Distinguishing the Schwarzschild black hole from the $\mathbb{RP}^3$ geon using local measurements

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The Question

- A detector can detect a reflecting surface (e.g. Casimir-Polder force [1])
- A detector can detect a local gravitational field (e.g. black hole in flat space [2] and AdS [3])
- The equivalence principle suggests that if all fields are in identical states, any two spacetimes should be locally indistinguishable by an observer
- However, the quantum field carries information about global structure, e.g. whether a detector is enclosed by a massive shell [4]
- **Can a detector distinguish a topological identification inside a black hole?**

[1] H. B. G. Casimir, D. Polder (1948)
[2] L. Hodgkinson, J. Louko, A. C. Ottewill (2014)
[3] K. K. Ng, L. Hodgkinson, J. Louko, R. B. Mann, E. Martín-Martínez (2014)
[4] K. K. Ng, Robert B. Mann, Eduardo Martin-Martinez (2016)

Topological Censorship?

- "Wormholes" are unstable in GR, unless the null energy condition is violated [5]
- Nontrivial topologies are hidden behind event horizons
- But this includes *past* event horizons, e.g. white holes...
- Classically, we cannot probe the white hole
- Quantum mechanics predicts the existence of Hawking radiation, which interacts with the topology, and *can* be detected
- Demonstrated using UdW detector transition rates by A. Smith[6] for an asymptotically AdS 2+1 dimensional spacetime (i.e. BTZ)

[5] Friedman, Schleich and Witt, 1993, Phys.Rev.Lett.75.1872

[6] A. Smith, 2014, Class. Quant.Grav. 31 (2014) 082001

The geon

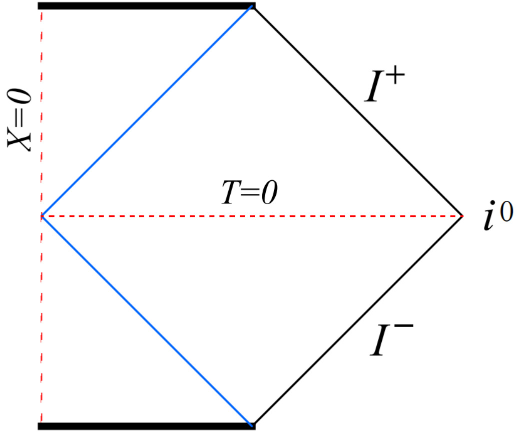


Figure: The $\mathbb{RP}^3$ geon. Picture taken from Louko and Marolf 1998.

The Unruh-DeWitt detector model [7] [8]

- Two-level pointlike system with gap Ω
- Initialized at state $|0\rangle$; other state is $|\Omega\rangle$
- Negative Ω means the detector is initially excited
- Coupled to a scalar field

$$H_I = \lambda\chi(\tau)\mu(\tau)\phi(x(\tau))$$

- Simple expression for transition probability at leading order

$$P(\Omega) = \lambda^2 |\langle 0_d| \mu(0) |\Omega_d\rangle|^2 F(\Omega)$$

$$F(\Omega) = \lim_{\epsilon\to 0} \int_{-\infty}^{\infty} d\tau' \int_{-\infty}^{\infty} d\tau'' \chi(\tau')\chi(\tau'') e^{-i\Omega(\tau''-\tau')} W_\epsilon(\tau', \tau'')$$

[7] W. G. Unruh (1976) Notes on Black Hole Evaporation. *Phys. Rev. D* 14:870

[8] B. S. DeWitt (1979) Quantum Gravity: The new synthesis. *General Relativity: an Einstein Centenary Survey*, ed. S. W. Hawking and W. Israel, Cambridge University Press (Cambridge). pp. 680-745

The $\mathbb{RP}^3$ geon

$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2 d\Omega_2^2$$

$$f(r) = 1 - \frac{2M}{r}$$

- External spacetime is same as in regular Schwarzschild
- Involution applied to remove parallel exterior

$$J : (T, X, \theta, \phi) \to (T, -X, \pi - \theta, \phi + \pi)$$

Making the geon

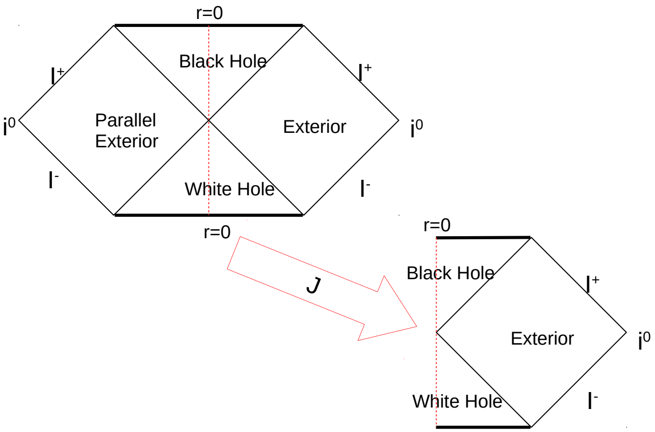


Figure: The Penrose diagram of the Schwarzschild spacetime and the $\mathbb{RP}^3$ geon.

The switching function

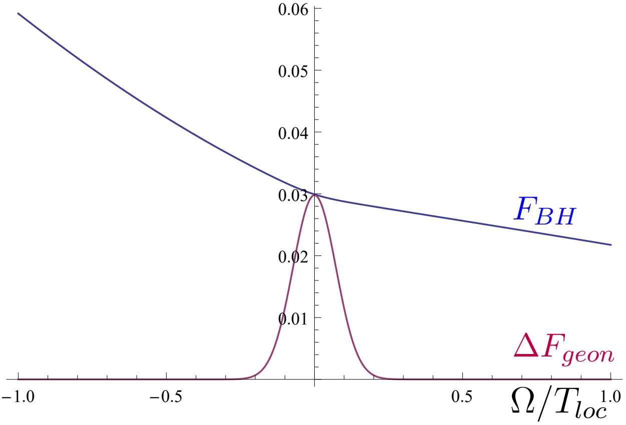
- Simple switching function

$$\chi(\tau) = e^{-\tau^2/2\sigma^2}$$

- Decreasing σ increases 'width' of $\tilde{\chi}(\tilde{\omega})$
- Geon response largest at small $\tilde{\omega}$, so we need large σ
- For computation, our 'default' choices were $\sigma = 100$, $r = 3r_S$, $\Omega \ll T_{loc}$

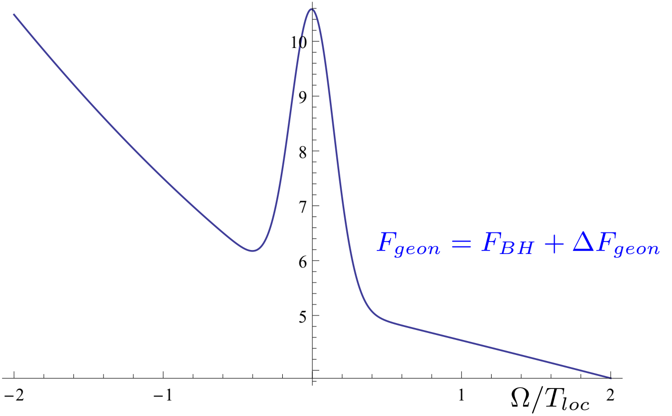
Responses vs. detector gap (r=3)

Figure: BH (blue) and Geon-part (red) response vs. detector gap



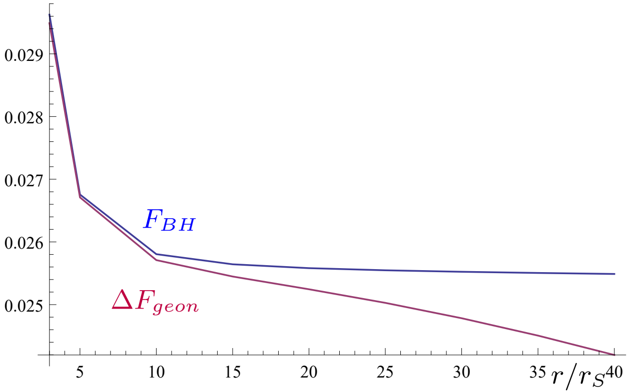
Total response vs. detector gap (r=3)

Figure: Total geon response vs. detector gap



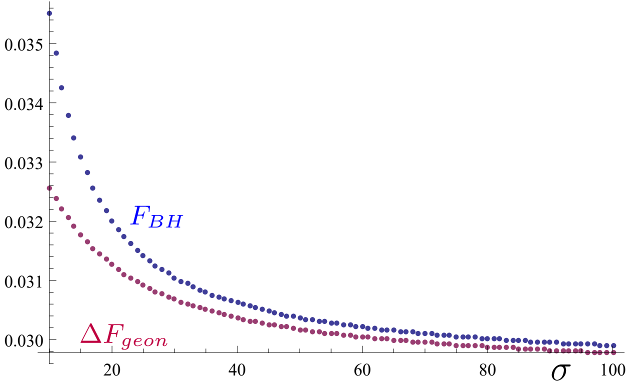
Detector response vs. detector location

Figure: BH (blue) and Geon-part (red) response vs. detector position



Responses vs. width

Figure: BH (blue) and Geon (red) response vs. σ



Unruh detector response vs. detector location

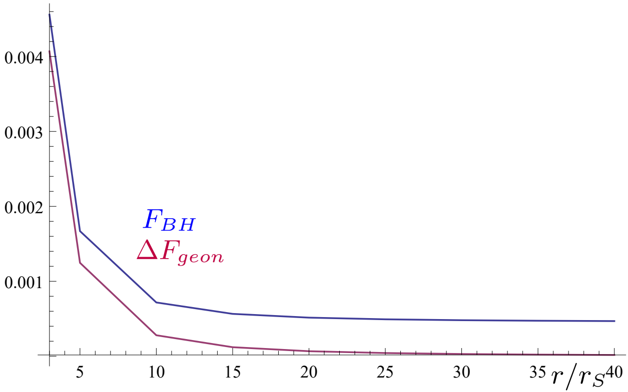
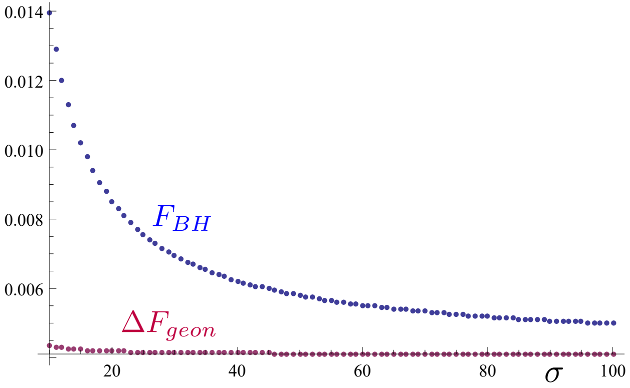


Figure: BH (blue) and Geon-part (red) response vs. detector position

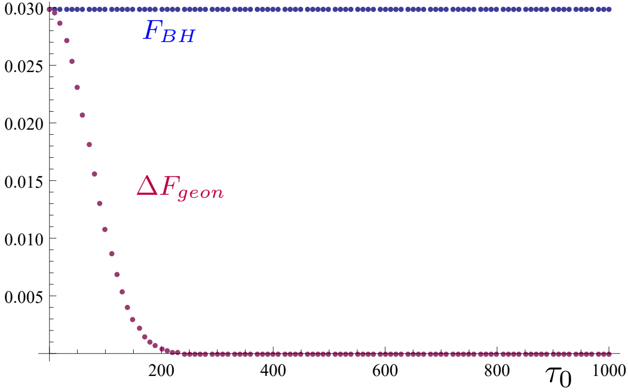
Unruh responses vs. width

Figure: BH (blue) and Geon (red) response vs. σ



Responses vs. peak switching time

Figure: BH (blue) and Geon (red) response vs. peak switching time



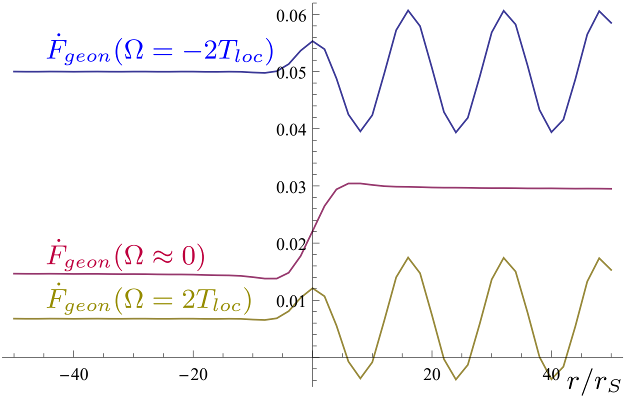
Transition Rates?

$$\chi_{\tau_0} = \Theta(\tau_0 - \tau), \dot{F}(\Omega) = \partial_{\tau_0} F_{\tau_0}(\Omega)$$

- Sudden switching leads to divergences: $\hat{\chi}_{\tau_0}(\tilde{\omega}) = \sqrt{\frac{\pi}{2}}\delta(\tilde{\omega}) - \frac{ie^{i\tau_0\tilde{\omega}}}{\sqrt{2\pi}\tilde{\omega}}$
- These divergences are independent of the field, and depend only on the switching function[9]
- Under certain conditions, these divergences can be eliminated, e.g. when calculating the *difference* between responses, or a time-independent response
- $\dot{F}_{BH}$ and $\Delta\dot{F}_{geon}$ (and therefore $\dot{F}_{geon} = \dot{F}_{BH} + \Delta\dot{F}_{geon}$) can be found

[9] Louko and Satz (Class.Quant.Grav.25:055012, 2008)

$\mathbb{RP}^3$ transition rates



Conclusions

⋆ Classically no experiment can distinguish a geon from a black hole

⋆ But a UdW detector can distinguish them:

- Geon contribution to transition rate is detectable
- Geon contribution vanishes at asymptotic past and future
- Geon contribution vanishes at asymptotic infinity
- **For long switching times, geon contribution equals BH contribution at low energies**

⋆ A quantum detector can distinguish geons from regular black holes
$\Rightarrow$ Even when the only difference is behind the horizon!

The Klein-Gordon equation

Solving the Klein-Gordon equation proceeds as usual in the Schwarzschild case, with an in/up basis.
See Hodgkinson, Louko and Ottewill, Phys. Rev. D 89, 104002 (2014)

$$r^* = -\int_r^\infty \frac{dr'}{f(r')}$$

$$w_{\omega lm} = (4\pi\omega)^{-1/2} e^{-i\omega t} Y_{lm}(\theta, \phi) r^{-1} \Phi_{\omega lm}(r)$$

$$[\partial_{r^*}^2 + \omega^2 - \tilde{V}(r^*)]\Phi = 0$$

$$\tilde{V}(r) = f(r)V(r),\ V(r) = \frac{l(l+1)}{r^2} + \frac{r_0}{r^3}$$

$$R_{\omega lm} = \Phi_{\omega lm}/r$$

Hartle-Hawking vacuum

$$
\begin{aligned}
F_{BH}(\Omega) =& \sum_{l=0}^{\infty} \int_0^{\infty} \frac{(2l+1)d\tilde{\omega}}{8\pi\omega\sinh(2\pi\omega)} \left(|R_{\omega l}^{\mathrm{in}}|^2 + |R_{\omega l}^{\mathrm{up}}|^2\right) \\
& \times \sqrt{\frac{\sigma^2}{\pi}} e^{-\sigma^2(\tilde{\omega}^2+\Omega^2)} \cosh(2\pi\omega - 2\sigma^2\tilde{\omega}\Omega) \\
F_J(\Omega) =& \sum_{l=0}^{\infty} \int_0^{\infty} \frac{(-1)^l(2l+1)d\tilde{\omega}}{8\pi\omega\sinh(2\pi\omega)} \left(|R_{\omega l}^{\mathrm{in}}|^2 + |R_{\omega l}^{\mathrm{up}}|^2\right) \\
& \times \sqrt{\frac{\sigma^2}{\pi}} e^{-\sigma^2(\tilde{\omega}^2+\Omega^2)} \\
=& e^{-\sigma^2\Omega^2} F_J(0)
\end{aligned}
$$

Unruh vacuum

The Unruh vacuum represents a radiating black hole that does *not* receive radiation from afar.

$$F_{BH,U} = \sum_{l=0}^{\infty} \int_0^{\infty} \frac{(2l+1)d\tilde{\omega}}{8\pi\omega} \sqrt{\frac{\sigma^2}{\pi}} \left(e^{-\sigma^2(\tilde{\omega}-\Omega)^2} |R^{\mathrm{in}}_{\omega l}|^2 \right.$$
$$\left. + \; e^{-\sigma(\tilde{\omega}^2+\Omega^2)} \frac{\cosh(2\pi\omega - 2\sigma\tilde{\omega}\Omega)}{\sinh(2\pi\omega)} |R^{\mathrm{up}}_{\omega l}|^2 \right) \quad (1)$$

$\mathbb{RP}^3$ transition rates

$$
\begin{aligned}
\dot{F}_{BH}(\Omega) =& \sum_{l=0}^{\infty} \frac{2l+1}{16\pi\tilde{\Omega}\sinh(2\pi\tilde{\Omega})}(|R^{in}_{\tilde{\Omega}l}|^2 + |R^{up}_{\tilde{\Omega}l}|^2) \\
& \times e^{-2\pi\tilde{\Omega}} \\
\Delta\dot{F}_{geon}(\Omega) =& \sum_{l=0}^{\infty} \frac{(-1)^l(2l+1)}{16\pi\tilde{\Omega}\sinh(2\pi\tilde{\Omega})}(|R^{in}_{\tilde{\Omega}l}|^2 + |R^{up}_{\tilde{\Omega}l}|^2) \\
& \times \cos(2\tau_0\Omega) \\
& + \sum_{l=0}^{\infty} \int_0^{\infty} \frac{(-1)^l(2l+1)d\tilde{\omega}}{16\pi\omega\sinh(2\pi\omega)} \left(|R^{\mathrm{in}}_{\omega l}|^2 + |R^{\mathrm{up}}_{\omega l}|^2\right) \\
& \times \frac{2\tilde{\omega}\sin(2\tau_0\tilde{\omega})}{\pi(\tilde{\omega}^2 - \Omega^2)} \\
\tilde{\Omega} =& \Omega\sqrt{1 - 1/r}
\end{aligned}
$$