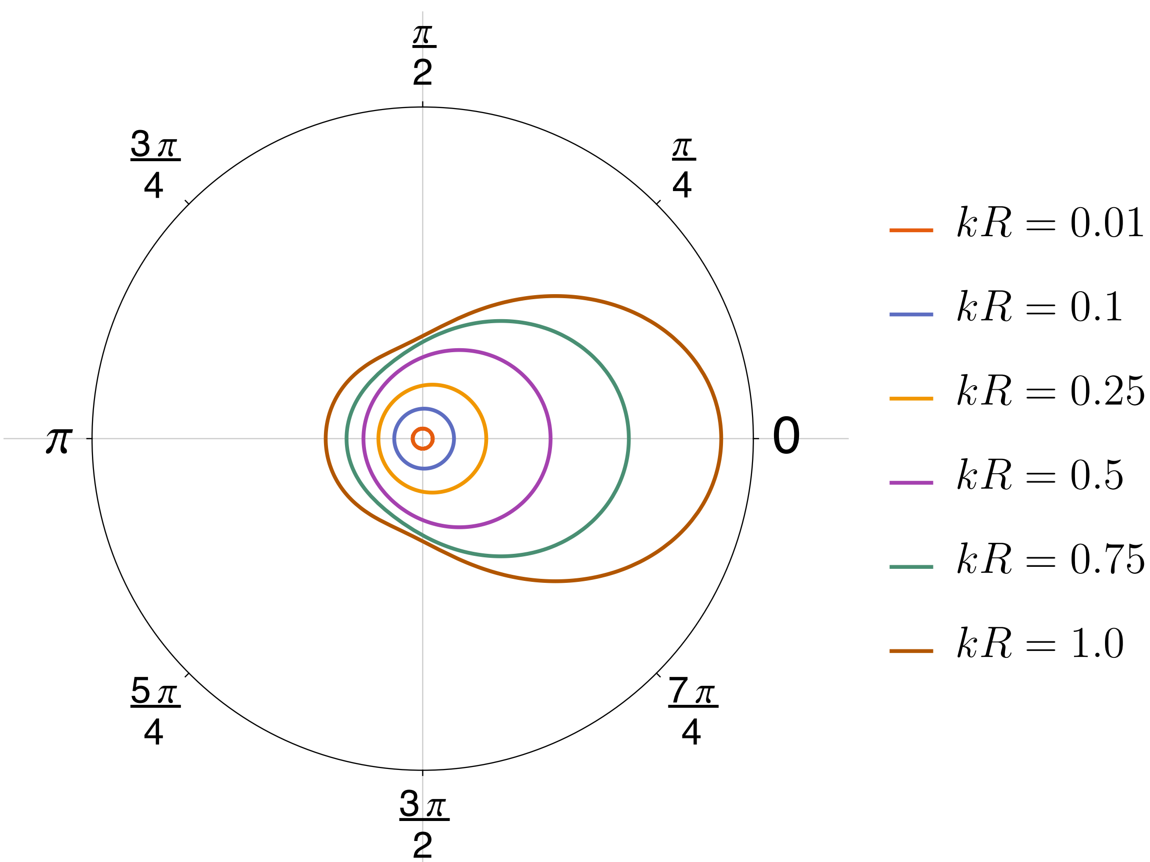
- Wavefunction computed analytically from matching conditions.

$$\Psi(r,\theta) = \sum_{l=-\infty}^{\infty} e^{il\theta} \left[a_l \begin{pmatrix} H_l^+(k_< r) \\ -H_{l+1}^+(k_< r) e^{i\theta} \end{pmatrix} + b_l \begin{pmatrix} H_l^-(k_< r) \\ -H_{l+1}^-(k_< r) e^{i\theta} \end{pmatrix} + c_l \begin{pmatrix} H_l^+(k_> r) \\ -H_{l+1}^+(k_> r) e^{i\theta} \end{pmatrix} + d_l \begin{pmatrix} H_l^-(k_> r) \\ -H_{l+1}^-(k_> r) e^{i\theta} \end{pmatrix} \right]$$

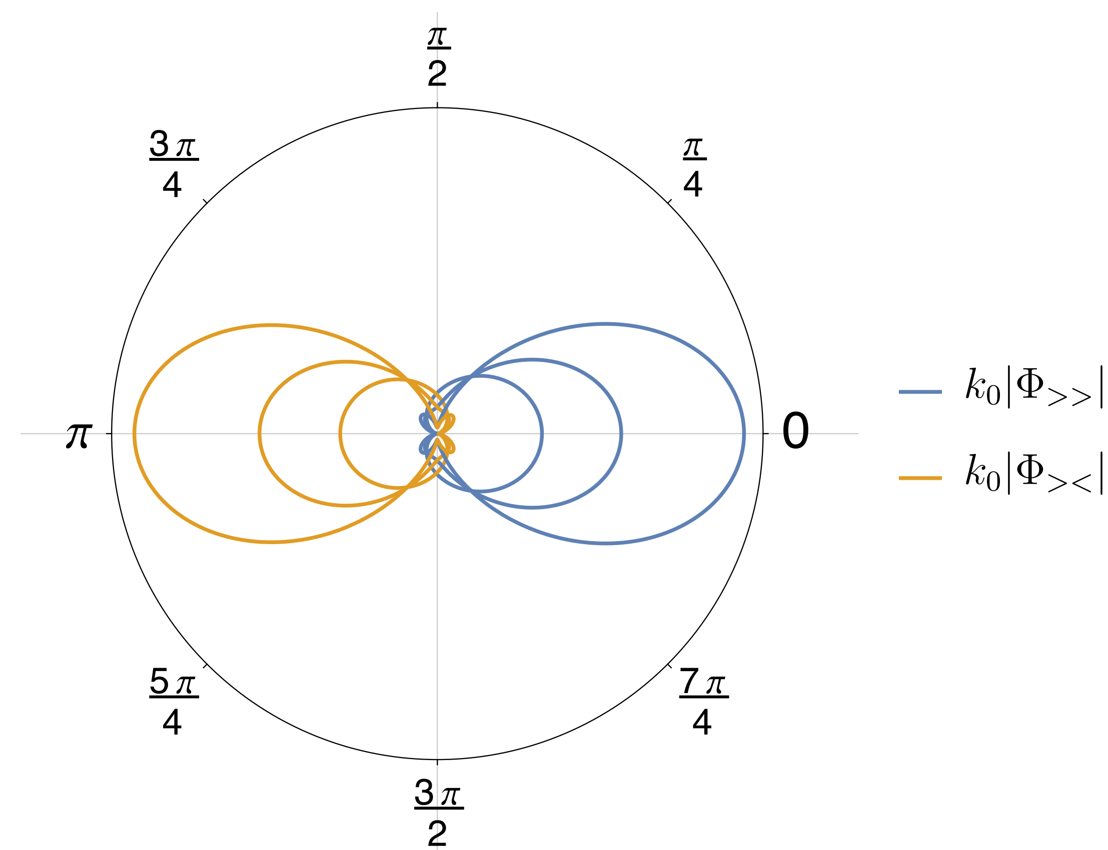
- Cross-sections and S-matrix extracted.

Example: Hard Disk

- Differential cross-section in conventional 2D system (no Rashba):



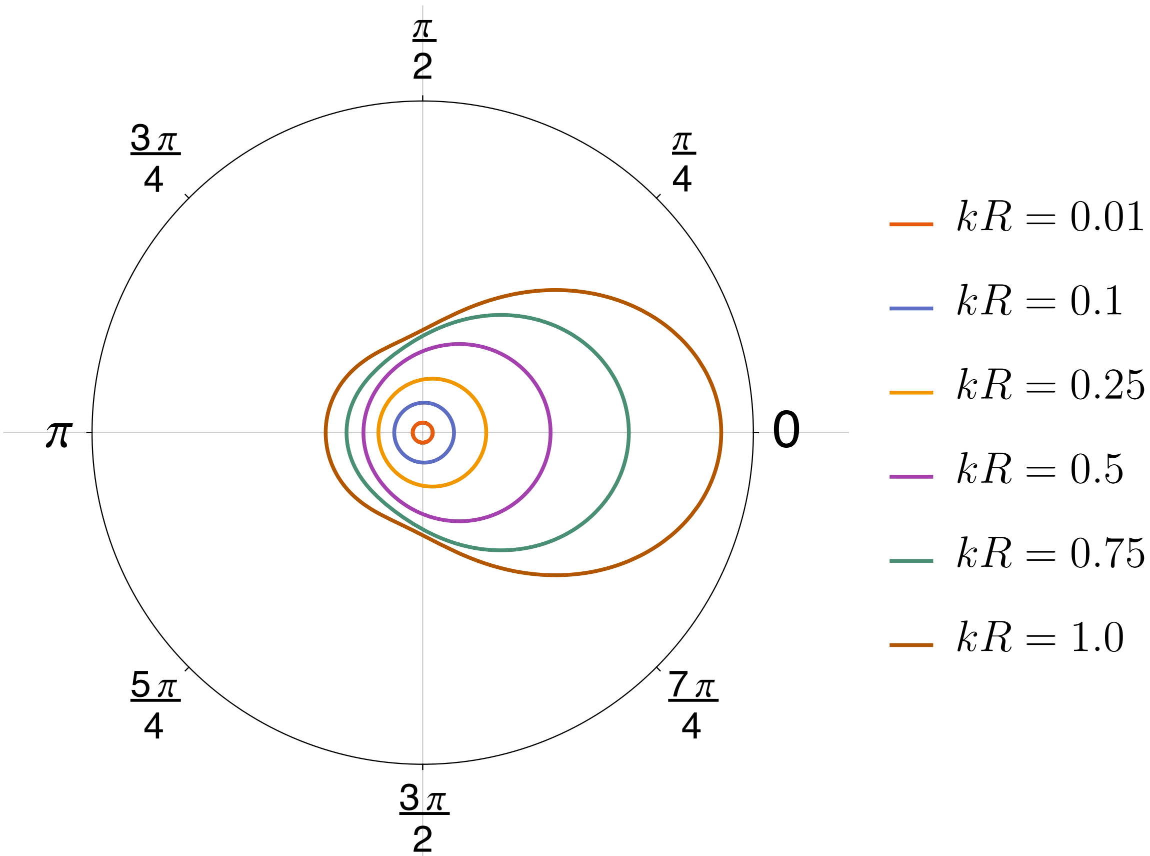
- Differential cross-section in Rashba system:



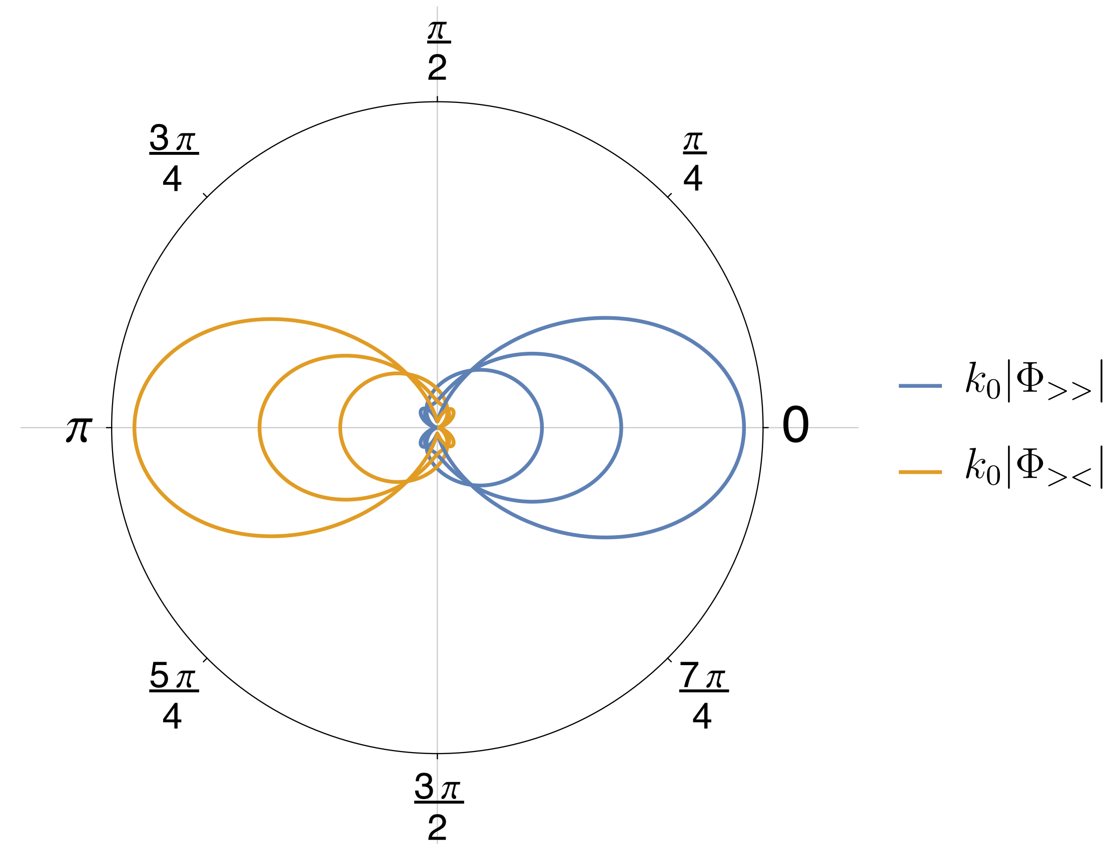
J. Hutchinson and J. Maciejko, Phys. Rev. B **93**, 245309 (2016).

Example: Hard Disk

- Differential cross-section in conventional 2D system (no Rashba):



- Differential cross-section in Rashba system:

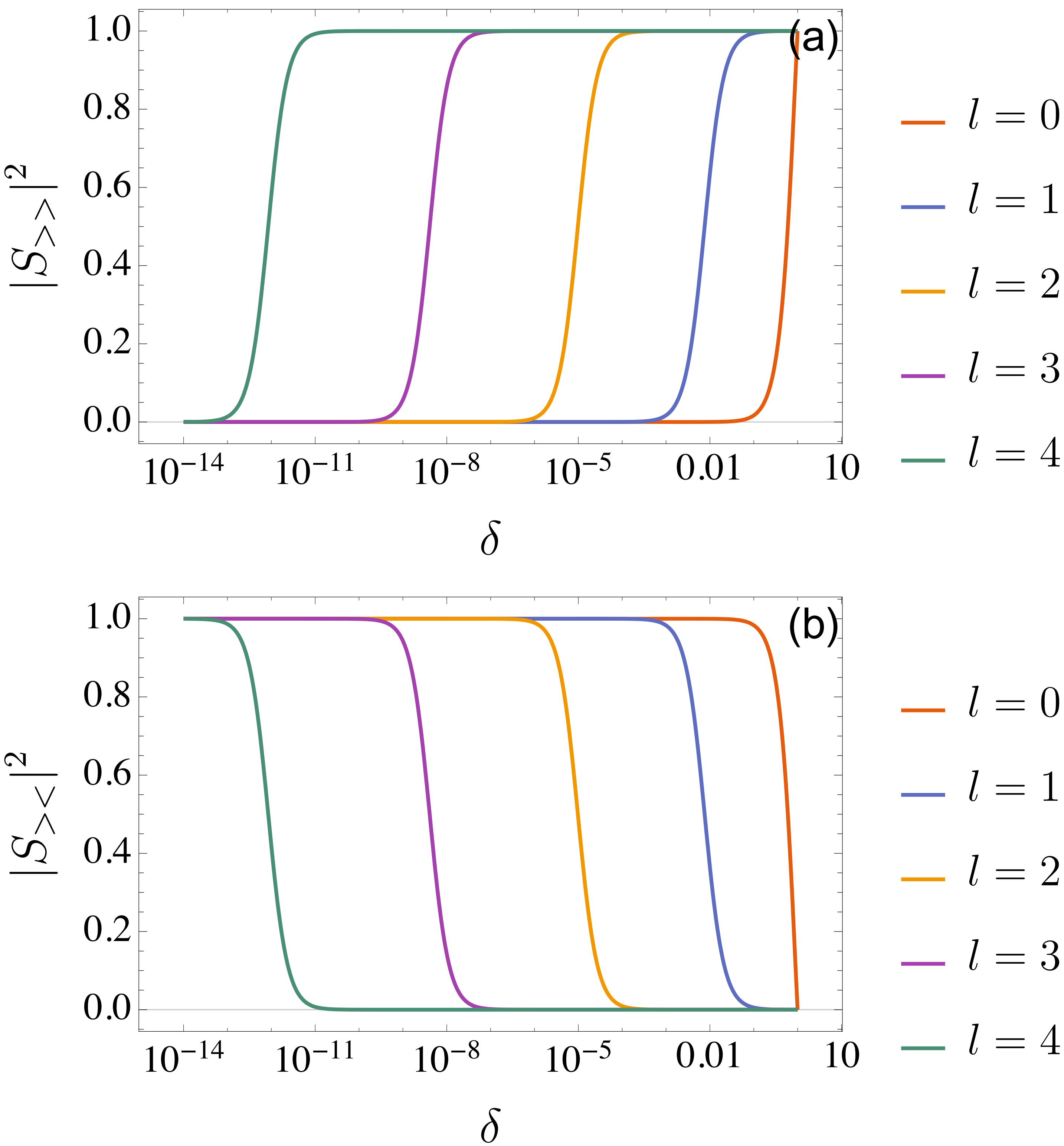


- In the low energy limit, scattering looks 1D!

$$\left(\frac{d\sigma}{d\theta}\right)_{\gtrless}\Bigg|_{E=-E_0} = \frac{2\pi}{k_0}\left[\delta^2(\theta) + \delta^2(\theta - \pi)\right]$$

J. Hutchinson and J. Maciejko, Phys. Rev. B **93**, 245309 (2016).

Example: Hard Disk

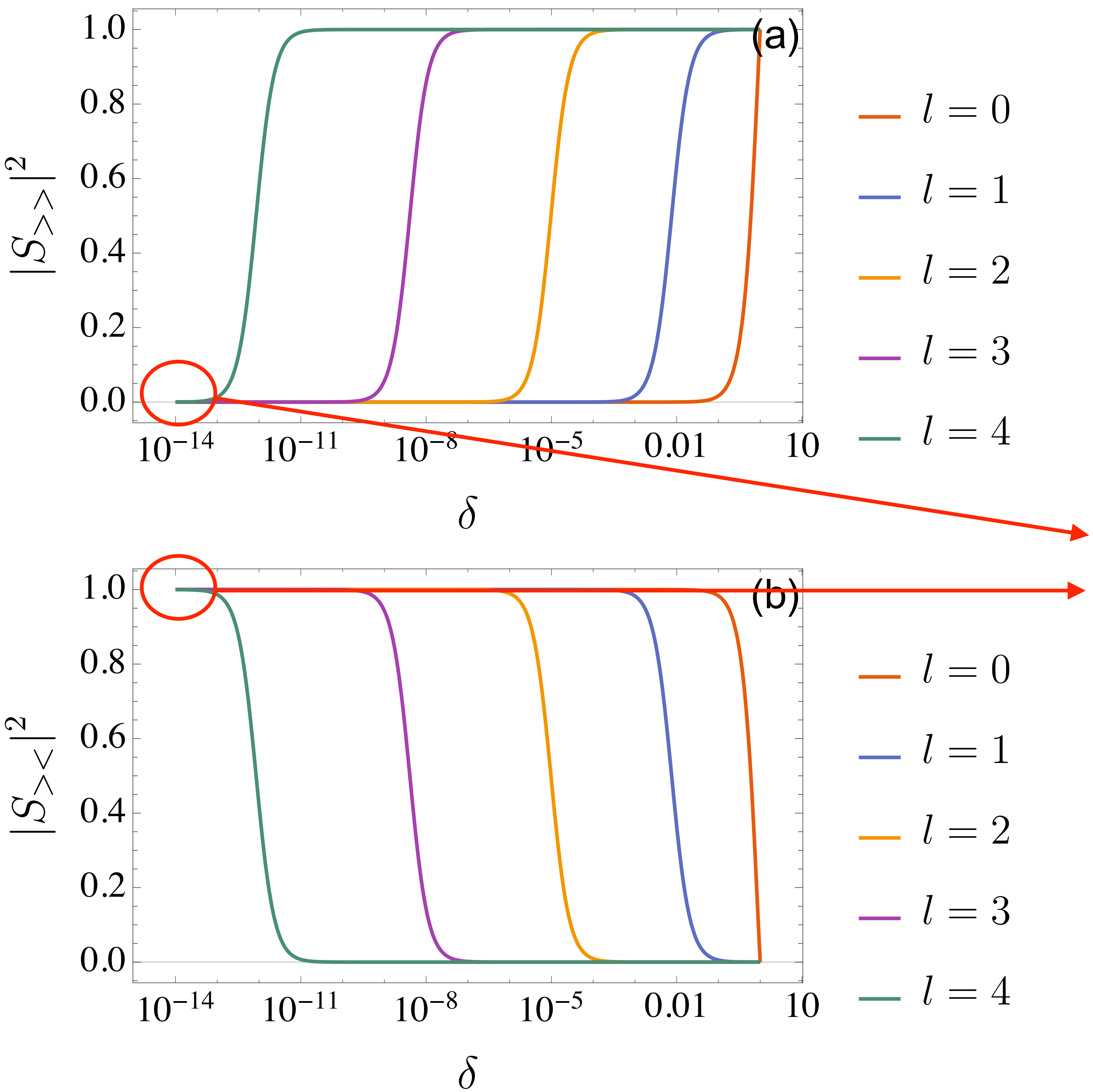


$$V = \begin{cases} \infty & r < R \\ 0 & r > R \end{cases}$$

- S-matrix decomposed in partial waves.

J. Hutchinson and J. Maciejko, Phys. Rev. B **93**, 245309 (2016).

Example: Hard Disk



$$V = \begin{cases} \infty & r < R \\ 0 & r > R \end{cases}$$

- S-matrix decomposed in partial waves.

Low energy limit:

$$S^l = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$$

- Independent of l
- Off diagonal
- Universal?

J. Hutchinson and J. Maciejko, Phys. Rev. B **93**, 245309 (2016).

Scattering Formalism

- Relate T and S matrices through Lippmann-Schwinger equation:

$$\psi_{\boldsymbol{k}\sigma}(\boldsymbol{r};E) = \psi^{\text{in}}_{\boldsymbol{k}\sigma}(\boldsymbol{r};E) + \sum_{\sigma'\sigma''} \int d^2\boldsymbol{r}' \int \frac{d^2\boldsymbol{k}'}{(2\pi)^2} G^{+}_{\sigma\sigma'}(\boldsymbol{r},\boldsymbol{r}';E)\, T^{\boldsymbol{k}'\boldsymbol{k}}_{\sigma'\sigma''} e^{i\boldsymbol{k}'\cdot\boldsymbol{r}'} \eta^{-}_{\sigma''}(\theta_k)$$

- For negative energies:

partial wave expansion

$$S^{l}_{\mu\nu} = \mathbb{I}_{\mu\nu} - \frac{im}{k_0\delta}\sqrt{k_\mu k_\nu}\, T^{l}(k_\nu, k_\mu)$$

lower helicity S matrix

$k_{\lessgtr}$ indices

lower helicity T-matrix

Scattering Formalism

- Cross-sections from Fermi's golden rule:

$$\left.\frac{d\sigma}{d\theta}\right|_{\mu\nu} = \frac{w_{\mu\to\nu}}{|\boldsymbol{j}_{\boldsymbol{\mu}}|}$$

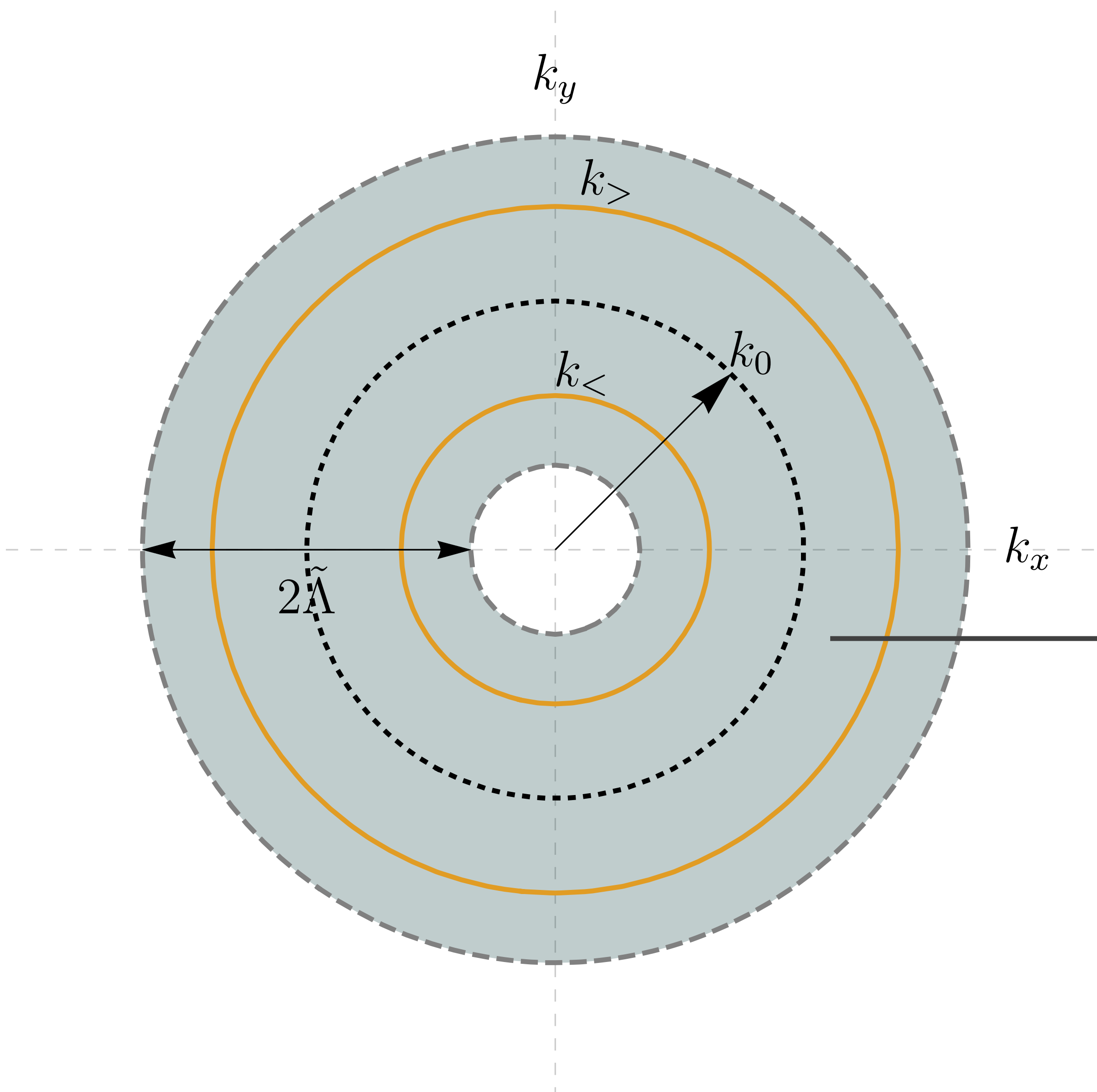
$$\sigma_\mu = \frac{2}{k_\mu} \sum_{l=-\infty}^{\infty} (1 - \mathrm{Re}(S^l_{\mu\mu}))$$

- Optical theorem: $\mathrm{Im}(T^{\boldsymbol{k}_{\boldsymbol{\mu}}\boldsymbol{k}_{\boldsymbol{\mu}}}_{--}(\theta = 0)) = -\frac{k_0\delta}{2m}\sigma_\mu$

→ Is there a generic form for the T-matrix?

Generic Rashba T-matrix

- Claim: The low-energy T-matrix takes a universal form for any circular-symmetric, finite range, spin-independent potential.



$\delta << \Lambda << 1$

Allowed virtual transitions within cutoff

Generic Rashba T-matrix

- Claim: The low-energy T-matrix takes a universal form for any circular-symmetric, finite range, spin-independent potential.

Born series:

"On-shell"

"Off-shell"

$$T_{ji}^{\boldsymbol{k}_\nu \boldsymbol{k}_\mu} = \boxed{V_{ji}(\boldsymbol{k_\nu}, \boldsymbol{k_\mu})} + \boxed{\sum_{n=+,-} \int \frac{d^2 q}{(2\pi)^2} V_{jn}(\boldsymbol{k_\nu}, \boldsymbol{q}) G_{nn}^{+}(q) T_{ni}^{\boldsymbol{q}\boldsymbol{k}_\mu}}$$

Generic Rashba T-matrix

$$V_{ji}(\boldsymbol{k_\nu}, \boldsymbol{k_\mu}) \approx V_{ji}(k_0\hat{k_\nu}, k_0\hat{k_\mu}) + O(\delta)$$

$$T^l_{--} \approx \frac{1}{m}\frac{\delta^*_l}{1 + i\delta^*_l/\delta} = -\frac{i\delta}{m} + O(\delta^2)$$

With $$\delta^*_l \equiv \frac{m}{2}(V^l(k_0, k_0) + V^{l+1}(k_0, k_0))$$

$$V^l(k, k') = \int_0^{2\pi} \frac{d\theta}{2\pi} \int_0^\infty drrV(r)J_0(|\boldsymbol{k} - \boldsymbol{k'}|r)e^{-il\theta}$$

Generic Rashba T-matrix

- Remarks:

1) To lowest order, T-matrix is independent of potential and partial wave!

$$T^{l}_{--} = -\frac{i\delta}{m} + O(\delta^2)$$

2) We obtain previous S-matrix limit.

$$S^{l} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$$

3) The energy dependence is fundamentally different than in conventional 2D scattering.

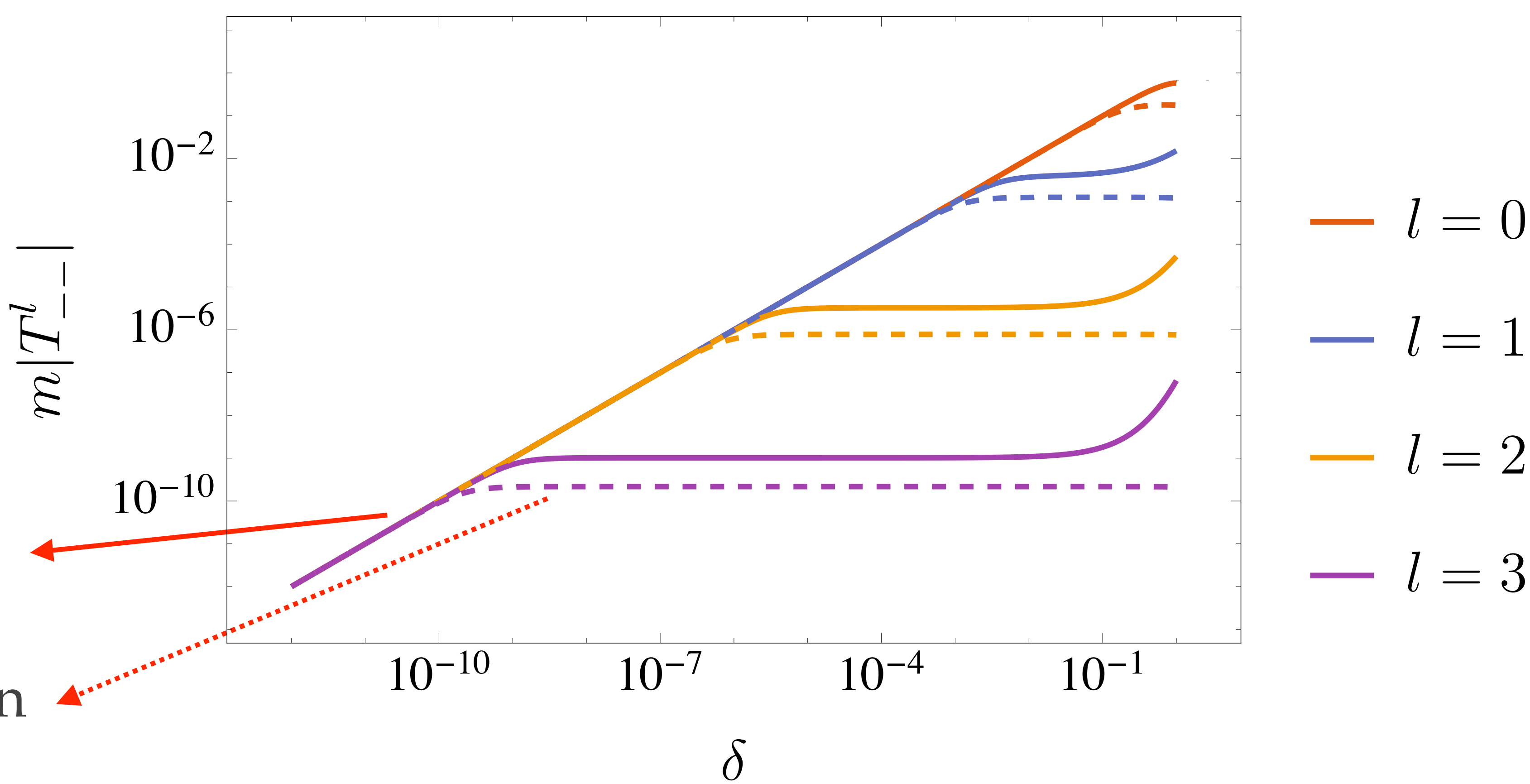
$$T^{\boldsymbol{k}\boldsymbol{k}'} \approx T^{0}(E) \sim \frac{1/m}{i - \frac{1}{\pi}\ln(E/E_a)}$$

4) The energy dependence is that of a 1D T-matrix!

$$T_{1D} \approx \frac{i}{m}\sqrt{2mE}$$

Example: Delta-shell

$$V(r) = V_0 \delta(r - R)$$

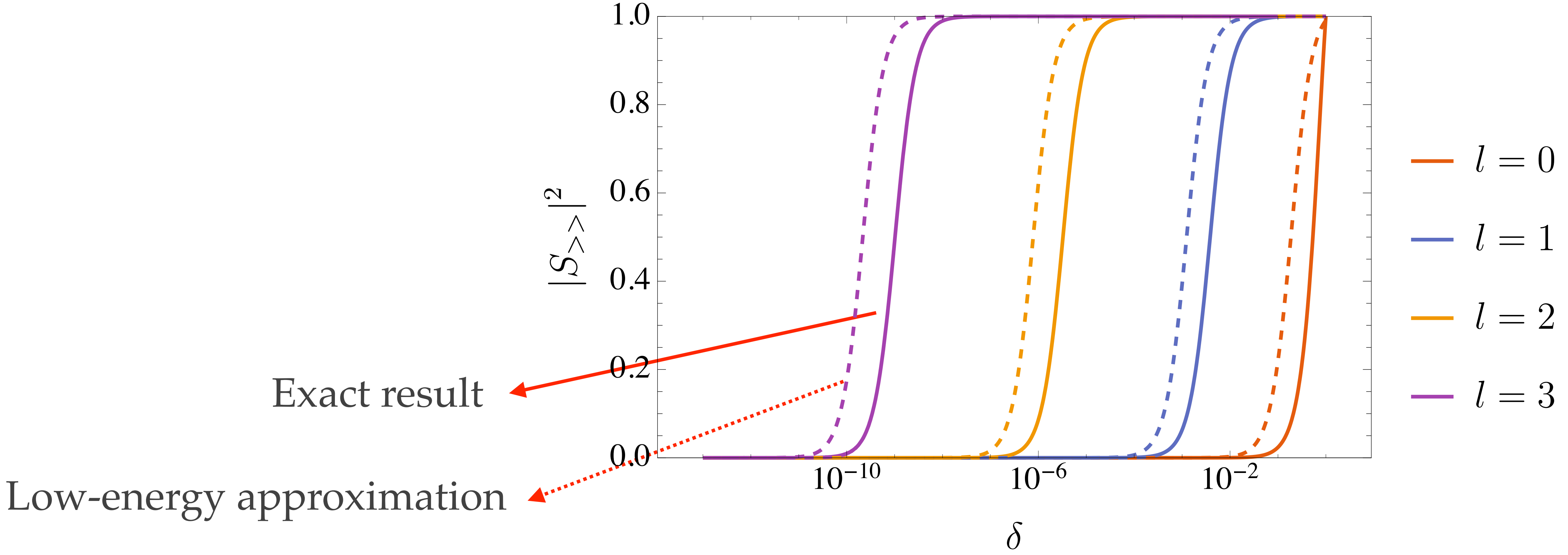


Exact result

Low-energy approximation

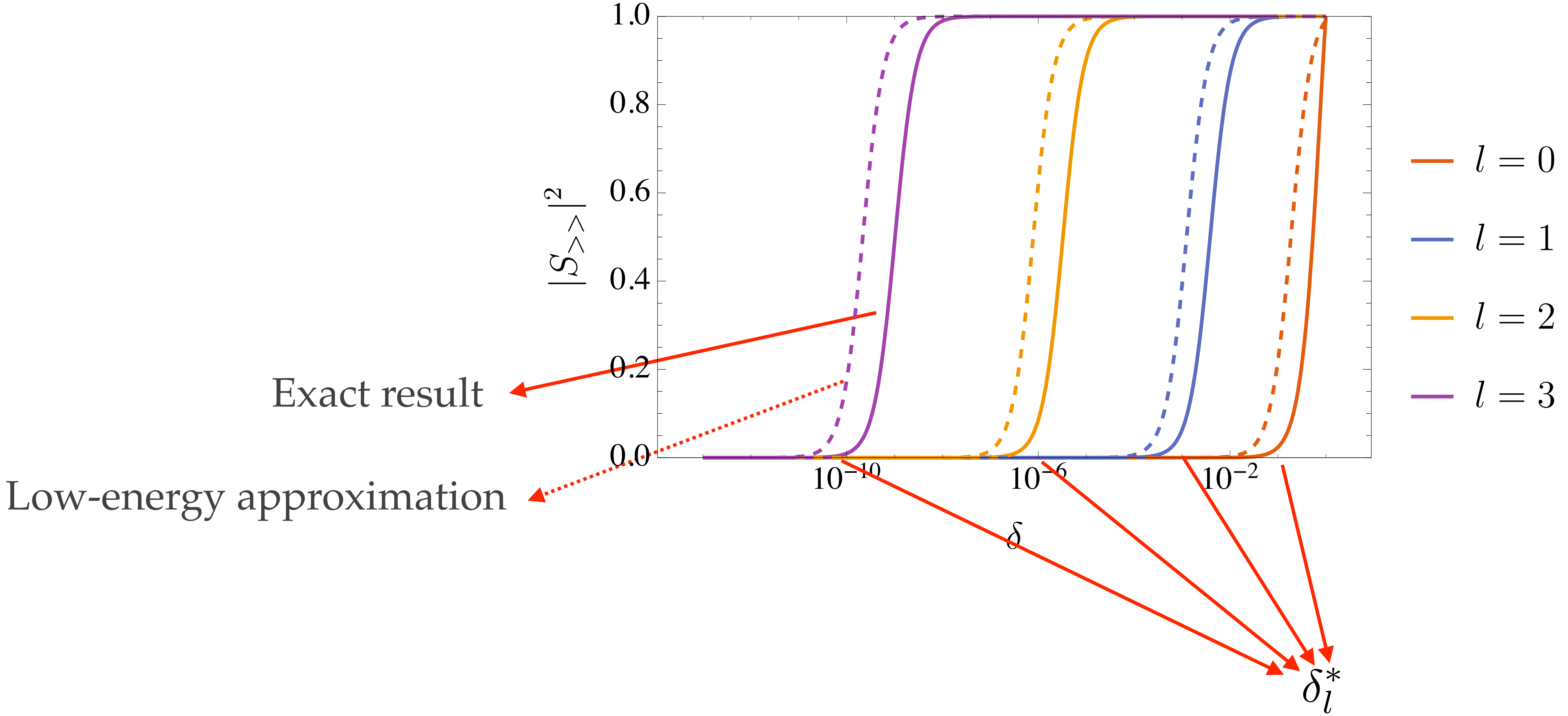
Example: Delta-shell

$$V(r) = V_0\delta(r - R)$$



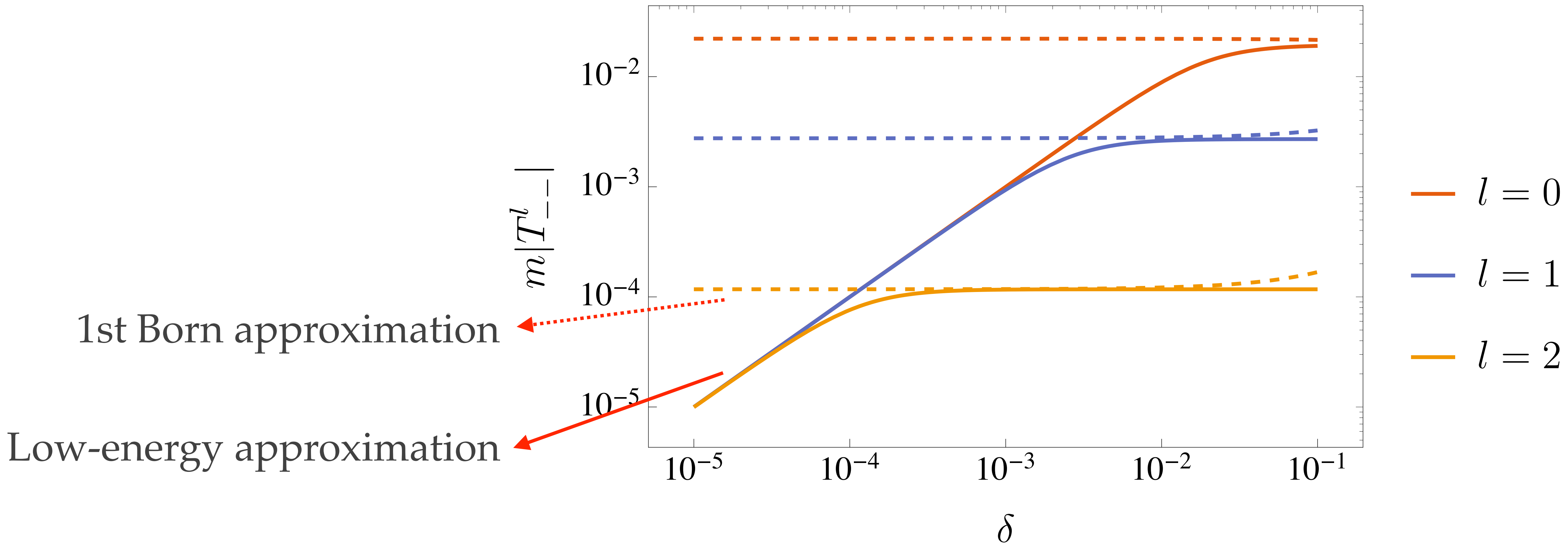
Example: Delta-shell

$$V(r) = V_0\delta(r-R)$$



Example: Circular Barrier

$$V(r) = \begin{cases} V_0 & r < R \\ 0 & r > R \end{cases}$$



Conductivity

- Optical theorem gives low-energy cross section:

$$\sigma \approx \frac{2}{k_0} \sum_{l=-\infty}^{\infty} \frac{\delta_l^{*2}/\delta^2}{1+\delta_l^{*2}/\delta^2}$$

- Semi-classical Boltzmann:

$$0 = \partial_t n_{\mathbf{k}} + \dot{\mathbf{k}} \cdot \nabla_{\mathbf{k}} n_{\mathbf{k}} + \mathbf{v} \cdot \nabla_{\mathbf{r}} n_{\mathbf{k}} - \left(\frac{\partial n_{\mathbf{k}}}{\partial t}\right)_{\text{collisions}}$$

Conductivity

- Optical theorem gives low-energy cross section:

$$\sigma \approx \frac{2}{k_0} \sum_{l=-\infty}^{\infty} \frac{\delta_l^{*2}/\delta^2}{1+\delta_l^{*2}/\delta^2}$$

- Semi-classical Boltzmann:

$$0 = \partial_t n_{\mathbf{k}} + \dot{\mathbf{k}} \cdot \nabla_{\mathbf{k}} n_{\mathbf{k}} + \mathbf{v} \cdot \nabla_{\mathbf{r}} n_{\mathbf{k}} - \left(\frac{\partial n_{\mathbf{k}}}{\partial t} \right)_{\text{collisions}}$$

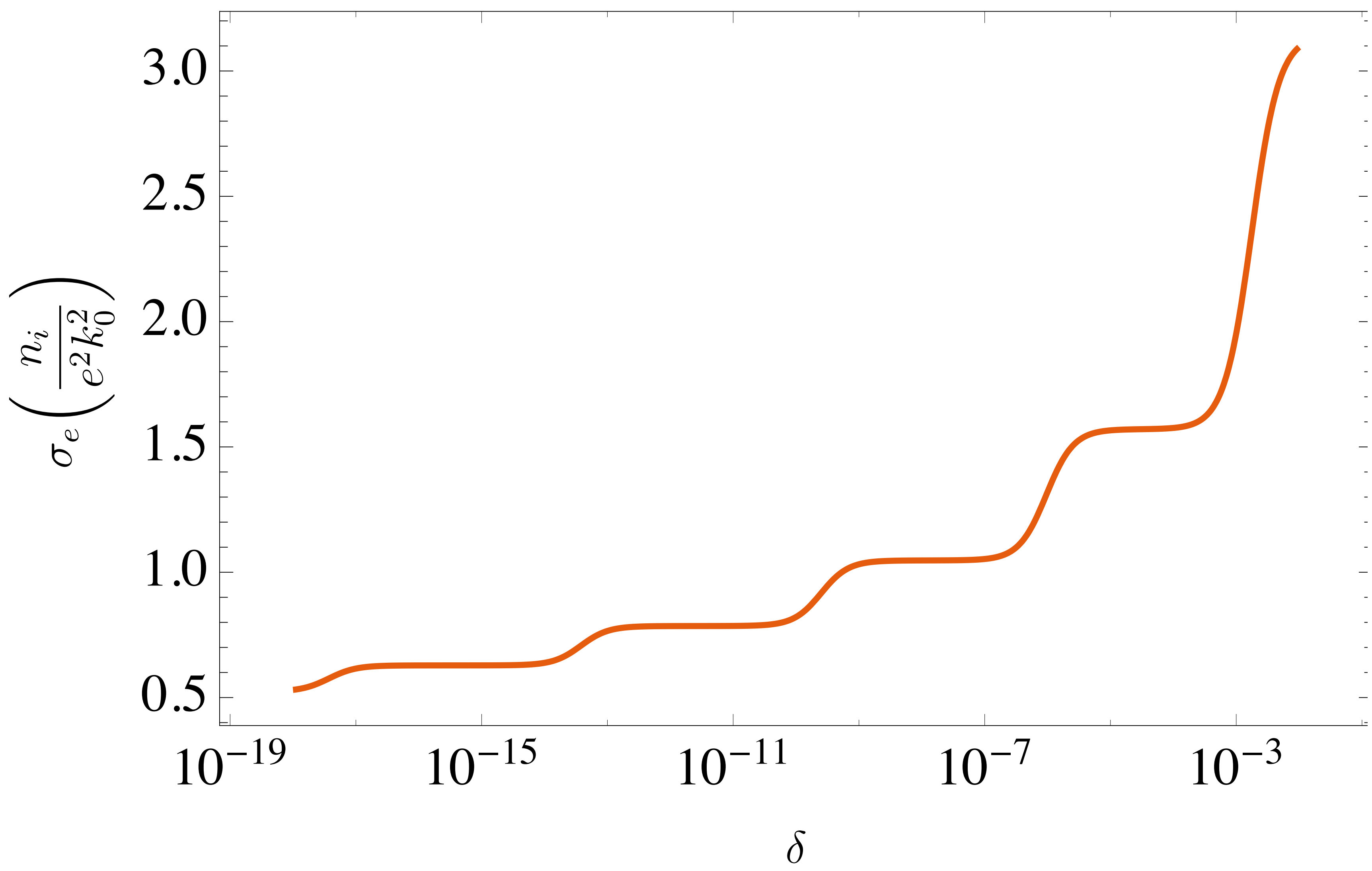
- Current density:

$$\mathbf{j} = -e \sum_{\nu} \int d\phi \int dE \rho_\nu(E) n_{\mathbf{k}_\nu}(E) \mathbf{v}_\nu(E,\phi)$$

- Conductivity:

$$\boxed{\sigma_e = \frac{e^2 k_0}{2\pi n_i \sigma}}$$

Conductivity



- Conductivity:

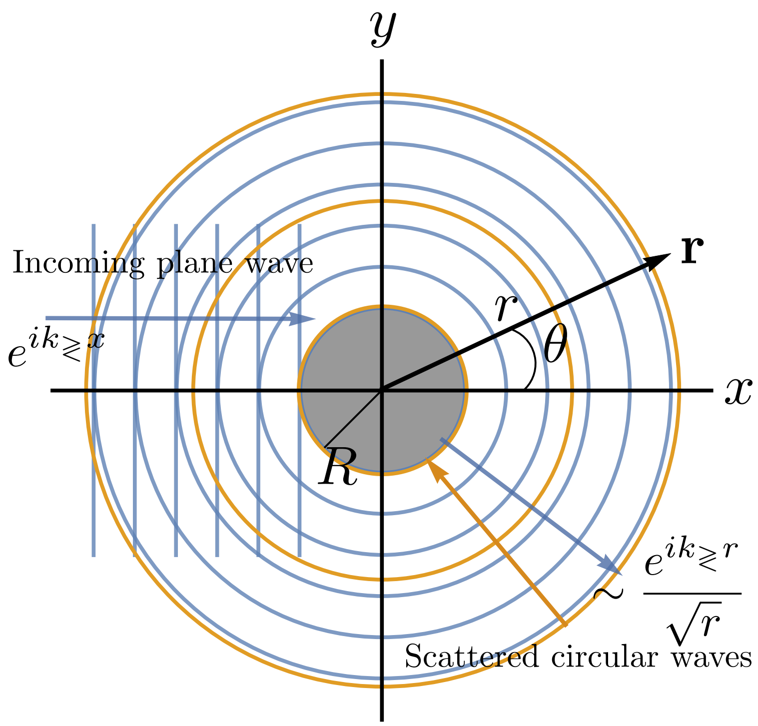
$$\sigma_e = \frac{e^2 k_0}{2\pi n_i \sigma}$$

Summary

- The low energy limit of a Rashba system contains interesting physics not seen at energies above the Dirac point:
  - Change in the topology of the Fermi surface (Lifschitz transition).
- Low energy scattering quantities have a 1D character:
  - Differential cross sections become confined to a line (incident wave axis).
  - T matrix has an energy dependence inherent to 1D systems.
- Low energy T matrix is universal - independent of potential features
- Low energy $\neq$ s-wave!
- Conductivity displays quantized plateaus.

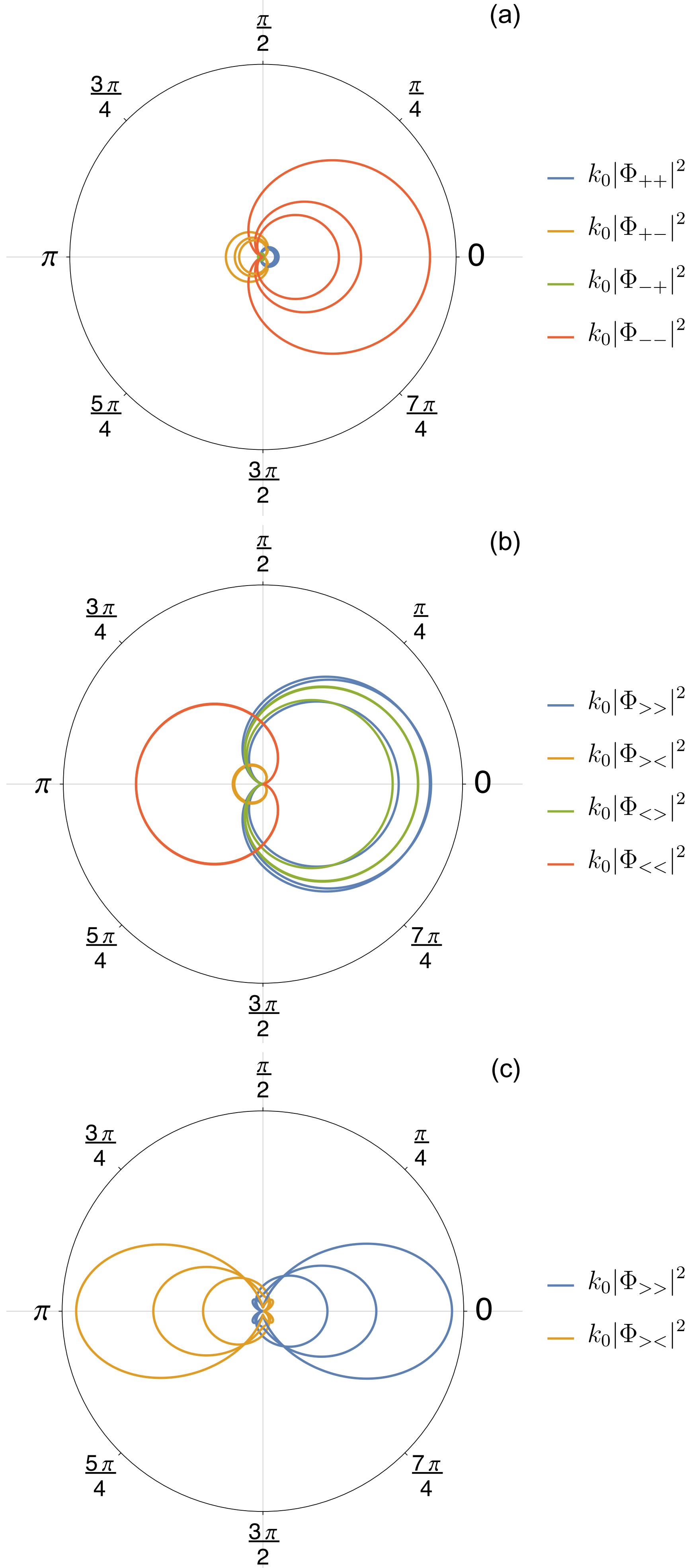
Thank you!

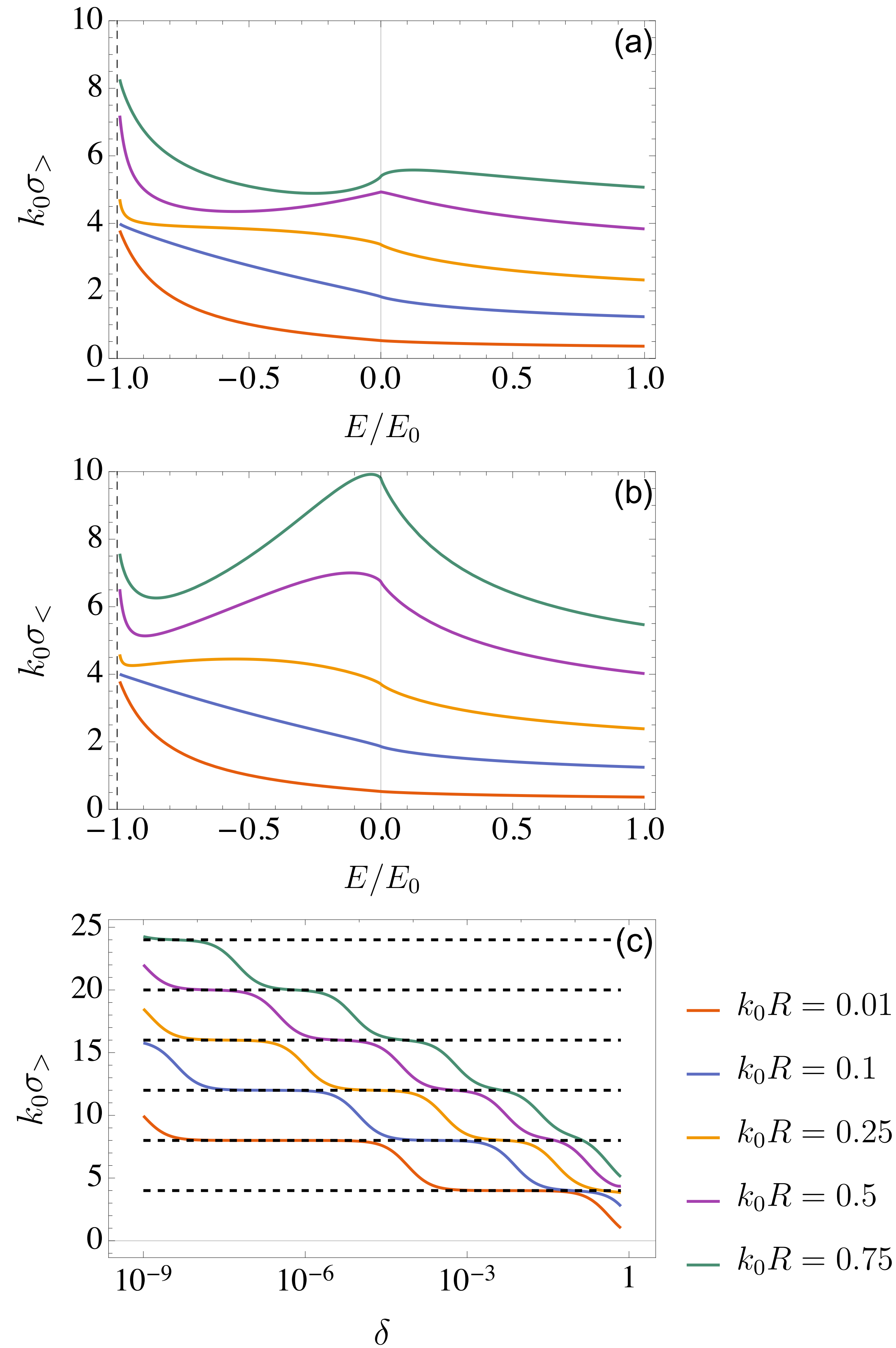
$$\psi_\mu(\boldsymbol{r};E) \approx \psi_\mu^{\mathrm{in}}(\boldsymbol{r};E) - \frac{m}{(k_> - k_<)}\sqrt{\frac{2i}{\pi r}}\Big(\sqrt{k_>}e^{ik_> r}\eta^-(\theta_r)\eta^-(\theta_r)^\dagger T^{\boldsymbol{k}_> \boldsymbol{k}}\eta^-(0) + i\sqrt{k_<}e^{-ik_< r}\eta^+(\theta_r)\eta^+(\theta_r)^\dagger T^{-\boldsymbol{k}_< \boldsymbol{k}}\eta^-(0)\Big)$$

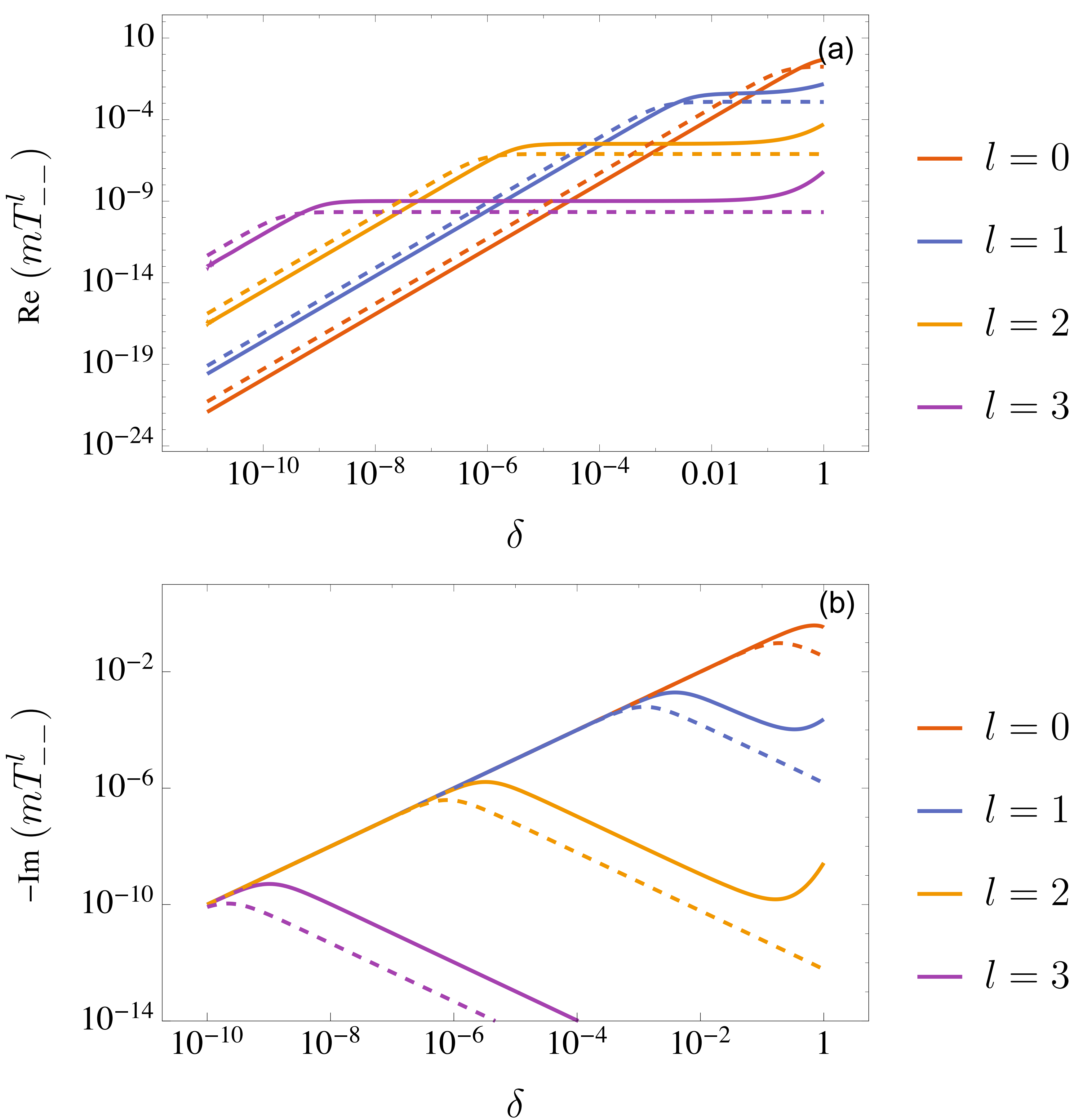


$$|\boldsymbol{k_\mu} - \boldsymbol{k_\nu}|r = r\sqrt{k_\mu^2 + k_\nu^2 - 2k_\mu k_\nu \cos\theta_{k'-k}}$$
$$= \sqrt{2}k_0 r\sqrt{1-\cos\theta_{k'-k}} + O(\delta)$$

$$\Psi(r,\theta) = \sum_{l=-\infty}^{\infty} e^{il\theta}\left[a_l\begin{pmatrix} H_l^+(k_< r) \\ -H_{l+1}^+(k_< r)e^{i\theta}\end{pmatrix} + b_l\begin{pmatrix} H_l^-(k_< r) \\ -H_{l+1}^-(k_< r)e^{i\theta}\end{pmatrix} + c_l\begin{pmatrix} H_l^+(k_> r) \\ -H_{l+1}^+(k_> r)e^{i\theta}\end{pmatrix} + d_l\begin{pmatrix} H_l^-(k_> r) \\ -H_{l+1}^-(k_> r)e^{i\theta}\end{pmatrix}\right]$$


(a)
$k_0|\Phi_{++}|^2$
$k_0|\Phi_{+-}|^2$
$k_0|\Phi_{-+}|^2$
$k_0|\Phi_{--}|^2$
(b)
$k_0|\Phi_{>>}|^2$
$k_0|\Phi_{><}|^2$
$k_0|\Phi_{<>}|^2$
$k_0|\Phi_{<<}|^2$
(c)
$k_0|\Phi_{>>}|^2$
$k_0|\Phi_{><}|^2$


(a)
$k_0\sigma_>$
E/E_0
(b)
$k_0\sigma_<$
E/E_0
(c)
$k_0\sigma_>$
δ
$k_0R = 0.01$
$k_0R = 0.1$
$k_0R = 0.25$
$k_0R = 0.5$
$k_0R = 0.75$


(a)
Re (mT^l_{--})
10
10^{-4}
10^{-9}
10^{-14}
10^{-19}
10^{-24}
10^{-10}
10^{-8}
10^{-6}
10^{-4}
0.01
1
δ
$l = 0$
$l = 1$
$l = 2$
$l = 3$
(b)
-Im (mT^l_{--})
10^{-2}
10^{-6}
10^{-10}
10^{-14}
10^{-10}
10^{-8}
10^{-6}
10^{-4}
10^{-2}
1
δ
$l = 0$
$l = 1$
$l = 2$
$l = 3$