On the Rotation of Celestial Bodies: an Emerging Phenomenon

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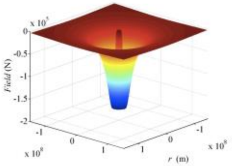
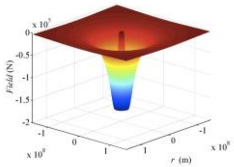


General Motivation

- Main theoretical research interests are in Emerging Phenomena and Systems.
- A large part of my works on biological systems (neuromuscular systems and neural networks).
- Focus, in the last decade or so, on physical systems: emergence of physical laws…



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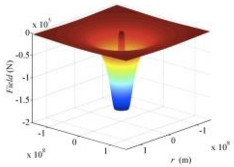
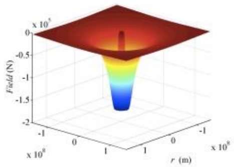


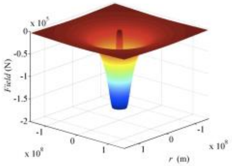
Topics

- **Emerging gravity: a recap.**
- ***Erfc*** **symmetric space-time geometry.**
- **An Axisymmetric Interpretation.**



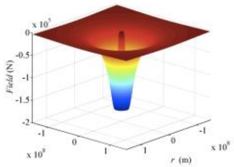
Why are the celestial bodies rotating?

The studies of massive rotating bodies in the context of General Relativity are mostly based, directly or indirectly, on the Kerr metric. The rotation is assumed from the start and investigators try to fit various parameters to mimic the observed phenomena. Can we find a metric from which the rotation terms intrinsically emerge?



Topics

- **Emerging gravity: a recap**
  What is the conducting thread?
- ***Erfc* symmetric space-time geometry.**
- **An Axisymmetric Interpretation.**

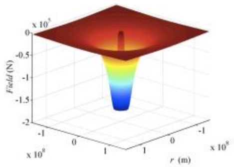


1-Einstein Gravitation Equation

$$G = KT$$

« Spacetime tells matter how to move; matter tells spacetime how to curve ».

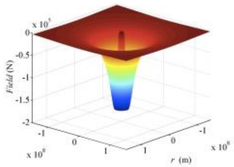
Wheeler, J.A., Ford, K.W., *Geons, Black Holes, and Quantum Foam: A Life in Physics*. W. W. Norton & Company, 2000.



2-Interdependence principle

Spacetime curvature (S) and matter-energy density (E) are two inextricable information spaces defining the physically observable probabilistic universe (U); they must be mutually exploited to describe any subset U_i of this universe. The probability of observing a subset (U_i) is:

$$\mathrm{P}(U_i) = \mathrm{P}(S_i, E_i) = \mathrm{P}(S_i / E_i)\mathrm{P}(E_i) = \mathrm{P}(\mathrm{E}_i / \mathrm{S}_i) P(S_i)$$



3-Modifying Einstein Equation

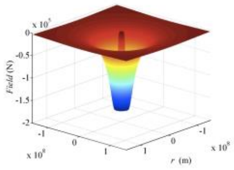
BAYESIAN JOINT PROBABILITIES

$$\mathrm{P}(U_i) = \mathrm{P}(S_i, E_i) = \mathrm{P}(S_i / E_i)\mathrm{P}(E_i) = \mathrm{P}(\mathrm{E}_i / \mathrm{S}_i) P(S_i)$$

$$f(S_i / E_i) = \frac{f(S_i)}{f(E_i)} \times f(E_i \; / \; S_i)$$

For a weak field low speed symmetric static system:

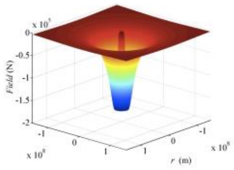
$$G_{00} = K \; T_{00} \times f(E_i \; / \; S_i)$$



4-A simple stochastic model of a star formation…

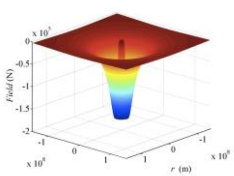
Assuming that in a far remote and isolated part of the Universe, a star is slowly building up from the gradual agglomeration of chunks of matter-energy.

Whatever the physical processes involved, these chunks of matter-energy can be considered as random variables described by their own density functions and, from a probabilistic point of view, the whole process is equivalent to adding random variables, i.e. making the convolution of their corresponding probability density functions.



5- Gaussian Convergence

- These densities are real, normalized, non-negative functions with a finite third moment and a scaled dispersion.
- The central limit theorem predicts that in a Euclidean flat spacetime, when the number of random chunks is very large ($N \rightarrow \infty$), the ideal form of the global probability density will be a Gaussian multivariate.



6-Moving from flat to curved space

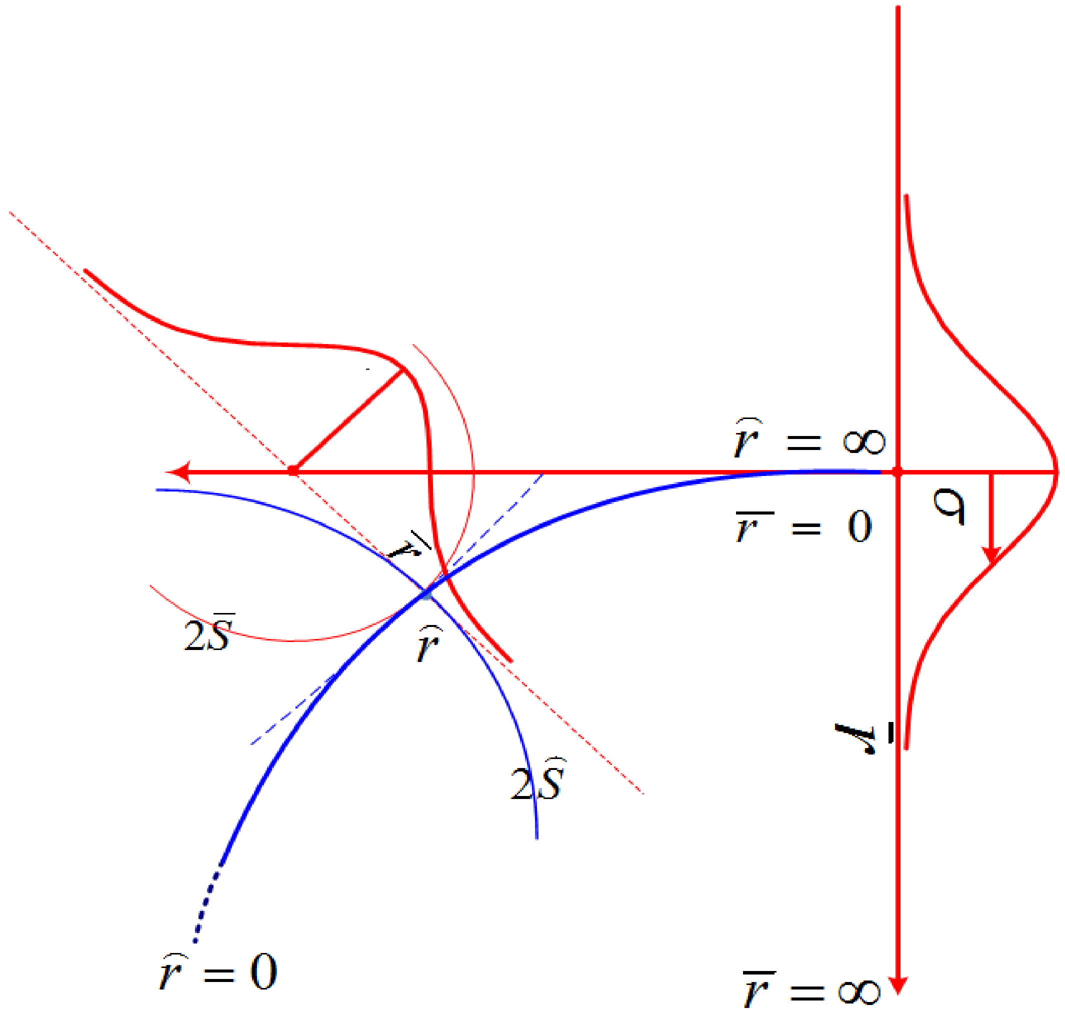
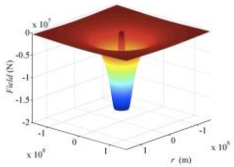
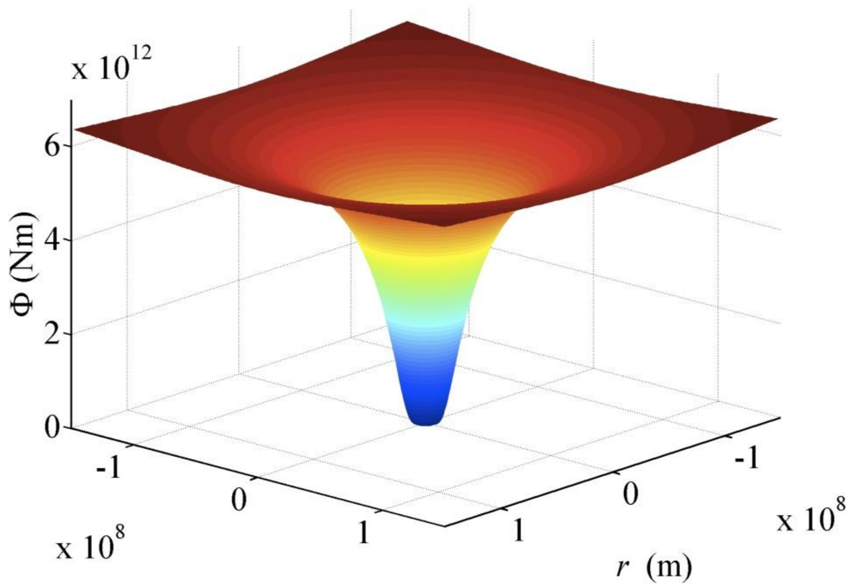


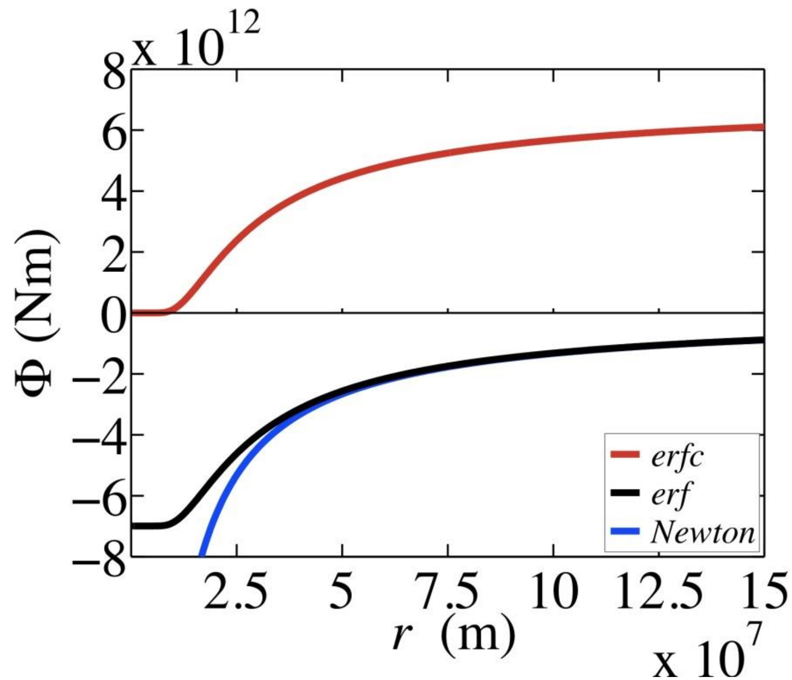
Figure 1

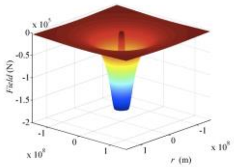


7-Emergence of Newton's law of gravitation: the potential

$$\Phi_{erfc}(r) = \frac{2KMc^4}{4\pi\sigma^3}\left(\frac{\sqrt{\pi}}{\sqrt{2}\sigma}\right)\mathit{erfc}\left(\frac{\sigma}{\sqrt{2}r}\right) = \mathit{\Phi}_{erfc}(r)$$







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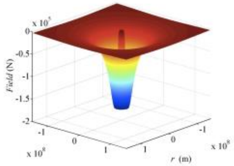


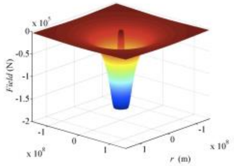
A New Symmetric Metric

$$ds^2 = \left[1 + \frac{2K_\sigma}{c_{th}^2} erfc\left(\frac{\sigma}{\sqrt{2r}}\right)\right] c_{th}^2 dt^2$$

$$- \left[1 + \frac{2K_\sigma}{c_{th}^2} erfc\left(\frac{\sigma}{\sqrt{2r}}\right)\right]^{-1} dr^2$$

$$- r^2 d\theta^2 - r^2 \sin^2 \theta d\phi^2$$

where: $K_\sigma = \sqrt{\pi} GM \ / \ \sigma\sqrt{2}$



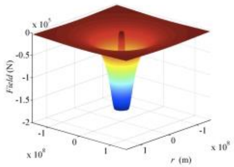
A New Symmetric Metric

No Singularities,
(neither coordinate or intrinsic).

Two types of corrections:

1- Due to the constant offset of the *erfc* potential

2- Due to the difference between an *erf* and a $1/r$ potential



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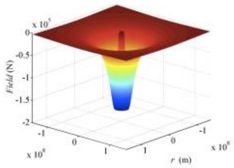
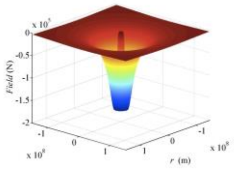


QUESTION

Can we convert this symmetric metric into into an asymetric one?



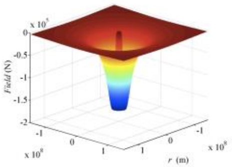
YES…here is how it goes:

- Algebrical Equivalence

$$ds^2 = \left[1 - \frac{2K_\sigma}{c_{th}^2} erf\left(\frac{\sigma}{\sqrt{2}r}\right)\right] c_{th}^2 dt^2 + 2K_\sigma dt^2$$

$$+ \frac{2K_\sigma}{c_{th}^2}\left[1 - \frac{2K_\sigma}{c_{th}^2} erf\left(\frac{\sigma}{\sqrt{2}r}\right)\right]^{-1} \times \left[1 + \frac{2K_\sigma}{c_{th}^2} erfc\left(\frac{\sigma}{\sqrt{2}r}\right)\right]^{-1} dr^2$$

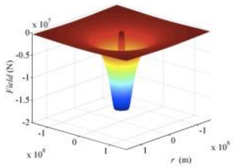
$$- \left[1 - \frac{2K_\sigma}{c_{th}^2} erf\left(\frac{\sigma}{\sqrt{2}r}\right)\right]^{-1} dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta d\phi^2$$



Defining a Rotation ω_{st} and a Space-time Expansion v_{st} Speed Components

$$\omega_{st} = \frac{d\phi}{dt}$$

$$v_{st} = \frac{dr}{dt}$$



A New Axisymmetric Metric

$$ds^2 = \left[1 - \frac{2K_\sigma}{c_{th}^2} erf\left(\frac{\sigma}{\sqrt{2}r}\right)\right] c_{th}^2 dt^2 + \frac{2K_\sigma}{\omega_{st}} d\phi dt$$

$$+ \frac{2K_\sigma v_{st}}{c_{th}^2}\left[1 - \frac{2K_\sigma}{c_{th}^2} erf\left(\frac{\sigma}{\sqrt{2}r}\right)\right]^{-1} \times \left[1 + \frac{2K_\sigma}{c_{th}^2} erfc\left(\frac{\sigma}{\sqrt{2}r}\right)\right]^{-1} drdt$$

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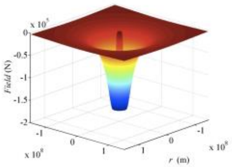

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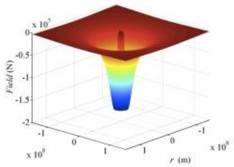
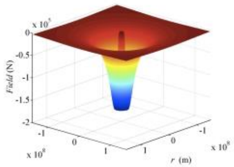
A general format describing a rotating and expanding manifold:

$$ds^2 = g_{00}dt^2 + 2g_{03}d\phi dt + 2g_{01}drdt + g_{11}dr^2 + g_{22}d\theta^2 + g_{33}d\phi^2$$



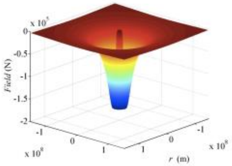
An interpretation…

In other words, the static energy associated with the offset of the *erfc* potential can be seen as the source of the body rotation and its associated space-time expansion.



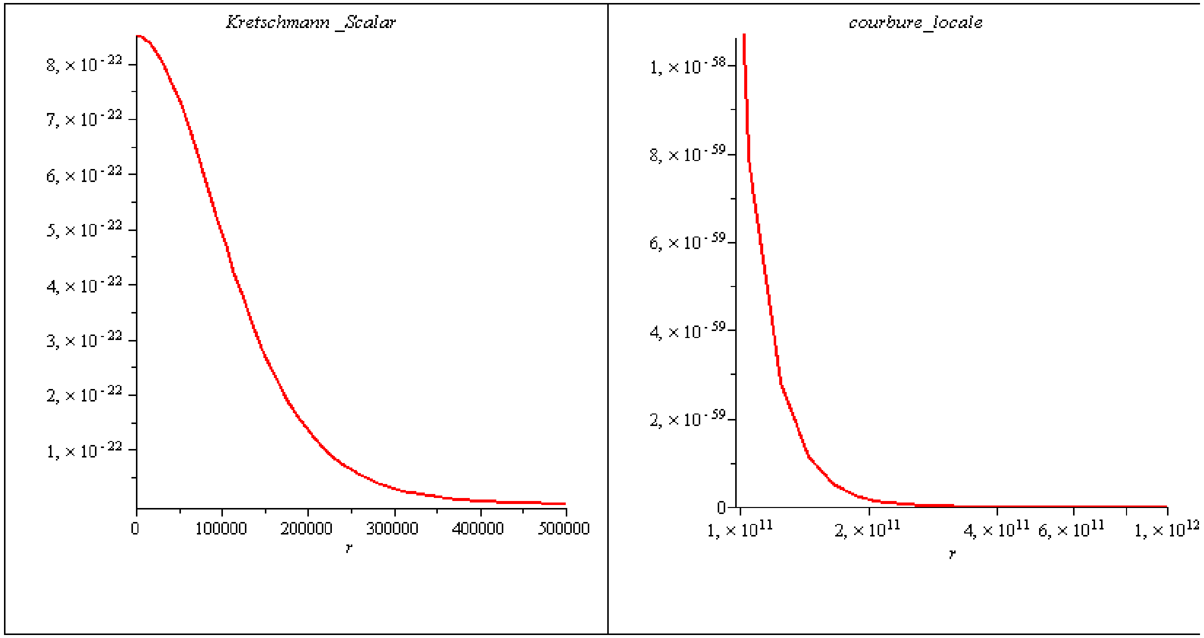
A complex axisymetric metrics

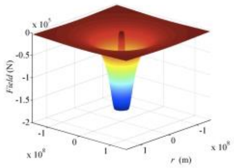
	Non zero components
Covariant metric components (6)	$g_{00}, g_{03}, g_{01}, g_{11}, g_{22,} g_{33}.$
Contravariant metric components (7)	$g^{00}, g^{01}, g^{03}, g^{11}, g^{13}, g^{22}, g^{33}.$
Christoffel symbols of the first kind (10)	$\Gamma_{00,1}, \Gamma_{01,0}, \Gamma_{11,0}, \Gamma_{11,1}, \Gamma_{12,2}, \Gamma_{13,3},$ $\Gamma_{22,1}, \Gamma_{23,3}, \Gamma_{33,1}, \Gamma_{33,2}.$
Christoffel symbols of the second kind (21)	$\Gamma^0_{00}, \Gamma^0_{01}, \Gamma^0_{11}, \Gamma^0_{13}, \Gamma^0_{22}, \Gamma^0_{23}, \Gamma^0_{33},$ $\Gamma^1_{00}, \Gamma^1_{01}, \Gamma^1_{11}, \Gamma^1_{13}, \Gamma^1_{22}, \Gamma^1_{23}, \Gamma^1_{33},$ $\Gamma^3_{00}, \Gamma^3_{01}, \Gamma^3_{11}, \Gamma^3_{13}, \Gamma^3_{22}, \Gamma^3_{23}, \Gamma^3_{33}.$
Riemann tensor covariant components (16)	$R_{0101}, R_{0103}, R_{0113}, R_{0202}, R_{0203}, R_{0212}, R_{0213},$ $R_{0303}, R_{0312}, R_{0313}, R_{1212}, R_{1213}, R_{1223}, R_{1313},$ R_{1323}, R_{2323}
Ricci tensor components (9)	$R_{00}, R_{01}, R_{03}, R_{11}, R_{12},$ $R_{13}, R_{22}, R_{23}, R_{33}.$
Einstein tensor components (9)	$G_{00}, G_{01}, G_{03}, G_{11}, G_{12},$ $G_{13}, G_{22}, G_{23}, G_{33}.$



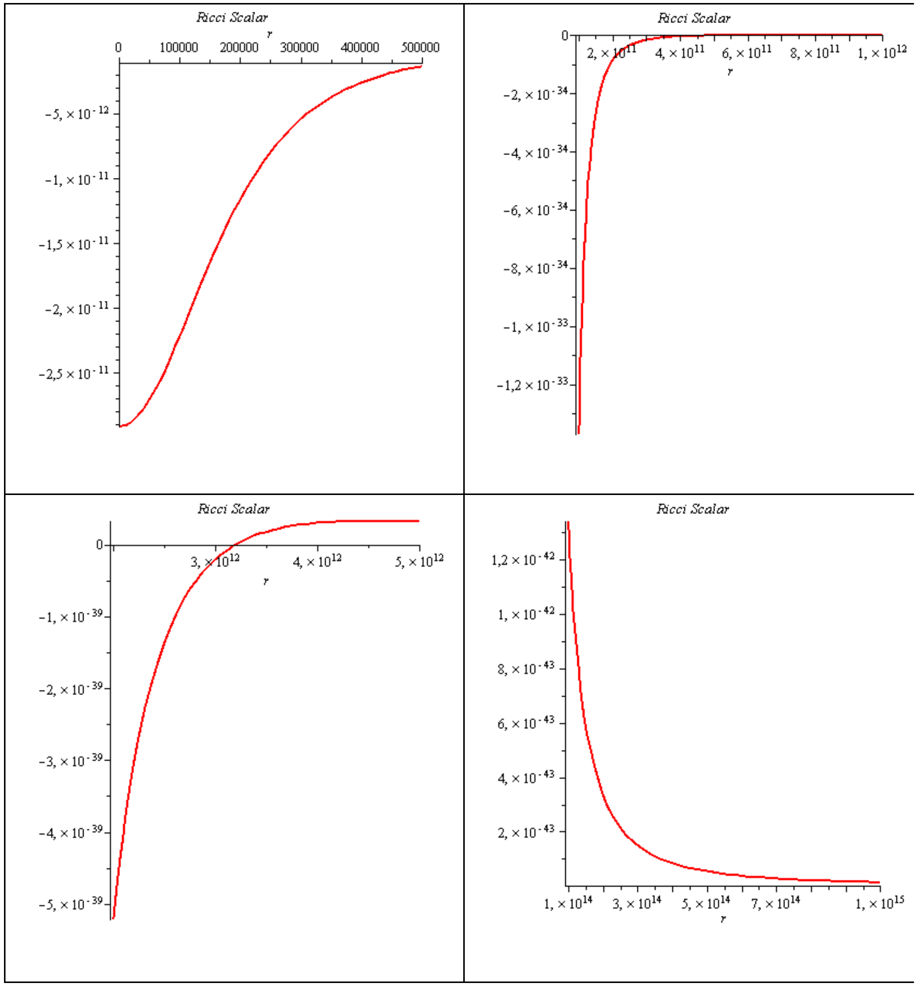
Kretschmann Scalar

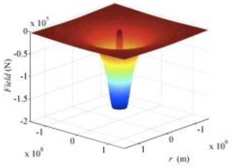
- A typical example…





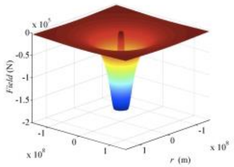
Its Corresponding Ricci Scalar…





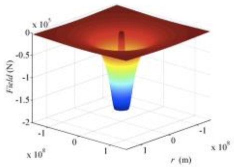
Food for thoughts

Could it be that according to this model, in some regions, the Star would appear as surrounded by a repulsive ring when only the trace of the Ricci tensor is taken into account???



Generally speaking

- One intrinsic singularity ?
- One coordinate singularity ?
- No gravitationnal collapse
- The intrinsic singularity is never reached
- There are conditions for a horizon
- A new black hole model?



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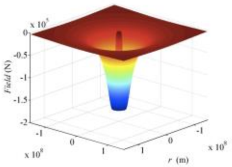
Complexity

The overall geometry is quite complicated, with various zero crossings which points out the complexity of the space-time associated to a rotating mass as seen from within the system.

All the $G_{\mu\nu}$ curves are continuous since, in general, the metric is valid from *r* equals zero to infinity.



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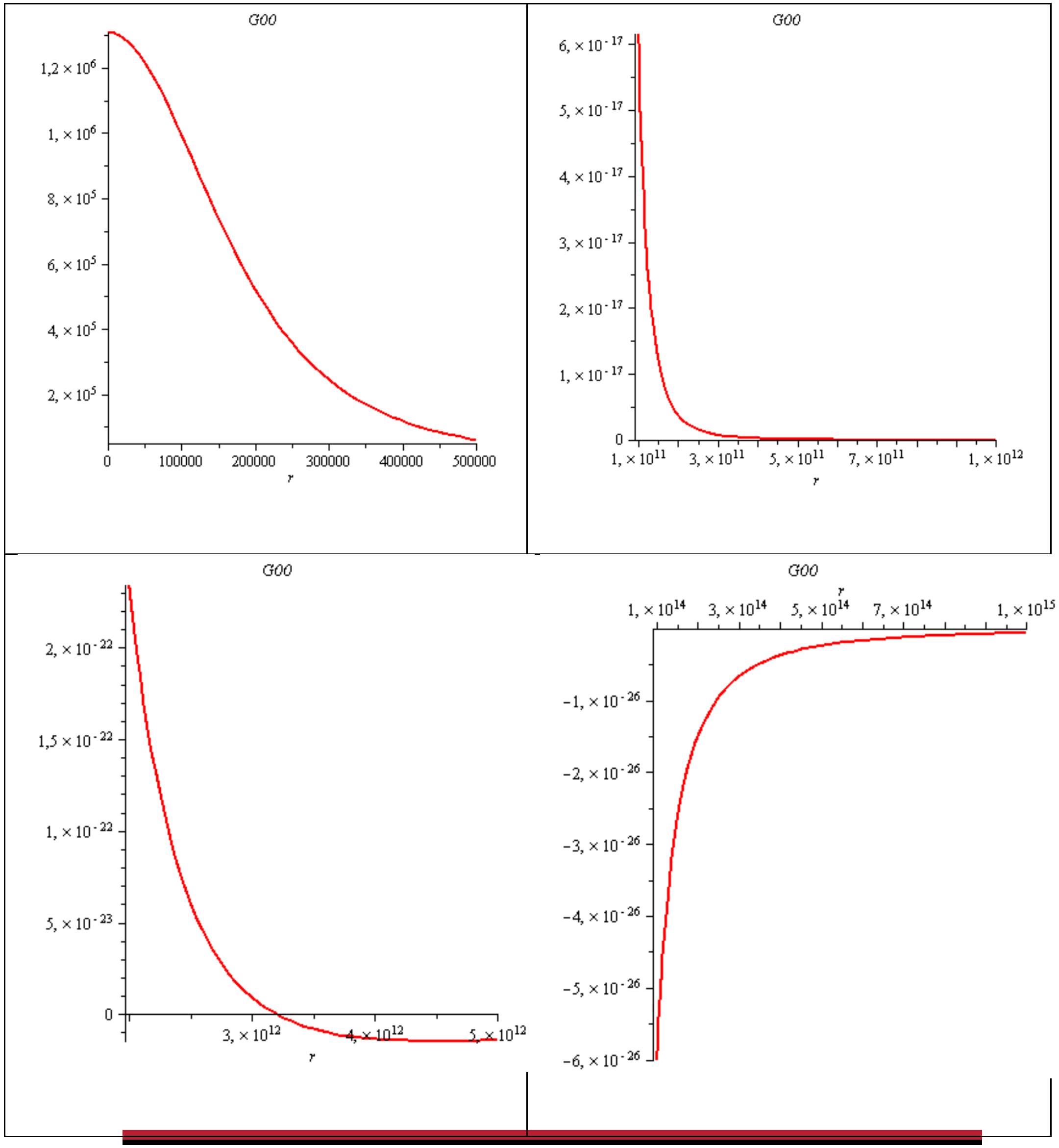
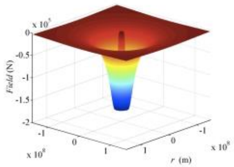
The energy density component





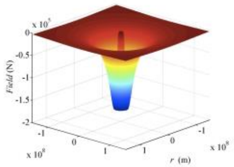
Complexity

The non-null diagonal components make it difficult to interprete these plots since the links of the covariant $G_{\mu\nu}$ with the corresponding contravariant $T^{\mu\nu}$ are not as direct as it is for the symmetric metric. For example:

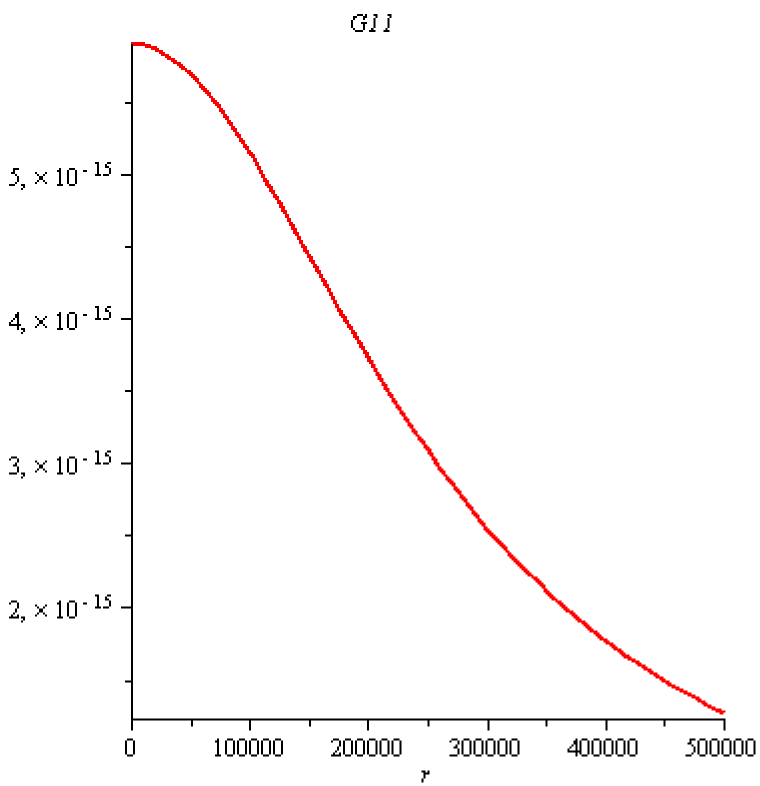
$$G_{00} = g_{00}^2 T^{00} + 2g_{00}g_{01}T^{01} + 2g_{00}g_{03}T^{03} + g_{01}^2 T^{11} + 2g_{01}g_{03}T^{13} + g_{03}^2 T^{33}$$

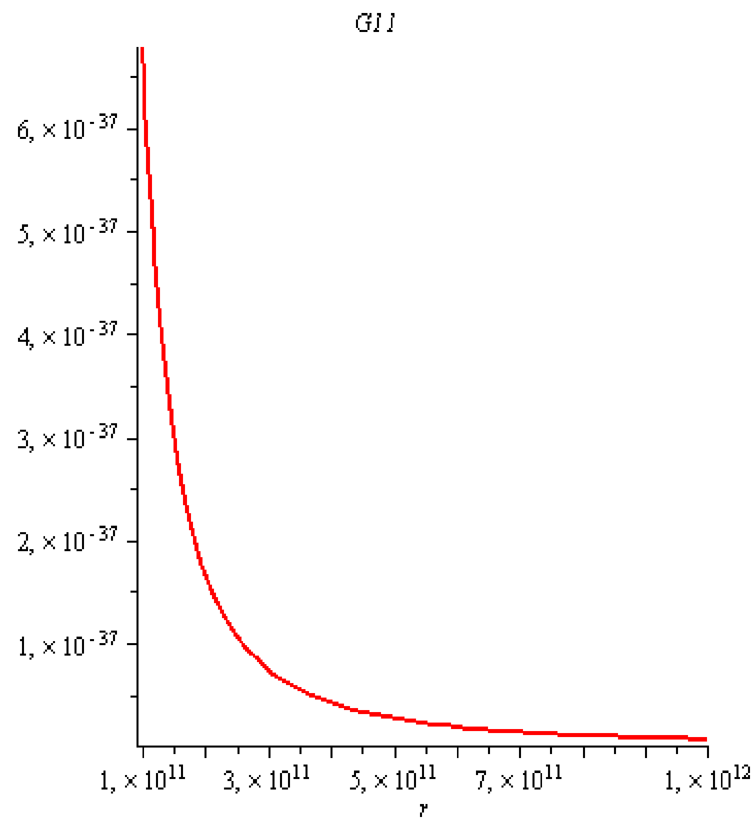
as compared to the direct symmetric interpretation:

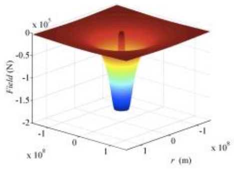
$$G_{00} = KT_{00} = Kg_{00}^2 T^{00}$$



The radial component



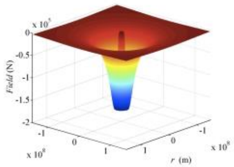




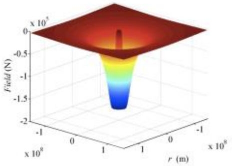
Take home message

Converting an emergent *erfc* symmetric metric into an algebrically equivalent *erf* axisymetric metric provides some insights on the potential origin of the rotation of celestial bodies

…



Plenty of interesting problems for CAP 2018…



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Questions?