# Non-perturbative calculations in scalar field theories

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# Introduction

strong coupling $\rightarrow$ can't use perturbation theory

different approaches (for example):

• lattice calculations
$\rightarrow$ *continuum and infinite volume limits*

• continuum methods
- Schwinger-Dyson equations
- *n*-particle irreducible (*n*pi) effective theories
- renormalization group (RG)

$\rightarrow$ *truncation*

my work: npi using a renormalization group approach

*I will discuss mostly symmetric scalar* $\varphi^4$ *theory*

# Introduction to npi

2pi for scalar theories:

generating functional with local and bi-local sources

$$Z[J,B] = e^{iW[J,B]} = \int \mathcal{D}\varphi e^{i(S[\varphi]+J_i\varphi_i+\frac{1}{2}\varphi_i B_{ij}\varphi_j)}$$

short-hand notation:

$$\int dx \int dy \; \varphi(x)B(x,y)\varphi(y) \to \varphi_i B_{ij}\varphi_j \to B\varphi^2$$

# Legendre transform:

$$\Gamma[\phi, G] = W[J, B] - J_i\phi_i - \frac{1}{2}B_{ij}\phi_i\phi_j$$

$$= S_{\rm cl}[\phi] + \frac{i}{2}{\rm Tr}\ln G^{-1} + \frac{i}{2}{\rm Tr}\, G_0^{-1}(G - G_0) + \Gamma_2[\phi, G]$$

$\Gamma[\phi, G]$ is a functional of the 1- and 2-point functions

$\phi$ and $G$ determined self-consistently from equations of motion

variational principle (in the absence of sources)

$$\frac{\delta\Gamma}{\delta\phi} = \frac{\delta\Gamma}{\delta G} = 0$$

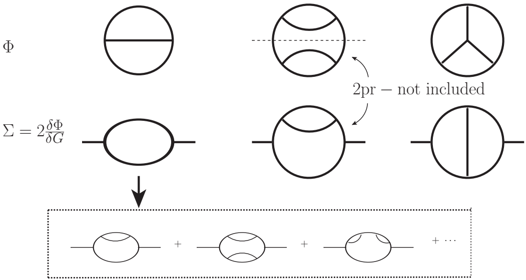
## compare to $\Gamma[\phi]$ = 1pi effective action:

- $\Gamma[\phi, G]$ depends on the self consistent propagator

$\rightarrow$ truncated $\Gamma[\phi, G]$ includes an infinite resummation of diagrams

$\rightarrow$ non-perturbative

- $\Gamma[\phi, G]$ is 2pi - no double counting



# npi effective action

npi $\Gamma$ is a functional of $n$-point functions

3pi $\Gamma[\phi, G, U]$, 4pi $\Gamma[\phi, G, U, V] \cdots$

$n$-point functions determined self-consistently from the eom's

$\Rightarrow$ hierarchy of coupled equations

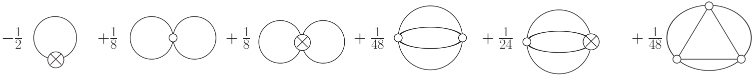
- no exact solution method is available
- approximation $\rightarrow$ truncate the effective action

# 4 loop 2pi effective action

$$\Phi = \Phi_{\text{no}\cdot\text{int}} + \Phi_{\text{int}} \qquad\qquad (\Phi = i\Gamma)$$

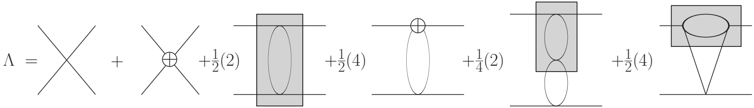
$$\Phi_{\text{no}\cdot\text{int}} = -\frac{1}{2}\phi G^{-1}_{\text{no}\cdot\text{int}}\phi - \frac{1}{2}\text{Tr}\ln G^{-1} - \frac{1}{2}\text{Tr}\, G^{-1}_{\text{no}\cdot\text{int}} G$$

$\Phi_{\text{int}}$ 4-loop 2pi (symmetric)



# Npi renormalization – 4 dimensions

4-loop 2pi 4-kernel $\Lambda$ defined as $\Lambda = 4 \frac{\delta^2 \Phi_{\rm int}}{\delta G^2}\Big|_{\substack{\phi=0 \\ G=\tilde{G}}}$




the 2-loop diagrams contain nested 1-loop subdivergences
$\rightarrow$ two 1-loop counter-terms must cancel two different 1-loop boxes

$\Rightarrow$ can see there is no one $\delta\lambda_1$ that works

# Resolution for 2pi

introduce 2 ct's . . . sounds bad . . . BUT

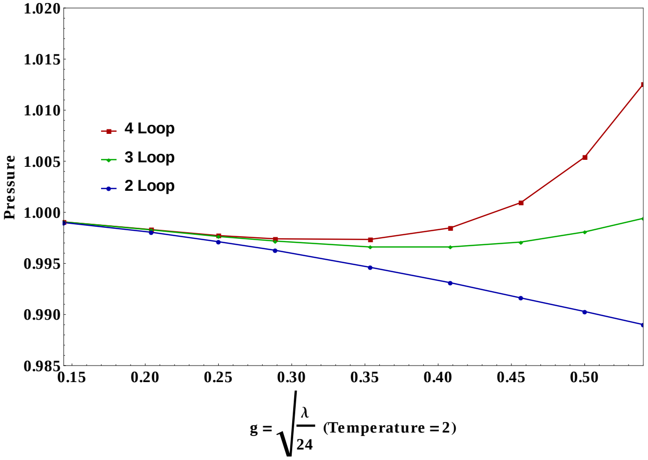
1) they both come from the action

2) at $L \to \infty$ loops they are equal

*H. van Hees, J. Knoll, Phys. Rev.* **D65**, *025010 (2002);*
*J-P Blaizot, E. Iancu, U. Reinosa, Nucl. Phys.* **A736**, *149 (2004);*
*J. Berges, Sz. Borsányi, U. Reinosa, J. Serreau, Annals Phys.* **320**, *344 (2005).*

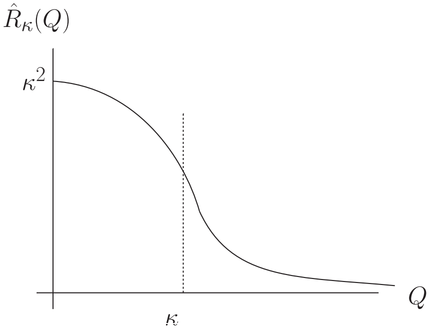
must develop another method to renormalize at higher orders

# Numerical Results - arXiv:1603.02085



# Renormalization group method

add to the action a non-local regulator term $\Delta S_\kappa[\varphi] = -\frac{1}{2}R_\kappa\varphi^2$




$$R_\kappa = \frac{Q^2}{e^{Q^2/\kappa^2} - 1}$$

$R_\kappa(Q) \sim \kappa^2$ for $Q \ll \kappa$

fluctuations $Q \ll \kappa$ suppressed

$R_\kappa(Q) \to 0$ for $Q \geq \kappa$

fluctuations $Q \gg \kappa$ unaffected

# Method

$n$-point functions depend on $\kappa$

(1) choose an uv scale $\kappa = \mu$ (defn of bare parameters)

theory is classical at this scale (all fluctuations suppressed)

$\rightarrow$ $n$-point functions are known functions of the bare parameters

(2) derive a hierarchy of differential 'flow' equations

$\rightarrow$ relate $\kappa$ dependent $n$-point functions and their derivatives wrt $\kappa$

(3) solve flow equations starting from bc's at $\kappa = \mu$

$\rightarrow$ obtain the $n$-point fcns at $\kappa = 0$ (the quantum solutions)

# Hierarchy of flow equations

definitions of kernels: $\Phi^{(m)}_{\text{int}\cdot\kappa} = 2^m \frac{\delta^m \Phi_{\text{int}}}{\delta G^m}\Big|_{\substack{G=G_\kappa \\ \phi=o}}$

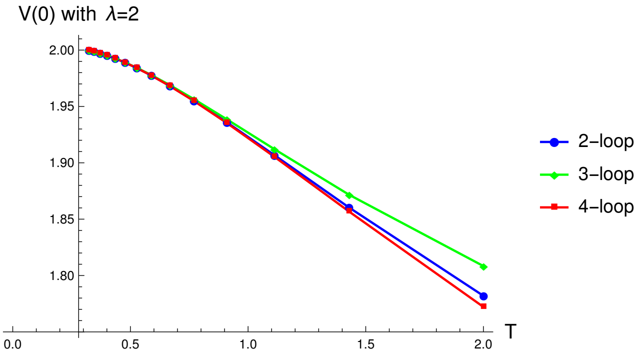
flow equations

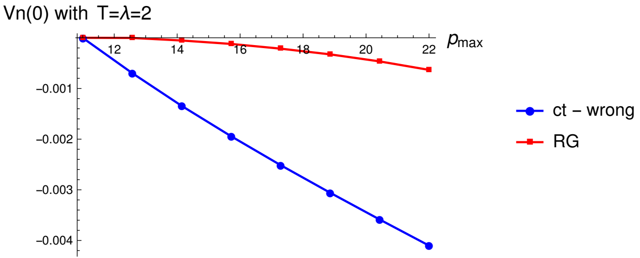
$$\partial_\kappa \Phi^{(m)}_{\text{int}\cdot\kappa}\Big|_{\substack{G=G_\kappa \\ \phi=o}} = \frac{1}{2}\int dQ\, \partial_\kappa \left(R_\kappa + \Sigma_\kappa\right) G^2_\kappa(Q)\, \Phi^{(m+1)}_{\text{int}\cdot\kappa}(Q,\quad)\Big|_{\substack{G=G_\kappa \\ \phi=o}}$$

$\Rightarrow$ infinite hierarchy of coupled flow eqns for the n-point kernels

BUT: the hierarchy truncates when the action is truncated

# Preliminary Results – 4-loop 2pi




Vn(0) with T=λ=2
12
14
16
18
20
22
$p_{max}$
-0.001
-0.002
-0.003
-0.004
ct – wrong
RG

# Conclusions

the 4 loop 2pi calculation can be done in two ways:

i) counterterm renormalization

ii) functional renormalization group regulator

we've shown that the two methods give the same answer

- verifies the RG approach works

significance:

- CT calculation:

requires the introduction of two counterterms
can't be generalized to higher order theories

- RG method:

all divergences are absorbed into one bare coupling which is introduced at the level of the lagrangian
*can be generalized to higher order nPI*
we've derived the equations for the 4pi theory
- and shown that the consistency requirements are satisfied

numerical calculations are in progress