The holographic Schrödinger Equation

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Overview

1. Theory
   - Conformal QCD
   - Emerging confinement
   - Light front holography
   - Chiral symmetry
   - A fundamental AdS/QCD scale
2. Phenomenology
   - Restoring quark masses and helicities
   - Diffractive vector meson production
   - Non-perturbative effects in B physics
   - Pion physics
3. Summary and Outlook

A very comprehensive review for the theory

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Light-front holographic QCD and emerging confinement

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Conformal invariance of QCD

QCD Lagrangian

$$\mathcal{L}_{QCD} = \bar{\Psi}(i\gamma^\mu D_\mu - m)\Psi - \frac{1}{4} G^a_{\mu\nu} G^{a\mu\nu}$$

with

$$D_\mu = \partial_\mu - i g_s A^a_\mu T^a$$

and

$$G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g_s c^{abc} A^b_\mu A^c_\nu$$

Neglecting

1. Quark masses ($m \to 0$)
2. Quantum loops (no Λ_{QCD})

then QCD action

$$S_{\mathrm{QCD}} = \int \mathrm{d}^4 x \mathcal{L}_{\mathrm{QCD}}$$

is conformally invariant

Generating a confinement scale perturbatively

- Conformal symmetry is broken (anomalously) by short distance quantum effects
- Quantum loops $\to$ running α_s with Λ_{QCD}

$$\alpha_s^{1-\text{loop}}(Q^2) = \frac{4\pi}{\beta_0 \ln\left(\frac{Q^2}{\Lambda^2_{\text{QCD}}}\right)}$$

$Q^2 \gg \Lambda^2_{\text{QCD}}$: asymptotic freedom
$Q^2 \sim \Lambda^2_{\text{QCD}}$: confinement
$m_p \sim \Lambda_{\text{QCD}}$: hadronic scale

- Λ_{QCD} is scheme-dependent
- World average (2016): $\Lambda_{\overline{\text{MS}}}^{(n_f=3)} = 0.332 \pm 0.017$ GeV

Generating a confinement scale non-perturbatively

- Can a more fundamental QCD confinement scale be generated non-perturbatively ?
- Hint comes from conformal symmetry breaking in QM

de Alfredo, Furbini and Furlan. Nuovo. Cim. A34 (1976) 569

In conformal QM, the evolution parameter can be transformed so as to generate a mass scale in the Hamiltonian while retaining the conformal invariance of the underlying action

Strategy:

- Reformulate QCD on the light-front ($ct \to x^+$)
- Reduce the many-parton bound state QCD problem to an effective two-parton problem and use dAFF mechanism

LF coordinates and momenta

- LF coordinates

$$x^{\pm} = x^0 \pm x^3 \qquad\qquad (\text{LF time} : x^+)$$

- LF energy and momentum

$$P^{\pm} = P^0 \pm P^3 \qquad\qquad (\text{LF Hamiltonian} : P^-)$$

- Zero transverse momentum frame: $P_\perp = 0$
- LF Hamiltonian generates LF time translations

$$i \frac{\partial}{\partial x^+} |\Psi(P)\rangle = P^- |\Psi(P)\rangle$$

LF Schrödinger Equation

Lorentz invariant LF Hamiltonian

$$H_{LF} = P^{\mu} P_{\mu} = P^{+} P^{-} = M^2 \qquad (P_{\perp}^2 = 0)$$

LF Schrödinger Equation

$$H_{LF} |\Psi(P)\rangle = M^2 |\Psi(P)\rangle$$

Fock expansion (legitimate only on LF)

$$|\Psi(P^+, S_z)\rangle = \sum_{n,\lambda_i} \int [\mathrm{d}x_i][\mathrm{d}^2\mathbf{k}_{\perp,i}] \Psi_n(x_i, \mathbf{k}_{\perp,i}, \lambda_i) |n : x_i P^+, \mathbf{k}_{\perp,i}, \lambda_i\rangle$$

- $\Psi_n(x_i, \mathbf{k}_{\perp,i}, \lambda_i)$ are the LF wavefunctions
- x_i are the LF momenta fractions: $x_i = \frac{k_i^+}{P^+}$
- λ_i are the LF helicities

Predicting the meson mass

$$\langle\Psi(P')|P^\mu P_\mu|\Psi(P)\rangle = M^2\langle\Psi(P')|\Psi(P)\rangle$$

$$\begin{aligned} M^2 &= \sum_{n,\lambda_i}\int [\mathrm{d}x_i][\mathrm{d}^2\mathbf{k}_{\perp,i}]\sum_{\alpha=1}^{n}\left(\frac{\mathbf{k}_{\perp,\alpha}^2 + \cancelto{0}{m_\alpha^2}}{x_\alpha}\right)|\underbrace{\Psi_n(x_i,\mathbf{k}_{\perp,i},\lambda_i)}_{\text{LF wavefunctions}}|^2 \\ &+ \text{(interactions)} \\ &+ [q \to \{\bar{q}, g\}] \end{aligned}$$

- Interactions: terms $\propto$ strong coupling α_s
- $n = 2$: $q\bar{q}$
- $n > 2$: higher Fock states (hFs)

Reduction of the LF SE

1. Assume LF wavefunctions depend only on the total invariant mass squared in each Fock sector
2. Assume spin effects decouple from dynamics (suppress helicity indices)

Then absorb higher Fock states in interactions (no truncation)

$$\Psi_n(x_i, \mathbf{k}_{\perp,i}, \lambda_i) \to \Psi_n((\underbrace{k_1 + k_2 + \ldots k_n}_{\text{Invariant mass}})^2)$$

Then

$$M^2 = \int \mathrm{d}x \int \frac{\mathrm{d}^2 k_\perp}{16\pi^3} \frac{k_\perp^2}{x(1-x)} |\Psi(M_{q\bar{q}}^2)|^2 + (\underbrace{\text{interactions}}_{\text{incl. hFs}})$$

where

$$M_{q\bar{q}}^2 = \frac{k_\perp^2}{x(1-x)}$$

The holographic Schrödinger Equation

Introduce Fourier transform of $M_{q\bar{q}}$

$$\zeta = \sqrt{x(1-x)}b_\perp$$

and separate variables

$$\Psi(x,\zeta,\varphi) = \frac{\phi(\zeta)}{\sqrt{2\pi\zeta}} X(x) e^{iL\varphi}$$

Then

$$M^2 = \int \mathrm{d}\zeta \phi^*(\zeta)\sqrt{\zeta}\left(-\frac{\mathrm{d}^2}{\mathrm{d}x^2} - \frac{1}{\zeta}\frac{\mathrm{d}}{\mathrm{d}\zeta} + \frac{L^2}{\zeta^2}\right)\frac{\phi(\zeta)}{\sqrt{\zeta}} + \overbrace{\int \mathrm{d}\zeta \phi^*(\zeta) U(\zeta)\phi(\zeta)}^{\text{interactions}}$$

so that

$$\left(-\frac{\mathrm{d}^2}{\mathrm{d}\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right)\phi(\zeta) = M^2\phi(\zeta)$$

Holographic SE versus Ordinary SE

Holographic SE

$$H_{\mathrm{hLF}}\phi(\zeta) = M^2\phi(\zeta)$$

$$H_{\mathrm{hLF}} = -\frac{\mathrm{d}^2}{\mathrm{d}\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U(\zeta)$$

- Simple
- Lorentz invariant
- Unique effective potential
- Meson with massless quarks

Ordinary SE in 2d

$$H_{\mathrm{nr}}\phi(r) = E\phi(r)$$

$$H_{\mathrm{nr}} = \frac{\mathrm{d}^2}{\mathrm{d}r^2} - \frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r} - \frac{L^2}{r^2} - \frac{2m}{\hbar}V(r)$$

- Simple
- Not Lorentz invariant
- No unique potential
- Non relativistic system

Simple: analytical solutions can be found

The dAFF mechanism

Conformal QM action

$$A = \frac{1}{2}\int \mathrm{d}t \left(\dot{Q} - \frac{g}{Q^2}\right) \leftrightarrow H = \dot{Q} + \frac{g}{Q^2}$$

Change evolution parameter

$$\mathrm{d}t \to d\tau = \frac{\mathrm{d}t}{u + vt + wt^2} \qquad \mathrm{dim}(w) = [\mathrm{mass}]^2$$

Action remains conformally invariant but Hamiltonian does not

$$H \to G = \dot{q} + \frac{g}{q^2} + \underbrace{\left(\frac{4uw - v^2}{4}\right) q^2}_{\text{breaks conformal invariance}}$$

Harmonic LF oscillator

- The set $\{u = 2, v = 0, w = 2\kappa^2\}$ maps G onto H_{LF}

dAFF Hamiltonian

$$G(x) = \left(-\frac{d^2}{dx^2} + \frac{g}{x^2} + \kappa^4 x^2\right)$$

LF holographic Hamiltonian

$$H_{\text{hLF}} = \left(-\frac{d^2}{d\zeta^2} - \frac{1 - 4L^2}{4\zeta^2} + U(\zeta)\right)$$

$$\boxed{U(\zeta) = \kappa^4 \zeta^2}$$

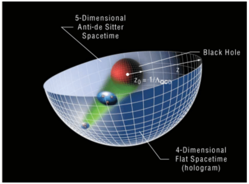
- dAFF constrains LF confinement potential to be harmonic

Mapping to AdS gravity

Physical spacetime	Anti de Sitter spacetime
LF transverse distance $\longmapsto$	Fifth dimension in AdS
ζ	z_5
Angular momentum $\longmapsto$	Spin + (AdS mass×radius)
L^2	$(\mu R)^2 + (2 - J)^2$

LF hSE maps onto wave equation for spin-J modes in AdS

Geometry drives Dynamics



The dilaton field that distorts pure AdS geometry drives the confinement dynamics in physical spacetime

$$U(\zeta) = \frac{1}{2}\varphi''(\zeta) + \frac{1}{4}\varphi'(\zeta)^2 + \frac{2J-3}{2\zeta}\varphi'(\zeta)$$

A quadratic dilaton $\varphi = \kappa^2 z_5^2$ gives

$$U(\zeta) = \kappa^4\zeta^2 + \overbrace{2\kappa^2(J-1)}^{\text{AdS/QCD}}$$

Predicting a massless pion

Solving the holographic LF Schrödinger Equation

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right)\phi(\zeta) = M^2\phi(\zeta)$$

with

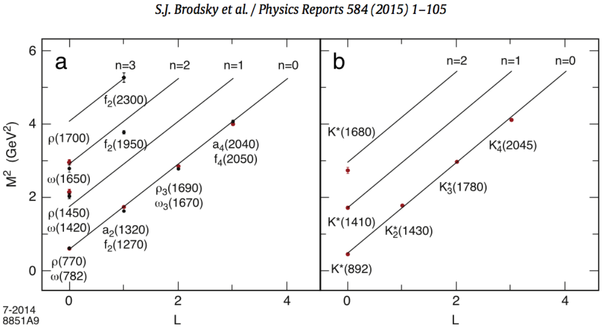
$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(J-1)$$

gives

$$M^2 = (4n + 2L + 2)\kappa^2 + 2\kappa^2(J-1)$$

- Lightest bound state ($n = L = J = 0$) is massless ($M = 0$)
- Identify with pion (as expected by chiral symmetry)
- If $U = \kappa^4\zeta^p$, $M_\pi = 0$ only for $p = 2$

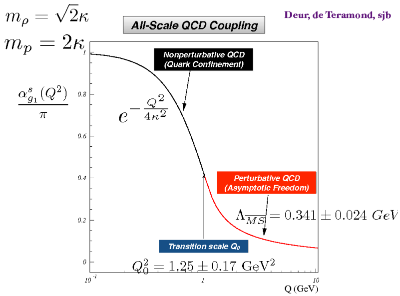
Fixing the confinement scale



Regge trajectories of light vector mesons with $\kappa = 0.54$ GeV

Predicting Λ_{QCD} using κ

Figure from Stan Brodsky



- Running α_s from AdS/QCD

$$\alpha_s^{\mathrm{AdS}}(Q^2) = \alpha_s^{\mathrm{AdS}}(0) e^{-\frac{Q^2}{4\kappa^2}}$$

- Using $\kappa = 0.523 \pm 0.024$ GeV and matching to $\alpha_s^{\overline{\mathrm{MS}}}$ to 5-loop accuracy:

$$\Lambda_{\overline{\mathrm{MS}}}^{n_f=3} = 0.339 \pm 0.019 \text{ GeV}$$

Excellent agreement with world average $\Lambda_{\overline{\mathrm{MS}}}^{(n_f=3)} = 0.332 \pm 0.017$ GeV [Brodsky, Deur, de Téramond, arXiv:1608.04933 (2016)]

Chiral symmetry breaking

Non-zero quark masses appear in two ways

1. By "completing" invariant mass of $q\bar{q}$ pair

$$M_{q\bar{q}}^2 = \frac{k_\perp^2}{x(1-x)} \to \frac{k_\perp^2 + m^2}{x(1-x)}$$

2. Via dynamical spin effects in vector mesons

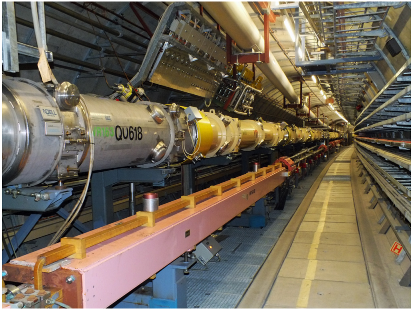
$$\Psi_{\lambda,\lambda'}(x, k_\perp) \to \Psi(x, k_\perp, \lambda, \lambda') = \Psi(x, k_\perp) S_{\lambda\lambda'}(x, k_\perp)$$

with a photon-like spin wavefunction

$$S_{\lambda\lambda'}(x, k_\perp) = \frac{\bar{v}_{\lambda'}(x, k_\perp)}{\sqrt{(1-x)}} [\gamma^\mu \cdot \epsilon_\mu] \frac{u_\lambda(x, k_\perp)}{\sqrt{x}}$$

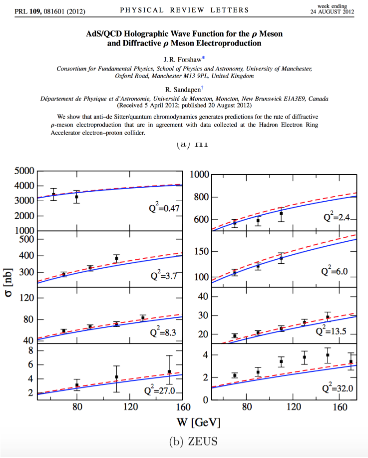
Additional quark mass terms appear in spin wavefunction

Diffractive processes at HERA



- Extensively measured at HERA collider: $\gamma^* p \to (\rho, \phi) p$
- Probe the perturbative to non-perturbative transition
- Sensitive to the QCD confinement scale κ

Successful predictions for diffractive ρ production


PRL 109, 081601 (2012) PHYSICAL REVIEW LETTERS week ending 24 AUGUST 2012

AdS/QCD Holographic Wave Function for the ρ Meson and Diffractive ρ Meson Electroproduction

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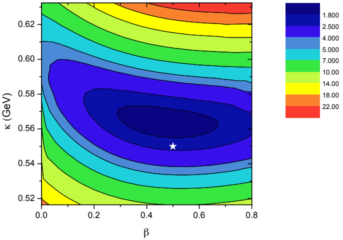
(Received 5 April 2012; published 20 August 2012)

We show that anti–de Sitter/quantum chromodynamics generates predictions for the rate of diffractive ρ-meson electroproduction that are in agreement with data collected at the Hadron Electron Ring Accelerator electron–proton collider.

(b) ZEUS



White star: AdS/QCD prediction

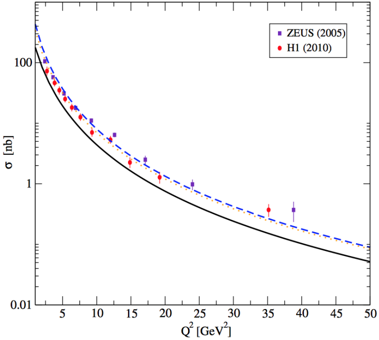
$\kappa = 0.54$ GeV

$M_\rho = \sqrt{2}\kappa$

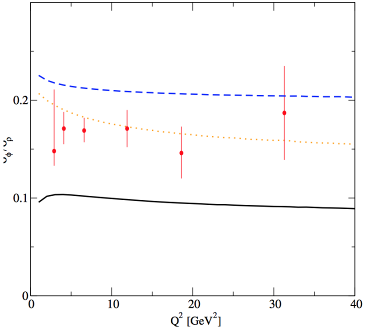
$m_{u/d} = 0.14$ GeV

Simultaneous ρ and ϕ description

M. Ahmady, RS, N. Sharma (PRD, 2016)



PHYSICAL REVIEW D **94,** 074018 (2016)



Simultaneous prediction of ρ and ϕ diffractive production with $\kappa = 0.54$ GeV and $m_{u/d} = 0.046$ GeV, $m_s = 0.14$ GeV

New Physics at LHCb

NATURE | NEWS

Physicists excited by latest LHC anomaly

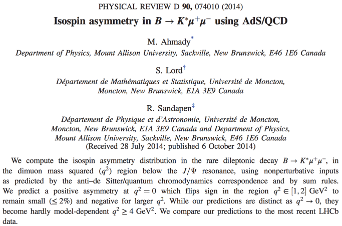
A series of odd findings have theorists hoping for new particles.

Davide Castelvecchi

19 April 2017

- Exclusive semileptonic B decays are sensitive probes of NP
- Clean to measure
- Theory uncertainties due to QCD bound state effects

Isospin asymmetry in $B \to K^* \mu^+ \mu^-$


PHYSICAL REVIEW D **90**, 074010 (2014)

Isospin asymmetry in $B \to K^* \mu^+ \mu^-$ using AdS/QCD

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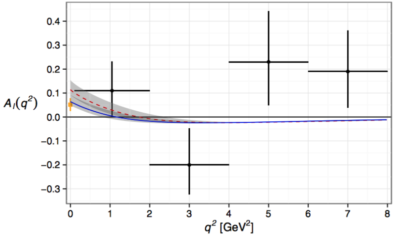
(Received 28 July 2014; published 6 October 2014)

We compute the isospin asymmetry distribution in the rare dileptonic decay $B \to K^* \mu^+ \mu^-$, in the dimuon mass squared (q^2) region below the J/Ψ resonance, using nonperturbative inputs as predicted by the anti–de Sitter/quantum chromodynamics correspondence and by sum rules. We predict a positive asymmetry at $q^2 = 0$ which flips sign in the region $q^2 \in [1, 2]$ GeV^2 to remain small ($\lesssim 2\%$) and negative for larger q^2. While our predictions are distinct as $q^2 \to 0$, they become hardly model-dependent $q^2 \geq 4$ GeV^2. We compare our predictions to the most recent LHCb data.



- We attempt to quantify non-perturbative uncertainties in theory predictions for the isospin asymmetry
- For forward-backward asymmetry, see M. Ahmady, D. Hatfield (student), S. Lord (student) and RS, PRD 92 (2015) 114028

New predictions for pion observables

Mohammad Ahmady, Farrukh Chishtie, and Ruben Sandapen
Phys. Rev. D **95**, 074008 – Published 7 April 2017

See talk by M. Ahmady in this session

Summary and Outlook

Summary

- A fundamental confinement scale κ emerges in the holographic SE, governing the strength of its LF harmonic confinement potential
- Successful phenomenology of light mesons with a universal κ but quark mass remains a free parameter

Outlook

- In principle, κ can be used to predict low energy constants of effective theories of the strong interaction

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