Doppler shift lifetime measurements using the TIGRESS Integrated Plunger at ISAC-II/TRIUMF

K. Starosta for TIGRESS/TIP collaboration

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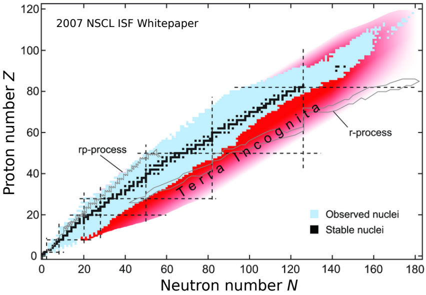
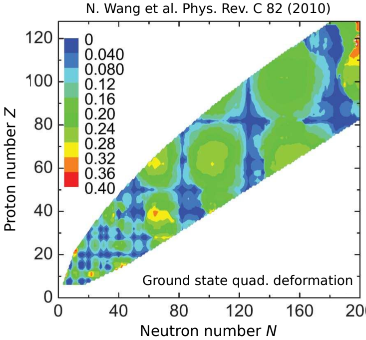
May 29, 2017

Selected goals of nuclear science research



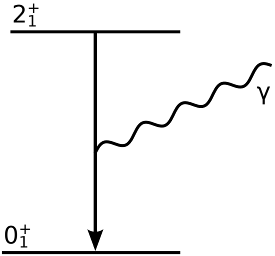


- Understand the mechanisms of shell evolution in medium-mass and heavy nuclei as a function of isospin.
- Develop a theoretical framework that is able to make accurate predictions of nuclear properties.

TRIUMF 5 Year Plan 2015–2020.

Studying nuclear structure using the electromagnetic force

- The electromagnetic force provides a convenient non-intrusive probe of nuclear systems bound by the strong force.
- Lifetime measurements using gamma-ray spectroscopy provide:
  1. An observable sensitive to nuclear structure.
  2. A useful benchmark for nuclear model calculations.

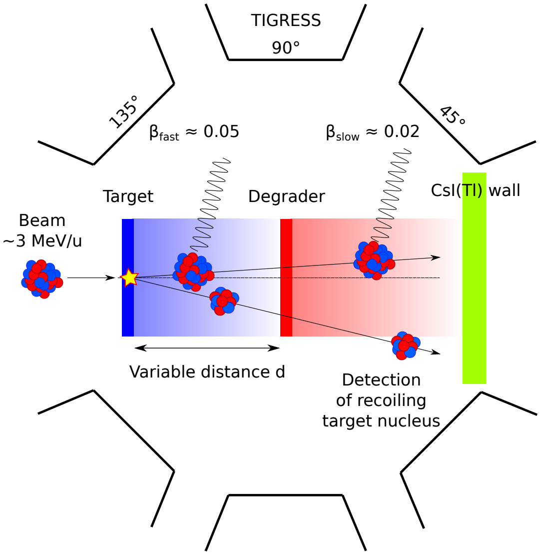


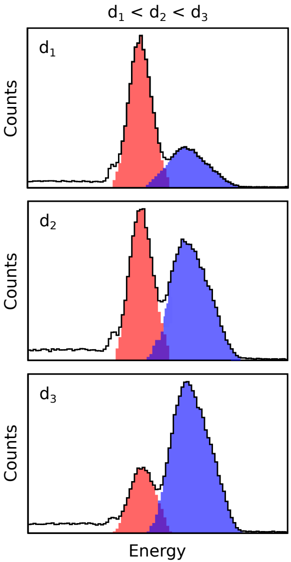
$$\tau(E2; 2_1^+ \rightarrow 0_1^+) = \lambda(E2; 2_1^+ \rightarrow 0_1^+)^{-1}$$

$$\lambda(E2; 2_1^+ \rightarrow 0_1^+) \propto E(2_1^+)^5 \times B(E2; 2_1^+ \rightarrow 0_1^+)$$

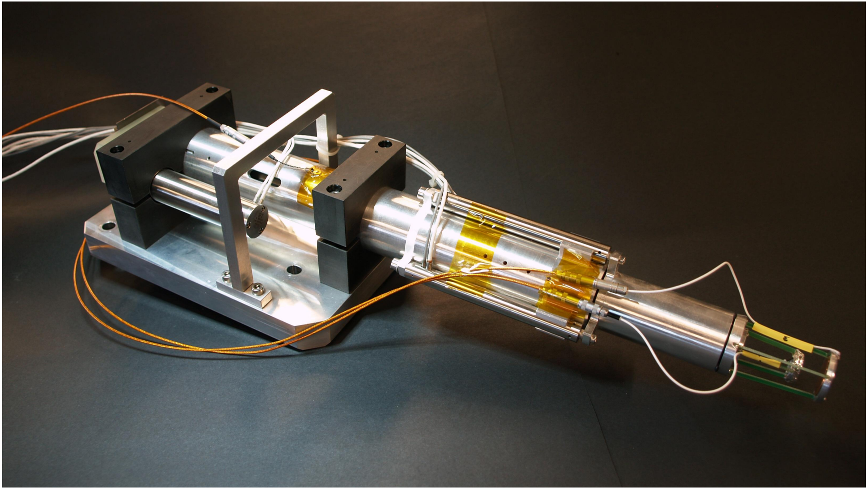
$$B(E2; 2_1^+ \rightarrow 0_1^+) = \frac{1}{5} \langle 2_1^+ || E2 || 0_1^+ \rangle^2 \propto \beta^2$$

The Recoil Distance Method with TIP and TIGRESS



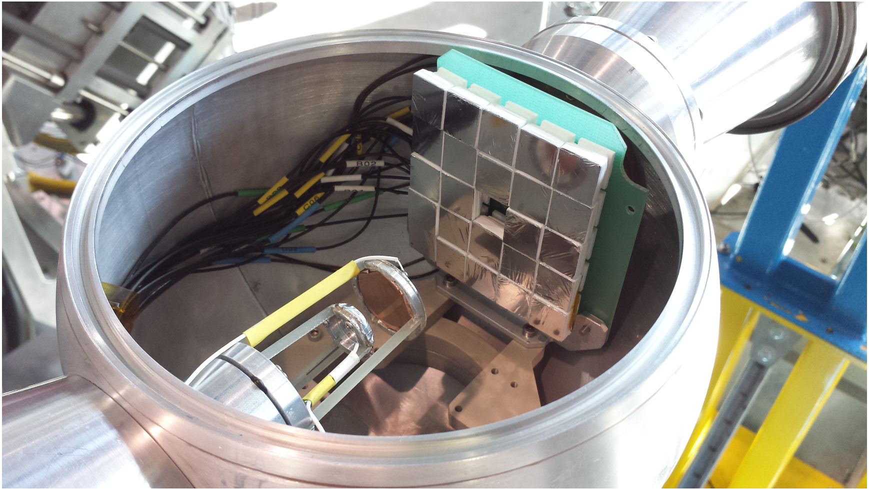


The TIGRESS Integrated Plunger (TIP) device



P. Voss et al. Nucl. Inst. and Meth. A 746 (2014) 87, P. Voss et al. Phys. Proc. 66 (2015) 524.

TIP with the CsI(TI) wall

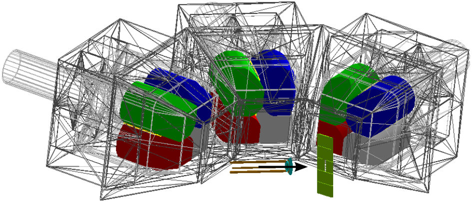


TIP commissioning experiment details

- Measurement of the $2_1^+ \rightarrow 0_1^+$ lifetime in ^{84}Kr.
- Previous Coulex experiment reports $\tau = 5.84 \pm 0.18$ ps.
- TIP and 11 TIGRESS detectors in a 3/5/3 configuration with 24-element CsI(Tl) wall for particle identification (PID).
- Excited state populated via partially unsafe Coulex reaction (beam energy of 250 MeV or 2.979 MeV/u).
- A total of 13 target/degrader separation distances from 20–400 μm were analyzed.
- Data analysis via a comparison to Geant4-simulated lineshapes developed for low-statistics experiment analysis.

T. J. Mertzimekis et al. Phys. Rev. C 64 (2001) 024314.

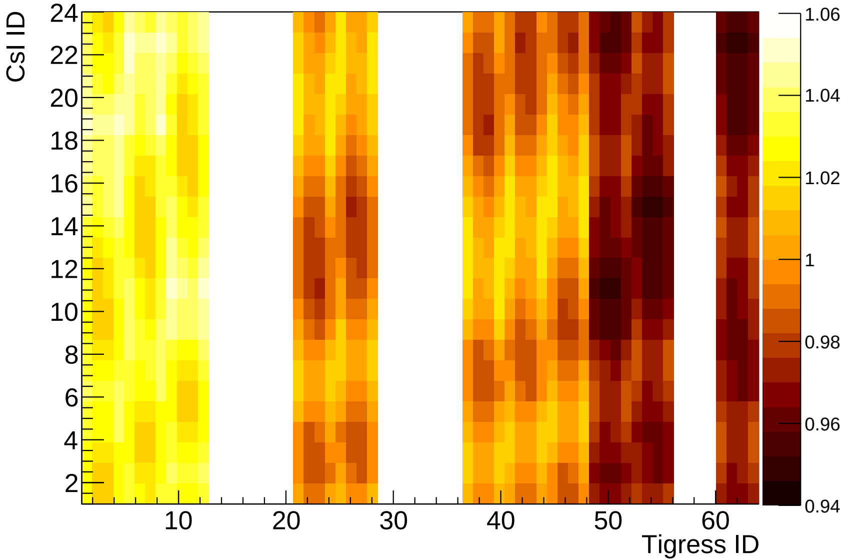
Geant4 simulation framework



- Coulomb excitation followed by gamma-ray decay.
- Analytic solutions for single step $E2$ process (Coulex kinematics, angular distributions, etc.) with track weighting to handle thick target integration.
- Gamma-ray sensitive detectors ported from GRIFFIN/TIGRESS code originating from Guelph.

Adrich et al. Nucl. Inst. and Meth. A 598 (2009) 454, Alder et al. Rev. Mod. Phys. 28 (1956) 432.

Geant4-facilitated data analysis: Doppler-shift factors



Histogram analysis using the likelihood ratio χ^2

- For a Poisson likelihood function, the likelihood ratio χ^2 is given by

$$\chi^2 = 2 \sum_{i=1}^{k} y_i - n_i + n_i \ln(n_i/y_i)$$

  where y_i is the model and n_i the observed data.
- Pros:
  - Versatile (goodness of fit, point estimation, error analysis).
  - Control over parent distribution.
  - Minimizing likelihood ratio $\chi^2 \equiv$ maximizing likelihood function.
  - **No variance estimation!**
- Cons:
  - Non-linear.
  - ...

Baker and Cousins. Nucl. Inst. and Meth. A 221 (1984) 437.

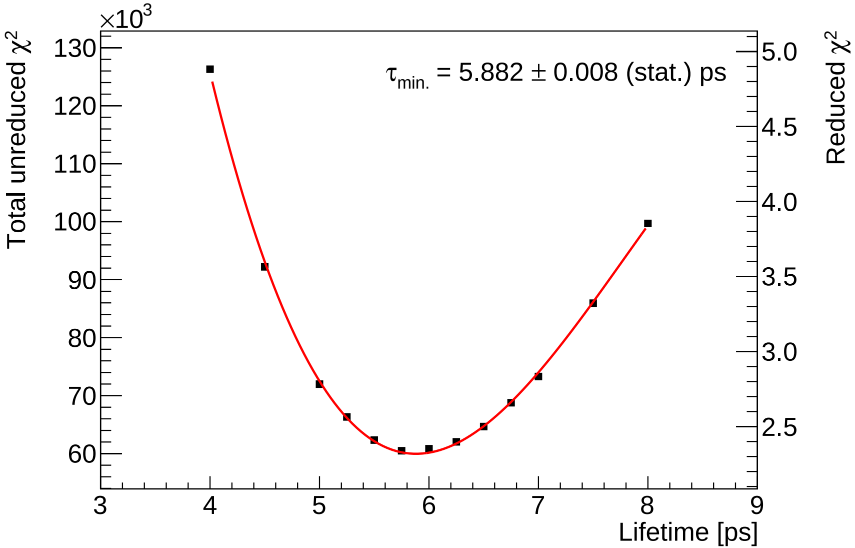
Data analysis procedure

- For a given input lifetime τ, simulate gamma-ray spectra grouped by the Doppler-shift factor at all distances.
- Model data using

$$y_i = \alpha_0 s_i + \alpha_1 + \alpha_2 \operatorname{erfc}\left(\frac{i-c}{w\sqrt{2}}\right),$$

  where s_i is the simulated data and the α's are free parameters.
- Minimize $\chi^2_{d,g}$ for each distance and group.
- Minimum in total $\chi^2 = \sum_{\text{dist.}} \sum_{\text{gr.}} \chi^2_{d,g}$ corresponds to best fit lifetime $\tau_{\text{min.}}$

Best fit lifetime from χ^2 analysis



Comparison of fitting methods

1. Likelihood ratio χ^2

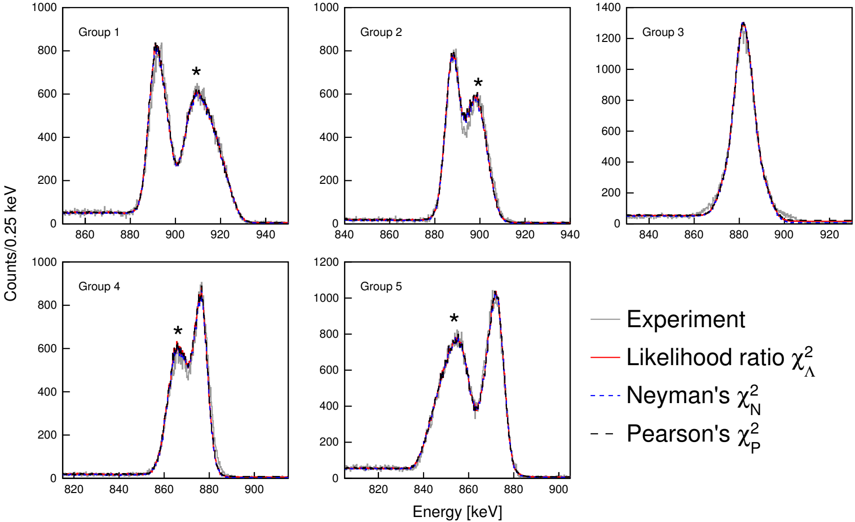
$$\chi^2_\Lambda = 2\sum_{i=1}^{k} y_i - n_i + n_i \ln(n_i/y_i)$$

2. Neyman χ^2

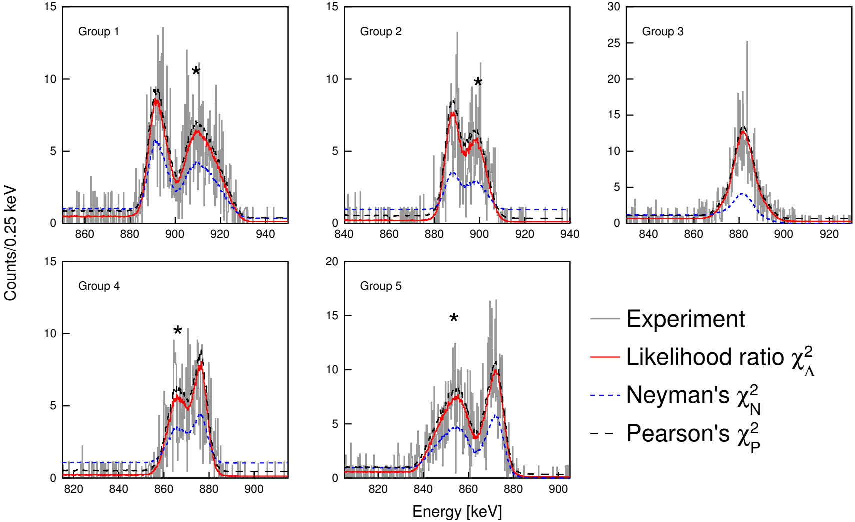
$$\chi^2_N = \sum_{i=1}^{k} \frac{(n_i - y_i)^2}{n_i}.$$

- Probably the most common fitting statistic (ROOT default for histogram fits, for example).
- Uses data to estimate variance.
- Assumes bin error can be estimated by $\sqrt{n_i}$: what if $n_i = 0$?

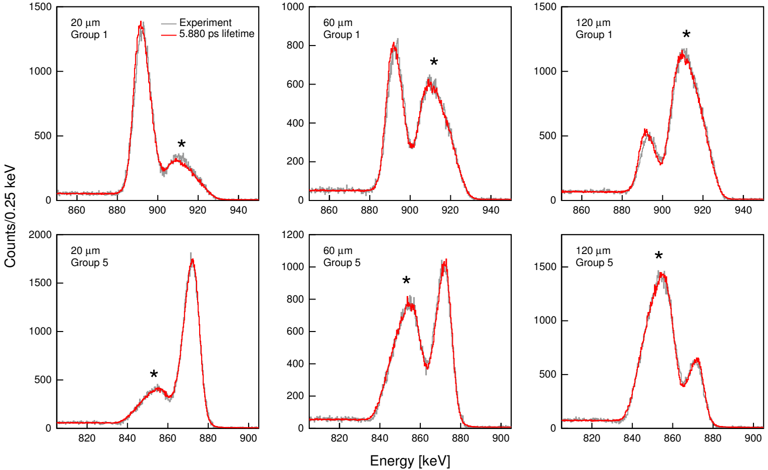
Lineshape fits for ^{84}Kr, 60 μm: all data



Lineshape fits for ^{84}Kr, 60 μm: 1% of data



Simulated lineshapes: groups 1 and 5



Best fit lifetime: 5.880 ± 0.013 (stat.) ± 0.070 (sys.) ps

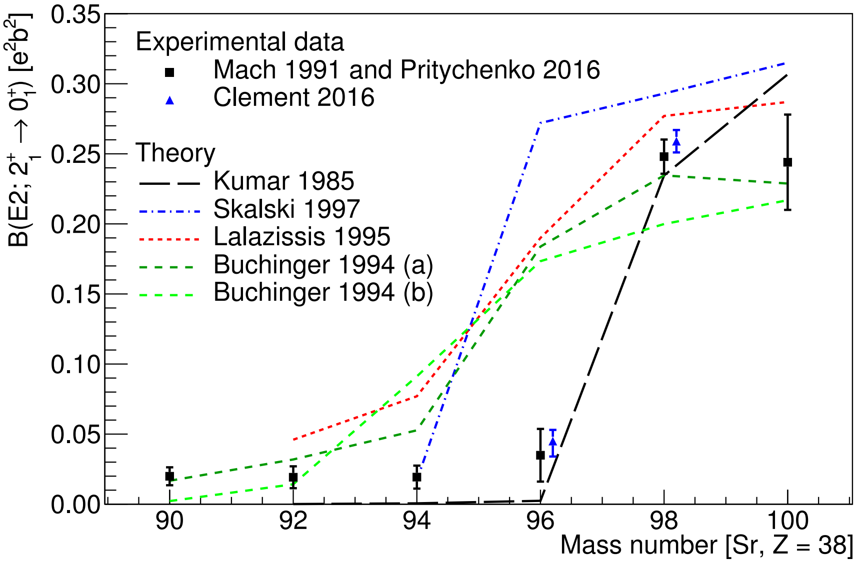
Literature value: 5.84 ± 0.18 ps [Mertzimekis 2001]

Commissioning experiment summary

- Systematic uncertainties from the following 3 sources were identified:
  1. Transitions from higher-lying (feeding) states,
  2. Misalignment of the target and degrader foils,
  3. Choice of fit range for the χ^2 analysis.
- No deorientation effect observed in the data.
- Final reported lifetime: $\tau = 5.880 \pm 0.008$ (stat.) $\pm\, 0.070$ (sys.) ps.
- Excellent agreement with literature value of 5.84 ± 0.18 ps with factor of $\sim$2 reduction in uncertainty.
- A robust and flexible framework has been developed for the planning and analysis of RDM experiments using TIP.
- Paper for submission to Nucl. Inst. and Meth. A in preparation.

T. J. Mertzimekis et al. Phys. Rev. C 64 (2001) 024314.

TIP RIB experiment motivation

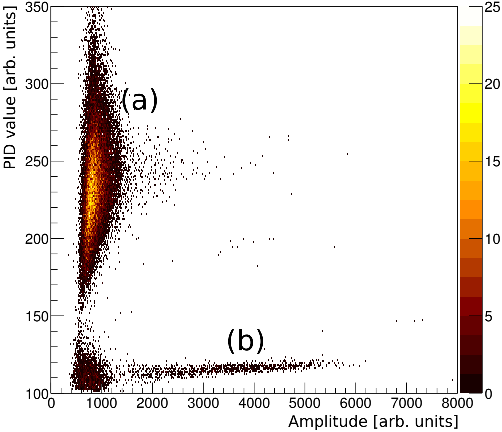


TIP RIB experiment details

- Performed December 9–14, 2015.
- Measurement of the $2_1^+ \rightarrow 0_1^+$ lifetime in ^{94}Sr.
- Previous fast timing experiment reports 10 ± 4 ps.
- TIP and 16 TIGRESS detectors with 24-element CsI(Tl) wall.
- Running conditions same as commissioning experiment.
- Except the beam rate: factor of $\sim 10^4$ lower for RIB.
- Data was recorded at three target/degrader separation distances: 50, 100, 150 μm.

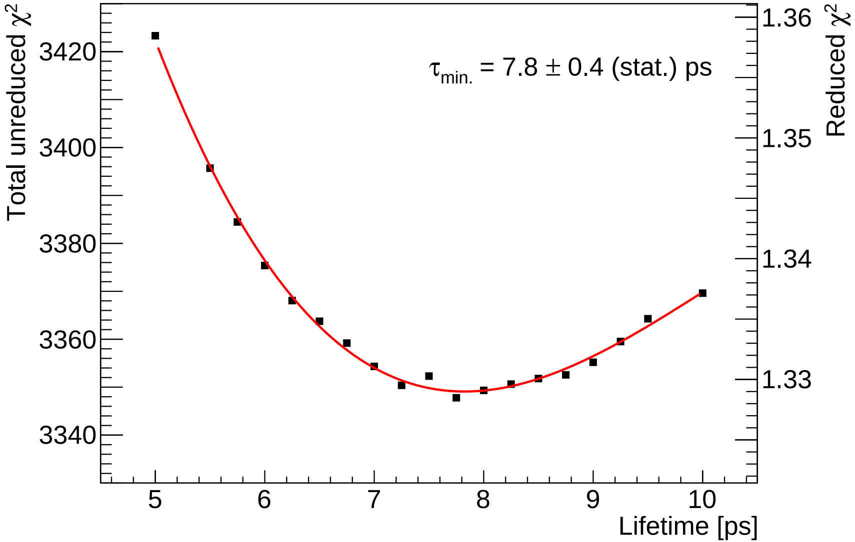
Mach et al. Nuc. Phys. A 523 (1991) 197.

Particle identification with RIBs

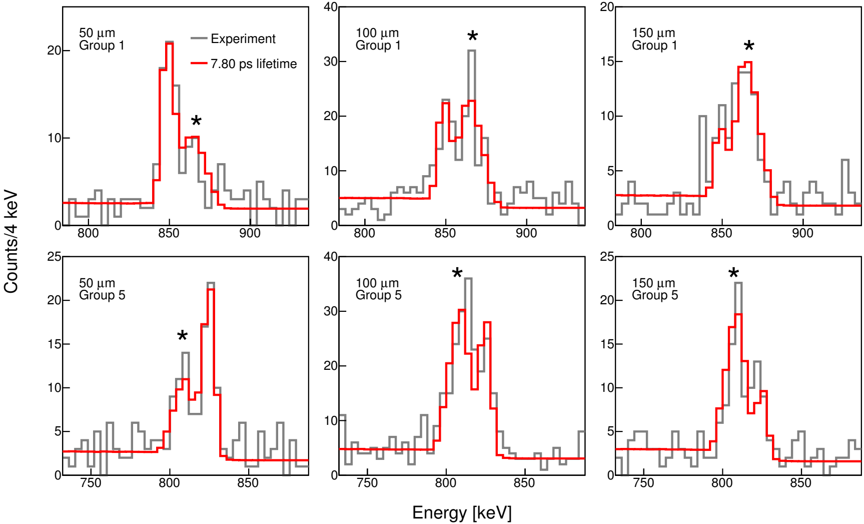


(a) Electrons from β^- decay (b) Al recoils from Coulex reaction

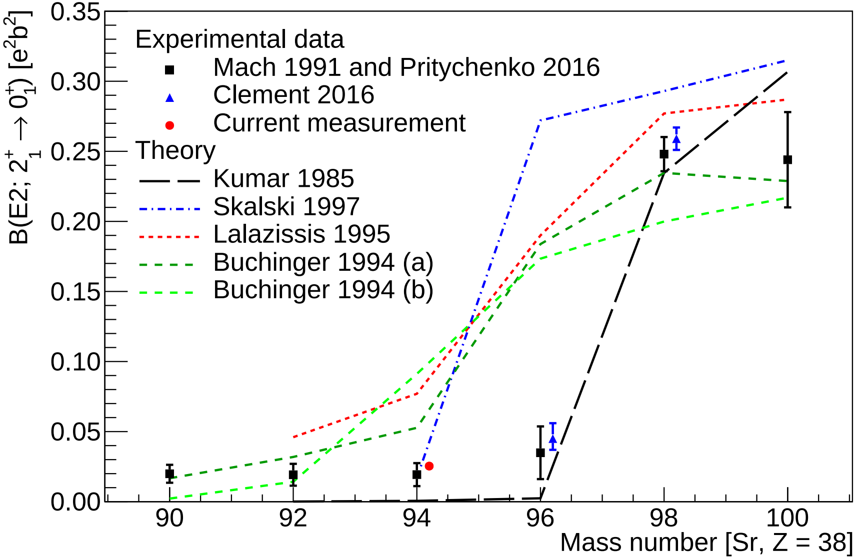
Best fit lifetime from χ^2 analysis



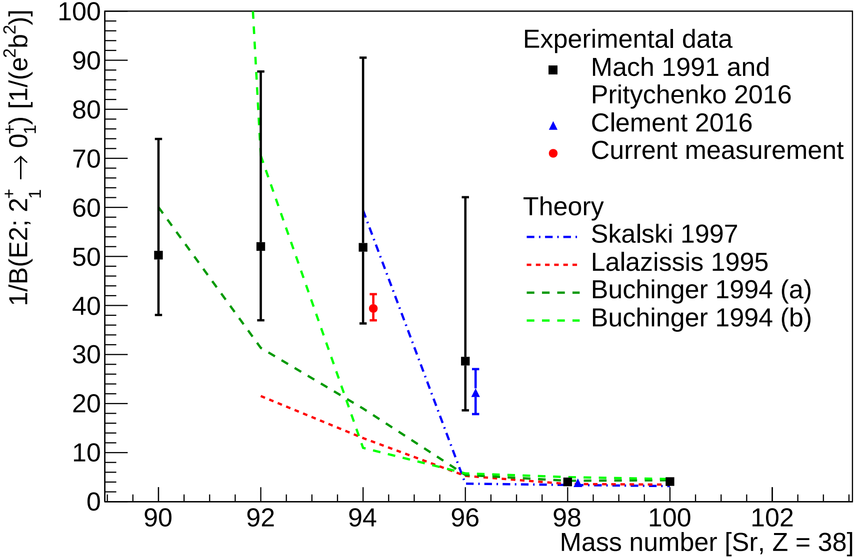
Simulated lineshapes: groups 1 and 5



$B(E2: 2_1^+ \rightarrow 0_1^+)$ measurement



Impact of high-precision $B(E2: 2_1^+ \rightarrow 0_1^+)$ measurement



Summary of results

Current work

Lifetime τ	$7.80^{+0.5}_{-0.4}$ (stat.) $\pm$ 0.07 (sys.) ps
$B(E2; 2_1^+ \rightarrow 0_1^+)$	$0.0254^{+0.0015}_{-0.0014}$ (stat.) $\pm$ 0.0002 (sys.) e^2b^2

Mach (1991)

Lifetime τ	10 ± 4 ps
$B(E2; 2_1^+ \rightarrow 0_1^+)$	0.020 ± 0.008 e^2b^2

- A robust and flexible framework has been developed for the planning and analysis of RDM experiments using TIP.
- Details of simulation framework, data analysis, and results for the ^{84}Kr commissioning experiment for submission to Nucl. Inst. and Meth. A in preparation.
- ^{94}Sr lifetime measurement paper submitted to Phys. Rev. C.

T. J. Mertzimekis et al. Phys. Rev. C 64 (2001) 024314.

Acknowledgments

TIP Design
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Theory references

- Kumar 1985: Quadrupole plus pairing Hamiltonian.
- Skalski 1997: Nilsson-Strutinsky method with the Woods-Saxon potential.
- Lalazissis 1995: Relativistic mean field.
- Buchinger 1994 (a): Finite range liquid drop.
- Buchinger 1994 (b): Strutinsky energy theorm.

A. Kumar and M. R. Gunye, Phys. Rev. C 32, 2116 (1985).
J. Skalski, S. Mizutori, and W. Nazarewicz, Nuclear Physics A 617, 282 (1997).
G. Lalazissis and M. Sharma, Nuclear Physics A 586, 201 (1995).

F. Buchinger *et al.*, Phys. Rev. C 14 49, 1402 (1994).

TIP commissioning experiment details

^{84}Kr properties	
E_γ	881.615 keV
$\tau_{\text{lit.}}$	5.84 ± 0.18 ps

Plunger setup.

	Material	Thickness [mg/cm^2]	Thickness [µm]
Target	Al	1.07 ± 0.04	3.96 ± 0.16
Degrader	Cu	3.90 ± 0.16	4.35 ± 0.18

Beam properties	
Beam energy	250 MeV
Safe Coulex	200 MeV
Rate	$\sim 2 \times 10^8$ pps

TIP RIB experiment details

^{94}Sr properties	
E_γ	836.9 keV
$\tau_{\text{lit.}}$	10 ± 4 ps

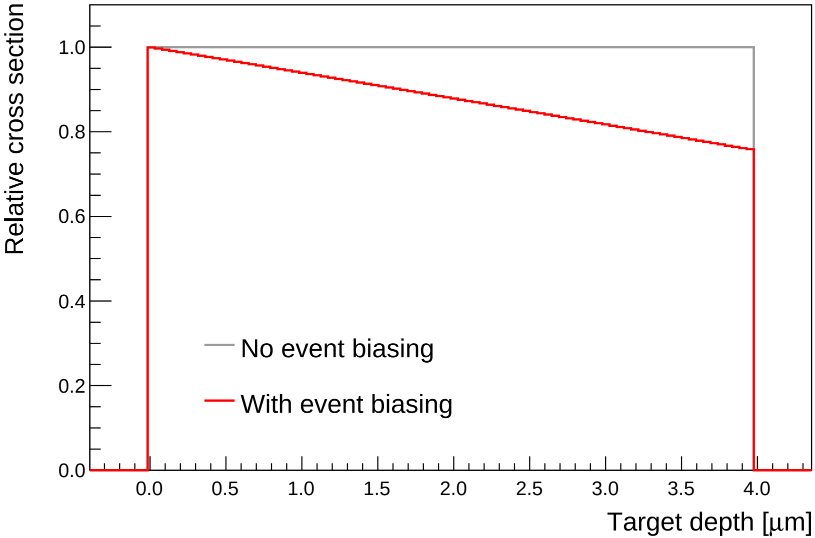
Plunger setup.

	Material	Thickness [mg/cm^2]	Thickness [µm]
Target	Al	1.09 ± 0.04	4.05 ± 0.17
Degrader	Cu	3.69 ± 0.15	4.12 ± 0.17

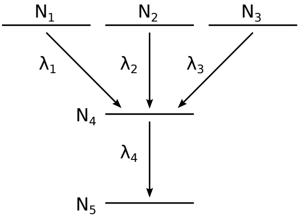
Beam properties

Beam energy	280 MeV
Safe Coulex	227 MeV
Rate	$\sim$ 2–5 $\times 10^4$ pps

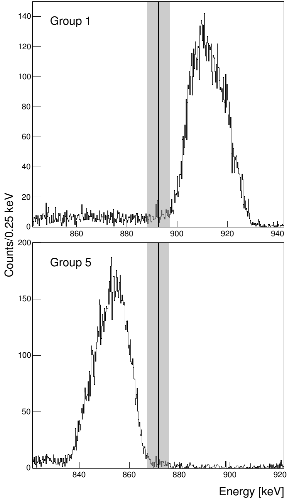
Thick target integration with Geant4



Feeding analysis

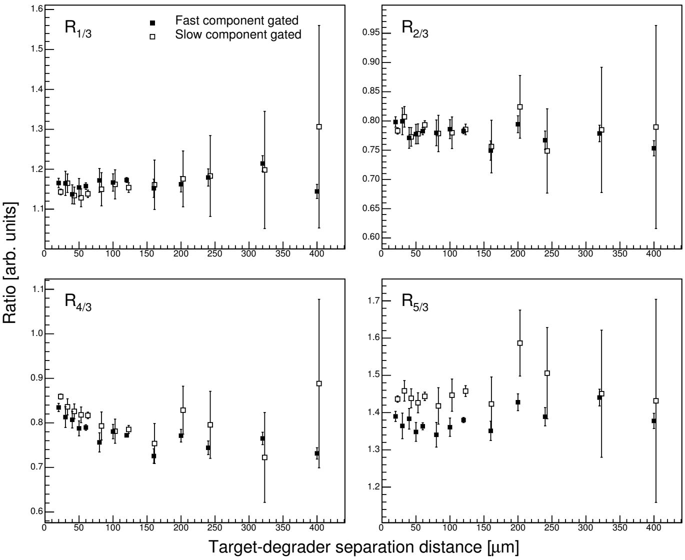


$$N_5(t) = N_4(0)\left(1 - e^{-\lambda_4 t}\right)$$
$$+ \left[\sum_{i=1}^{3} N_i(0)\frac{\lambda_i}{\lambda_4 - \lambda_i}\right]\left(e^{-\lambda_4 t} - 1\right)$$
$$+ \sum_{i=1}^{3}\left[N_i(0)\frac{\lambda_4}{\lambda_4 - \lambda_i}\left(1 - e^{-\lambda_i t}\right)\right]$$



No observed feeding at 400 μm

Deorientation effect



Analysis of low-statistics data sets

Challenges associated with the comparison of simulated data sets from Geant4 to low statistics data:

- Typical least squares analysis requires an estimate of the variance either from the model (Pearson χ^2) or from the data (Neyman χ^2).
- These are derived under the implicit assumption of a Gaussian error distribution.
- With few counts, errors are poorly estimated.
- Even worse, if there are 0 counts in the data, the very commonly used Neyman χ^2 statistic is undefined!
- Inconsistent normalization (which depends on choice of statistic!) without inclusion of explicit normalization parameter.

Basic quantities for histogram analysis

Consider a histogram with k bins labeled by the index i running from 1 to k and a model with J parameters labeled by index j. Define the following quantities:

- $n_i =$ number of events in bin i.
- $\mathbf{n} = (n_1, n_2, \ldots, n_k)$.
- $\boldsymbol{\alpha} = (\alpha_1, \alpha_2, \ldots, \alpha_j)$.
- $y_i =$ number of events predicted by the model (via $\boldsymbol{\alpha}$) in bin i.
- $\mathbf{y} = (y_1, y_2, \ldots, y_k)$.

Baker and Cousins. Nucl. Inst. and Meth. A 221 (1984) 437.

Likelihood ratio χ^2 derivation

Assume the data in each bin are independently Poisson distributed. Then the likelihood function $\mathcal{L}$ is given by

$$\mathcal{L}(\mathbf{y};\mathbf{n}) = \prod_{i=1}^{k} \frac{y_i^{n_i}\exp(-y_i)}{n_i!}.$$

Let $\mathbf{m}$ be the true (unknown) values of $\mathbf{n}$. The likelihood ratio Λ is

$$\Lambda = \frac{\mathcal{L}(\mathbf{y};\mathbf{n})}{\mathcal{L}(\mathbf{m};\mathbf{n})},$$

and the likelihood ratio test theorem says that the "likelihood chi-square"

$$\chi^2_\Lambda = -2\ln\Lambda = -2\ln\mathcal{L}(\mathbf{y};\mathbf{n}) + 2\ln\mathcal{L}(\mathbf{m};\mathbf{n})$$

asymptotically obeys a χ^2 distribution.

Baker and Cousins. Nucl. Inst. and Meth. A 221 (1984) 437, Wilks. Ann. Math. Stat. 9 (1938).

Likelihood ratio χ^2 derivation

Replace the unknown $\mathbf{m}$ with its bin-by-bin maximum likelihood estimator $\mathbf{n}$. A bit of algebra yields

$$\begin{aligned}\chi^2_\Lambda &= -2\ln\mathcal{L}(\mathbf{y};\mathbf{n}) + 2\ln\mathcal{L}(\mathbf{m};\mathbf{n}) \\ &= -2\ln\mathcal{L}(\mathbf{y};\mathbf{n}) + 2\ln\mathcal{L}(\mathbf{n};\mathbf{n}) \\ &= 2\sum_{i=1}^{k} y_i - n_i + n_i\ln(n_i/y_i).\end{aligned}$$

Some attractive features:

- It's a χ^2 statistic (familiarity, versatility, error analysis).
- No variance estimation.
- Self-normalizing.
- A clear way to handle bins with $n_i = 0$.
- Minimizing χ^2_Λ is equivalent to maximizing $\mathcal{L}(\mathbf{y};\mathbf{n})$.

Baker and Cousins. Nucl. Inst. and Meth. A 221 (1984) 437.