Foliation dependence of black hole apparent horizons in spherical symmetry

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Outline

INTRO

A black hole is *defined by its horizons*: the "frontier between things observable and things unobservable". Black hole mechanics and thermodynamics developed in the 1970s are based on *event horizons* (null surfaces) for stationary black holes.
Realistic black holes are ultimately non-stationary:

- astrophysical BH: interacts with accretion disk/fluid, with a companion in a binary system; BH mergers (*LIGO* detections of grav. waves)
- mathematical BH: Hawking radiation, backreaction; embedded in dynamical universe.

An event horizon is a (connected component of) the causal past of future null infinity $\mathscr{I}^+$. Trace back all the light rays which make it to infinity until they hit the boundary of the black hole region (from which light cannot escape).
It is a global and teleological concept, requires the knowledge of the entire causal structure of spacetime. Virtually useless for highly dynamical situations.

The apparent/trapping horizon is a quasilocal concept.

Apparent/trapping horizons defined using congruences of null rays (no reference to global structure):

- outgoing null geodesics (tangent l^a)
- ingoing null geodesics (tangent n^a)
- their expansions $\theta_{(+)} = \nabla^c l_c$, $\theta_{(-)} = \nabla^c n_c$

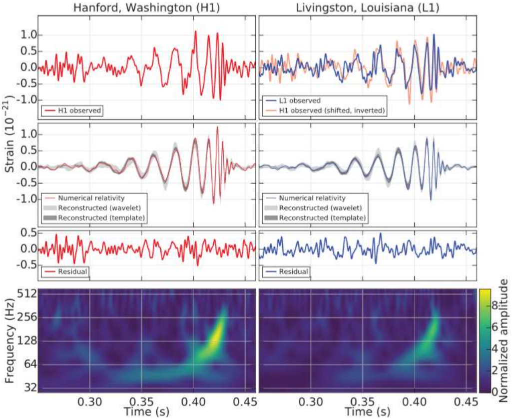
a **future apparent horizon** is a surface defined by

$$\theta_{(+)} = 0 \quad \text{and} \quad \theta_{(-)} < 0$$

a future outer trapping horizon is characterized by

$$\theta_{(+)} = 0$$
$$\theta_{(-)} < 0$$
$$n^c \nabla_c \theta_{(+)} < 0$$

It is much easier to locate *quasilocal* AHs than event horizons. The two recent *LIGO* detections of black hole mergers measured masses/orbital parameters by comparing data with templates of grav. waveforms built using AHs



in astrophysics, we use AHs, not event horizons

Problem: **apparent horizons depend on the foliation** ($3+1$ spacetime splitting, or observer). Epitomized by the fact that in Schwarzschild space there exist (contrived) foliations with no AHs (Wald & Iyer 1991; Schnetter & Krishnan 2006).

Does the existence of a (dynamical) BH depend on the observer?

A real problem, but no better candidate than AHs (unless BH is completely isolated). Try to live with this problem.

The problem would be alleviated (not solved) if there were “preferred foliations” (*e.g.*, fixed by symmetries). Fixing the observer is already necessary, *e.g.*, when computing the BH temperature in QFT.

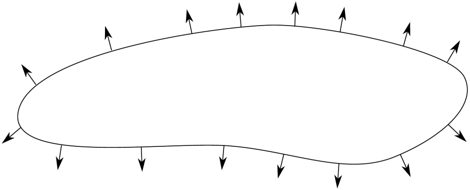
Restrict to GR (no theories with preferred frames and Lorentz violation).

In the presence of symmetry, the symmetric solution breaks the invariance of the general theory, so a symmetric foliation seems reasonable. Compare with FLRW and comoving observers, which are preferred in some physical sense–they are the only ones who see the cosmic microwave background homogeneous and isotropic (apart from small fluctuations $\delta T/T \sim 10^{-5}$).

A natural solution?

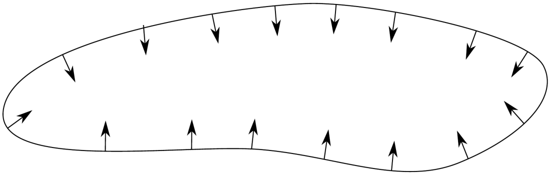
Consider a spacetime $(\mathcal{M}, g_{ab})$, let $\mathcal{S}$ be a spacelike, embedded, compact, orientable 2-surface which lies in some hypersurface $\mathcal{H}$ that is a surface of simultaneity for some family of observers u^a. Let

h_{ab} = 2-metric induced by g_{ab} on $\mathcal{S}$;
$\mathcal{R}^{(h)}$ = Ricci scalar of h_{ab};
$\pm$ denote outgoing/ingoing null geodesic congruences from $\mathcal{S}$;
$\theta_{(\pm)}$ = their expansions;
$\sigma^{(\pm)}_{ab}$ = their shear tensors;
ω^a = anholonomicity (projection onto $\mathcal{S}$ of the commutator of the null normals to $\mathcal{S}$);
μ = volume 2-form on $\mathcal{S}$; A = area of $\mathcal{S}$.



The Hawking-Hayward energy used in BH studies is

$$M_{HH} = \frac{1}{8\pi}\sqrt{\frac{A}{16\pi}}\int_{\mathcal{S}} \mu \left(\mathcal{R}^{(h)} + \theta_{(+)}\theta_{(-)} - \frac{1}{2}\sigma_{ab}^{(+)}\sigma_{(-)}^{ab} - 2\omega_a\omega^a \right)$$



A possible (maybe natural) choice is identifying the observer's 4-velocity u^a with the (timelike) normal n^a to $\mathcal{S}$, *i.e.*, picking the observers who see $\mathcal{S}$ and the matter on it at rest. M_{HH} is the 0-component of a (timelike) 4-momentum vector P^a and is naturally gauge-dependent. As in SR, the "mass" of a particle is the rest mass, so the observer defining the AH and its quasilocal mass would be the one that sees the AH at rest.

Does it work when applied to AHs? **NO**, because:

- AHs can be time/space/light-like and $\mathcal{S}$ is the 2-D intersection of the 3-D AH worldtube (generated by a vector field t^a) with a time slice. This is always a 2-D spacelike surface, but setting $u^a = n^a$ implies $u^a \propto t^a$, which can be null or spacelike and the definition of M_{HH} is invalid.
- In practice the AH is never at rest in the frames used in numerical collapse studies (*e.g.*, comoving gauge, the AH is never comoving).

Need to fix $\mathbf{u^a \neq n^a}$ but how? No preferred u^a in general spacetimes, but there are preferred u^a in spherical symmetry.

SPHERICAL SYMMETRY

Adopt a pragmatic approach: no cure for the foliation-dependence problem, but it can be alleviated in spherical symmetry (SS). There is a *restricted* gauge-independence of the AHs if we limit to spherically symmetric foliations.

In SS the AHs are located by the *scalar* equation

$$\boxed{\nabla^c R \nabla_c R = 0}$$

where $R =$ areal radius (a geometric, gauge-invariant quantity).

All SS foliations produce the same AHs

AHs tied to the Misner-Sharp-Hernandez mass M_{MSH} (special case of Hawking-Hayward quasilocal energy, internal energy in the 1st law of BH thermodynamics), which satisfies

$$1 - \frac{2M_{MSH}}{R} = \nabla^c R \nabla_c R$$

and, on the apparent horizons,

$$R_{AH} = 2M_{MSH}(R_{AH})$$

like for the Schwarzschild black hole (but now dynamical)

Choose observers (foliations) adapted to SS:

$$u^{\mu} = \left(u^0, u^1, 0, 0\right)$$

in coordinates adapted to the SS. In general, 2-sphere $\mathcal{S}$ is in radial motion w.r.t observers u^a.
Check comoving gauge and Kodama gauge, the two most used in the literature.

COMOVING GAUGE

Mostly used in studies of gravitational collapse: u^a is the 4-velocity of the collapsing (perfect) fluid, described by the stress-energy tensor

$$T_{ab} = (P + \rho)\, u_a u_b + P g_{ab}$$

Line element is

$$\boxed{ds^2 = -e^{2\phi(t,r)} dt^2 + e^{\lambda(t,r)} dr^2 + R^2(t,r) d\Omega_{(2)}^2}$$

(no heat flux $u_a T^{ab} g^{(3)}_{bc}$ in this gauge). AHs are almost never comoving with the fluid and

$$u^\mu = \left(e^{-\phi}, 0, 0, 0 \right) \neq n^\mu \quad \text{of } \mathcal{S}$$

Tools adapted to comoving gauge:

$$D_t = e^{-\phi}\,\partial_t \quad \text{derivative w.r.t. proper time}$$

$$D_r = e^{-\lambda/2}\partial_r \quad \text{derivative w.r.t. proper radius}$$

and

$$\mathbf{U} \equiv D_t R = e^{-\phi}\dot{R} \equiv e^{-\phi}\partial_t R$$

$$\mathbf{\Gamma} \equiv D_r R = e^{-\lambda/2}R' \equiv e^{-\lambda/2}\partial_r R$$

The equation locating the AHs is (Helou, Musco & Miller 2016)

$$\nabla^c R \nabla_c R = 0 \;\Leftrightarrow\; \boxed{\Gamma^2 - U^2 = 0}$$

The line element is rewritten as

$$ds^2 = -\frac{\dot{R}^2}{U^2} dt^2 + \frac{R'^2}{\Gamma^2} dr^2 + R^2 d\Omega^2_{(2)}$$

If $\tau \equiv$ proper time of comoving observer,

$$\frac{dR}{d\tau} = U + \Gamma v$$

and, at the AH,

$$v_{AH} = \left.\frac{dR}{d\tau}\right|_{AH} = \frac{1 + 8\pi R^2_{AH} P}{1 - 8\pi R^2_{AH} \rho}$$

KODAMA GAUGE

Mostly used in the thermodynamics of dynamical horizons; based on the *Kodama vector*

$$K^a \equiv \epsilon^{ab}\nabla_b R$$

($\epsilon^{ab} =$ volume form of 2-metric h_{ab}), lies in (T, R)-subspace orthogonal to 2-spheres of symmetry, defined in gauge-independent way.
$J^a \equiv G^{ab}K_b$ is surprisingly conserved, $\nabla^b J_b = 0$ ("Kodama miracle"), and M_{MSH} is the corresponding Noether charge.
The line element is

$$ds^2 = g_{00}(T, R)dT^2 + g_{11}(T, R)dR^2 + R^2 d\Omega^2_{(2)}$$

The AHs are located by

$$g^{RR}(T,R)=0$$

$K^a = \frac{1}{\sqrt{|g_{TT}|g_{RR}}}\left(\frac{\partial}{\partial T}\right)^a$ identifies a preferred “Kodama time” and is a substitute for a timelike Killing vector where there is none.

K^a is

- timelike outside AH
- null on AH
- spacelike inside.

The "tunneling method" uses this time to compute the surface gravity κ and temperature $T = \kappa/2\pi$ of dynamical BHs.

Identify the surface $\mathcal{S}$ in the definition of $M_{HH} = M_{MSH}$ with the AH and use the Kodama observer. Then M_{HH} is the "Newtonian" mass in a frame in which there is no spatial flow of Kodama energy $\neq$ the frame in which $\mathcal{S}$ is at rest.

The AH is in radial motion in the Kodama foliation since $n^1 \neq 0$.

The Kodama current in Kodama coords. is

$$J^\mu = \left(G^{00} K_0, G^{10} K_0, 0, 0 \right)$$

RELATION BETWEEN COMOVING AND KODAMA GAUGES

Find transformation between comoving and Kodama gauges:

$$R = R(t,r) \rightarrow \ dr = \frac{dR - \dot{R}dt}{R'}$$

sub into comoving gauge line element

$$\begin{aligned} ds^2 &= -e^{2\phi}dt^2 + e^{\lambda}dr^2 + R^2 d\Omega^2_{(2)} \\ &= -\left(e^{2\phi} - \frac{\dot{R}^2}{R'^2}\right)dt^2 + \frac{e^{\lambda}}{R'^2}dR^2 - \frac{2\dot{R}e^{\lambda}}{R'^2}dtdR + R^2 d\Omega^2_{(2)} \end{aligned}$$

eliminate cross-term in $dtdR$ by redefining time coordinate $t \to T(t,r)$ such that

$$dT = \frac{1}{F}\left(dt + \beta dR\right)$$

where

$$F = \text{ integrating factor guaranteeing that } dT \text{ is exact}$$

$$\beta(t,R) = \text{ function to be determined}$$

Then

$$ds^2 = \ ... + 2F\left[\beta\left(e^{2\phi} - \frac{\dot{R}^2 e^{\lambda}}{R'^2}\right) - \frac{\dot{R}e^{\lambda}}{R'^2}\right] dTdR + \ ...$$

set

$$\beta(t,R) = \frac{\dot{R}\, e^{\lambda}}{R'^2\left(e^{2\phi} - e^{\lambda}\dot{R}^2/R'^2\right)}$$

Then line element is diagonalized

$$
\begin{aligned}
ds^2 &= -\left(e^{2\phi} - \frac{\dot{R}^2 e^{\lambda}}{R'^2} \right) F^2 dT^2 + \frac{e^{\lambda+2\phi} dR^2}{R'^2 e^{2\phi} - \dot{R}^2 e^{\lambda}} + R^2 d\Omega^2_{(2)} \\
&= -e^{2\phi} \left(1 - \frac{U^2}{\Gamma^2} \right) F^2 dT^2 + \frac{dR^2}{\Gamma^2 - U^2} + R^2 d\Omega^2_{(2)}
\end{aligned}
$$

the equation locating the apparent horizons is now

$$\nabla^c R \nabla_c R = 0 \Leftrightarrow g^{RR} = 0 \Leftrightarrow \boxed{\mathbf{U = \pm \Gamma}}$$

AHs in comoving gauge coincide with AHs in Kodama gauge

($-$ for black hole AH, $+$ for cosmological AH)

4-velocity of Kodama observers

$$u^{\mu}_{(K)} = \left(\frac{e^{-\phi}}{F\sqrt{1 - U^2/\Gamma^2}}, 0, 0, 0 \right)$$

Consider coordinate transformation
x^{μ} (comoving) $\to x^{\mu'}$ (Kodama), then

$$u^{\mu'}_{(C)} = \frac{\partial x^{\mu'}}{\partial x^{\mu}}\, u^{\mu}_{(C)} = e^{-\phi}\frac{\partial x^{\mu}}{\partial t} = \left(\frac{e^{-\phi}}{F\left(1 - U^2/\Gamma^2\right)}, U, 0, 0 \right)$$

and

$$u^{a}_{(K)} u^{(C)}_{a} = -\frac{1}{\sqrt{1 - U^2/\Gamma^2}} = -\gamma\left(v_{relative}\right)$$

$$|v_{relative}| = \left| \frac{U}{\Gamma} \right| , \qquad |v_{relative}|_{AH} = 1$$

Even though AHs are the same for Kodama and comoving observers, these surfaces are perceived differently:

- they accelerate w.r.t. one another;
- vacuum state (Kodama) $\neq$ vacuum state (comoving);
- surface gravities $\kappa_{(K)} \neq \kappa_{(C)}$;
- black hole temperatures $T_{(K)} \neq T_{(C)}$. Thermodynamics remains fully gauge-dependent.

Example: 3-velocity of AH

$$\left(v_{AH}^{(K)}\right)^2 = \left(\frac{g_{00}\,dR}{g_{11}\,dt}\right)^2 = 1 \neq \left(v_{AH}^{(C)}\right)^2 = \frac{1 + 8\pi R_{AH}^2 P}{1 - 8\pi R_{AH}^2 \rho}$$

CONCLUSIONS

- Event horizons useless in dynamical situations. AHs used, but they depend on the foliation. Does the existence of a BH depend on the observer?
- In general, no solution yet to this problem $\rightarrow$ pragmatic approach.
- Spherical symmetric foliations natural in the presence of spherical symmetry. *With this restriction*, foliation-dependence problem is circumvented (but BH thermodynamics remains fully gauge-dependent). Checked explicitly for the two most used foliations.
- What should we do in the general situation (no symmetries)?

THANK YOU