Harvesting Entanglement with Inertially Moving Detectors

Laura Henderson
R. B. Mann
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What is "Entanglement Harvesting"?

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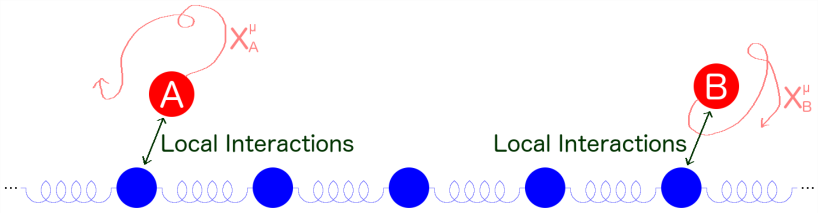

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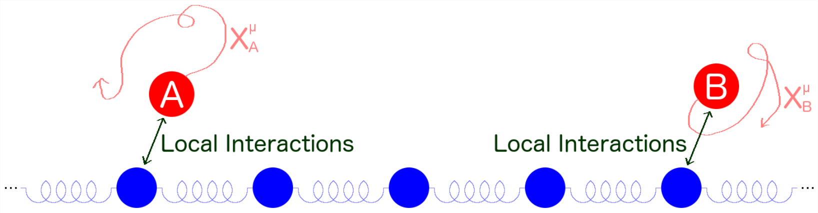

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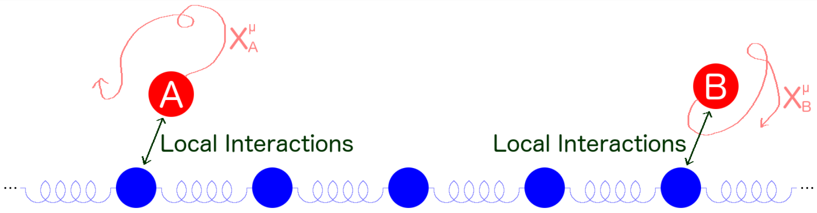
3. This entanglement has be swapped from the field to the detectors.



History

- Summers and Werner showed in 1985 that there are correlations between timelike and spacelike separated regions of a free quantum field in the vacuum state[1].

[1] S. J. Summers and R. F. Werner, Phys. Lett. A 110, 257 (1985).
[2] A. Valentini, Phys. Lett. A 153, 321 (1991).
[3] G. Salton, R. B. Mann, and N. C. Menicucci, New J. Phys. 17, 035001 (2015).
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- In 2016 Pozas-Kerstjens and Martín-Martínez calculated the entanglement for a pair of stationary detectors over a variety of spacial profiles and switching functions[4].

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What happens to entanglement harvesting when two detectors move with a **constant velocity**?

The Field and the Detector

- The field is a massless scalar field, $\hat{\phi}(\mathbf{x}, t)$, in vacuum state, $|0\rangle$.

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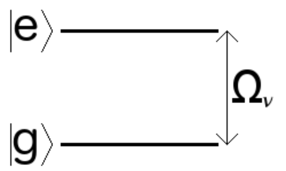

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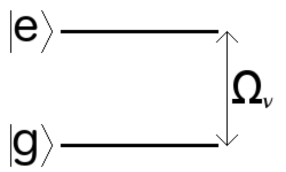

  - They detect a particle when they move from the ground, $|g\rangle$, to the excited state, $|e\rangle$.

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The Interaction Hamiltonian

The detectors (A and B) are coupled to the scalar field by the interaction Hamiltonian[6]

$$\hat{H}_I = \lambda \sum_{\nu \in \{A,B\}} \chi(t)\hat{\mu}_\nu(t)\hat{\phi}\big(\mathbf{x}_\nu(t)\big),$$

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- $\mathbf{x}_\nu(t)$ is the spacetime position of each detector.

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Evolution of the System

1. The detector-field system is initially in the state

$$\rho_0 = |0\rangle\langle 0| \otimes |g_A\rangle\langle g_A| \otimes |g_B\rangle\langle g_B| \,.$$

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3. Trace out the field to get the final state of the two detectors

$$\rho_{AB} = \mathrm{Tr}_\phi\left[\rho\right] = \begin{pmatrix} 1-\mathcal{L}_{AA}-\mathcal{L}_{BB} & 0 & 0 & \mathcal{M}^* \\ 0 & \mathcal{L}_{AA} & \mathcal{L}_{AB} & 0 \\ 0 & \mathcal{L}_{BA} & \mathcal{L}_{BB} & 0 \\ \mathcal{M} & 0 & 0 & 0 \end{pmatrix} + \mathcal{O}(\lambda_\nu^4).$$

Measuring Entanglement

The entanglement is measured through the the negativity $\mathcal{N}$ (an entanglement monotone)[7].

$$\mathcal{N} = \frac{\mathrm{Tr}\left|\rho^{T_A}\right| - 1}{2}.$$

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It is, to second order in λ

$$\mathcal{N}^{(2)} = -\frac{1}{2}\left(\mathcal{L}_{AA} + \mathcal{L}_{BB} - \sqrt{\left(\mathcal{L}_{AA} - \mathcal{L}_{BB}\right)^2 + 4|\mathcal{M}|^2}\right).$$

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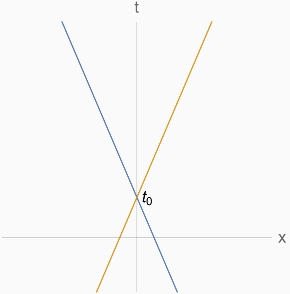
Uniform Velocity

The detectors have opposite velocities (equal speeds) and pass by each other at $t = t_0$.

$$x_A(t) = v(t - t_0) \qquad\qquad x_B(t) = -v(t - t_0).$$



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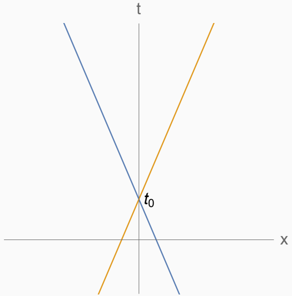


This is a 1+1 dimensional problem, so it requires an infrared (IR) cutoff, Λ.

$\mathcal{L}$ and $\mathcal{M}$

$$\mathcal{L}_{AA} = \frac{\lambda^2\sigma^2}{2}\left(1 - v^2\right)\int_\Lambda \frac{dk}{|k|} e^{-\sigma^2 G_-(k)^2}$$

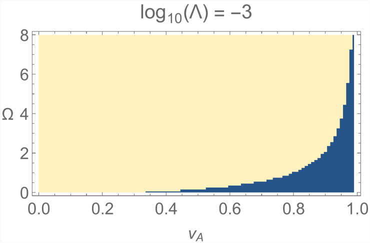
$$\mathcal{L}_{BB} = \frac{\lambda^2\sigma^2}{2}\left(1 - v^2\right)\int_\Lambda \frac{dk}{|k|} e^{-\sigma^2 G_+(k)^2}$$

$$\mathcal{M} = \frac{\lambda^2\sigma^2}{4}(1 - v^2)\int_\Lambda \frac{dk}{|k|}\,\mathrm{erf}\left(i\sigma|k|\right) \times \left[e^{-2ikvt0} e^{-\frac{\sigma^2}{2}\left(G_+(k)^2 + P_-(k)^2\right)} + e^{2ikvt0} e^{-\frac{\sigma^2}{2}\left(G_-(k)^2 + P_+(k)^2\right)}\right]$$

where

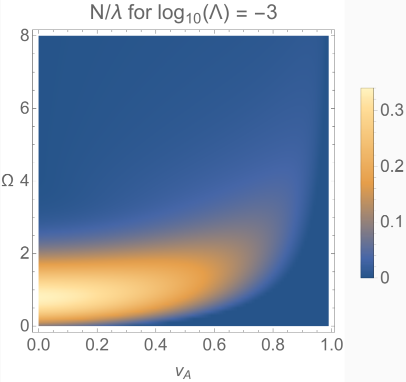
$$G_\pm(k) = |k| + (1 - v^2)\Omega \pm vk, \quad P_\pm(k) = |k| - (1 - v^2)\Omega \pm vk.$$

Negativity

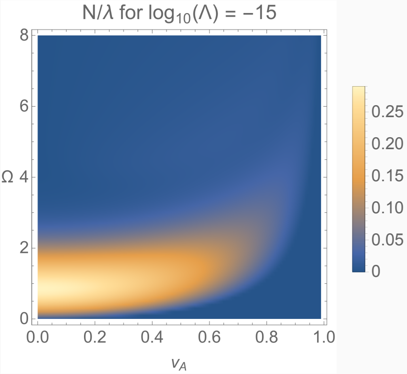


Yellow = Entanglement; Blue = No Entanglement

Negativity

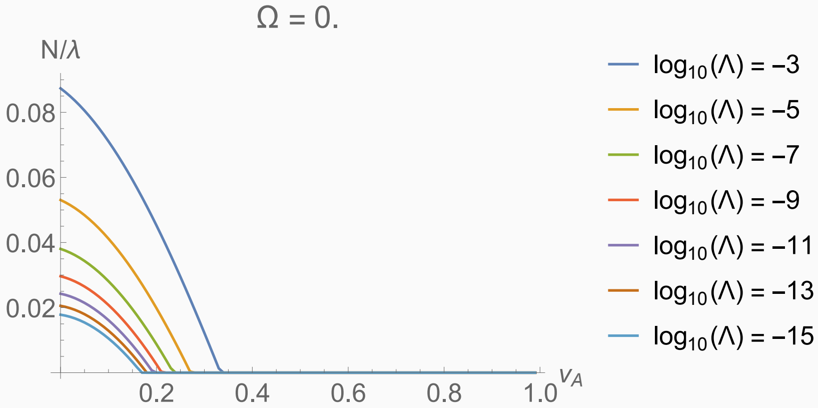


N/λ for log10(Λ) = −3
Ω
v_A
0
2
4
6
8
0.0
0.2
0.4
0.6
0.8
1.0
0.3
0.2
0.1
0



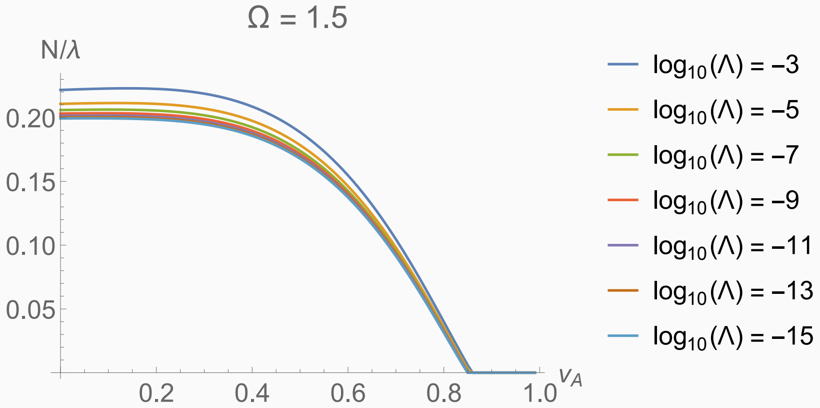
N/λ for log10(Λ) = −15
Ω
v_A
0
2
4
6
8
0.0
0.2
0.4
0.6
0.8
1.0
0.25
0.20
0.15
0.10
0.05
0

$\Omega = 0$



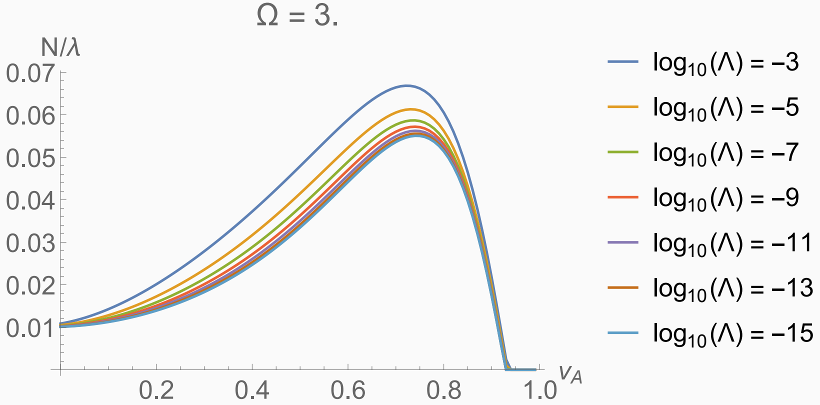
Ω = 0.
N/λ
0.08
0.06
0.04
0.02
0.2
0.4
0.6
0.8
1.0
v_A
log10(Λ) = −3
log10(Λ) = −5
log10(Λ) = −7
log10(Λ) = −9
log10(Λ) = −11
log10(Λ) = −13
log10(Λ) = −15

$\Omega = 1.5$



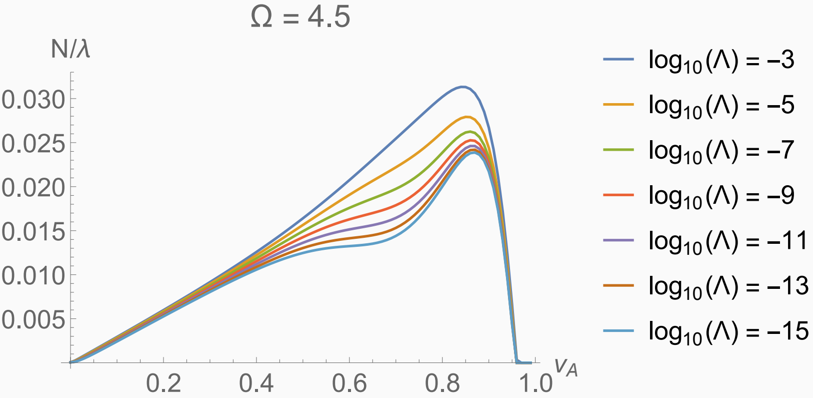
Ω = 1.5
N/λ
0.20
0.15
0.10
0.05
0.2
0.4
0.6
0.8
1.0
v_A
log10(Λ) = −3
log10(Λ) = −5
log10(Λ) = −7
log10(Λ) = −9
log10(Λ) = −11
log10(Λ) = −13
log10(Λ) = −15

$\Omega = 3$

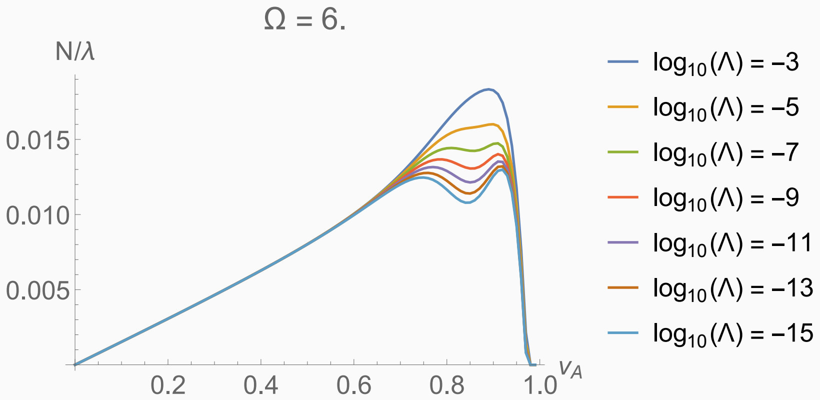


Ω = 3.
N/λ
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0.2
0.4
0.6
0.8
1.0
v_A
$\log_{10}(\Lambda) = -3$
$\log_{10}(\Lambda) = -5$
$\log_{10}(\Lambda) = -7$
$\log_{10}(\Lambda) = -9$
$\log_{10}(\Lambda) = -11$
$\log_{10}(\Lambda) = -13$
$\log_{10}(\Lambda) = -15$

$\Omega = 4.5$

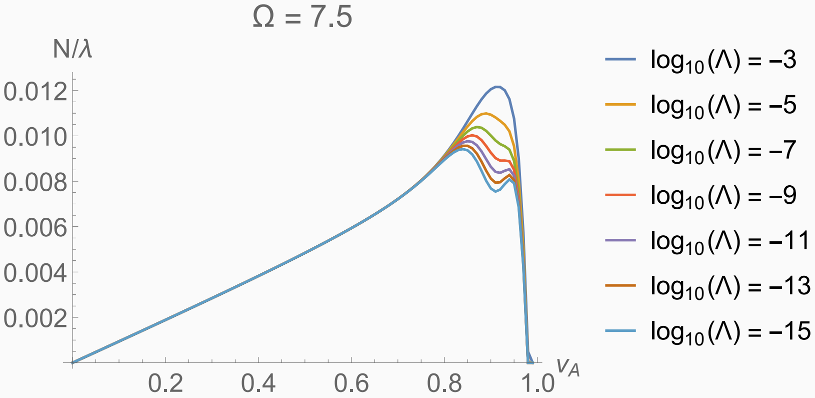

Ω = 4.5
N/λ
0.030
0.025
0.020
0.015
0.010
0.005
0.2
0.4
0.6
0.8
1.0
vA
log10(Λ) = −3
log10(Λ) = −5
log10(Λ) = −7
log10(Λ) = −9
log10(Λ) = −11
log10(Λ) = −13
log10(Λ) = −15

$\Omega = 6$



Ω = 6.
N/λ
0.015
0.010
0.005
0.2
0.4
0.6
0.8
1.0
v_A
log10(Λ) = −3
log10(Λ) = −5
log10(Λ) = −7
log10(Λ) = −9
log10(Λ) = −11
log10(Λ) = −13
log10(Λ) = −15

$\Omega = 7.5$



Ω = 7.5
N/λ
0.012
0.010
0.008
0.006
0.004
0.002
0.2
0.4
0.6
0.8
1.0
v_A
log10(Λ) = −3
log10(Λ) = −5
log10(Λ) = −7
log10(Λ) = −9
log10(Λ) = −11
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- For each energy gap, there is a different velocity which maximizes the entanglement.
- As the energy gap increases more local maxima form in the velocity curves.
- A sufficiently small IR cutoff is required to observe this extra structure.