Gravity and dust in 2+1 dimensions

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V. Husain and JZ, `arXiv:1506.00725` (2015), to appear in PRD

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A tractable model of quantum gravity

- Little is known about quantum interactions between matter and gravity.
- To learn more about this, we consider a 3d model with gravity and dust.
- We find a rich solution space including conical singularities, wave solutions and the BTZ (Bañados-Teitelboim-Zanelli) black hole.
- The wave solutions permit a non-perturbative, canonical quantization.
- This allows us to investigate questions such as:
  - What is a quantum geometry?
  - What is a quantum horizon?

Full theory

- We begin with the action

$$S = \int dx^3 \sqrt{g} \left({}^{(3)}R - 2\Lambda \right) - \int dx^3 \sqrt{g} m \left(g^{\mu\nu} \partial_\mu \psi \partial_\nu \psi + 1 \right)$$

- The Hamiltonian consists of the ‘scalar’ and ‘diffeomorphism’ constraints.
- The configuration variables are the space metric q_{ab} and dust-field ψ.
- These degrees of freedom are interdependent due to the constraints.

Physical theory

- A fully reduced theory is obtained by:
  - imposing circular symmetry $q_{ab} = q_{ab}(r)$
  - choosing the dust-time gauge $t = \psi$
- Gravity and matter degrees of freedom are combined into a single configuration variable $\Omega(t, r)$.
- A physical Hamiltonian $H[\Omega, P]$ remains to generate dynamics.

Spacetime

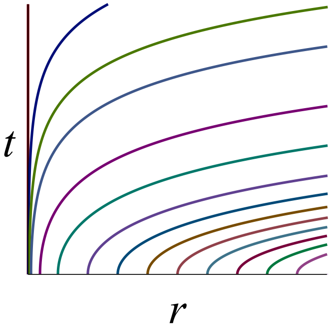
- The spacetime metric is

$$ds^2 = -\left(1 - \Omega^2 P^2\right) dt^2 + \Omega^2 P \; dr dt + \Omega^2 dr^2 + r^2 d\theta^2$$

- Constant-time slices are spatially-flat.
- When $\Omega \neq 1$ at the origin, there is a conical singularity.
- Dynamical apparent horizons may be present.

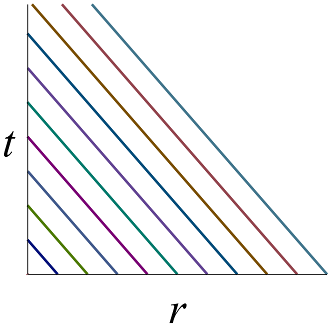
General solutions

- The momentum P defines characterstic curves; Ω is constant along each curve.
- Below are the characteristics for $\Lambda = 1$ and $P(0, r) = 0$.



Particular solutions

- Setting $\dot{\Omega} = \dot{P} = 0$ yields the BTZ black hole.
- When $\Lambda = 0$ and $P = v$ is constant, there are travelling wave solutions for Ω.



Quantum theory

- Travelling wave solutions satisfy the advection equation:

$$\dot{\Omega} = v\Omega'$$

- A quantum theory can be developed from the space of solutions to this equation[1].
- The operator $\widehat{\Omega}$ can be written in terms of ladder operators $(\hat{a}_k^\dagger, \hat{a}_k)$.
- A Fock basis and coherent states are constructed from the n_k-particle states.

[1] Crnkovic and Witten (1986)

Metric operator

- Given v, the classical spacetime is completely determined by Ω^2.
- We take the metric operator to be

$$\hat{g}_{\mu\nu} = g_{\mu\nu}\left(: \widehat{\Omega^2} : \right)$$

- A quantum spacetime is given by the expectation value:

$$\langle \hat{g}_{\mu\nu} \rangle$$

- The variance is generally non-zero.

Quantum geometries

- n_k-particle states yield conical geometries with a discrete mass spectrum

$$2\pi \left(1 - \sqrt{\frac{k}{2n_k}}\right)$$

- Coherent states yield dynamical wave spacetimes which may contain travelling horizons.
- Non-zero variance of the metric operator gives rise to horizon fluctuations.

Summary

- We studied 3d gravity with dust in circular symmetry using the dust as the time coordinate.
- The space of wave solutions permit a non-perturbative quantization.
- Quantum geometries are determined by the expectation value of $\hat{g}_{\mu\nu}$ in a state.
- n_k-particle states have a discrete mass spectrum.
- Coherent states may have travelling horizons which are subject to fluctuations.

Thank you.

AARMS