Cavity-Induced Spin-Orbit-Coupled Bose-Einstein Condensation:

A New Approach for Exploring Ultracold Atoms

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CAP Congress, June 2015

UNIVERSITY OF CALGARY

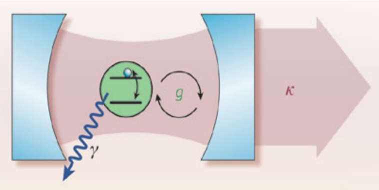
Outline

1. Motivation
   - Cavity QED
   - Intrinsic spin-orbit (SO) coupling for electrons
   - Synthetic laser-induced spin-orbit coupling for atoms
2. Cavity-induced SO-coupled Bose-Einstein condensates (BEC)
   - Model and the effective Hamiltonian : cavity-mediated long-range (LR) interactions
   - The ground state : mean-field theory
   - Collective excitations : Bogoliubov theory
3. Summary

Cavity QED

- Single-atom cavity QED : strong atom-photon interaction

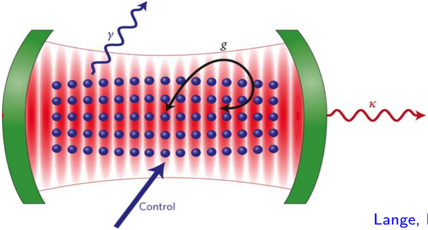
$$g \gg (\kappa, \gamma)$$



Schoelkopf & Girvin, Nature **451**,664 (2008)

- Many-atom cavity QED : **long-range** (LR) interactions between atoms

$$g_{\text{eff}} = \left(\sum_{j=1}^{N} g_j^2 \right)^{\frac{1}{2}} \sim \sqrt{N} g$$

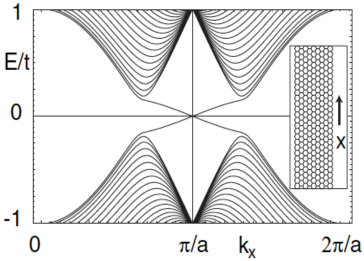


Lange, Nature Physics **5**,455 (2009)

Intrinsic Spin-Orbit Coupling for Electrons

- Spin-orbit (SO) coupling :
  coupling of an electron's
  center-of-mass momentum
  to its **spin** degrees of freedom

Graphene with SO coupling :
a '**topological insulator**'



Kane & Mele, PRL **95**, 226801 (2005)

- For 2D electron gases
  1. Rashba SO coupling : $H_R = \alpha(\boldsymbol{\sigma} \times \mathbf{p}) \cdot \hat{\mathbf{z}} = \alpha(\sigma_x p_y - \sigma_y p_x)$
  2. Dresselhaus SO coupling : $H_D = \beta(-\sigma_x p_y - \sigma_y p_x)$
  3. Rashba + Dresselhaus SO coupling :

$$H = \frac{1}{2m}\left[\left(p_x - \frac{e}{c}A_x(\sigma_x, \sigma_y)\right)^2 + \left(p_y - \frac{e}{c}A_y(\sigma_x, \sigma_y)\right)^2\right]$$

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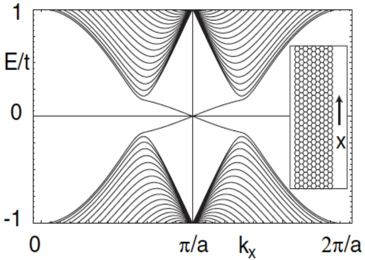

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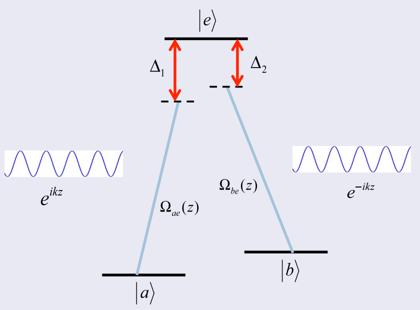
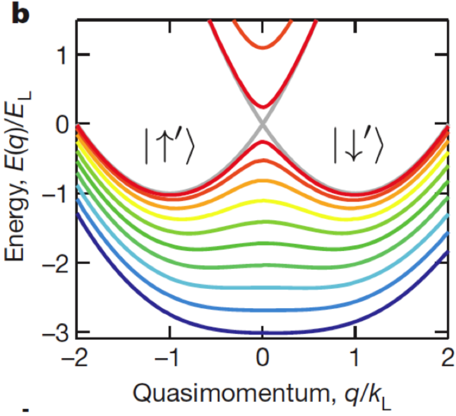
'*Synthetic*' Spin-Orbit Coupling for Atoms

- Laser-induced SO coupling : two-photon Raman process

Λ Scheme

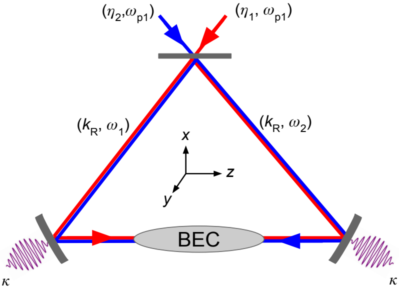




Lin, Jimenez-Garcia, & Spielman, Nature **471**, 83 (2011)

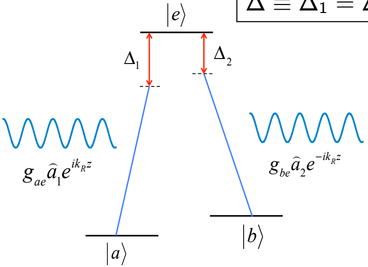
Cavity-Induced SO-Coupled BEC : Model

- Three-level atoms + two counter-propagating ring-cavity modes :



$$\Delta_c \equiv \omega_{p1} - \omega_1 = \omega_{p2} - \omega_2$$

$$\Delta \equiv \Delta_1 = \Delta_2$$



$$\mathcal{H}^{(1)} = \frac{\hbar^2 \mathbf{q}^2}{2m} I_{3\times 3} + \varepsilon_a \sigma_{aa} + \varepsilon_b \sigma_{bb} + \varepsilon_e \sigma_{ee}$$
$$+ \hbar \left(\omega_1 \hat{a}_1^\dagger \hat{a}_1 + \omega_2 \hat{a}_2^\dagger \hat{a}_2 \right) + i\hbar \left(\eta_1 \hat{a}_1 e^{-i\omega_{p1}} + \eta_2 \hat{a}_2 e^{-i\omega_{p2}} - \text{H.c} \right)$$
$$+ \hbar \left[e^{ik_R z} \mathcal{G}_{ae} \hat{a}_1 \sigma_{ea} + e^{-ik_R z} \mathcal{G}_{be} \hat{a}_2 \sigma_{eb} + \text{H.c.} \right]$$

Effective Hamiltonian : Cavity-Mediated LR Interactions

- Weak-coupling ($\kappa \gtrsim (\mathscr{G}_{ae}, \mathscr{G}_{be})$) limit : $\partial_t \hat{a}_j = \frac{1}{i\hbar}[\hat{a}_j, \mathcal{H}] - \kappa \hat{a}_j = 0$

To second order in $\gamma \equiv 2\mathscr{G}_{ae}\mathscr{G}_{be}/\Delta\Delta_c \ll 1$:

$$H_{\rm eff} = \int d^3r \left(\hat{\mathbf{\Psi}}^\dagger \mathcal{H}^{(1)}_{\rm SO} \hat{\mathbf{\Psi}} + \frac{1}{2} g_1 \hat{n}_1^2 + \frac{1}{2} g_2 \hat{n}_2^2 + g_{12} \hat{n}_1 \hat{n}_2 \right) + \sum_{\tau=1,2} U_\tau \hat{N}_\tau^2 + U_\pm \hat{S}_+ \hat{S}_- + U_\mp \hat{S}_- \hat{S}_+ + 2U_{\rm ds} \hat{N} \hat{S}_x$$

$$\mathcal{H}^{(1)}_{\rm SO} = \frac{1}{2m}\left[\mathbf{p}_\perp^2 + (p_z + \hbar k_R \sigma_z)^2 \right] + \hbar \Omega_R \sigma_x$$

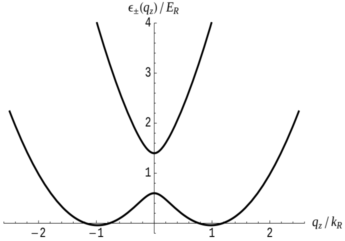
$$\hat{\mathbf{\Psi}}(\mathbf{r}) = (\hat{\psi}_1(\mathbf{r}), \hat{\psi}_2(\mathbf{r}))^{\rm T}, \qquad \hat{N}_\tau = \int \hat{n}_\tau(\mathbf{r}) d^3r = \int \hat{\psi}_\tau^\dagger(\mathbf{r}) \hat{\psi}_\tau(\mathbf{r}) d^3r$$

$$\hat{S}_+ = \hat{S}_-^\dagger = \int \hat{\psi}_1^\dagger(\mathbf{r}) \hat{\psi}_2(\mathbf{r}) d^3r, \quad \hat{S}_x = \frac{1}{2}(\hat{S}_+ + \hat{S}_-), \quad \{1,2\} \equiv \{b,a\}$$

Single- and Many-Particle Physics

- Single-particle Hamiltonian $\mathcal{H}_{\rm SO}^{(1)}$:
  single-particle energy dispersion

two minima located at $\boxed{q_z = \pm k_0 \simeq \pm k_R}$



- Many-particle variational wavefunction ansatz :

$$\begin{bmatrix}\psi_1\\ \psi_2\end{bmatrix} = \sqrt{\bar{n}}\left\{c_1 e^{-ik_0 z}\begin{bmatrix}\cos\theta_{\mathbf{k}_0}\\ -\sin\theta_{\mathbf{k}_0}\end{bmatrix} + c_2 e^{ik_0 z}\begin{bmatrix}\sin\theta_{\mathbf{k}_0}\\ -\cos\theta_{\mathbf{k}_0}\end{bmatrix}\right\} \Longrightarrow E[c_1, c_2]$$

$$n(\mathbf{r}) = \bar{n}\left[1 + \tilde{\Omega}_R |c_1 c_2| \cos(2k_0 z + \gamma)\right], \quad s_z(\mathbf{r}) = \left(|c_1|^2 - |c_2|^2\right) k_0/k_R$$

$(c_1, c_2) = (1, 0)$ or $(0,1)$: **plane-wave phase** (PWP)	$c_1 \neq 0$ & $c_2 \neq 0$: **stripe phase** (SP)

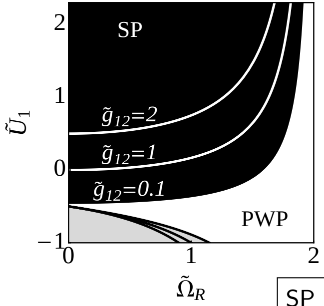
The Ground State : Variational Approach

- Phase diagrams : $U_1 = U_2 \Rightarrow U_1 \hat{N}_1^2 + U_2 \hat{N}_2^2$

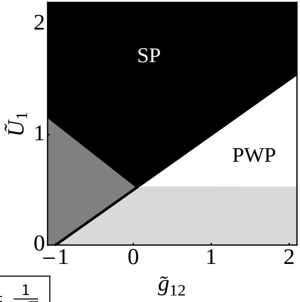
Re-scalling

$$\tilde{\Omega}_R \equiv \hbar\Omega_R/E_R, \qquad \tilde{g}_{12} \equiv g_{12}/|g_1|, \qquad \tilde{U}_1 \equiv VU_1/|g_1|$$

$\mathrm{sgn}(g_1) = \tilde{g}_2 = 1$



$\mathrm{sgn}(g_1) = \tilde{g}_2 = -1$



SP : $c_1 = c_2 = \frac{1}{\sqrt{2}}$

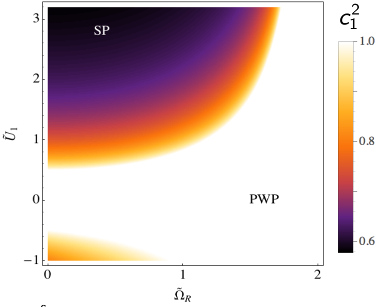
The Ground State : Variational Approach & GP Equations

- Phase diagram : $U_1 \neq U_2$

$$\mathrm{sgn}(g_1) = \tilde{g}_2 = \tilde{U}_2 - \tilde{U}_1 = 1$$

$$\tilde{g}_{12} = 2$$

$$\boxed{\mathrm{SP} : c_1 \neq c_2}$$



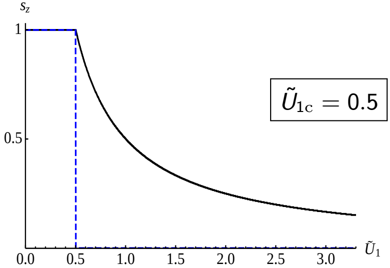
- Magnetization :

$$s_z = \left(|c_1|^2 - |c_2|^2\right) k_0/k_R$$

$$\tilde{\Omega}_R = 0.1$$

Black solid : $\tilde{U}_2 - \tilde{U}_1 = 1$

Blue dashed : $\tilde{U}_2 - \tilde{U}_1 = 0$



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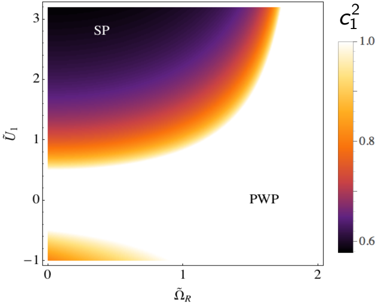
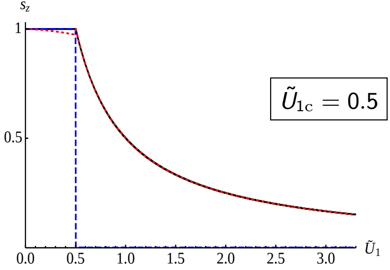


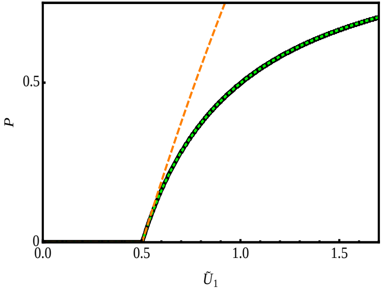


SP-PWP Quantum Phase Transition

- Order parameter : $P = 1 - s_z k_R / k_0 = 2(1 - c_1^2)$

$\mathrm{sgn}(g_1) = \tilde{g}_2 = \tilde{U}_2 - \tilde{U}_1 = 1$

$\tilde{g}_{12} = 2$ & $\tilde{\Omega}_R = 0.1$



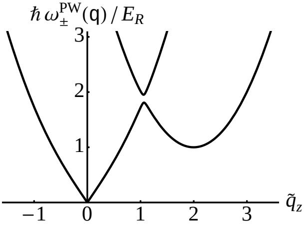
$\tilde{U}_{1c} = 0.5$

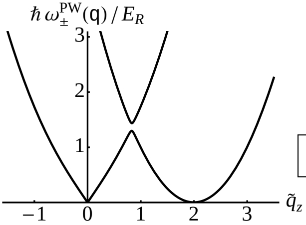
$P_{\mathrm{MF}} = 2(\tilde{U}_1 - \tilde{U}_{1c})^{\beta} / (\tilde{U}_2 - \tilde{U}_1)$ with the mean-field exponent $\beta = 1$

Elementary Excitations : Bogoliubov Theory

- PWP :

$\tilde{U}_1 = \tilde{U}_2 = 0$





$\tilde{U}_1 = 0.5$

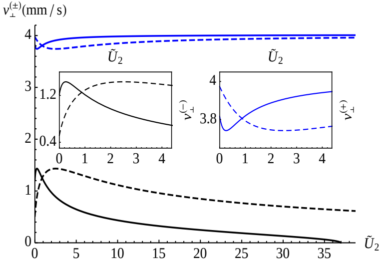
$\tilde{U}_2 = 1.5$

$\tilde{U}_{1c} = 0.5$

- SP : speed of the sound at long wavelength ($\mathbf{q} \to 0$)

$\mathrm{sgn}(g_1) = \tilde{g}_2 = 1$
$\tilde{g}_{12} = 0.7 \;\; \& \;\; \tilde{\Omega}_R = 0.4$

Solid curve : $\tilde{U}_1 = 1/4$
Dashed curve : $\tilde{U}_1 = 5/2$



Summary

- **SO coupling** : the essential ingredient for topological insulators
- **Cavity-induced SO-coupled BEC** : SO coupling + cavity mediated LR interactions between atoms
- **Ground state** : SP or PWP, determined by the interplay between the two-body and LR interactions
- **SP-PWP transition** : a second order quantum phase transition
- **Collective excitations** : superfluid and roton-type feature in PWP
- **References** :
  - F. Mivehvar & D. L. Feder, Phys. Rev. A **89**, 013803 (2014)
  - F. Mivehvar & D. L. Feder, http://arxiv.org/abs/1505.02189
- **Acknowledgement of funding agencies** :